Dense Matter and Functional Renormalization Group

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Motivation

Relativistic heavy ion collisions provide an important means to explore the phase diagram of QCD at finite temperature and chemical potential. Despite its conceptual simplicity, the **statistical model** is in good agreement with experiments as concerning particle yields as well as temperature and baryon chemical potential at chemical freeze-out.
It was conjectured [1] that only multi-particle processes/collective effects can maintain chemical equilibrium (at least for low baryon densities)

$$\Rightarrow \quad T_{\text{chemical freeze-out}} \simeq T_{\text{chiral crossover}} \, .$$

Question: What happens at large densities?
Since $\mu \simeq 900$ MeV is the territory of nuclear physics, an effective **nucleon-meson model** is useful.

Lagrangian:

$$\mathcal{L} = \bar{\psi}\Big(i\partial\!\!\!/ + g_v \omega\!\!\!/ + \mu\gamma^0 + g_s(\sigma + i\gamma^5 \boldsymbol{\pi}\cdot\boldsymbol{\tau})\Big)\psi + \\ + \tfrac{1}{2}(\partial\sigma)^2 + \tfrac{1}{2}(\partial\boldsymbol{\pi})^2 + \partial_{[\mu}\omega_{\nu]}\partial^{[\mu}\omega^{\nu]} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu + U_{\text{mic}}(\sigma,\boldsymbol{\pi}).$$

contains protons and neutrons, as well as vector-mesons, a scalar meson and pion degrees of freedom.

Extension I: Treatment of **Neutron Stars**. Recently, a two-solar mass neutron star was observed [2], so one might ask if the model is compatible with the latest data.
Extension II: Treatment of mesonic fluctuations with help of the **functional renormalization group** (FRG).

Mean Field Approximation [3]

At the mean field level, σ and ω_0 acquire finite expectation values. The nucleons are integrated out $\rightarrow$ **free-gas pressure**. The effective potential is

$$U = U_{\text{vac}}(\sigma,\boldsymbol{\pi},\omega_0) - 4P_{\text{free gas}}(T,\mu,\sigma,\omega_0),$$
$$P_{\text{free gas}} = \int \frac{d^3p}{(2\pi)^3}\log\left[1 + \mathrm{e}^{-\beta(\sqrt{p^2+m^2}-\mu_{\text{eff}})}\right] + (\mu_{\text{eff}} \to -\mu_{\text{eff}}),$$
$$m = g_s\sigma, \qquad \mu_{\text{eff}} = \mu + g_v\omega_0.$$

The vacuum potential is expanded around its form near the **liquid-gas phase transition** ($T = 0,\ \mu = \mu_c = 922.7$ MeV);

$$U_{\text{vac}} = -m_\pi^2 f_\pi(\sigma - f_\pi) + \sum_{n=1}^{N_{\max}} a_n(\rho - \rho_0)^n - \frac{m_\omega^2}{2}\omega_0^2, \qquad \rho = \tfrac{1}{2}\sigma^2 + \tfrac{1}{2}\boldsymbol{\pi}^2.$$

Procedure:

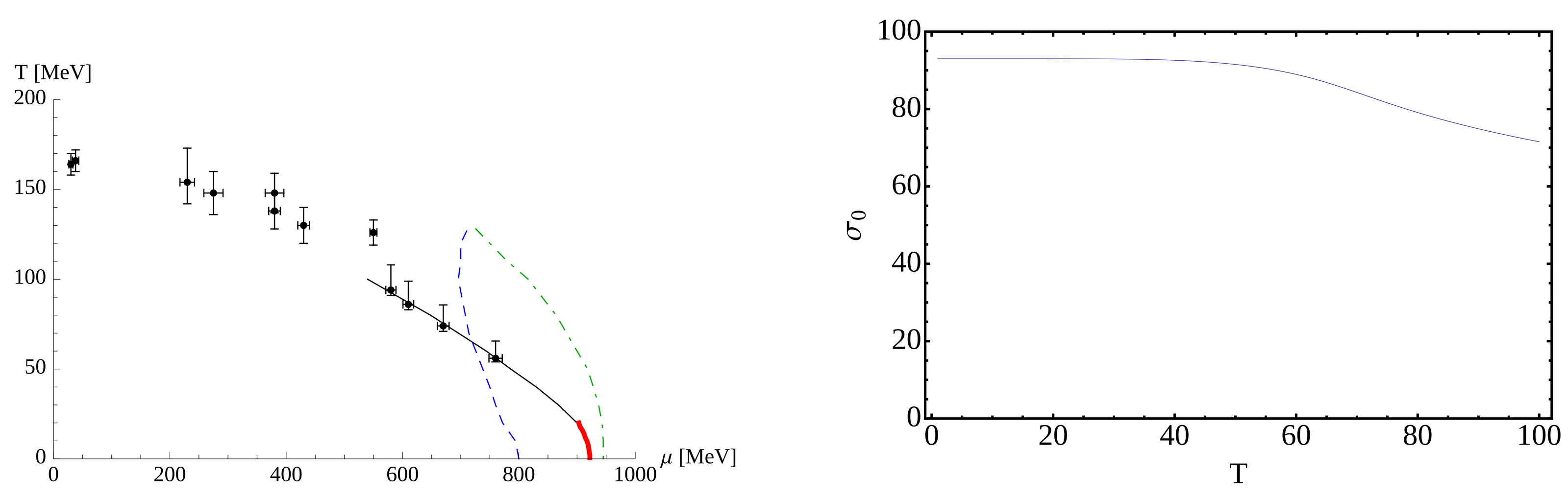
1. Minimize the potential

$$\partial_\sigma U = 0 \quad \Rightarrow \bar{\sigma},$$
$$\partial_{\omega_0} U = 0 \quad \Rightarrow \quad \omega_0 = -\frac{g_v}{m_\omega^2}\cdot n_B.$$

2. Compute the **grand-canonical potential** and the thermodynamical observables

$$\Omega = U(\bar{\sigma},\bar{\omega}_0), \qquad n_B = \partial_\mu\Omega, \qquad s = -\partial_T\Omega,$$
$$p = -\Omega, \qquad \epsilon = \Omega + Ts + \mu n_B.$$

Result: There is **no indication of a chiral phase transition** at chemical freeze-out [3, 4]:



(a) Crosses: chemical freeze-out from experiment [4]. Black Line: constant $n_B = .15 n_{\text{nucl}}$. Red Line: liquid-gas transition. Blue/green dashed lines: applicability of the model.

(b) Chiral order parameter as a function of temperature (both in MeV) at chemical potential $\mu = 750$ MeV.

There is no discontinuity in the chiral order parameter. This would be the case if there were indeed a phase transition.

Neutron Stars I

A study of **neutron-star matter** requires additional ingredients:

1. There is an excess of neutrons over protons. Therefore different **chemical potentials** μ_p, μ_n for protons and neutrons are needed.
2. **Electrons** with chemical potential μ_e.

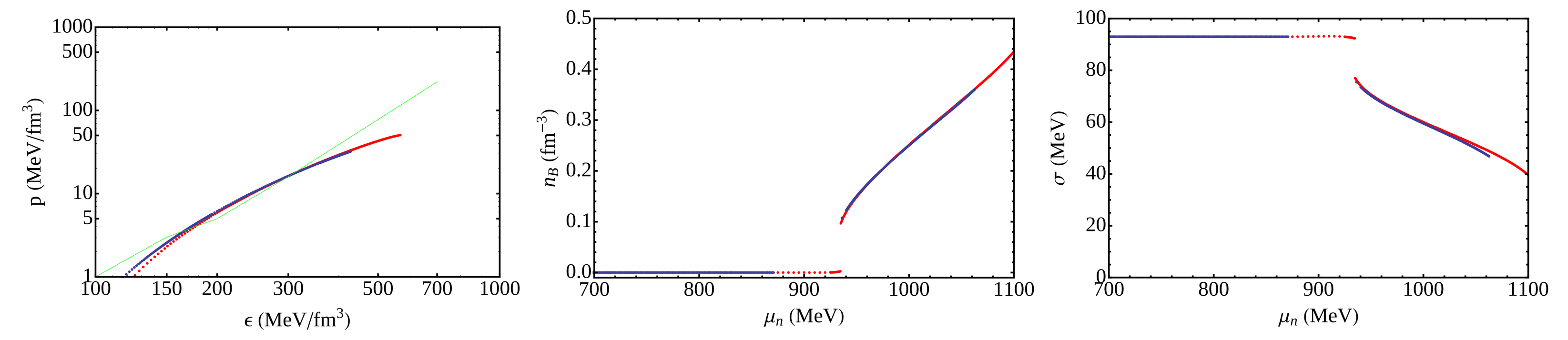
Procedure in mean field approximation: Solve simultaneously:

$$\partial_\sigma U = 0,$$
$$\partial_{\omega_0} U = 0 \quad \Leftrightarrow \quad \omega_0 = -\frac{g_v}{m_\omega^2}\cdot n_B,$$
$$\mu_n = \mu_p + \mu_e, \ \textbf{beta equilibrium},$$
$$n_p = n_e, \ \textbf{charge neutrality}.$$

Here $\mu_e = \sqrt{p_F^2 + m_e^2}$ and $p_F^3 = 3\pi^2 n_e$.

Neutron Stars II

Comparison: Blue: Beta Equilibrium, Red: Pure Neutron Matter, Green: Akmal et al. [5]



(a) Equation of state. (b) Baryon density as a function of the neutron chemical potential. (c) Chiral order parameter.

There still appears a first order liquid-gas transition, which is unphysical for neutron stars. The repulsion is not sufficient, which is due to the fact that a **vector-isovector degree of freedom ρ** is still missing.

Neutron Stars III: Extension by the ρ-field

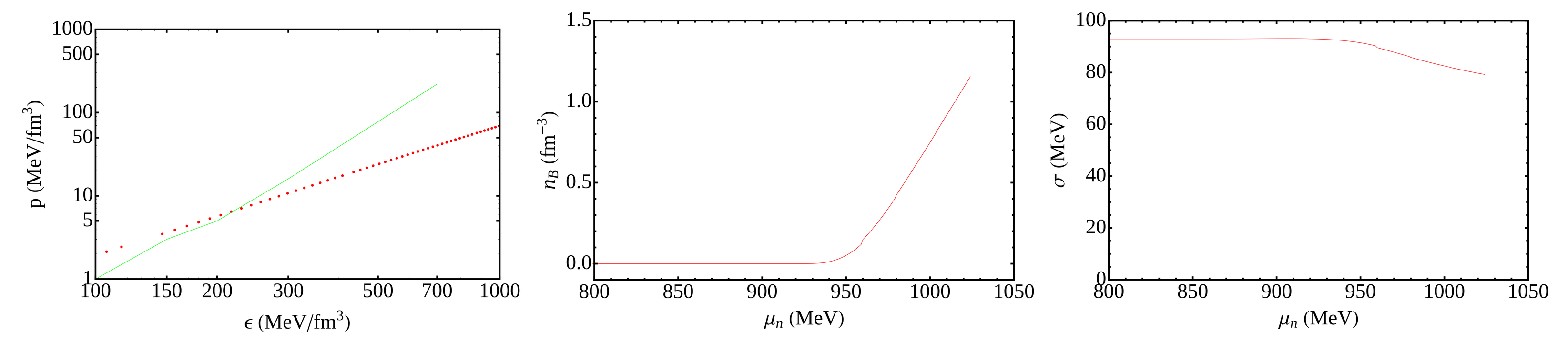
Taking the $\boldsymbol{\rho}$ into account, the improved Lagrangian is

$$\mathcal{L} = \bar{\psi}\Big(i\partial\!\!\!/ + g_v(\omega\!\!\!/ + \boldsymbol{\rho\!\!\!/}\cdot\boldsymbol{\tau}) + g_s(\sigma + i\gamma_5\boldsymbol{\pi}\cdot\boldsymbol{\tau}) + \left(\begin{smallmatrix}\mu_p & \\ & \mu_n\end{smallmatrix}\right)\gamma^0\Big)\psi + \\ + \bar{\psi}_e(i\partial\!\!\!/ + \mu_e\gamma^0)\psi_e + \tfrac{1}{2}(\partial\sigma)^2 + \tfrac{1}{2}(\partial\boldsymbol{\pi})^2 + U(\rho,\sigma) + \\ + \partial_{[\mu}\omega_{\nu]}\partial^{[\mu}\omega^{\nu]} + \partial_{[\mu}\boldsymbol{\rho}_{\nu]}\partial^{[\mu}\boldsymbol{\rho}^{\nu]} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \tfrac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu.$$

Now σ, ω_0 and ρ_0^3 acquire finite expecation values. The **effective chemical potentials** are

$$\mu_{\text{eff},p} = \mu + g_\omega(\omega_0 + \rho_0^3),$$
$$\mu_{\text{eff},n} = \mu + g_\omega(\omega_0 - \rho_0^3).$$

Comparison: Red: Pure Neutron Matter, Green: Akmal et al. [5]



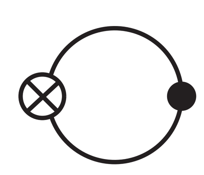
(a) Equation of state. (b) Baryon density. (c) Chiral order parameter.

The equation of state is still **not sufficiently stiff** to support a two-solar-mass neutron star.

Renormalization Group

So far, the model was studied in the mean field approximation, without any mesonic fluctuations. They can be included with help of the **Functional Renormalization Group**. The effective action Γ_k is studied that includes all fluctuations above the renormalization scale k. Its flow is governed by Wetterich's Equation [6] that interpolates between an **UV-action** at $k = \Lambda_{\text{UV}}$ and a **full effective action** at $k = 0$:

$$k\partial_k\Gamma_k[\Phi_k] = \frac{1}{2}\mathrm{Tr}\, k\partial_k R_k\left(\Gamma_k^{(2)}[\Phi_k] + R_k\right)^{-1} = \frac{1}{2}$$



$\Gamma_k^{(2)}$ is its the second derivative of the effective action with respect to the fields. R_k is the **regulator function**, which cuts off low momenta. For the model at hand, Wetterich's equation is

$$\partial_t\Gamma_k = \beta V\frac{k^5}{12\pi^2}\left[3\frac{1 + 2\frac{1}{\mathrm{e}^{\beta E_\pi}-1}}{E_\pi} + \frac{1 + 2\frac{1}{\mathrm{e}^{\beta E_\sigma}-1}}{E_\sigma} - \frac{8}{E_q}\left(1 - \frac{1}{\mathrm{e}^{\beta(E_q-\mu_{\text{eff}})}+1} - \frac{1}{\mathrm{e}^{\beta(E_q+\mu_{\text{eff}})}+1}\right)\right],$$
$$E_\pi^2 = k^2 + U', \quad E_\sigma^2 = k^2 + U' + 2\rho U'', \quad E_q^2 = k^2 + 2g_s^2\rho.$$

Conclusions

1. The study of a **nucleon-meson** model expanded around the liquid-gas first-order phase transition is generalized to **neutron star matter**.
2. A **vector-isovector** degree of freedom has to be taken into account in order to get rid of the phase transition for neutron star matter.
3. The equation of state appears to be to soft to support a two solar-mass neutron star.
4. Studies of fluctuation effects are in progress.

References

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