# Multigluon correlations in the color glass condensate

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## Abstract

We describe recent progress in understanding two-particle correlations in the dilute-dense system, e.g. in forward dihadron production in deuteron-gold collisions. This evaluating higher point Wilson line correlators from the JIMWLK equation. We find that the large $N_c$ approximation used so far in the phenomenological literature is not very accurate, but a Gaussian approximation is a surprisingly close. We then calculate the dihadron correlation.

# Outline

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1. Introduction: $d$Au collisions at forward rapidity
2. Quadrupole from JIMWLK Gaussian approximation
3. Calculating the dihadron correlation

References:


1. A. Dumitru, J. Jalilian-Marian, T. L., B. Schenke and R. Venugopalan, PLB 706 (2011) 219
2. T. Lappi and H. Mäntysaari, arXiv:1207.6920 and in preparation

# Gluon saturation, Glass and Glasma

Small $x$: the hadron/nucleus wavefunction is characterized by **saturation scale** $Q_s \gg \Lambda_{QCD}$.

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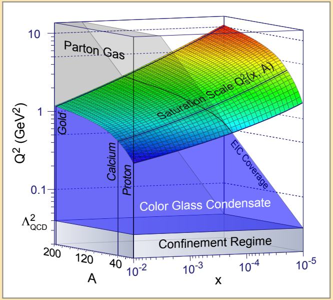
$\mathbf{p}_T \sim Q_s$: strong fields $A_\mu \sim 1/g$

- occupation numbers $\sim 1/\alpha_s$
- classical field approximation.
- small $\alpha_s$, but nonperturbative



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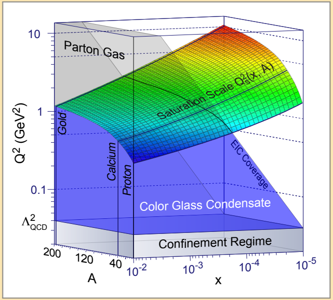


## CGC: Effective theory for wavefunction of nucleus

- Large $x$ = color charge $\rho$, **probability** distribution $W_y[\rho]$
- Small $x$ = classical gluon field $A_\mu$ + quantum flucts.

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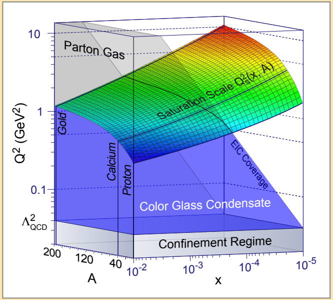


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**Glasma** field configuration of two colliding sheets of CGC.

# JIMWLK evolution

## Heart of CGC formalism: RGE for energy/$x$ dependence

More convenient degrees of freedom:

$\rho^a(\mathbf{x}_T, x^-) \implies$ Wilson line $U(\mathbf{x}_T) = P \exp\left\{ i \int \mathrm{d}x^- \frac{1}{\nabla_T{}^2} \rho(\mathbf{x}_T, x^-) \right\}$

## Wilson lines from probability distribution $W_y[U]$.

JIMWLK gives Energy/rapidity dependence of $W_y[U]$:

$$\partial_y W_y[U(\mathbf{x}_T)] = \mathcal{H} W_y[U(\mathbf{x}_T)]$$

Here the JIMWLK Hamiltonian is:

$$\mathcal{H} \equiv \frac{1}{2} \int_{\mathbf{x}_T \mathbf{y}_T \mathbf{z}_T} \frac{\delta}{\delta \widetilde{\mathcal{A}}_c^+(\mathbf{y}_T)} \mathbf{e}_T^{ba}(\mathbf{x}_T, \mathbf{z}_T) \cdot \mathbf{e}_T^{ca}(\mathbf{y}_T, \mathbf{z}_T) \frac{\delta}{\delta \widetilde{\mathcal{A}}_b^+(\mathbf{x}_T)},$$

$$\mathbf{e}_T^{ba}(\mathbf{x}_T, \mathbf{z}_T) = \frac{1}{\sqrt{4\pi^3}} \frac{\mathbf{x}_T - \mathbf{z}_T}{(\mathbf{x}_T - \mathbf{z}_T)^2} \left(1 - U^\dagger(\mathbf{x}_T) U(\mathbf{z}_T)\right)^{ba}$$

Numerics using Langevin formulation

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Numerics using Langevin formulation

# 2-particle correlation in forward pA/dA

- Quark from p/d (large $x$) from pdf
- Radiate gluon
- Propagate eikonally in color field of target $A$ $\Longrightarrow$ Wilson lines $U$

$$\frac{\mathrm{d}\sigma^{qA\to qgX}}{\mathrm{d}^3k_1\ \mathrm{d}^3k_2} \propto \int_{\mathbf{x}_T,\bar{\mathbf{x}}_T,\mathbf{y}_T,\bar{\mathbf{y}}_T} e^{-i\mathbf{k}_{T1}\cdot(\mathbf{x}_T-\bar{\mathbf{x}}_T)}\ e^{-i\mathbf{k}_{T2}\cdot(\mathbf{y}_T-\bar{\mathbf{y}}_T)}\ \mathcal{F}(\bar{\mathbf{x}}_T-\bar{\mathbf{y}}_T,\mathbf{x}_T-\mathbf{y}_T)$$
$$\times\left\langle \hat{Q}(\mathbf{y}_T,\bar{\mathbf{y}}_T,\bar{\mathbf{x}}_T,\mathbf{x}_T)\ \hat{D}(\mathbf{x}_T,\bar{\mathbf{x}}_T) - \hat{D}(\mathbf{y}_T,\mathbf{x}_T)\hat{D}(\mathbf{x}_T,\bar{\mathbf{z}}_T) + \ldots \right\rangle_{\text{target}}$$

$(\mathbf{z}_T = z\mathbf{x}_T + (1-z)\mathbf{y}_T,\ \bar{\mathbf{z}}_T = z\bar{\mathbf{x}}_T + (1-z)\bar{\mathbf{y}}_T)$ Marquet 2007

**Need target averages of Wilson line operators**

$$\hat{D}(\mathbf{x}_T-\mathbf{y}_T) \equiv \frac{1}{N_c}\,\mathrm{Tr}\,U(\mathbf{x}_T)U^\dagger(\mathbf{y}_T)$$
$$\hat{Q}(\mathbf{x}_T,\mathbf{y}_T,\mathbf{u}_T,\mathbf{v}_T) \equiv \frac{1}{N_c}\,\mathrm{Tr}\,U(\mathbf{x}_T)U^\dagger(\mathbf{y}_T)U(\mathbf{u}_T)U^\dagger(\mathbf{v}_T)$$

# Approximations for 4-point function $\langle \hat{Q} \rangle$

A. Dumitru, J. Jalilian-Marian, T. L., B. Schenke and R. Venugopalan, PLB 706 (2011) 219

## Get $\langle \hat{D}(\mathbf{x}_T, \mathbf{y}_T) \rangle$ from BK, easier than JIMWLK

$\Longrightarrow$ would be nice to reconstruct higher correlators from dipole

1. "Naive large $N_c$" E.g. Marquet 2007, Albacete & Marquet 2010 $Q \approx DD$
2. "Gaussian" approximation E.g. "GT" of Kuokkanen et al., see also Iancu, Triantafyllopoulos : relate all to $\left\langle \hat{D} \right\rangle$ assuming Gaussian correlators for Wilson lines, as in MV model.

## Compare these to JIMWLK

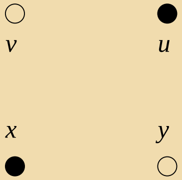
Study operator

$$\hat{Q} = \frac{1}{N_c} \operatorname{Tr} U(\mathbf{x}_T) U^\dagger(\mathbf{y}_T) U(\mathbf{u}_T) U^\dagger(\mathbf{v}_T)$$

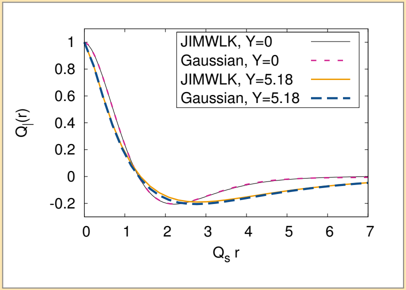
for two particular coordinate configurations:

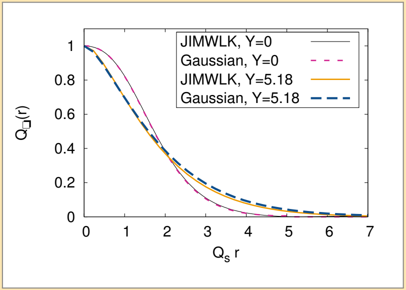
Line $\mathbf{u}_T = \mathbf{x}_T$; $\mathbf{v}_T = \mathbf{y}_T$

Square



# Gaussian is good



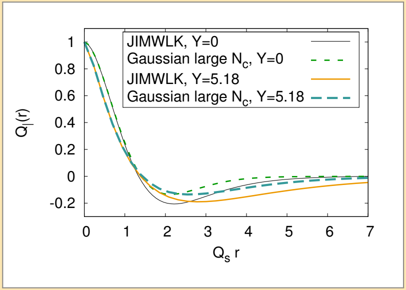

Line

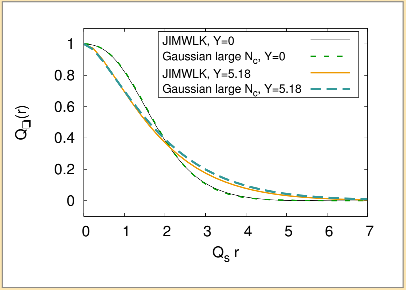
Square

$y = 0$ is MV $\Longrightarrow$ satisfies Gaussian by construction.

— But JIMWLK stays very close at later rapidities.

# Gaussian is quite good, even at large $N_c$

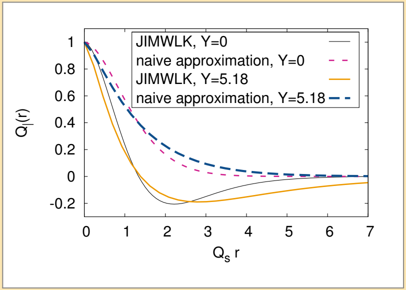


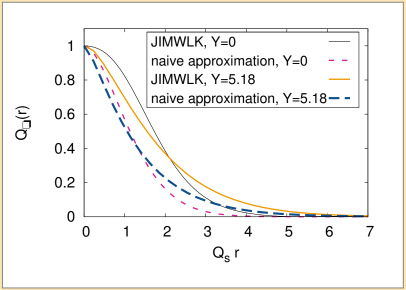




Line Square

(Concentrate your eyes on $Q_s r \lesssim 1 \Longleftrightarrow p_T \gtrsim Q_s$)

# Naive large $N_c$ is not good




Line

Square

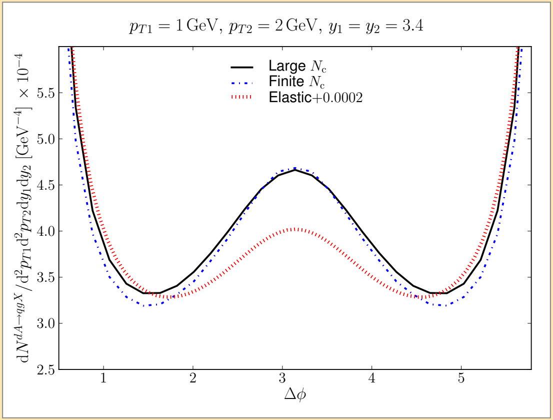
Even characteristic length/momentum scale differs by factor $\sim 2$.

How does this show up in dihadron cross section?

# Effect on the cross section

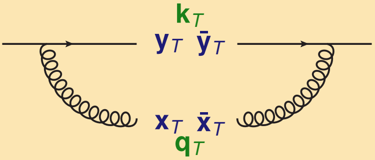
T. Lappi, H. Mäntysaari, in preparation

Use "factorized Gaussian": $\left\langle \hat{Q}\hat{D} \right\rangle \approx \left\langle \hat{Q} \right\rangle \left\langle \hat{D} \right\rangle$, with Gaussian $\left\langle \hat{Q} \right\rangle$




- Enhances $\Delta\varphi \approx \pi$peak by a factor $\sim 2$ "elastic" = cf. Albacete & Marquet
  $\Longrightarrow$ **essential for data comparison**
- The correct large $N_c$ limit OK (Not the "naive" $Q \approx DD$.)

# DPS contribution




DPS limit:

- $|\mathbf{x}_T - \bar{\mathbf{x}}_T| \sim |\mathbf{y}_T - \bar{\mathbf{y}}_T| \sim \frac{1}{Q_s}$
- $|\mathbf{x}_T - \mathbf{y}_T| \gg 1/Q_s$

CGC formula for $\sigma^{qA \to qgX}$ becomes

$$\left\langle \frac{1}{N_c} \operatorname{Tr} V^\dagger(\mathbf{x}_T) V(\bar{\mathbf{x}}_T) V^\dagger(\mathbf{y}_T) V(\bar{\mathbf{y}}_T) \frac{1}{N_c} \operatorname{Tr} V^\dagger(\bar{\mathbf{y}}_T) V(\mathbf{y}_T) \right\rangle$$

$$\xrightarrow{DPS} \left\langle \frac{1}{N_c} \operatorname{Tr} V^\dagger(\mathbf{x}_T) V(\bar{\mathbf{x}}_T) \right\rangle \left\langle \frac{1}{N_c{}^2} \overset{\text{ad}}{\operatorname{Tr}} U^\dagger(\mathbf{y}_T) U(\bar{\mathbf{y}}_T) + 1 \right\rangle$$

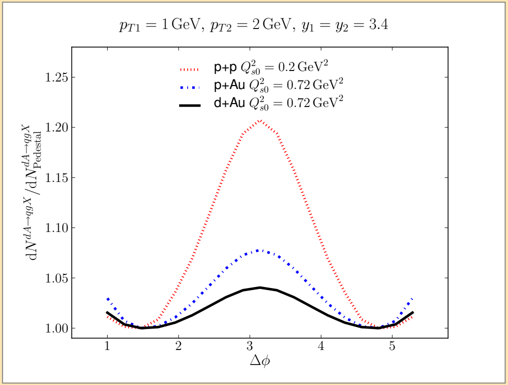
- $\Delta\varphi$-independent contribution from q,g scattering independently.
- IR divergence from $q \to qg$ split $\Longrightarrow$ regulate
- Nonperturbative input from **probe**: double parton distribution.

# Effect of DPS contribution

Plotting

$$\frac{\text{peak} + \text{pedestal}}{\text{pedestal}}$$

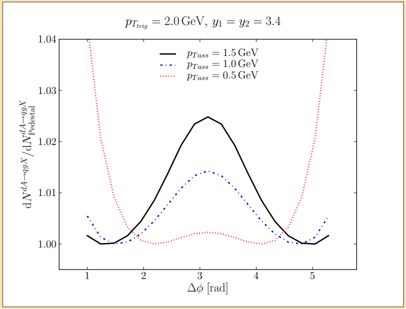
Plot is parton level



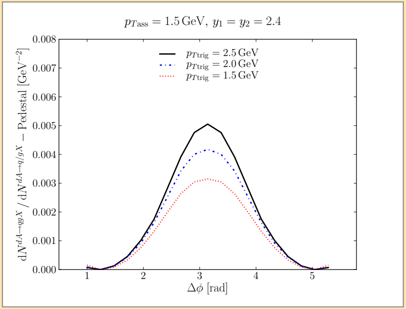

- Pedestal larger in dAu compared to pAu (peak same)
- Pedestal agreement with data still “order of magnitude”

| Data | $p_T$ range | pedestal | exp. |
|---|---|---|---|
| PHENIX dAu | $1.1\text{GeV} < p_T^{\text{trig}} < 1.6\text{GeV}$ | 0.104 | 0.176 |
| PHENIX dAu | $1.6\text{GeV} < p_T^{\text{trig}} < 2.0\text{GeV}$ | 0.081 | 0.163 |
| STAR dAu | $2\text{GeV} < p_T^{\text{trig}},\ 1\text{GeV} < p_T^{\text{ass}} < p_T^{\text{trig}}$ | 0.011 | 0.0145 |

# Dependence on associate and trigger $p_T$




Vary associate




Vary trigger

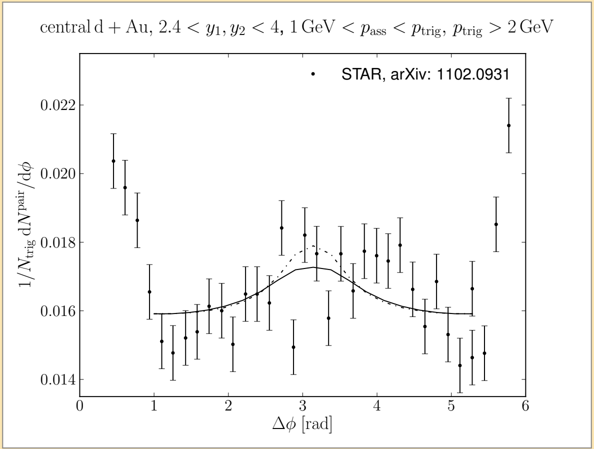
More details in poster by H. Mäntysaari

# Conclusion

- Studied renormalization group evolution of multiparticle correlators in CGC/JIMWLK formalism.
- Application: dihadron correlations in dAu / pPb
  - Including full Wilson line operator
    $\Longrightarrow$ factor $\sim 2$ difference compared to earlier literature.
  - Gaussian approximation works well.
  - Also calculate $\Delta\varphi$-independent pedestal

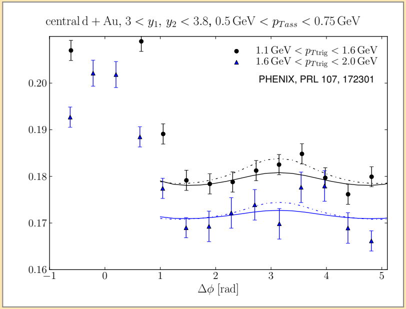
Data comparisons

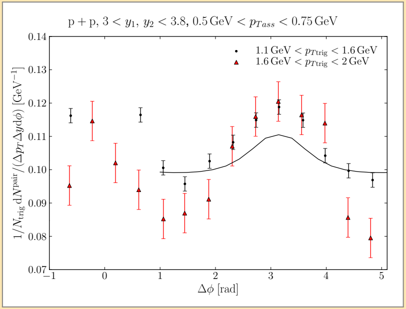
# STAR




(Pedestal adjusted to data, initial $Q_s^2 = 0.72\text{GeV}^2$ (dashed) $1.51\text{GeV}^2$ (solid) )

# PHENIX



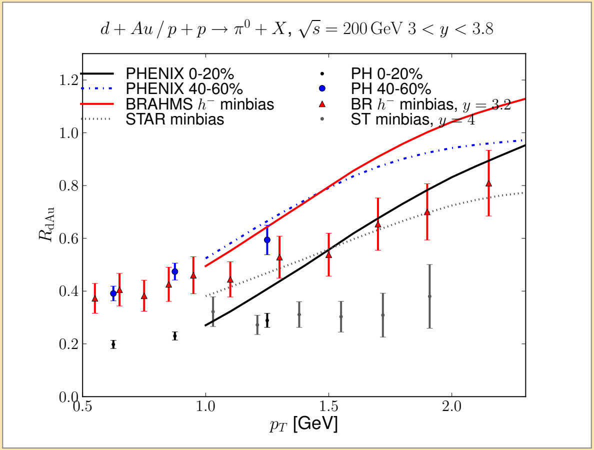




dAu pp

(Pedestal adjusted to data, initial $Q_s^2 = 0.72\text{GeV}^2$ (dashed) $1.51\text{GeV}^2$ (solid) )

# Single inclusive

Use MV, simple scaling of $Q_s^2 \sim N_{bin}$ $\Longrightarrow$ nothing has been fitted to dAu data




STAR $y = 4$ data hard to understand: this is common to all CGC calculations.