Chiral symmetry breaking in QCD and related theories

(" INSTANTON LIQUID 2")

Edward Shuryak (Stony Brook)

For about a decade it is known that topological fluctuations -- instantons -- are modified by the nonzero Polyakov line VEV and split into Nc dyons. By now there is extensive lattice literature confirming this fact and explaining certain observations by properties of such dyons, mostly at T=(1-2)Tc. This talk report the first direct simulations of the statistical mechanics of the ``dyonic vacuum", using one-loop partition function. We found that chiral symmetry breaking and Dirac eigenvalues spectra are strongly affected by the LLbar dyon clustering. Among many consequences explaining lattice data is the dependence on the chiral transition on the number of fermion species Nf and the fermionic periodicity condition.

Nambu-Jona-Lasinio versus the instanton liquid model

- NJL (1961) introduced hypothetical 4-fermion interaction (chirally symmetric)
- 2 parameters, G and cutoff Lambda (about 1 GeV)
- Good chiral physics, pions etc (no confinement)
- Eta' also massless
- Other particles like sigma, rho,N = 2 const.quarks
- Higher orders undefined

The Instanton Liquid Model ES (1982) also has 2 parameters

$$n \approx 1fm^{-4}, \rho \approx 1/3fm, n\rho^4 \sim 10^{-2}$$

It also describes all the chiral physics correctly

It can be and was solved to all orders
Rho and nucleon are bound and many correlators are well described

eta' is now correct (repulsive 4-fermi term from 't Hooft makes it heavy)

Instantons emerging from vacuum quantum noise by ``cooling”
(MIT group, 1993), S and Q

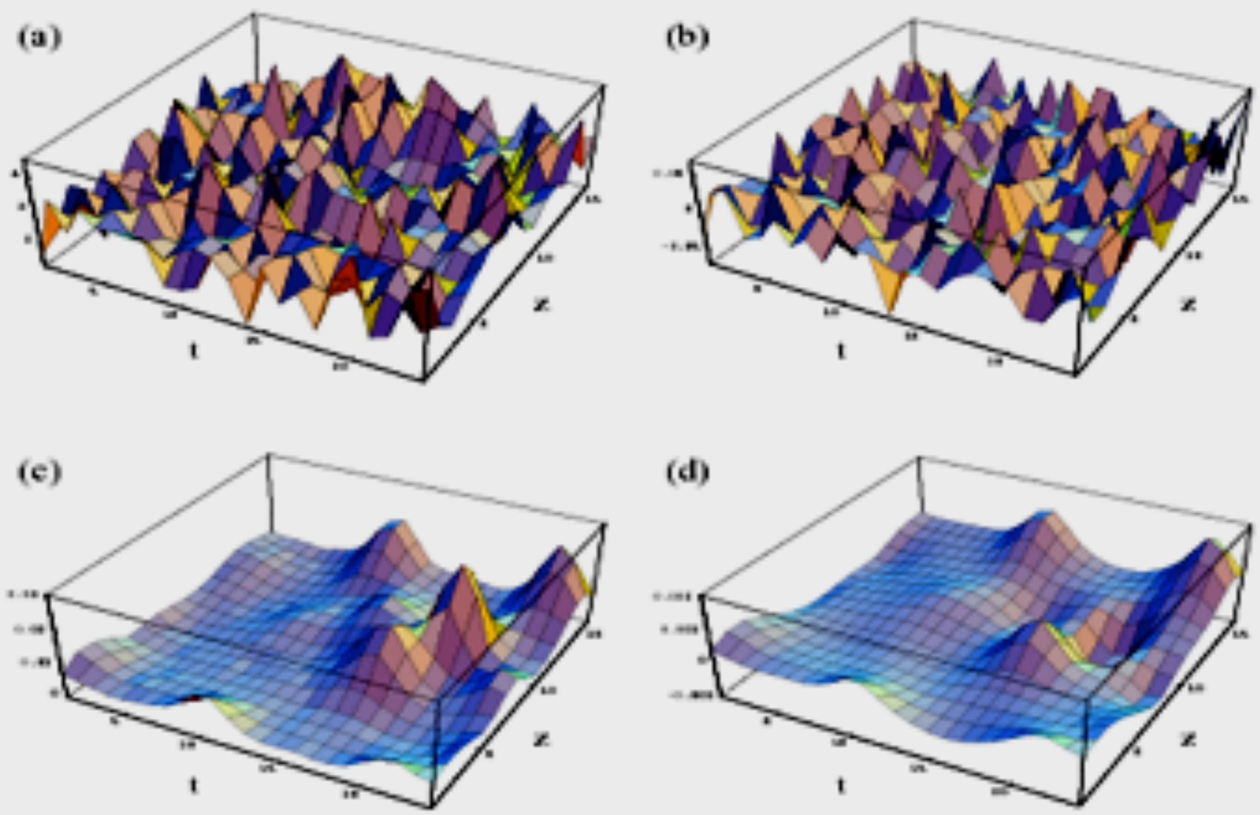
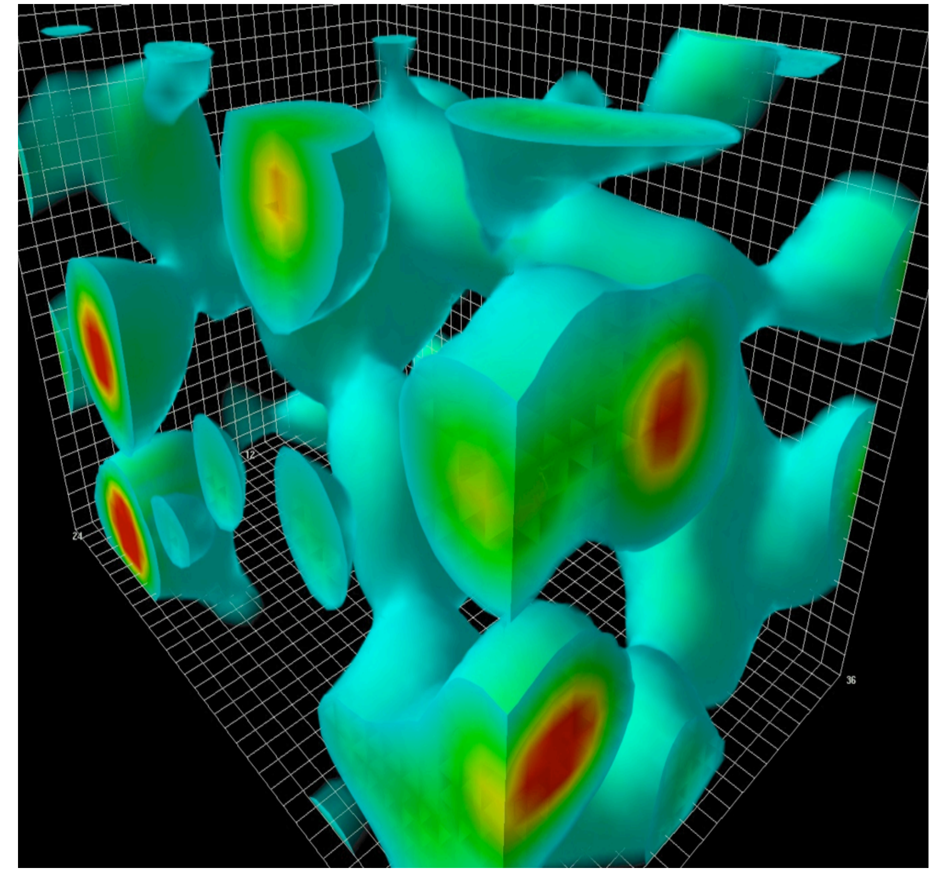


Fig. 5.8 Examples of cooling of lattice configurations



Instantons induce forces between light quarks which are qualitatively different from gluon exchanges

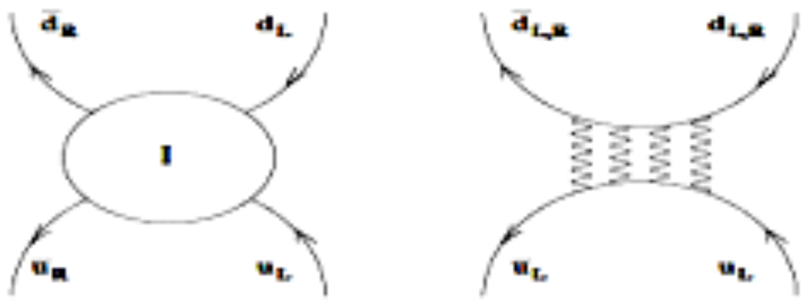
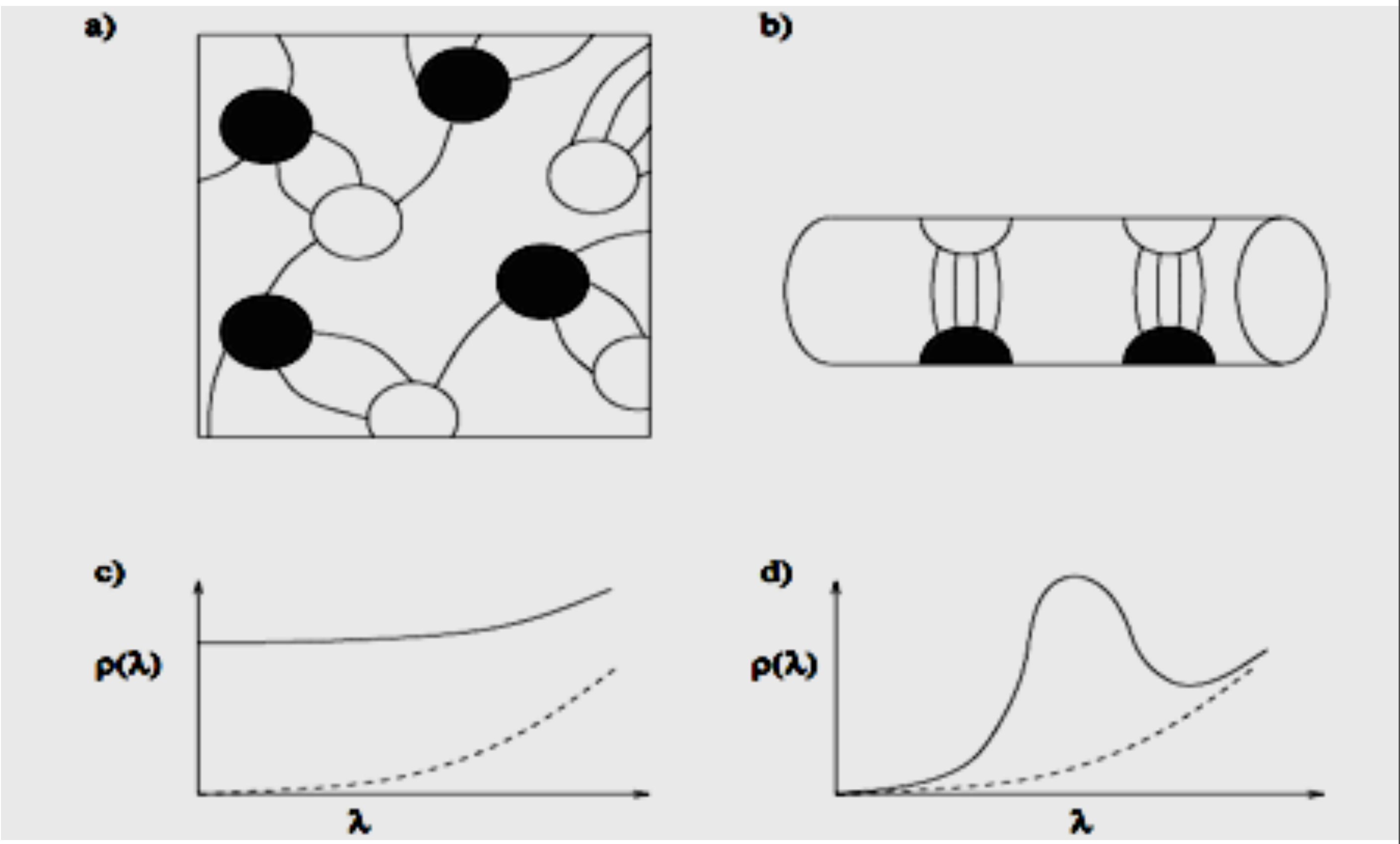


Fig. 4.1 The instanton-induced 't Hooft vertex (a) for 2 flavor QCD versus the ordinary gluon exchange diagrams (b). Note a very different chiral structure of the two: the latter does not violate any chiral symmetry because chirality is conserved along each line.

For $N_f = 2$, the result is

$$\mathcal{L}_{N_f=2} = \int d\rho\, n_0(\rho) \Big[\prod_f \left(m\rho - \frac{4}{3}\pi^2\rho^3 q_{f,R} q_{f,L} \right) + \frac{3}{32}\left(\frac{4}{3}\pi^2\rho^3\right)^2 \left(\bar{u}_R\lambda^a u_L \bar{d}_R \lambda^a d_L - \frac{3}{4}\bar{u}_R\sigma_{\mu\nu}\lambda^a u_L \bar{d}_R\sigma_{\mu\nu}\lambda^a d_L \right) \Big]. \quad (4.14)$$

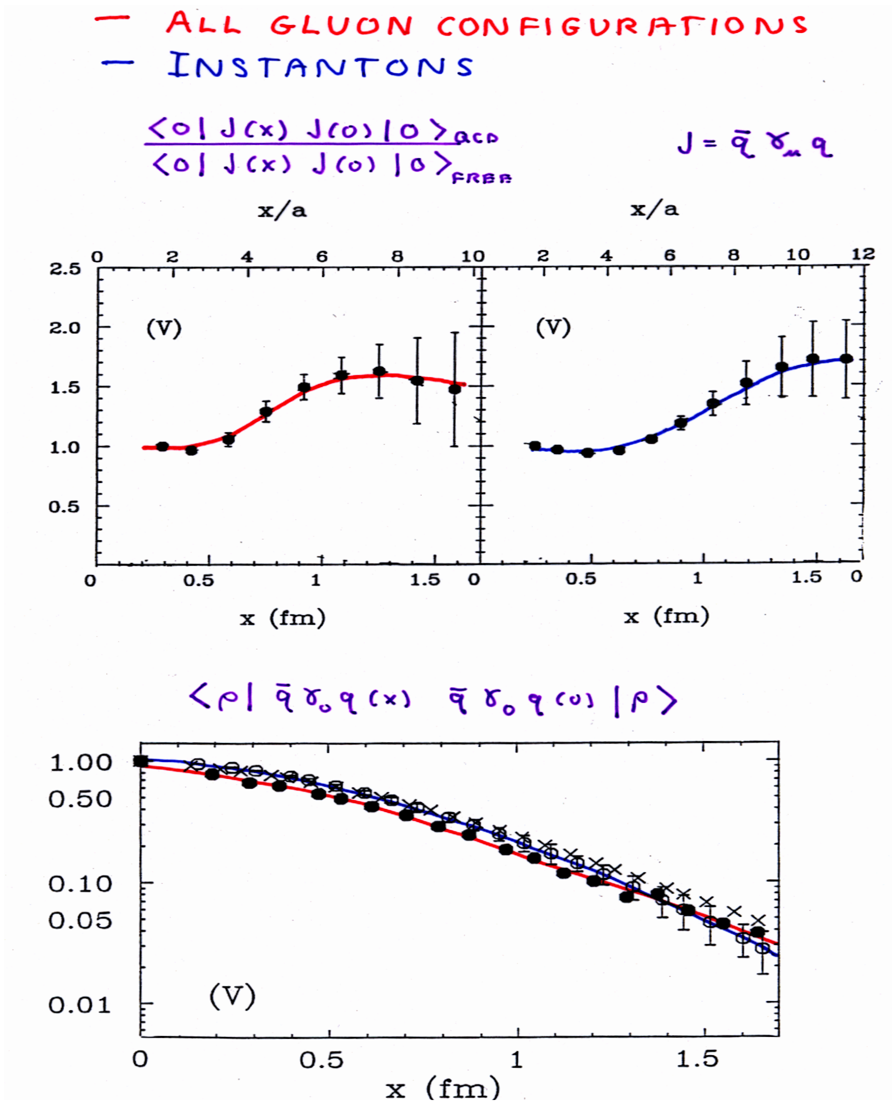
Instanton liquid at T=0 and T>Tc (schematic pictures)

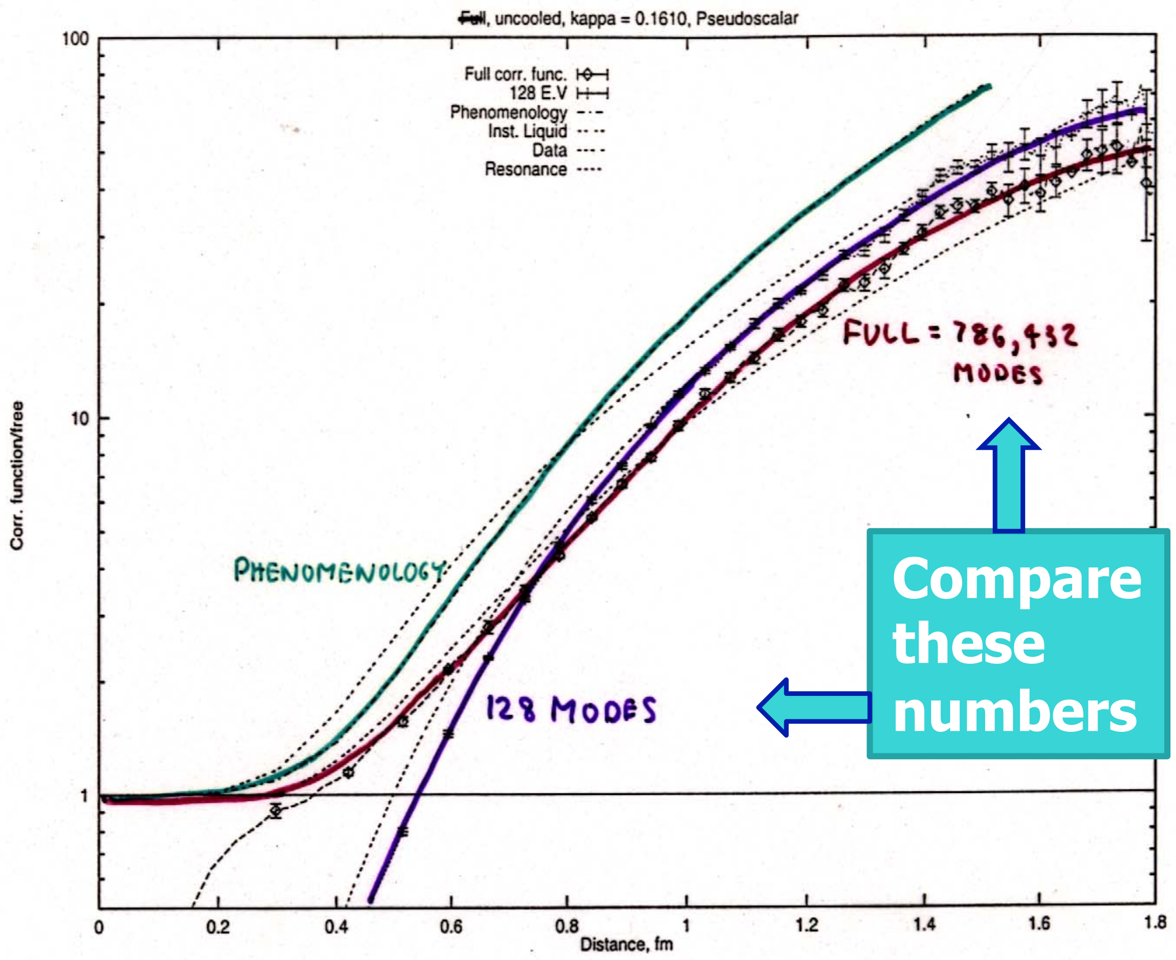


prediction of ILM: the Zero Mode Zone

The tiny fraction 0.01% of fermionic states (about 1 per instanton) is indeed enough to reproduce most of light quark hadronic physics

J.Negele,T.DeGrand,A.Hazenfratz

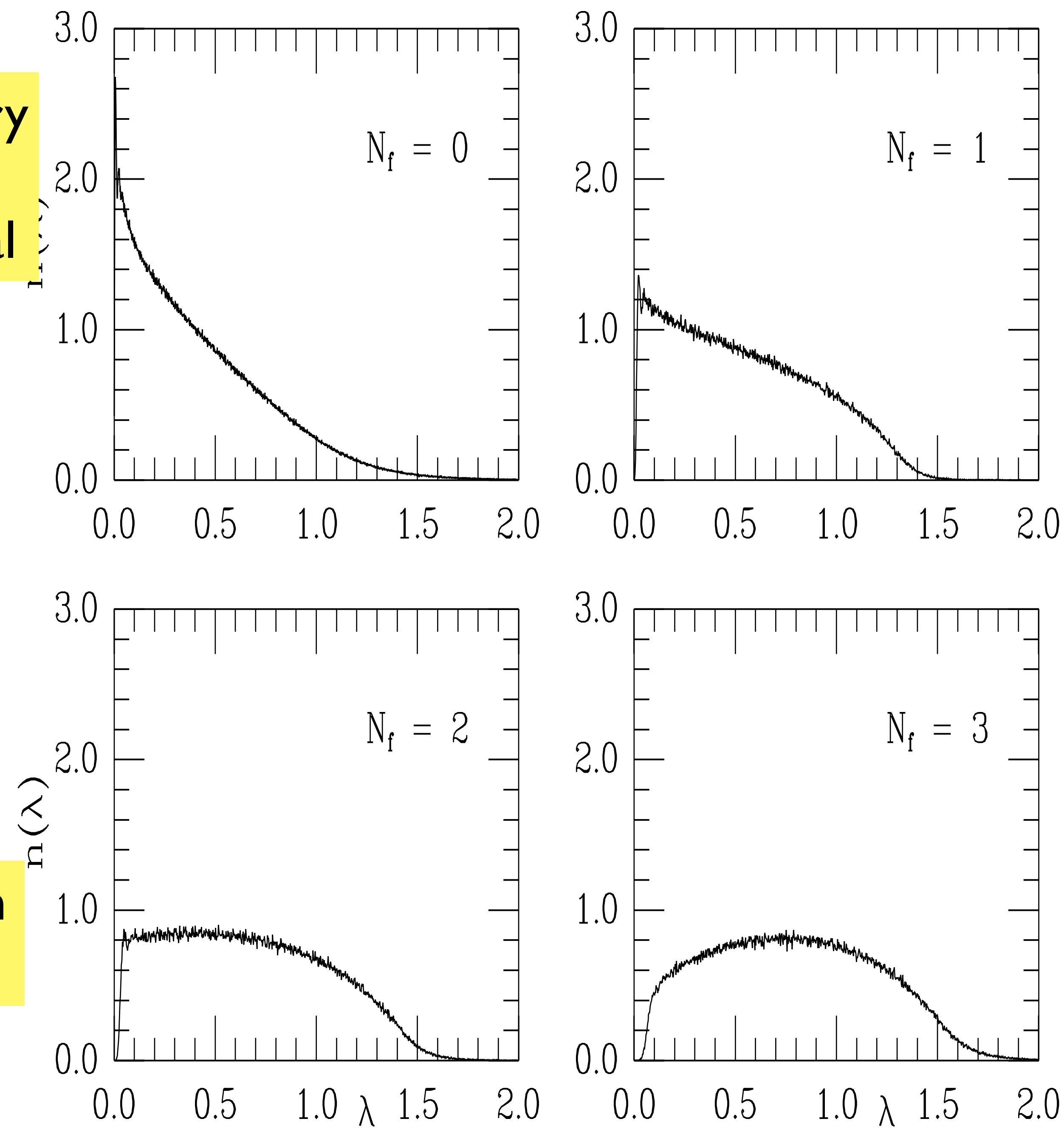




The spectrum of the Dirac eigenvalues

the quenched theory has a singularity: it is not really physical

Smilga-Stern theorem -|lambda|(Nf-2)



Dyons in SU(2)

name	E	M	mass
M	$+$	$+$	v
$\bar{M}$	$+$	-	v
L	-	-	$2\pi T - v$
$\bar{L}$	-	$+$	$2\pi T - v$

TABLE I: The charges and the mass (in units of $8\pi^2/e^2T$) for 4 SU(2) dyons.

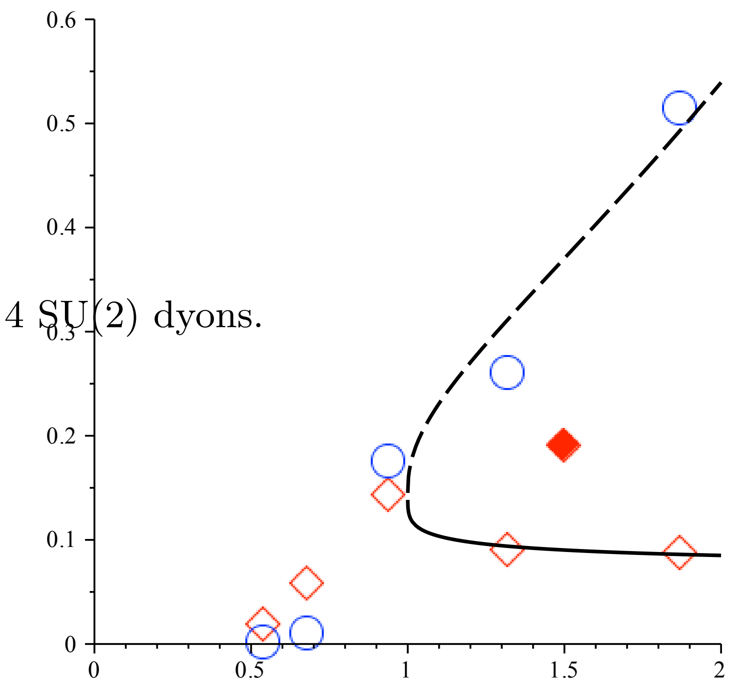


FIG. 1: (color online) The density of the topological clusters (in fm^{-4}) versus the temperature T/T_c of the SU(2) pure gauge theory. The open blue circles show static dyons, identified as M-type, while the open red diamonds are for calorons or L-type dyons [14]. The closed diamond at $T/T_c = 1.5$ is from [15], in which L-type dyons were identified directly by fermionic zero modes and the value of the Polyakov loop at its center. The dashed and solid lines correspond to semiclassical expectations for M, L dyon density, with parameters defined in the text.

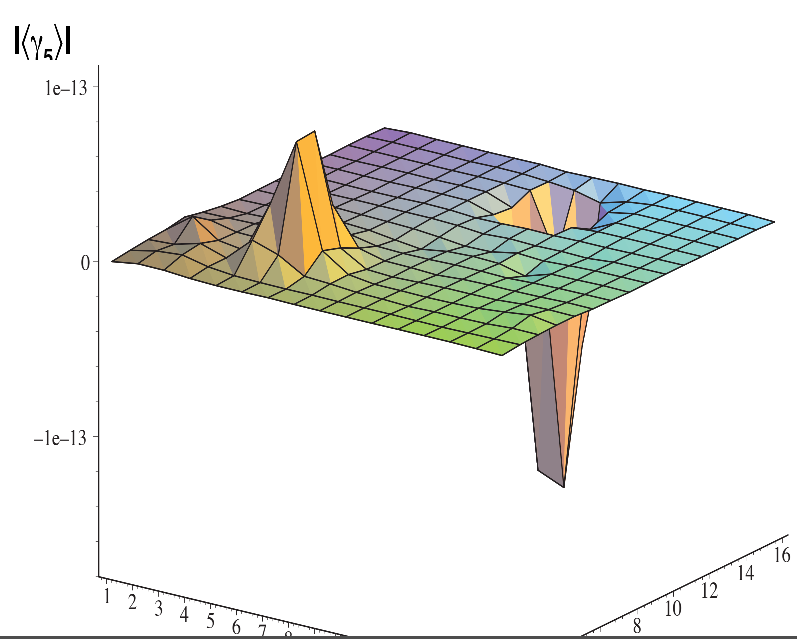
When v is small,
L is heavy and M is light

Nf is the number of fund.quarks

M is the fermion mass
Which is also dependent on holonomy

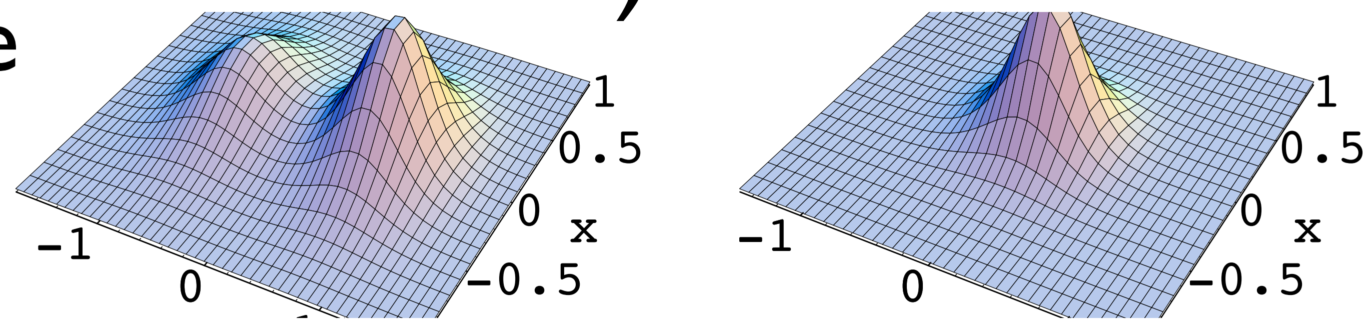
At high T the density is small =>
Neutral ``molecules" of 2Nc dyons
Here are its shape for SU(2)

In fact seen on the lattice as (G Gdual)
Where the field is``filtered" using
``on the gap" modes
Gattringer, PRL 2002



Instantons => Nc selfdual dyons (van Baal et al)

<P> nonzero Polyakov line
=> <A_4> nonzero
=> new solutions



Instanton liquid
4d+short range

Dyonic plasma
3+1d long range

1. The chiral symmetry breaking/restoration in the dyonic vacuum, ES and T.Sulejmanpasic, arXiv:1201.5624
2. The chiral... II. Adjoint fermions, ES and T.Sulejmanpasic
3. QCD Topology at finite temperature: Statistical Mechanics of Selfdual Dyons, P.Faccioli and ES, in progress

Fermionic zero modes for arbitrary periodicity parameter z

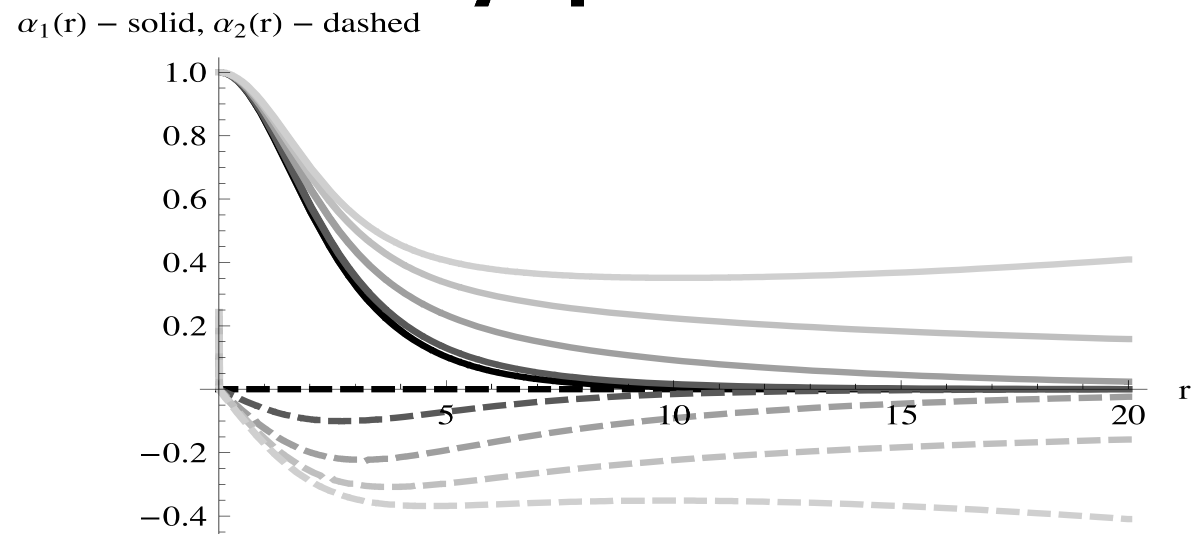


FIG. 6: Plot shows profile of zeromode components $\alpha_{1,2}$, for four different values of $z = 0, 0.2v/\beta, 0.4v/\beta, 0.5v/\beta, 0.55v/\beta$. Note that the zero mode delocalizes at $z = 0.5v/\beta$

The Dirac operator in the background of the dyon cubic lattice would connect only between the left and the right chiral fermions, i.e. it would look like

$$\not{D} = \begin{pmatrix} 0 & \dots & \dots & 0 & f(r_{11}) & f(r_{12}) & \dots & f(r_{1N}) \\ \vdots & \ddots & \dots & 0 & f(r_{21}) & f(r_{22}) & \dots & f(r_{2N}) \\ \vdots & & & & & & & \\ \vdots & \ddots & \dots & 0 & f(r_{N1}) & f(r_{N2}) & \dots & f(r_{NN}) \\ f(r_{11}) & f(r_{12}) & \dots & f(r_{1N}) & 0 & \dots & \dots & 0 \\ f(r_{21}) & f(r_{22}) & \dots & f(r_{2N}) & 0 & \dots & \dots & 0 \\ \vdots & & & & & & & \\ f(r_{N1}) & f(r_{N2}) & \dots & f(r_{NN}) & 0 & \dots & \dots & 0 \end{pmatrix} \tag{150}$$

where $f(r)$ is given by

$$f(r) = \int d^4x \, \psi_R^\dagger(\vec{x} - \vec{r}) \not{D} \psi_L(\vec{x}) \; , \tag{151}$$

where $\psi_{L,R}(\vec{x})$, are zero mode solutions on top of dyons localized at $\vec{x} = 0$. Since the decay of

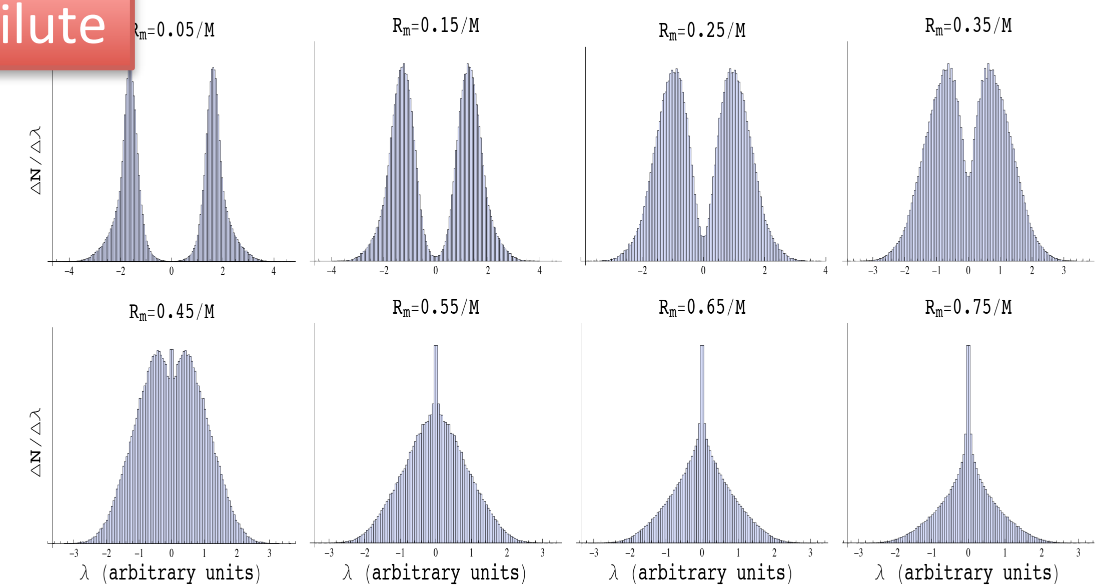
As a first step toward the understanding of the dyonic ensembles, and their role in chiral symmetry breaking/restoration, we had formulated some simplified models.

For calculation purposes it is convenient for these models to treat the dyon density

$$n_d = n_L = n_M = n_{\bar{L}} = n_{\bar{M}} \quad (137)$$

(which is also the same as the instanton density $n_{inst} + n_{antiinstanton}$) as the basic dimensional quantity, providing the units of length $n_d^{-1/3}$. Using such length units we put $n_d = 1$ for a while, and will be expressing other dimensional quantities in these units. We will be working with traditional periodic boxes of some size $L \times L \times L$, with L "large", and thus put into such boxes $N_d = L^3$ dyons of each kind.

dilute

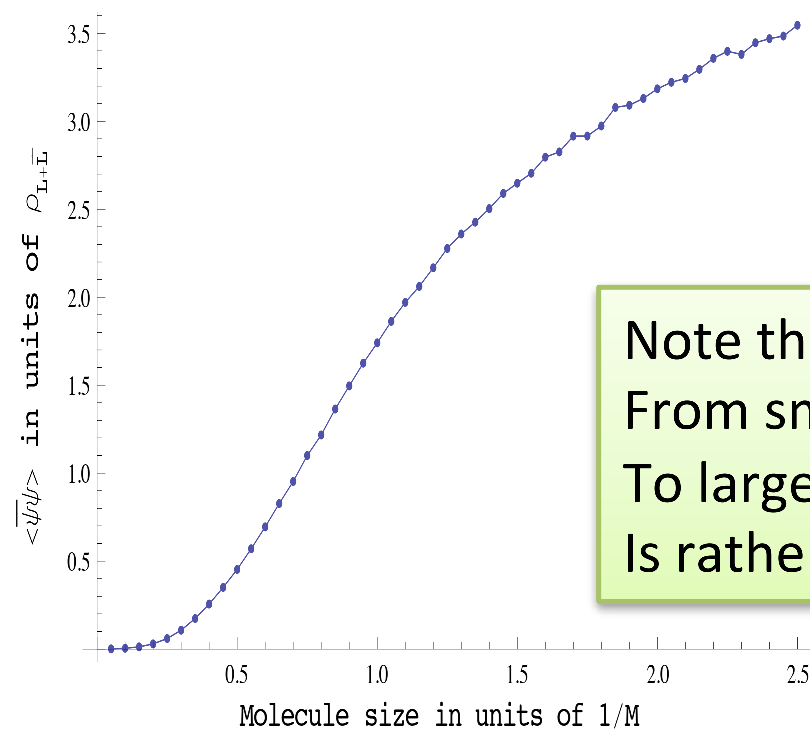


dense

FIG. 9: The spectrum of eigenvalues for several values λ of a Dirac operator with in an ensemble of 108 $L-\bar{L}$ pairs, with molecule sizes ranging from $R_m = 0.05/M \ldots 0.75/M$, with $M = 0.5\pi$ and $\rho_{L+\bar{L}} = 1 = 2\rho_L$.

The re-weighting including molecule
fermionic interaction
is in progress

Potentially stat.mech of dyonic ensemble using Metropolis
Like we did for instantons in 1980's



Note that transition
From small condensate
To large condensate
Is rather rapid

FIG. 10: Chiral condensate in the random molecule model. The

Many lattice observations are explained

Sensitivity to the fermionic boundary conditions

Introducing an arbitrary phase on the fermionic boundary condition, one can switch the fermionic zero mode between the dyons: this has been demonstrated using artificial configurations for calorons in [13].

The sensitivity appears at T>Tc: **antiperiodic fermions restore chiral symmetry**

While periodic ones don't! (in the quenched ensemble!)

This can be explained by the observation (Bruckmann ?) that

antiperiodic fermions have zero modes with heavy L, bar-L dyons which form ``clusters" or nuclei,

while periodic one have zero modes with the "non-twisted" M dyons which are nearly randomly distributed

Statistical mechanics of the dyons: 3 elements

- The moduli space metric (Atiyah,Manton,Diakonov)
- The screening (ES,Pisarski-Yaffe, Diakonov)
- Fermion-induced factor (ES,Sulejmanpasic)
- First numerical implementation, 64 dyons on S^3, Faccioli+ES

moduli volume element

Following Gibbons and Manton [11] and Diakonov [6], the invariant volume element for moduli metric can be approximated by the (first power) of the determinant of certain matrix G

$$\sqrt{\det \hat{g}} \approx \det \hat{G} \tag{3}$$

The Jacobian determinant for one instanton, or the LM dyon pair, has been calculated by Diakonov, Gromov, Petrov and Slizovskiy (DGPS) [9]. The nonzero modes lead to the so called screening phenomenon to which we turn in section II B. For SU(2) gauge group there are two types of dyons $m, n = L, M$ and $\hat{G}$ reads

$$\hat{G} = \begin{pmatrix} 4\pi\nu_L + \frac{1}{r_{LM}} & -\frac{1}{Tr_{LM}} \\ -\frac{1}{Tr_{LM}} & 4\pi\nu_M + \frac{1}{Tr_{LM}} \end{pmatrix}, \tag{4}$$

Let us now discuss the short-distance behavior. Now, the $V_{eff} = exp(-\log G)$ has multiparticle terms with all powers of $1/r$. To make sense of it, it is instructive to calculate it for some examples of configurations. We can use e.g. a "square" made of 2 L and 2 M dyons and found that in all cases their combined effect can be described by a weakening of the Coulomb.

screening

$$M_D^2 = g^2 T^2 (N_c/3 + N_f/6) \tag{11}$$

In this form one also finds it in the instanton screening term, calculated by Pisarski and Yaffe long ago [13]. The instanton size ρ and the $L-M$ separation are related in the well known way

$$\pi \rho^2 T = r_{ML} \tag{12}$$

relating the 4-d dipole of the instanton to the 3-d dipole of the dyon pair.

Let us now work out the corresponding general formula for screening potential which holds in the many-body case. The sum over all dyonic contributions to A_4 can be written as

$$\langle (A_4)^2 \rangle = \int d^3x \left| \sum_i \frac{Q_i}{r_i} \right|^2 \tag{13}$$

where now the sum runs over all dyons with $Q_i = \pm 1$ is the charge and $r_i = |\vec{x} - \vec{z}_i|$.

$$< (A_4)^2 >= 4\pi \sum_{i>j} Q_i Q_j r_{ij}$$

fermionic determinant

For the non-vanishing matrix element of the Dirac operator between $\bar{L}L$, the ij element in each block is given by the (approximate) formula [7]:

$$T^{ij}_{\bar{L}L} = c\frac{e^{-Mr_{ij}}}{\sqrt{1+Mr_{ij}}} \quad (17)$$

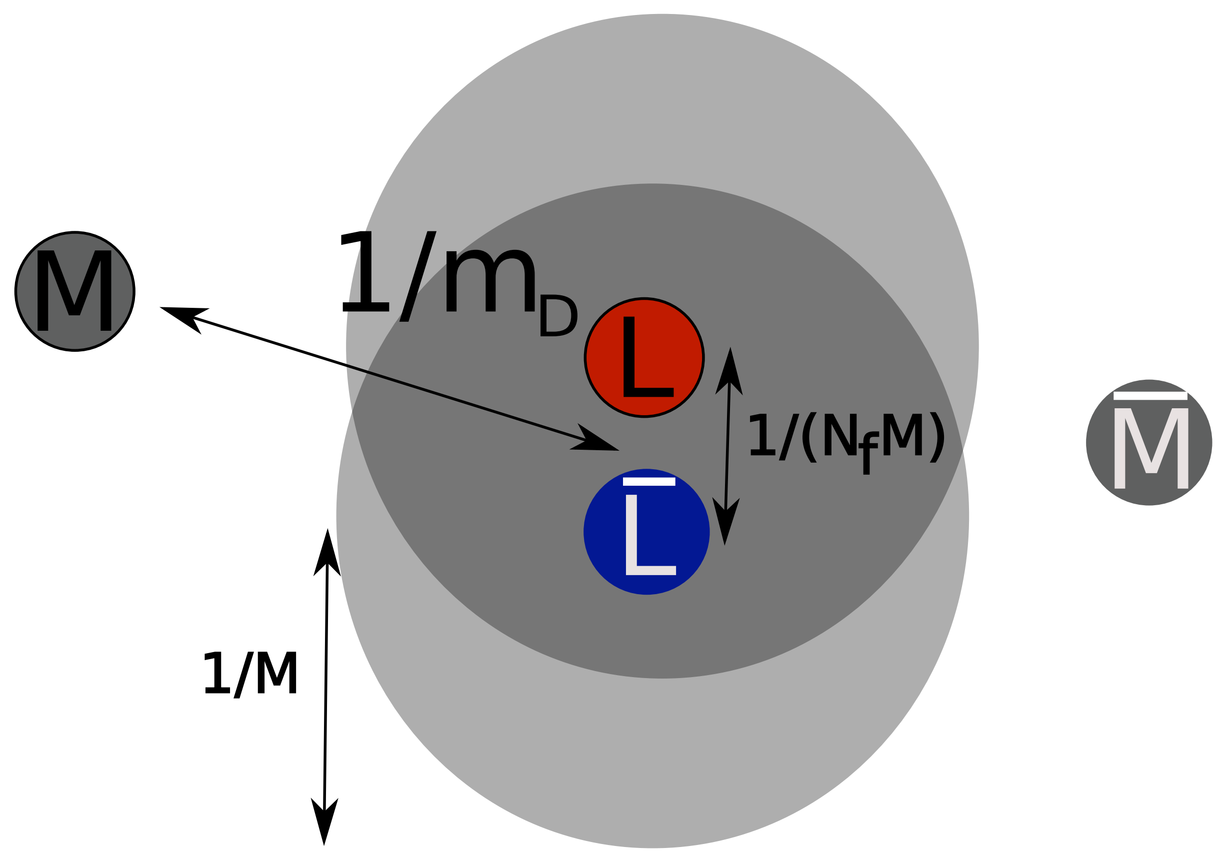
A. Single molecule case

Let us first consider the case in which there is only one molecule, and denote with L and $\bar{L}$ the index of the L−type dyons. This can be viewed as the second order one-loop diagram in 't Hooft effective Lagrangian, with two vertices with $2N_f$ fermionic propagators in between. From Eq. (19) and (17) we get

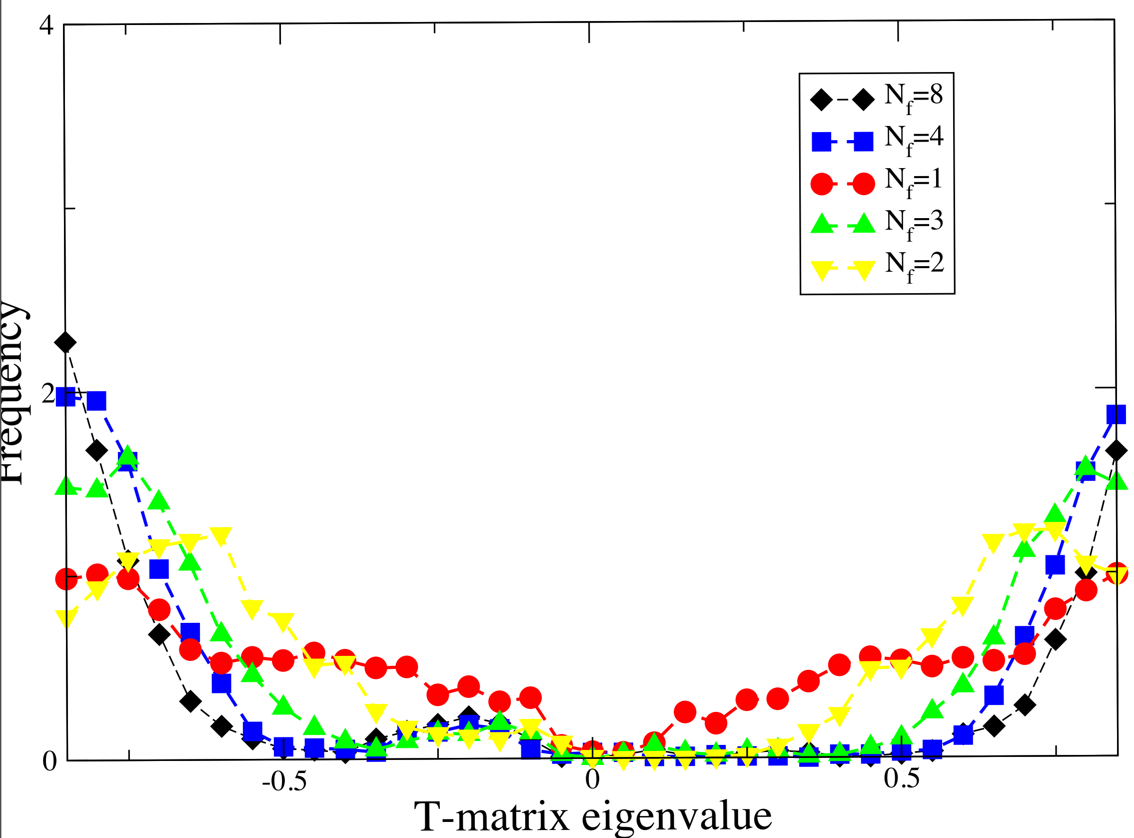
$$\det i\hat{D} = |T_{L\bar{L}}|^{2N_f} = e^{-2N_f M r_{L\bar{L}} - \log(1+Mr_{L\bar{L}})} \quad (20)$$

Neglecting the logarithmic dependence, the formula for the determinant for 1 molecule $(L, M, \bar{L}, \bar{M})$ reads as en effective potential

$$V = -\log\det i\hat{D} = 2N_f M\ r_{L\bar{L}} \quad (21)$$

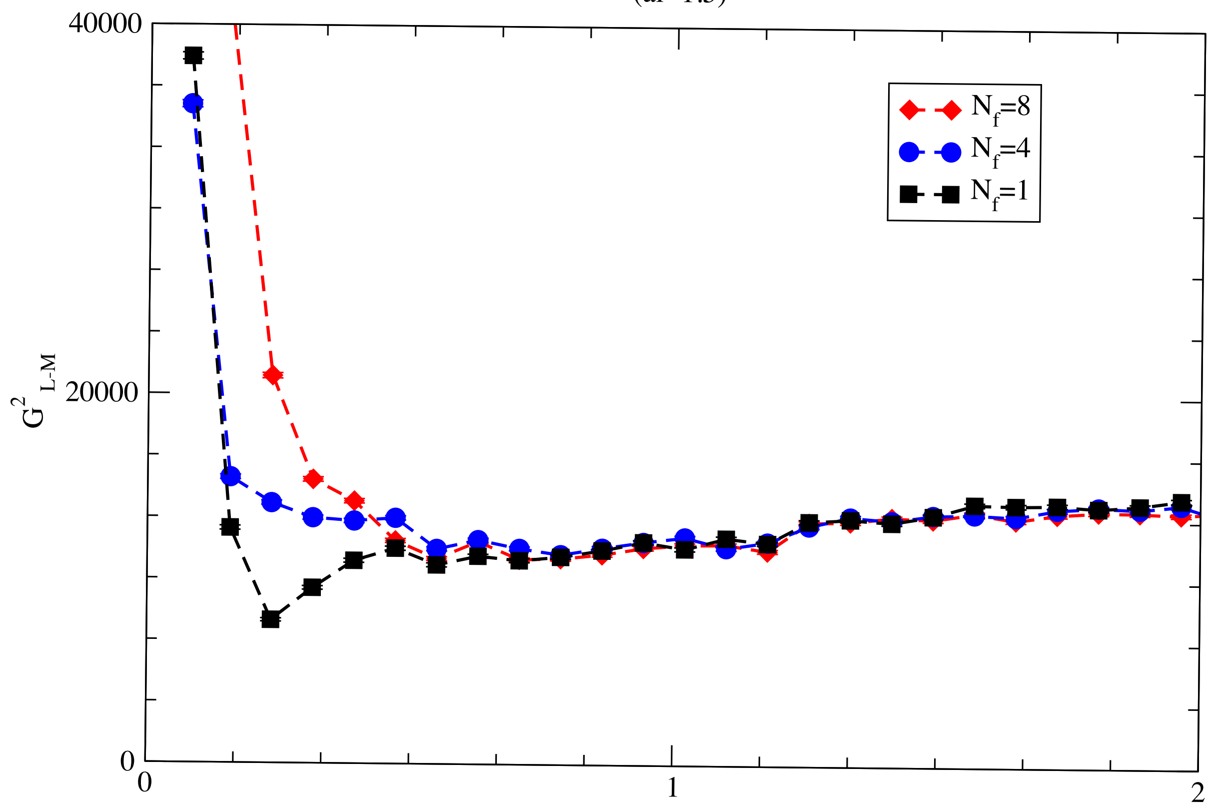


Al=1.5



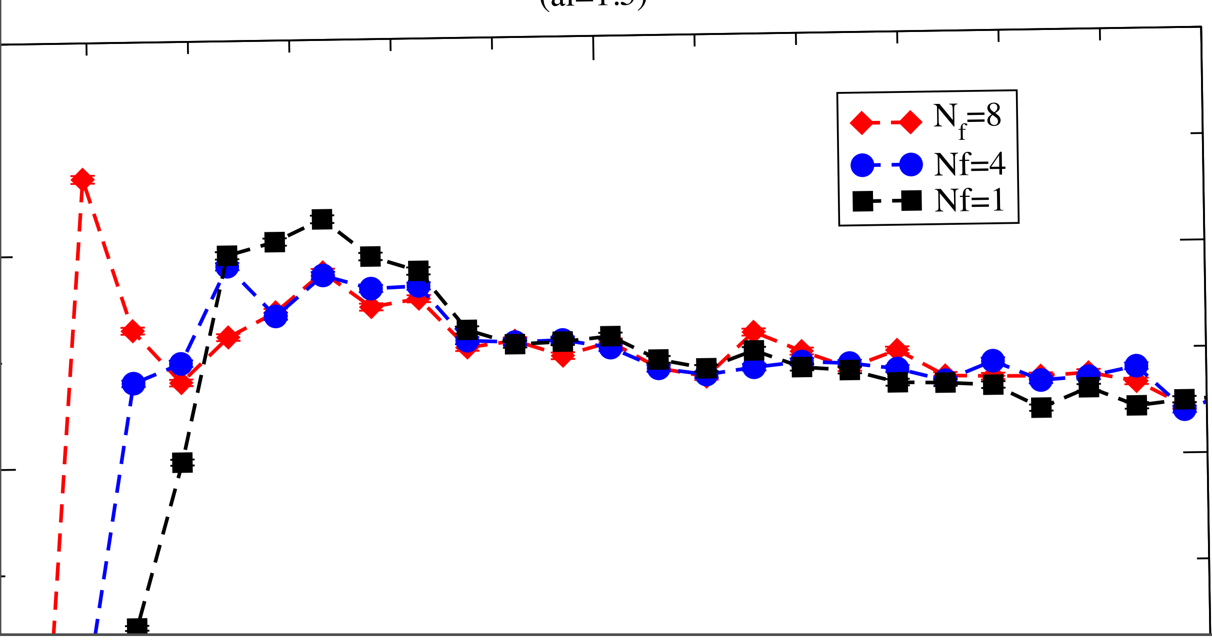
L-M dyon correlator T=T_c

(al=1.5)



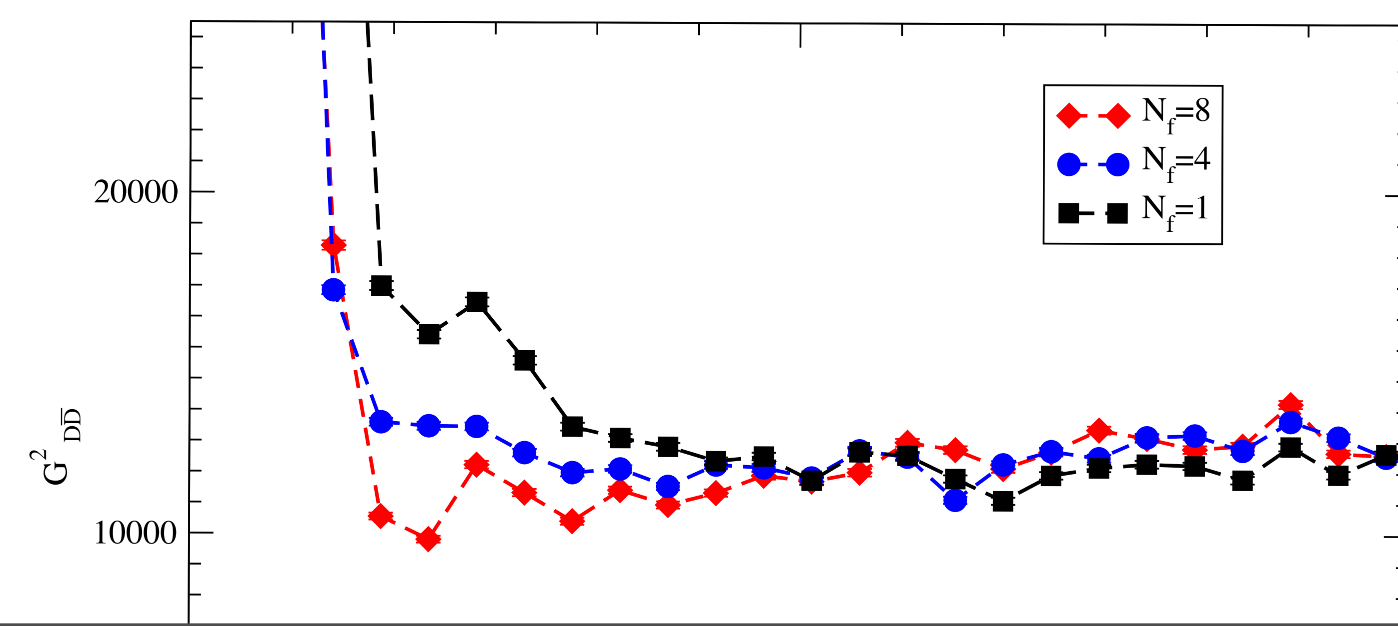
Diagonal dyon correlator at T= T_c

(al=1.5)



Dyon-antidyon correlator at T=T_c

(al=1.5)



summary

- first numerical simulation of the dyonic system are under way
- moduli (modified Coulomb)+ screening (quasi-confinement O(r))+ fermions
- LLbar clusters in a sea of M dyons at low density (high T)
- complicated liquid with chiral symmetry breaking at high density