A Non-AdS/CFT bound on η/s

P. Romatschke

CU Boulder

reporting on work with P. Kovtun and G. Moore: 1104.1586

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Low Viscosity over Entropy Density: Various Results

- Water 300 K , P=0.1MPa: $\frac{\eta}{s} \simeq 280$
- Water 800 K, P=100MPa: $\frac{\eta}{s} \simeq 3.6$
- Helium 4.4 K, P=0.1MPa: $\frac{\eta}{s} \simeq 2.1$
- Ultracold Atomic Gases: $\frac{\eta}{s} \simeq 0.5$
- RHIC/LHC: $\frac{\eta}{s} \lesssim 0.2(?)$

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Or is it possible to reach $\frac{\eta}{s} = 0$?

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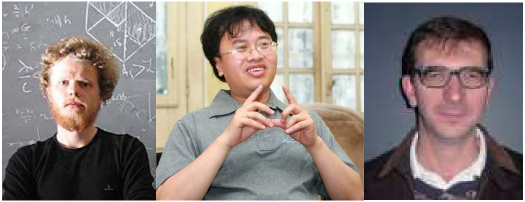
The Viscosity Bound from String Theory



In 2005, Kovtun, Son and Starinets realize that the viscosity calculation from gauge/gravity duality makes only very few assumptions. They conjecture

$$\frac{\eta}{s} > \frac{1}{4\pi} \simeq 0.08$$

for all relativistic quantum field theories

The Viscosity Bound from String Theory

- $\eta/s = \frac{1}{4\pi}$ for $g \to \infty$ and $g^2N \to \infty$
- $1/g$ corrections known to *increase* viscosity

  [Buchel, 0805.2683]
- $1/N$ corrections known to *decrease* viscosity

  [Kats& Petrov, 0712.0743]

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A viscosity bound from fluid dynamics

Possible resolution of the viscosity puzzle:

- Fluid dynamics is long wavelength effective theory
- Fluid dynamics contains collective modes (e.g. sound waves)
- These modes can be excited by thermal noise and self-interact
- Sound modes can scatter off another -> dissipation
- This dissipation will create an effective viscosity
- Effective viscosity large if "ordinary" viscosity small and vice versa
- There is a lower limit on η/s from fluid dynamics alone!

Mathematical Setup

- For physical systems, hydro equations need to be supplmemented by noise terms:

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu} + \Pi^{\mu\nu} \to \epsilon u^\mu u^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu} + \xi^{\mu\nu}$$

  where the noise fulfills

$$\langle \xi^{\alpha\beta} \rangle = 0\,, \quad \langle \xi^{\alpha\beta}(X)\xi^{\gamma\delta}(Y) \rangle = C\Delta^{\alpha<\beta}\Delta^{\gamma>\delta}\delta(X-Y)$$

- Then

$$\langle \partial_\mu T^{\mu\nu} = 0 \rangle$$

  correspond to "usual" hydro equations but now also

$$\langle T^{\alpha\beta} T^{\gamma\delta} \rangle$$

  may be calculated (in hydro perturbation theory)

Hydrodynamic Perturbation Theory

- Perturb around a known solution (e.g. $\epsilon_0 = \text{const.}\,, u_0^i = 0$)
- Express δu^i through noise via $\partial_\mu T^{\mu\nu} = 0$
- This leads to

$$\langle u^i u^j \rangle(\omega, \mathbf{k}) = \frac{2T}{\epsilon + P} \frac{k^i k^j}{k^2} \frac{\gamma_s^2 k^2 \omega^2}{(\omega^2 - c_s^2 k^2)^2 + \gamma_s^2 k^4 \omega^2} + \text{shear} + \mathcal{O}(\delta^2)$$

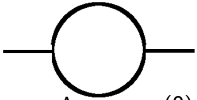
where $\gamma_s = \frac{4\eta}{3(\epsilon+P)}$ is sound-attenuation length

- Can calculate $\langle T^{xy} T^{xy} \rangle$ in hydro perturbation theory

$$\langle T^{xy} T^{xy} \rangle = \langle T^{xy} T^{xy} \rangle_0 + \langle T^{xy} T^{xy} \rangle_1 + \ldots$$

Hydro Perturbation Theory — remarks

$$G_R^{(0)} = \Delta_R = < u_x u_x >$$



$$G_R^{(1)} = \int_0^{\Lambda} d^4K \, G_R^{(0)} G_R^{(0)}$$

Employ hydro effective theory. Λ is effective theory cut-off. Can estimate Λ from where fluid dynamics is breaking down:

$$\Lambda \simeq \frac{1}{2\tau_R} \simeq \frac{(\epsilon + P)}{2\eta}$$

Energy-Stress Tensor Correlators in Hydro Perturbation Theory

Results to first order in hydro perturbation theory

- Tree level

$$\langle T^{xy} T^{xy} \rangle_0 = P - i\omega\eta + \mathcal{O}(\omega^2)$$

- Tree level + one loop correction

$$\langle T^{xy} T^{xy} \rangle_0 + \langle T^{xy} T^{xy} \rangle_1 = P - i\omega\eta - i\omega \frac{17 T \Lambda(\epsilon + P)}{120\pi^2\eta} + \mathcal{O}(\omega^{3/2})$$

One loop result cut-off dependent. Use estimate $\Lambda\gamma_\eta \sim 0.5$.

A viscosity bound from fluid dynamics

This leads to

$$\eta_{\text{physical}} = \eta + \frac{17T(\epsilon + P)^2}{240\pi^2\eta^2}$$

which implies a lower bound on the physical viscosity:

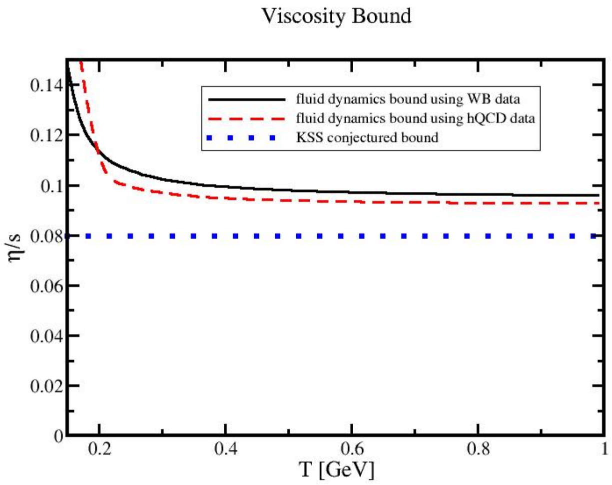
$$\frac{\eta_{\text{physical}}}{s} > \left(\frac{153}{320\pi^2}\frac{T^3}{s}\right)^{1/3} \simeq 0.09(\text{QGP})$$

Physical Origin: sound waves scatter and create dissipation (contributes to viscosity). Sound waves live long only if viscosity is small, so contribution becomes large at small viscosity.

Caveat: bound is (weakly) dependent on choice of cut-off Λ

A viscosity bound from fluid dynamics

For QCD one finds for η/s in units of $\hbar = k_B = 1$



[Kovtun, Moore & PR, arXiv:1104.1586]

Conclusions and Summary

- For nearly perfect fluids ($\eta/s \ll 1$), hydrodynamic modes are long-lived
- Hydrodynamic modes are populated by thermal noise and self-interact
- Self-interaction *increases* physical viscosity of fluid
- Because of this self-interaction it is *impossible* to have $\eta/s = 0$ for *any* fluid
- There is a lower limit on the physical fluid viscosity
- For the QGP, this limit is surprisingly close to the string theory bound (even though origins are totally different)