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Relativistic Fluctuating Hydrodynamics and its Application to Heavy Ion Collisions

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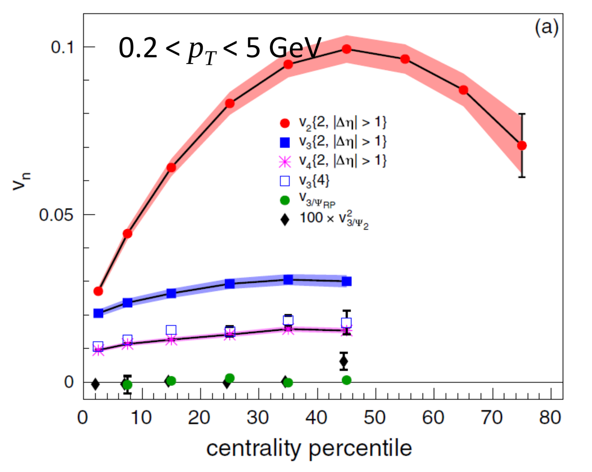
Abstract: While the initial state fluctuations are actively studied in dynamical models to investigate event-by-event fluctuation, there are other sources of fluctuations such as hydrodynamic fluctuations during the space-time evolution of QGP. To treat the hydrodynamic fluctuations in dynamical models, we formulated the relativistic fluctuating hydrodynamics in the context of the second-order causal theory.

1. Introduction

Event-by-Event Fluctuations in Harmonic Analysis

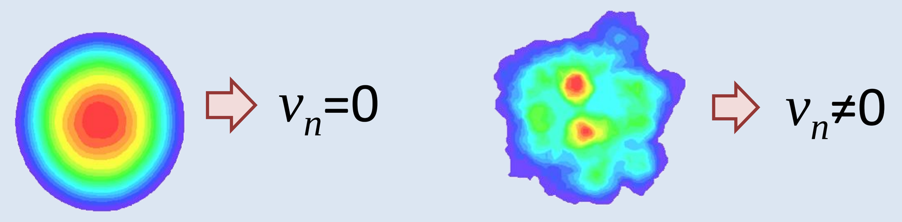
Anisotropic flows



ALICE Collaboration: Phys. Rev. Lett. 107 (2011) 032301 [arXiv:1105.3865]

Central collision,

The averaged initial condition | An initial condition of a single event



Event-by-event fluctuations →
- Non-zero harmonics at the central collision
- Non-zero odd harmonics

Hydrodynamic Fluctuations in Heavy Ion Collision Process

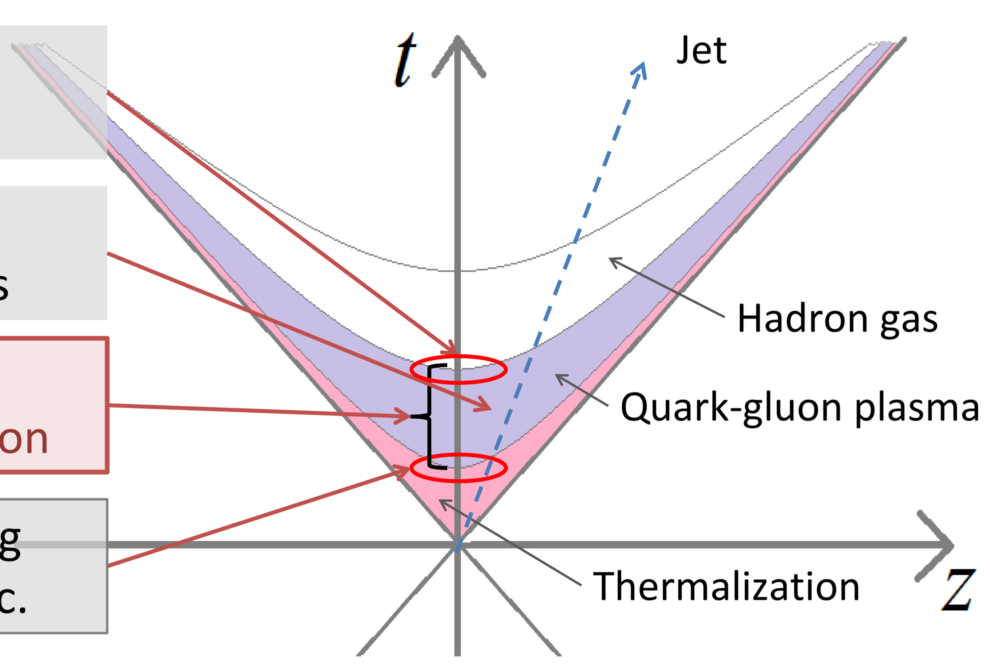
Various Event-by-Event Fluctuations in Heavy Ion Collisions

(d) *Thermal fluctuation* during the particlization

(c) *Disturbance* by Interaction between a fluid and (mini-)jets

(b) *Hydrodynamic fluctuation* arising together with dissipation

(a) *Initial state fluctuations* arising from the nucleon distribution, etc.



Hydrodynamic Fluctuations in First-Order Dissipative Hydrodynamics

= **Conservation Law**

$$\partial_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} = eu^\mu u^\nu - (P+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu},$$
$$\partial_\mu N^\mu = 0, \quad N^\mu = nu^\mu + \nu^\mu$$

14 degrees of freedom / 5 equations

$$g^{\mu\nu} := \mathrm{diag}(+,-,-,-),\quad u^\mu : \text{Landau frame},\quad \Delta^{\mu\nu} := g^{\mu\nu} - u^\mu u^\nu,$$
$$A^{(\mu\nu)} := \frac{1}{2}(A^{\mu\nu} + A^{\nu\mu}),\quad A^{\langle\mu\nu\rangle} := \Delta^{\mu\nu}_{\alpha\beta}A^{\alpha\beta} := [\Delta^{(\mu}{}_\alpha \Delta^{\nu)}{}_\beta - \tfrac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}]A^{\alpha\beta},\quad \theta = \partial_\alpha u^\alpha.$$

+ **Thermodynamics** (eq. of state and constitutive eqs.)

$$P + \Pi = P(e,n) - \zeta\theta + \delta\Pi,$$
$$\pi^{\mu\nu} = 2\eta\partial^{\langle\mu}u^{\nu\rangle} + \delta\pi^{\mu\nu},$$
$$\nu^\mu = \kappa T \nabla^\mu \frac{\mu}{T} + \delta\nu^\mu.$$

9 equations

Ensemble Average: Expected value

Hydrodynamic Fluctuations: Stochastic Process — Local deviation from the expected value in finite system

Hydro equation becomes *stochastic differential equation* like *Langevin equation*

Fluctuation Dissipation Relation → White Noise

$$\langle\delta\Pi(x)\delta\Pi(x')\rangle = 2T\zeta\,\delta^{(4)}(x-x'),$$
$$\langle\delta\pi^{\mu\nu}(x)\delta\pi_{\alpha\beta}(x')\rangle = 4T\eta\,\delta^{(4)}(x-x')\cdot g^{\langle\mu}{}_{\langle\alpha}g^{\nu\rangle}{}_{\beta\rangle},$$
$$\langle\delta\nu^\mu(x)\delta\nu^\alpha(x')\rangle = 2T\kappa\,\delta^{(4)}(x-x')\cdot(-\Delta^{\mu\alpha}).$$

J.I. Kapusta, B. Muller, M. Stephanov: Phys.Rev. C85 (2012) 054906 [arXiv:1112.6405]

Hydrodynamical Fluctuations in a Dynamical Model for Simulations

Single-shot simulation (averaged picture)
→ Linear contribution of hydrodynamic fluctuations vanishes.

Event-by-Event simulations
→ Effects of hydrodynamic fluctuations is important.

- *Hydrodynamic fluctuations in second-order dissipative hydrodynamics*
  → *Determine transport coefficients including memory effects*
  → Implement the hydrodynamic fluctuations in a dynamical model

2. Dissipative Hydrodynamics with Memory Functions and Colored Noises

First-order dissipative hydrodynamics: *acausal modes*

⇨ Second- or higher-order dissipative hydrodynamic equations with *n* conserved charges:
- finite relaxation time
- consistent with causality

$$\partial_\mu T^{\mu\nu} = 0,$$
$$\partial_\mu N_i^\mu = 0 \quad (i = 1,\ldots,n).$$

■ **Constitutive Equations** expressed in a more general form with *retarded Green functions*:

- $G(x, x')$: "***Memory function***" memory due to the non-zero relaxation time

$$\pi^{\mu\nu} = \int_{\substack{(x-x')^2>0\\ x^0>x'^0}} \mathrm{d}^4x' G_\pi(x,x')(\partial^{\langle\mu}u^{\nu\rangle}|_{x'}) + \delta\pi^{\mu\nu},$$
$$\Pi = \int_{\substack{(x-x')^2>0\\ x^0>x'^0}} \mathrm{d}^4x' G_\Pi(x,x')\theta(x') + \delta\Pi,$$
$$\nu_i^\mu = \int_{\substack{(x-x')^2>0\\ x^0>x'^0}} \mathrm{d}^4x' G_{ij}(x,x')(T\nabla^\mu \tfrac{\mu_j}{T}|_{x'}) + \delta\nu_i^\mu.$$

- The integrals ∫$G(x, y)$ $F(y)$, in general, contain most of terms appearing in the conventional second-order dissipative hydrodynamics. $G(x, y)$ can be a functional of the fluid fields in the past, thus this expression contains *more higher-order dissipative terms*.

$$G(x,y) = G(x,y;\ \{u^\mu(z), T(z), \mu_i(z)\}_z)$$
$$= G(x,y;\ u^\mu(x), T(x), \mu_i(x), \partial_\alpha u^\mu(x), \partial_\alpha T(x), \partial_\alpha \mu_i(x), \ldots)$$

- Consider a simpler case (near the equilibrium):

$$G(x,y) = G(x-y; T(x), \mu_i(x))$$

■ **Fluctuation Dissipation Relation**

$$\langle\delta\pi^{\mu\nu}(x)\delta\pi_{\alpha\beta}(x')\rangle = TG_\pi(x-x')\cdot g^{\langle\mu}{}_{\langle\alpha}g^{\nu\rangle}{}_{\beta\rangle},$$
$$\langle\delta\Pi(x)\delta\Pi(x')\rangle = TG_\Pi(x-x'),$$
$$\langle\delta\nu_i^\mu(x)\delta\nu_j^\alpha(x')\rangle = TG_{ij}(x-x')\cdot(-\Delta^{\mu\alpha}).$$

$G(x)$: Extended for x^0<0 as even functions

Non-zero correlation in different time → ***Colored Noise***

3. A Simple Case for Simulation

■ **Simple second-order dissipative hydro**

$$\tau_\pi D^*\pi^{\mu\nu} + \pi^{\mu\nu} = 2\eta\partial^{\langle\mu}u^{\nu\rangle},$$
$$\tau_\Pi D\Pi + \Pi = -\zeta\theta,$$
$$\tau_i D^*\nu_i^\mu + \nu_i^\mu = \kappa_{ij}T\nabla^\mu\frac{\mu_j}{T}.$$

$$D^*\pi^{\mu\nu} := \Delta^{\mu\nu}_{\alpha\beta}D\pi^{\alpha\beta} = D\pi^{\mu\nu} + 2u^{(\mu}\pi^{\nu)\alpha}Du_\alpha,$$
$$D^*\nu_i^\mu := \Delta^\mu{}_\alpha D N_i^\alpha = D\nu_i^\mu + u^\mu\nu_{i\alpha}Du^\alpha + n_i Du^\mu,$$
$$D = u^\alpha\partial_\alpha.$$

■ **Constitutive Equations with Memory functions**

$$\pi^{\mu\nu} = 2\eta\int_{-\infty}^{\tau} \mathrm{d}\tau' \frac{1}{\tau_\pi} e^{-\frac{\tau-\tau'}{\tau_\pi}} (\partial^{\langle\mu}u^{\nu\rangle}|_{\tau'}) + \delta\pi^{\mu\nu},$$
$$\Pi = -\zeta\int_{-\infty}^{\tau} \mathrm{d}\tau' \frac{1}{\tau_\Pi} e^{-\frac{\tau-\tau'}{\tau_\Pi}} \theta(\tau') + \delta\Pi,$$
$$\nu_i^\mu = \kappa_{ij}\int_{-\infty}^{\tau} \mathrm{d}\tau' \frac{1}{\tau_i} e^{-\frac{\tau-\tau'}{\tau_i}} (T\nabla^\mu \tfrac{\mu_j}{T}|_{\tau'}) + \delta\nu_i^\mu.$$

■ **Numerically generating colored noise**

Colored Gaussian noise can be created using white Gaussian noise

$$\xi(t) = \int_{-\infty}^{t} \mathrm{d}t' K(t-t')\cdot\xi_{\text{white}}(t')$$
$$\text{where } |\tilde{K}(\omega)|^2 = \tilde{G}(\omega).$$
$$\langle\xi_{\text{white}}(t)\xi_{\text{white}}(t')\rangle = \delta(t-t'). \quad \Rightarrow \quad \langle\xi(t)\xi(t')\rangle = G(t-t').$$

4. Summary

- In event-by-event dynamical simulations of heavy-ion collisions, the hydrodynamic fluctuations would have important effects on observables which should be investigated by using dynamical models.
- In second-order causal dissipative hydrodynamics, the hydrodynamic fluctuations have to be colored to be consistent with the finite relaxation time of the dissipative currents.