Mapping the Phase Structure of Cold Nuclei -- Gluon Saturation at small-x

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Refs: Mueller, Xiao, Yuan, to be published;
Stasto, Xiao, Yuan, arXiv:1109.1817;
Dominguiz, Xiao, Yuan, PRL 106, 022301 (2011);
Marquet, Dominguiz, Xiao, Yuan, PRD83, 105005 (2011)
Outlines

- Introduction
- Sudakov double logarithms in hard processes
- Two-particle correlations as probe for the phase structure of cold nuclei
QCD dynamics in cold nuclei

- Inclusive observables
  - DIS structure functions
  - $P+A \rightarrow h+x$
- Gluon density in integral form or limited access to the $k_t$-dependence
- Full exploration of the dynamics needs $k_t$-dependence in hard processes
Kt-dependent observables

- Hard processes probe the kt-dependent gluon distributions directly.
- Saturation phenomena manifest in the observables.

\[ P_J \gg K_T \]
Sudakov Double Logarithms

- Differential cross section depends on $Q_1$, where $Q^2 >> Q_1^2 >> \Lambda^2_{QCD}$

$$\frac{d\sigma}{dQ_1^2} = \frac{1}{Q_1^2} f_1 \otimes f_2 \otimes \sum_i \alpha_s^i \ln^{2i-1} \frac{Q^2}{Q_1^2} + \ldots$$

- We have to resum these large logs to make reliable predictions
  - $Q_T$: Dokshitzer, Diakonov, Trojan, 78; Parisi Petronzio, 79; Collins, Soper, Sterman, 85
  - Threshold: Sterman 87; Catani and Trentadue 89
Collins-Soper-Sterman Resummation

- Large Logs are re-summed by solving the energy evolution equation of the TMDs
  \[ \frac{\partial}{\partial \ln Q} f(k_\perp, Q) = (K(q_\perp, \mu) + G(Q, \mu)) \otimes f(k_\perp, Q) \]

- K and G obey the renormalization group eq.
  \[ \frac{\partial}{\partial \ln \mu} K = -\gamma_K = \frac{\partial}{\partial \ln \mu} G \]

- The large logs will be resummed into the exponential form factor
  \[ W(Q, b) = e^{-\int_{1/b}^{Q} \frac{d\mu}{\mu} \left( \ln \frac{Q}{\mu} A + B \right)} C \otimes f_1 C \otimes f_2 \]

- A,B,C functions are perturbative calculable.
Sudakov resummation at small-\(x\)

- Take massive scalar particle production \(p+A\to H+X\) as an example to demonstrate the double logarithms, and resummation.

\[
\frac{d\sigma^{(LO)}}{dy d^2 k_{\perp}} = \sigma_0 \int \frac{d^2 x_{\perp} d^2 x'_{\perp}}{(2\pi)^2} e^{i k_{\perp} \cdot r_{\perp}} x_0 g_p(x_0) S^{(WW)}(x_{\perp}, x'_{\perp})
\]

\[
S^{WW}_{Y}(x_{\perp}, y_{\perp}) = - \left\langle \text{Tr} \left[ \partial_{\perp}^\beta U(x_{\perp}) U^\dagger(y_{\perp}) \partial_{\perp}^\beta U(y_{\perp}) U^\dagger(x_{\perp}) \right] \right\rangle_Y
\]
Explicit one-loop calculations

\[ x_0 g_p(x_0) \int \frac{d\xi}{\xi} K_{DMMX} \otimes S^{WW}(x_\perp, y_\perp) + \left( -\frac{1}{e} \right) S^{WW}(x_\perp, y_\perp) P_{g/g} \otimes x_0 g(x_0), \]

- Collinear divergence \( \Rightarrow \) DGLAP evolution
- Small-x divergence \( \Rightarrow \) BK-type evolution

Dominguiz-Mueller-Munier-Xiao, 2011
Final result

- Double logs at one-loop order

\[
\frac{d\sigma^{(\text{LO+NLO})}}{dyd^2k_\perp}|_{k_\perp \ll Q} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{i\vec{k}_\perp \cdot \vec{r}_\perp} S_{Y=\ln 1/x_a}(x_\perp, x'_\perp) x g_p(x, \mu^2 = \frac{c_0^2}{r_{\perp}^2}) \left\{ 1 + \frac{\alpha_s}{\pi} C_A \left[ \beta_0 \ln \frac{Q^2 r_{\perp}^2}{c_0^2} - \frac{1}{2} \left( \ln \frac{Q^2 r_{\perp}^2}{c_0^2} \right)^2 + \frac{\pi^2}{2} \right] \right\},
\]

- Collins-Soper-Sterman resummation

\[
\frac{d\sigma^{(\text{resum})}}{dyd^2k_\perp}|_{k_\perp \ll Q} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{i\vec{k}_\perp \cdot \vec{r}_\perp} e^{-S_{\text{sud}}(Q^2, r_{\perp}^2)} S_{Y=\ln 1/x_a}(x_\perp, x'_\perp) x g_p(x, \mu^2 = \frac{c_0^2}{r_{\perp}^2}) \left[ 1 + \frac{\alpha_s}{\pi} \frac{\pi^2}{2 N_c} \right],
\]
Comments

- Sudakov double logs can be re-summed consistently in the small-x formalism.
- Kinematics of double logs and small-x evolution are well separated.
  - Soft vs collinear gluons
- If $Q_s$ is small, back to dilute region.
- If $Q_s$ is large ($\sim Q$), we can safely neglect the Sudakov effects.
Di-jet correlations in pA

- Effective kt-factorization is formulated

\[
\frac{d\sigma(pA\rightarrow \text{Dijet} + X)}{d\mathcal{P}.S.} = \sum_q x_1 q(x_1) \frac{\alpha_s^2}{\hat{s}^2} \left[ \mathcal{F}^{(1)}_{qq} H^{(1)}_{qq\rightarrow qq} + \mathcal{F}^{(2)}_{qq} H^{(2)}_{qq\rightarrow qq} \right] \\
+ x_1 g(x_1) \frac{\alpha_s^2}{\hat{s}^2} \left[ \mathcal{F}^{(1)}_{gg} \left( H^{(1)}_{gg\rightarrow q\bar{q}} + H^{(1)}_{gg\rightarrow gg} \right) \\
+ \mathcal{F}^{(2)}_{gg} \left( H^{(2)}_{gg\rightarrow q\bar{q}} + H^{(2)}_{gg\rightarrow gg} \right) + \mathcal{F}^{(3)}_{gg} H^{(3)}_{gg\rightarrow gg} \right],
\]

- In forward region of dAu collisions at RHIC and pA at LHC, Qs is large enough to neglect the Sudakov effects
Central dAu collisions

- $\eta_1 \sim \eta_2 \sim 3.2$
- $Q^2_{sA} \sim 0.85A^{1/3}Q_{sp}^2$
- GBW model
- No Sudakov effects
- No-BK evolution
- Geometric scaling
Mapping the phase structure

- RHIC
- pA at LHC
Universality of UGD

- Un-ambiguously study at the EIC
Conclusion

- Sudakov double logs are re-summed in the small-$x$ saturation formalism
- Hard processes are used to map the phase structure of cold nuclei at small-$x$
- Further developments shall follow to fully investigate the QCD dynamics at small-$x$ in dense medium
PHENIX $J_{dA}$
EIC predictions

![Graph showing EIC predictions for \( Q^2 = 1 \text{ GeV}^2 \) and \( Q^2 = 4 \text{ GeV}^2 \).](image)
pA at the LHC

\[ Y = \ln \frac{1}{x} \]

\[ \ln Q_s^2(Y) \]

\[ \ln \Lambda_{QCD}^2 \]

\[ \alpha_s \sim 1 \]

\[ \alpha_s \ll 1 \]

\[ P_T \]

\[-y=3.0 \]
\[-y=4.0 \]
\[-y=5.0 \]

\[ -y=3.0 \]
\[ -y=4.0 \]
\[ -y=5.0 \]
Violation effects
Compare to the STAR data

- $\eta_1 \sim \eta_2 \sim 3.1$
- GBW model for UGDs
- $Q_s^2 \sim (3.10^{-4}/x)^{0.28}\text{GeV}^2$
- $Q_{sA}^2 \sim 0.45A^{(1/3)}Q_{sp}^2$

- No Sudakov effects
- No-BK evolution
- Geometric scaling