Abstract: While the initial state fluctuations are actively studied in dynamical models to investigate event-by-event fluctuation, there are other sources of fluctuations such as hydrodynamic fluctuations during the space-time evolution of QGP. To treat the hydrodynamic fluctuations in dynamical models, we formulated the relativistic hydrodynamics in the context of the second-order causal relation.

1. Introduction

Event-By-Event Fluctuations in Harmonic Analysis

Hydrodynamic Fluctuations in Heavy Ion Collision Process

(a) Initial state fluctuations arising from the nucleon distribution, etc.
(b) Hydrodynamic fluctuation arising together with dissipation
(c) Disturbance by interaction between a fluid and (mini-)jets
(d) Thermal fluctuation during the particlization

Various Event-By-Event Fluctuations in Heavy Ion Collisions

- Hydrodynamic Fluctuations in First-Order Dissipative Hydrodynamics
  - Conservation Law
  - Thermodynamics
  - 14 degrees of freedom / 5 equations

2. Dissipative Hydrodynamics with Memory Functions and Colored Noises

Fluctuation Dissipation Relation

Gaitonde: Extended for $\xi(t)$ as even functions

3. A Simple Case for Simulation

Constitutive Equations with Memory Functions

Constitutive Equations expressed in a more general form with retarded Green functions:

Constitutive Equations with Memory Functions and Colored Noises

Fluctuation Dissipation Relation

Hydrodynamic Fluctuations in a Dynamical Model for Simulations

Hydrodynamic Fluctuations: Stochastic Process

Local deviation from the expected value in finite system

Hydrodynamic equations become stochastic differential equations like Langevin equation

4. Summary

- In event-by-event dynamical simulations of heavy-ion collisions, the hydrodynamic fluctuations would have important effects on observables which should be investigated by using dynamical models.
- In second-order causal dissipative hydrodynamics, the hydrodynamic fluctuations have to be colored to be consistent with the finite relaxation time of the dissipative currents.