A Way to Acquire Some Current Quark Mass from a General Relativistic Effect

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Abstract:

We calculate a way to acquire some current quark mass from a general relativistic effect. For a bare quark, we model that boundary conditions on the spacetime metric can plausibly couple the value of current quark mass to the charge, via external pressure (e.g., as supplied by a background field) at sub-fm length scales. This mechanism acquires some (up to ~ 40%) current quark mass "from" the charge. To construct an approximate metric, we model a bare quark as a spherically symmetric static perfect fluid with charge, using a recent exact Maxwell-Einstein metric for the interior, out to a radius r at which the boundary condition is to match to the Reissner-Nordström metric (for spacetime external to a charged mass). At r, the model produces internal pressure, which should be matched to external pressure. For reported values of quark charges q and bare masses m_q, this construction produces sub-fm radii. Although the metric at this radius differs only perturbatively from a flat spacetime, the matching condition is more significant and couples the current quark mass to the charge.

Outline

- We motivate interest in the following issue: Might there be a general relativistic effect beyond the Planck length scale?
- To consider general relativistic effects, we seek to calibrate an exact general relativistic metric for a system with mass and charge to produce mass density ρ_0 and pressure p_0 consistent with the physical Equation of State (EoS)
  - Use an exact metric that supports physically relevant EoS
- This construct enables us to explore properties away from interior in a relativistically covariant manner
  - In particular, can use this construct to look for a general relativistic effect beyond the Planck length
- We identify a potential candidate: pressure supplied by an external field at some radius generates a boundary condition that couples the externally observed mass to the interior charge
  - In our toy model calculation, this coupling is *nonzero* -- 0.06%-37%
  - Suggestive that it is possible that renormalized bare quark mass values could differ by this much from accepted values, due to this general relativistic effect

Are there General Relativistic (GR) metric effects beyond Planck length?

- To explore GR behaviors in & away from central region, we motivate interest in modeling quark matter using an exact general relativistic metric solution
- We are able to calibrate an exact metric to EoS for p(ρ) in interior
- This solution reproduces mass and charge of quarks and proton
- It therefore seems an interesting toy model to probe whether any general relativistic effects might appear at length scales beyond the Planck length
- A possible effect is identified -- in this model, an external pressure at a boundary creates a boundary condition in which the external mass has a *nonzero* 0.06%-37% contribution due to the internal charge.
  - A way to acquire some quark mass

Exact Maxwell-Einstein Metric

- For a system of matter and charge in a general relativistic treatment (in 3+1 dimensions), exact metrics are "Maxwell-Einstein" solutions to the field equations. They are few in number!
- Candidate metrics for spherically symmetric static charged perfect fluids for dimensionless Q/M >>1:
  - (Gupta and Maurya, 2011a): Astrophys. Space Sci. 332, 155-162
  - (Pant, 2011): Astrophys. Space Sci. 332, 403-408
  - (Gupta and Maurya, 2011b): Astrophys. Space Sci. 332, 415-421
  - (Maurya and Gupta, 2011): Astrophys. Space Sci. 332, 481-490
  - (Pant and Rajasekhara, 2011): Astrophys. Space Sci. 333, 161-168
  - (Bijalwan, 2011): Astrophys. Space Sci. 336, 413-418
  - (Gupta and Kumar, 2011): Astrophys. Space Sci. 336, 419-426
  - (Kiess, 2012): Astrophys. Space Sci. 339, 329-338
  - …
- We apply one of these metrics – (Kiess, 2012) - to quark matter, and calibrate it to the CEoS at interior
  - Use this toy model to explore properties away from interior

Metric and Notation Used Here

$$ds^2 = -e^{\nu(r)}c^2dt^2 + e^{\lambda(r)}dr^2 + r^2 d\Omega^2$$

This metric specified by 2 functions ν(r) & λ(r):

$$e^{\upsilon/2} = B\sin\left(\ln\left(\sqrt{\frac{\left(\sqrt{1-\frac{r^2}{R^2}+4\frac{r^4}{A^4}}+2\frac{r^2}{A^2}-\frac{A^2}{4R^2}\right)^{\sqrt{1-\varepsilon}}}{c_1}}\right)\right) \qquad e^{-\lambda} = 1-2m/r$$

for mass $m(r) = \frac{r^3}{2R^2} - \frac{2r^5}{A^4}$ mass density $\rho(r) = \frac{1}{4\pi\left(\frac{G}{c^2}\right)}\left[\frac{3}{2R^2} - \frac{(10+\varepsilon)r^2}{A^4}\right]$

charge density $\delta(r) \equiv \frac{|E_r|^2}{8\pi} = \frac{Q^2}{8\pi r^4} = \frac{1}{4\pi\left(\frac{G}{c^4}\right)}\left[\frac{\varepsilon r^2}{A^4}\right]$

& in terms of 5 constants: B, c_1, R, A, and ε

Pressure

in this model is

$$p(r) = \frac{1}{8\pi\left(\frac{G}{c^4}\right)A^2}\left[\frac{4\sqrt{1-\varepsilon}\sqrt{1-\frac{r^2}{R^2}+4\frac{r^4}{A^4}}}{\tan\left(\ln\left(\sqrt{\frac{\left(\sqrt{1-\frac{r^2}{R^2}+4\frac{r^4}{A^4}}+2\frac{r^2}{A^2}-\frac{A^2}{4R^2}\right)^{\sqrt{1-\varepsilon}}}{c_1}}\right)\right)} - \frac{A^2}{R^2} + 4\frac{r^2}{A^2}(1+\varepsilon/2)\right]$$

This metric solves $G_{\mu\nu} = 8\pi[G/c^4]\tau_{\mu\nu}$ for $\tau_{\mu\nu} = (\rho(r) + p(r)/c^2)u_\mu u_\nu + p(r)g_{\mu\nu}$

Calibrating metric to EoS at r=0

Value of mass and bounding radius r_0 fix ρ(r) for r< r_0.

Use physical EoS for cold hadronic matter [as shown, for example, in P. Danielewicz, R. Lacey, and W. Lynch, Science **298** (2002) 1592, and in G.F. Burgio, J. Phys. G: Nucl. Part. Phys. **35** (2008) 014048, and in K. Moghrabi, M. Grasso, X. Roca-Maza, and G. Colo, Phys. Rev. C**85** (2012) 044323] to deduce value of p(r).

Equate this value to p(r) in preceding equation to solve for value of c_1.

Behavior away from r=0

We model a quark or nucleon as a spherically symmetric static perfect fluid with mass and charge, out to a boundary radius r_0 at which internal pressure is matched to an external pressure (e.g., due to a field), in the spirit of models with confinement. Beyond r_0, we approximate the metric as that of the Reissner-Nordström form:

$$ds^2 = -\left(1-\frac{2M}{r}+\frac{G}{c^4}\frac{Q^2}{r^2}\right)c^2dt^2 + \frac{dr^2}{\left(1-\frac{2M}{r}+\frac{G}{c^4}\frac{Q^2}{r^2}\right)} + r^2 d\Omega^2$$

= the metric external to a system with mass M and charge Q

Boundary Condition at r_0

At radius r_0, we have a boundary condition:

Match internal metric to external metric (continuity)

$$M = m(r_0) + \frac{1}{2}\frac{G}{c^4}\frac{Q^2}{r_0}$$

… this couples interior mass $m(r_0)$ to external M, Q

Toy Model

Using this construct, we build a model for each quark and nucleon.

We find that r_0 can be varied over a range that is limited to be < $\sqrt{\frac{3}{2(10+\varepsilon)}}\frac{A^2}{R} = r_0^{\max}$

… set by a constraint to keep the mass density ≥ 0.

At each value of r_0 within $0 \le r_0 \le r_0^{\max}$, the constants c_1, R, A, & ε can be chosen to satisfy the EoS, the boundary condition, the charge, and the bare mass, for each quark or nucleon. The set of equations below show explicitly how this is done.

Equations – to model each nucleon/quark,

1. Choose ε within its range 0≤ε≤1
2. Choose r_0 such that mass density $\rho(r_0) = \frac{1}{4\pi\left(\frac{G}{c^2}\right)}\left[\frac{3}{2R^2} - \frac{(10+\varepsilon)r_0^2}{A^4}\right]$ is > 0 (this defines a maximum value of r_0)

   => Then ρ is known, & used in EoS to calculate the physical pressure, p
3. Constant c_1 is determined by matching pressure p(ρ) to that of EoS
4. Constant A is determined from $Q = \sqrt{\frac{2\varepsilon_0}{G}c^2\frac{r_0^3}{A^2}}$
5. Constant R is determined by solving for R in $M = m(r_0) + \frac{1}{2}\frac{G}{c^4}\frac{Q^2}{r_0}$ with $m(r_0) = \frac{r_0^3}{2R^2} - \frac{2r_0^5}{A^4}$
6. Constant B is determined by matching metric coefficient g_{tt} at r_0

Quantitative Results

	ρ(r=0) [1/fm³]	r_0 [fm]	Bare Mass [MeV/c²]	Fraction due to this effect [%]
	0.14	1.2	938	≈ 0.06
Proton	0.18	1.1	938	≈ 0.07
	0.24	1.0	938	≈ 0.08
Down	0.16	0.10-0.20	4.1-5.8	≈ 10
Quark	1.0	0.08-0.11	4.1-5.8	≈ 18
	10	0.04-0.05	4.1-5.8	≈ 37
Up	0.0011	0.4-0.8	1.7-3.3	≈ 24
Quark	0.0024	0.3-0.6	1.7-3.3	≈ 30
	0.0046	0.26-0.5	1.7-3.3	≈ 37

Discussion

- Of interest in this toy model is that the boundary constraint couples the internal (bare constituent) mass to the externally measured mass and charge.
- Some mass is acquired from this boundary constraint, which comes from matching the interior/exterior forms of the metric at the boundary r_0
- In this model, this effect is nonzero, and varies depending upon the value of r_0
- This effect is inherently a General Relativistic one.
- Next Steps: To improve upon this toy model, one could look at several effects:
  - A softer boundary, with a more gradual change in parameters as a function of r
  - More realistic model of quark/nucleon matter
  - Other exact Maxwell-Einstein metrics for interior solution
  - Numerical studies (which are always useful to complement exact metric solutions) – as with numerical relativistic hydrodynamic solvers
  - …

Conclusions

- A potential General Relativistic effect exists beyond the Planck length scale.
- To explore this issue, we built a toy model of a quark or nucleon by calibrating an exact Maxwell-Einstein metric to the physical EoS, as an internal metric, and matching this metric at some radius r_0 to the form of an external metric outside of a volume with mass and charge.
- This toy model is suggestive that a confining pressure (e.g., in the spirit of models with confinement) creates a boundary condition -- continuity of the metric, for different internal/external forms.
- This boundary condition has the effect of relating the value of the interior (bare) quark mass to the value of the external mass.
- The amount of mass "acquired" in this way is nonzero in this model and ranges from 0.06% to 37%
- Therefore, this fraction of bare mass could potentially come from this effect. This impacts the (renormalized) value used in QCD/QFT.