



Hydrodynamics and Transport

Björn Schenke

Physics Department, Brookhaven National Laboratory, Upton, NY

August 13 2012



Quark Matter 2012
Washington DC, USA



Fluid dynamics =

Conservation of energy and momentum for long wavelength modes

If the system is strongly interacting, i.e., has a short mean free path compared to the scales of interest, hydrodynamics should work

It was a surprise at RHIC that hydrodynamics worked so well (so well that we are still using it a lot)

I will try to give an overview of some of the important facts about relativistic hydrodynamics for heavy-ion collisions and explain different concepts that most speakers at QM2012 will assume to be known

Non-relativistic hydrodynamics

Equations of hydrodynamics can be obtained from a simple argument:

Variation of mass in the volume V is due to in- and out-flow through the surface ∂V :

$$\frac{\partial}{\partial t} \int \rho dV = - \int_{\partial V} \rho \mathbf{u} \cdot \mathbf{n} dS$$

Gauss' theorem:

$$\frac{\partial}{\partial t} \int \rho dV = - \int_V \nabla \cdot (\rho \mathbf{u}) dV$$

Conservation of mass: Continuity Equation

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$$

with mass density ρ and fluid velocity \mathbf{u} .

Conservation of momentum: Euler Equation

$$\partial_t \mathbf{u} + \mathbf{u}(\nabla \cdot \mathbf{u}) = -\frac{1}{\rho} \nabla p$$

Relativistic hydrodynamics

Relativistic system: **mass density is not a good degree of freedom**:
 Does not account for kinetic energy (large for motions close to c).

- Replace ρ by the total energy density ϵ .
- Replace \mathbf{u} by Lorentz four-vector u^μ .

Ideal energy momentum tensor is built from
 pressure p , energy density ϵ , flow velocity u^μ , and the metric $g^{\mu\nu}$.

Properties: symmetric, transforms like a Lorentz-tensor.
 So the most general form is

$$T^{\mu\nu} = \epsilon(c_0 g^{\mu\nu} + c_1 u^\mu u^\nu) + p(c_2 g^{\mu\nu} + c_3 u^\mu u^\nu)$$

Constraints:

$T^{00} = \epsilon$ and $T^{0i} = 0$ and $T^{ij} = \delta^{ij} p$ in the local rest frame.

It follows:

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - p(g^{\mu\nu} - u^\mu u^\nu)$$

Conservation of energy and momentum:

$$\partial_{\mu} T^{\mu\nu} = 0$$

together with

$$T^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - p(g^{\mu\nu} - u^{\mu} u^{\nu})$$

is ideal fluid dynamics.

In the non-relativistic limit ($u^2/c^2 \ll 1$ and $p \ll mc^2$):

$$\partial_{\mu} T^{\mu 0} = 0 \rightarrow \text{Continuity equation}$$

$$\partial_{\mu} T^{\mu i} = 0 \rightarrow \text{Euler equation}$$

Relativistic viscous hydrodynamics

Generally:

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + \pi^{\mu\nu}.$$

First order Navier Stokes theory (shear only):

$$\pi^{\mu\nu} = \pi_{(1)}^{\mu\nu} = \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha).$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Relativistic Navier Stokes is unstable (short wavelength modes become superluminal)

Second order theory:

$$\pi^{\mu\nu} = \pi_{(1)}^{\mu\nu} + \text{second derivatives.}$$

Israel-Stewart theory for a conformal fluid:

$$\pi^{\mu\nu} = \pi_{(1)}^{\mu\nu} - \tau_\pi \left(\frac{4}{3}\pi^{\mu\nu}\partial_\alpha u^\alpha + \Delta_\alpha^\mu \Delta_\beta^\nu u^\sigma \partial_\sigma \pi^{\alpha\beta} \right)$$

in flat space and neglecting vorticity and all terms that seem numerically unimportant

R. Baier, P. Romatschke, D. Son, A. Starinets, M. Stephanov, JHEP 0804:100 (2008)

η = shear viscosity

τ_π = shear relaxation time

Typical set of equations for heavy-ion physics

Using the set of equations

$$\partial_\mu T^{\mu\nu} = 0$$

and

$$\pi^{\mu\nu} = \pi_{(1)}^{\mu\nu} - \tau_\pi \left(\frac{4}{3} \pi^{\mu\nu} \partial_\alpha u^\alpha + \Delta_\alpha^\mu \Delta_\beta^\nu u^\sigma \partial_\sigma \pi^{\alpha\beta} \right)$$

is now standard.

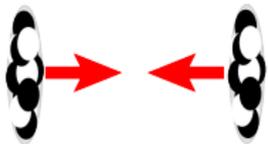
When **bulk viscosity** is included (non-conformal fluid)

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu - P g^{\mu\nu} + \pi^{\mu\nu} - \Pi \Delta^{\mu\nu}$$

see B. Betz, D. Henkel, D. Rischke, Prog.Part.Nucl.Phys.62, 556-561 (2009) for structure of bulk terms

Also heat flow and vorticity are sometimes included.

A heavy-ion collision

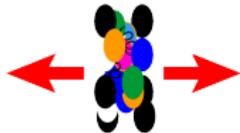


before collision

initial state

(e.g. color glass condensate)

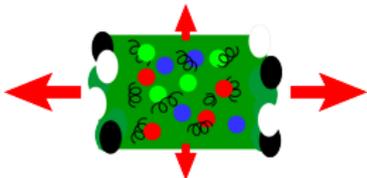
0 fm/c



pre-equilibrium

thermalization (glasma state)

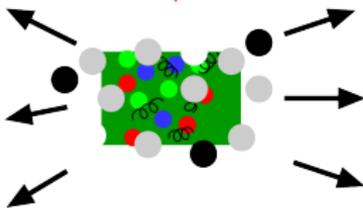
~ 0.5 fm/c



quark-gluon-plasma

Hydrodynamics, Jet quenching, ...

~ 3 – 5 fm/c



hadronization

Hydrodynamics

hadr.rescattering

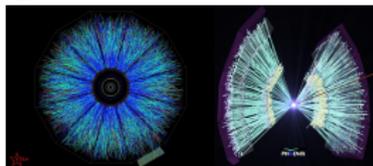
Hadronic transport

~ 10 fm/c

freeze-out

compare theory
to experiment

detection



Hydrodynamics works for all systems with short mean free path.
(comparing to size scales of interest)

How do we incorporate the physics of heavy-ion collisions?

- 1 Equation of state $p(\varepsilon, \rho_B)$
- 2 Initial conditions
- 3 Freeze-out and conversion of energy densities into particles
- 4 Values of transport coefficients (e.g. shear viscosity)

Is hydro useful for HICs?

Within hydro:

Equation of state	unknown	} ⇒ Predictive power?
Initial conditions	unknown	
Freeze-out	unknown	
Transport coefficients	unknown	

Is hydro useful for HICs?

Within hydro:

Equation of state	want to study	} ⇒ Predictive power?
Initial conditions	want to study	
Freeze-out	unknown	
Transport coefficients	want to study	

⇒ **Need more constraints!**

Hydrodynamics can provide the link from different models for the initial state, equation of state, etc. to experimental data

1 Use **another model** to fix unknowns:

- e.g. take initial conditions from color glass condensate
- Input equation of state from lattice QCD and hadron gas models

2 Use experimental data to **fix parameters**:

- use one set of data to fix parameters:

e.g. $\left. \frac{dN}{dy p_T dp_T} \right|_{b=0 \text{ fm}}$ and $\frac{dN}{dy}(b)$

Example parameters at RHIC:

$$\varepsilon_{0,\text{max}} \approx 30 \text{ GeV}/\text{fm}^3, \tau_0 \approx 0.6 \text{ fm}/c, T_{\text{fo}} \approx 130 \text{ MeV}$$

- predict another set of data:

Flow, photons and dileptons, HBT, ...

By the way: Initial energy density

The initial maximal energy densities needed to reproduce the experimental data are $\sim 30 \text{ GeV}/\text{fm}^3$. How much is that?

world energy consumption 2008: $474 \cdot 10^{18} \text{ J}$

$$474 \cdot 10^{18} \text{ J} \cdot \frac{1 \text{ eV}}{1.6 \cdot 10^{-19} \text{ J}} = 3 \cdot 10^{30} \text{ GeV}$$

$$3 \cdot 10^{30} \text{ GeV} / (30 \text{ GeV}/\text{fm}^3) = 10^{29} \text{ fm}^3$$

That's a box with dimensions

$$\begin{aligned} \sqrt[3]{10^{29} \text{ fm}^3} &= 4.6 \cdot 10^9 \text{ fm} \\ &= 4.6 \mu\text{m} \end{aligned}$$

Critical energy density to create quark-gluon-plasma: $1 \text{ GeV}/\text{fm}^3$
(lattice QCD).

By the way: Initial energy density

The initial maximal energy densities needed to reproduce the experimental data are $\sim 30 \text{ GeV}/\text{fm}^3$. How much is that?

$$\text{world energy consumption 2008: } 474 \cdot 10^{18} \text{ J}$$

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$$3 \cdot 10^{30} \text{ GeV} / (30 \text{ GeV}/\text{fm}^3)$$

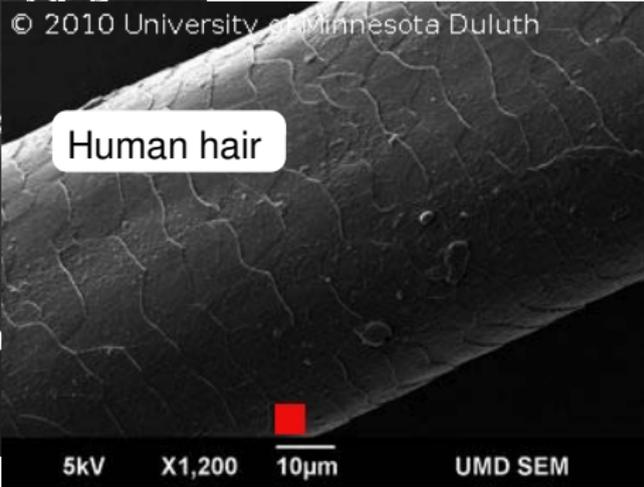
That's a box with dimensions

$$\sqrt[3]{10^{29} \text{ fm}^3} = 4.6 \cdot 10^9 \text{ fm}$$

$$= 4.6 \text{ } \mu\text{m}$$

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Human hair



5kV

X1,200

10 μ m

UMD SEM

Critical energy density to create quark-gluon-plasma: $1 \text{ GeV}/\text{fm}^3$
(lattice QCD).

- Landau hydrodynamics

- Initial fireball at rest: $u^\mu = (1, 0, 0, 0)$ everywhere
- Start with a slab of radius r_{nucleus} and thickness $2r/\gamma$
(γ is the γ -factor of the colliding nuclei)
- Assumption of $v_z = 0$ seems unrealistic

- Bjorken hydrodynamics

- At large energies $\gamma \rightarrow \infty$, Landau thickness $\rightarrow 0$
- No longitudinal scale \rightarrow scaling flow

$$v = \frac{z}{t}$$

Because all particles are assumed to have been produced at $(t, z) = (0 \text{ fm}/c, 0 \text{ fm})$
a particle at point (z, t) must have had average $v = z/t$

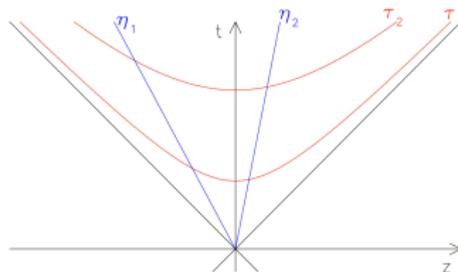
Practical coords. for scaling flow expansion

- Longitudinal proper time τ :

$$\tau = \sqrt{t^2 - z^2}$$

- Space-time rapidity η_s :

$$\eta_s = \frac{1}{2} \ln \frac{t+z}{t-z}$$



Inversely: $t = \tau \cosh \eta_s$ and $z = \tau \sinh \eta_s$

Boost-invariance: Results are independent of η_s .

This is assumed when you see 2+1D hydro calculations.

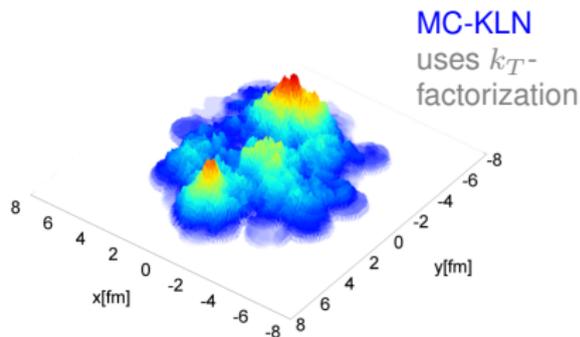
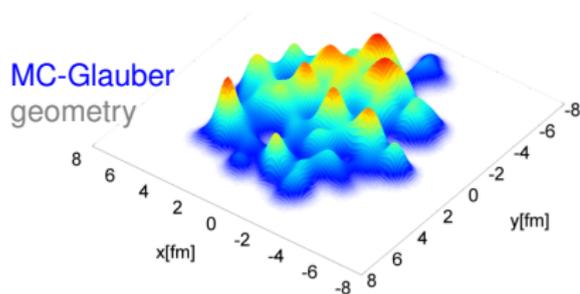
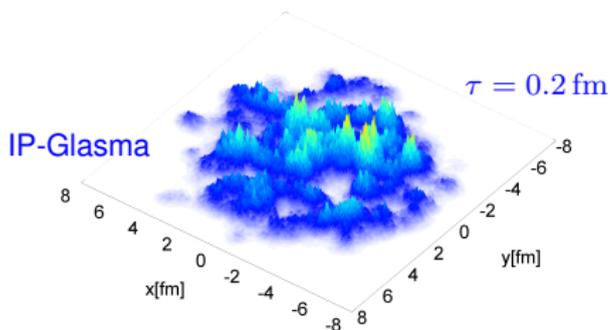
Good assumption when studying mid-rapidity
at highest RHIC and LHC energies.

Initial conditions - all including fluctuations

You will see different initial conditions being used:

- **MC-Glauber**: geometric model determining wounded nucleons based on the inelastic cross section (different implementations)
- **MC-KLN**: Color-Glass-Condensate (CGC) based model using k_T -factorization
Same fluctuations in the wounded nucleon positions as MC-Glauber
- **MCrcBK**: Similar to MC-KLN but with improved energy/rapidity dependence following from solutions to the running coupling Balitsky Kovchegov equation
- **IP-Glasma**: Recent CGC based model using classical Yang-Mills evolution of early-time gluon fields, including additional fluctuations in the particle production
- Also hadronic cascades **UrQMD** or **NEXUS** and partonic cascades (e.g. **BAMPS**) can provide initial conditions

Initial energy densities



MC-KLN: Drescher, Nara, nucl-th/0611017

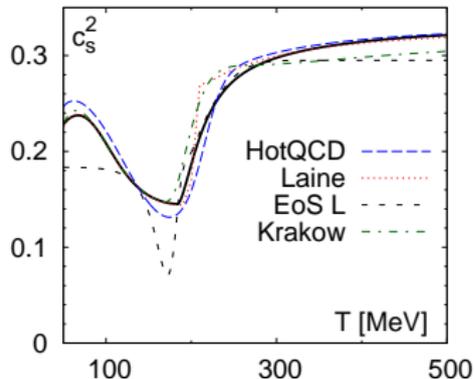
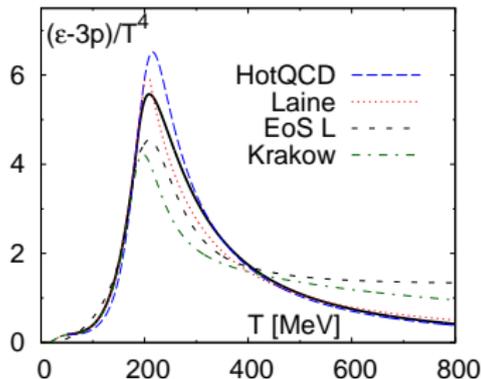
mckln-3.52 from http://physics.baruch.cuny.edu/files/CGC/CGC_IC.html with defaults, energy density scaling

- Initial time τ_0 : thermalization time - should be of order 1 fm/c
- Initial transverse flow: often set to zero
(cascade models provide initial flow, so does IP-Glasma)
- Assume boost-invariance: 2+1D hydrodynamics
The viscous 3+1D hydrodynamic simulations are
 - MUSIC B. Schenke, S. Jeon, and C. Gale, Phys.Rev.Lett.106, 042301 (2011)
 - P. Bozek, Phys.Rev. C85 (2012) 034901
- Initial $\pi^{\mu\nu}$: zero or Navier-Stokes value

Equation of State - QCD enters here

Need an equation of state $p(\varepsilon)$ to close the set of hydro equations

- Early days: 1st order phase transition EoS from MIT bag model
- Today: EoS from lattice QCD + hadron resonance gas model



Solid black: Parametrization from P. Huovinen, P. Petreczky, Nucl.Phys.A837:26-53 (2010) s95p-v*

HotQCD: HotQCD collaboration, Phys.Rev.D80:014504 (2009)

Laine: M. Laine and Y. Schröder, Phys. Rev. D73, 085009 (2006)

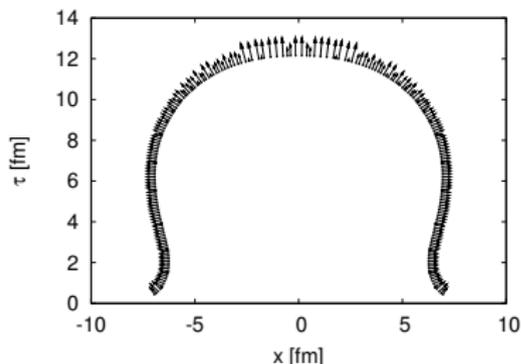
EoS L: H. Song and U. W. Heinz, Phys. Rev. C 78, 024902 (2008) using Wuppertal-Budapest results

Krakow: M. Chojnacki et al, Acta Phys. Polon. B 38, 3249 (2007) and Phys. Rev. C 78, 014905 (2008)
also S. Borsanyi et al, JHEP 1011:077 (2010)

The end of hydro...

Well - the end of the hydrodynamic evolution.

- Particles are observed. Not a fluid.
- How to convert fluid into particles?
- So how far is hydro valid - when to switch to a particle description?



- Kinetic equilibrium requires **scattering rate** \gg **expansion rate**
- scattering rate $\tau_{sc}^{-1} \sim \sigma n \sim \sigma T^3$
- expansion rate $\theta = \partial_\mu u^\mu$
 $= \tau^{-1}$ in 1+1D

Fluid description breaks down when $\tau_{sc}^{-1} \approx \theta$

→ **momentum distributions freeze out**

$\tau_{sc}^{-1} \propto T^3 \Rightarrow$ rapid transition to free streaming

Cooper-Frye freeze-out

Approximation: Decoupling takes place on **constant temperature** hypersurface Σ at $T = T_{\text{fo}}$

- Number of particles emitted = number of particles crossing Σ :

$$N = \int_{\Sigma} d\Sigma_{\mu} N^{\mu}$$

- We can compute the particle current:

$$\Rightarrow N^{\mu} = \int \frac{d^3p}{E} p^{\mu} f(x, \partial_{\mu} u^{\mu})$$

$$\Rightarrow N = \int \frac{d^3p}{E} \int_{\Sigma} d\Sigma_{\mu} p^{\mu} f(x, \partial_{\mu} u^{\mu})$$

So we get the **invariant inclusive momentum spectrum** (Cooper-Frye formula):

$$E \frac{dN}{d^3p} = \int_{\Sigma} d\Sigma_{\mu} p^{\mu} f(x, \partial_{\mu} u^{\mu})$$

Freeze-out in the viscous case

Viscous correction to the equilibrium distribution functions:

$$f \rightarrow f + \delta f$$

with

$$\delta f = f_0(1 \pm f_0)p^\alpha p^\beta \pi_{\alpha\beta} \frac{1}{2(\epsilon + \mathcal{P})T^2}$$

The choice $\delta f \sim p^2$ is not unique
 depends on microscopic interactions

Ambiguity in δf leads to uncertainty

see Dusling, Moore, and Teaney, Phys.Rev.C81:034907 (2010)

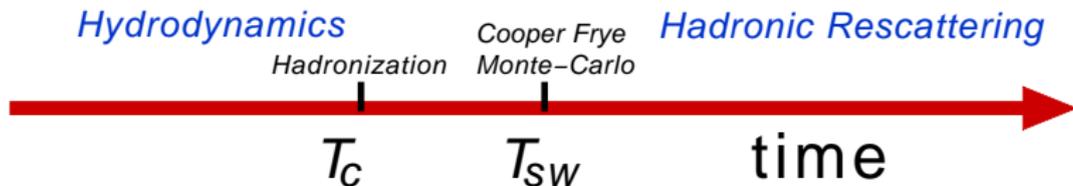
When you hear "afterburner": Late hadronic gas stage



©SEGA Enterprises Ltd.

Combining hydrodynamic evolution with microscopic hadronic transport models.

The alternative being to just take the thermal spectra and compute resonance decays.



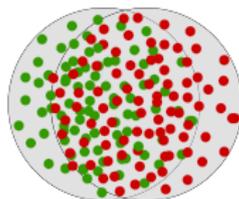
Use of a hadron cascade like UrQMD in hadron gas:

large dissipation and freeze-out naturally included

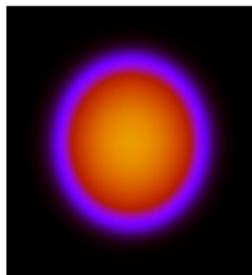
Less extreme transition than going from hydro right to free streaming

Different strategies

average, then evolve



evolve, then average

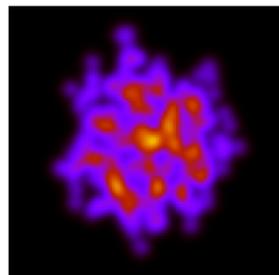


initial energy density

faster

more approximate

You will hear words like
single shot hydrodynamics



initial energy density

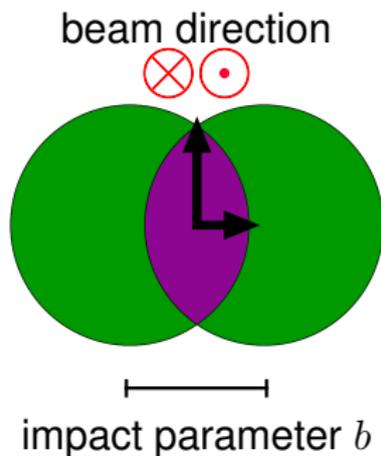
more precise

more costly

event-by-event hydrodynamics

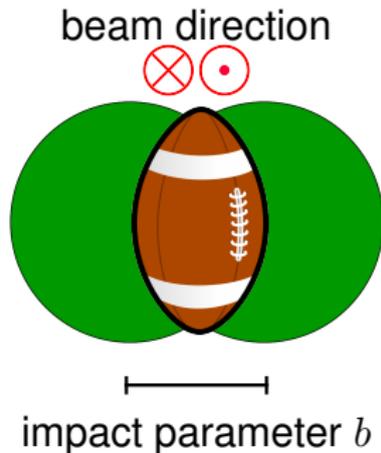
Successes of hydro: Describes anisotropic flow

Non-central collision



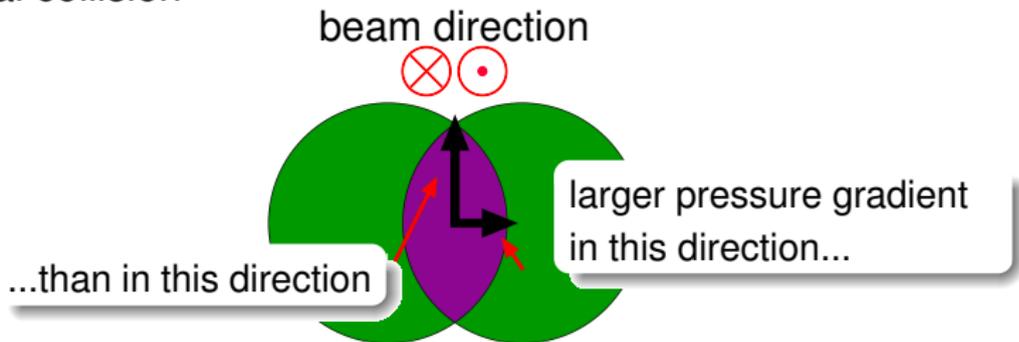
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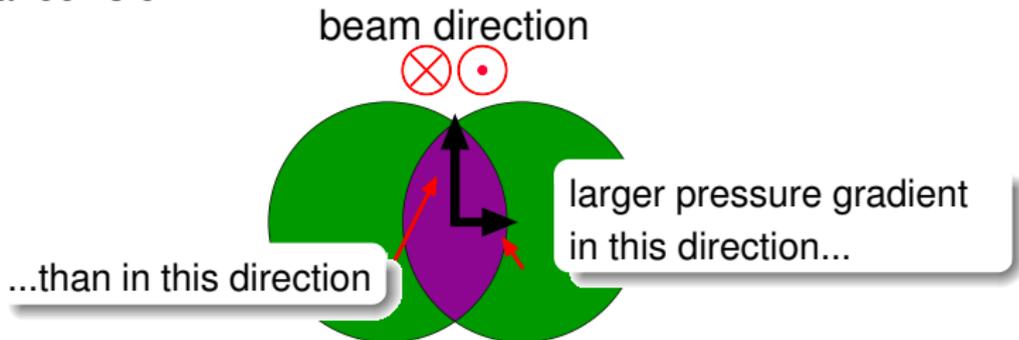
Successes of hydro: Describes anisotropic flow

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Successes of hydro: Describes anisotropic flow

Non-central collision



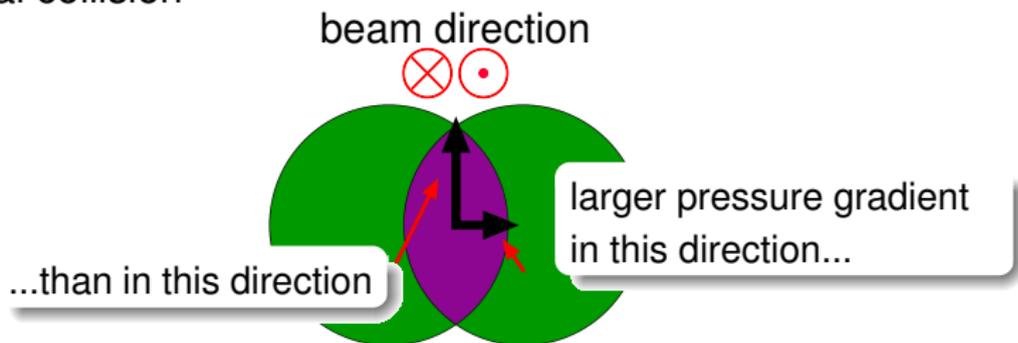
Similar behavior in very different system:

"A cigar-shaped cloud of fermionic ${}^6\text{Li}$ atoms is confined and rapidly cooled to degeneracy in a CO_2 laser trap [...] Upon abruptly turning off the trap, the gas exhibits a spectacular anisotropic expansion."

K. M. O'Hara et al., *Science* Volume 298, pp. 2179-2182 (2002)

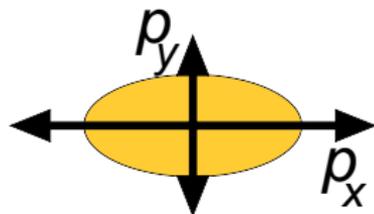
Successes of hydro: Describes anisotropic flow

Non-central collision



Particle distribution in **momentum space** will be anisotropic.

Quantify using a Fourier decomposition:



$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left(1 + \sum_n (2v_n \cos(n\phi)) \right) \Rightarrow v_2 \text{ characterizes elliptic flow}$$

Free streaming vs. hydro (IP-Glasma initial condition)

2+1D CYM

Hydro

after $\tau = 0.2 \text{ fm}/c$ (CYM before)

Free streaming vs. hydro (IP-Glasma initial condition)

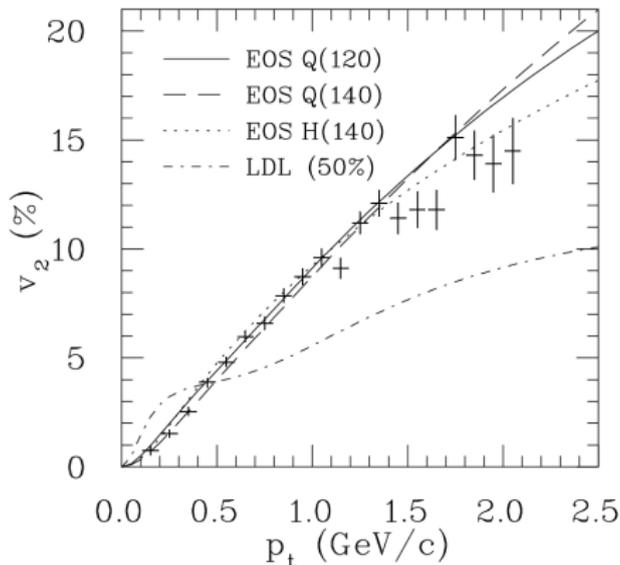
2+1D CYM

Hydro

after $\tau = 0.2 \text{ fm}/c$ (CYM before)

Success of hydro: charged hadron $v_2(p_T)$ at RHIC

Ideal hydro, first order phase transition (EOS Q), avg init cond



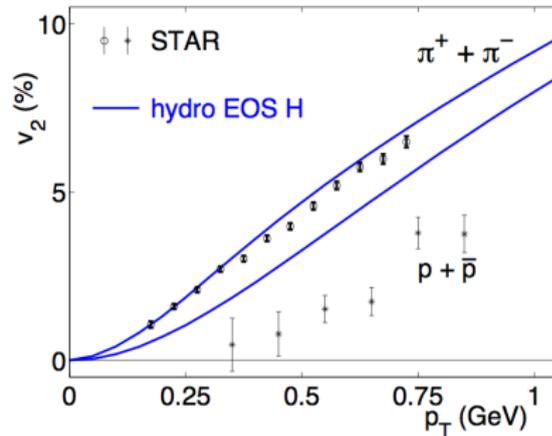
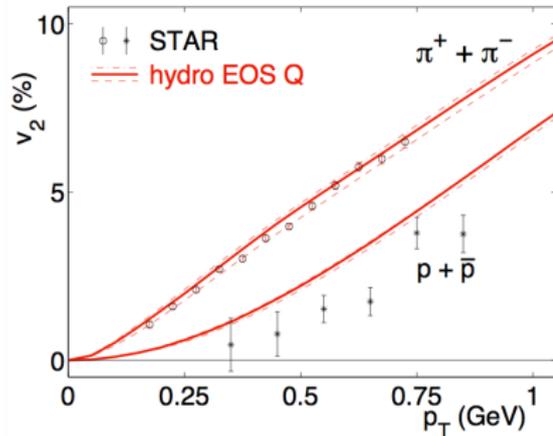
Au+Au, $\sqrt{s} = 130 A$ GeV, minimum bias

P. Huovinen et al, Phys.Lett. B503, 58-64 (2001)

LDL: low density limit - not hydro

identified particle $v_2(p_T)$ at RHIC

Ideal hydro, average Glauber initial conditions



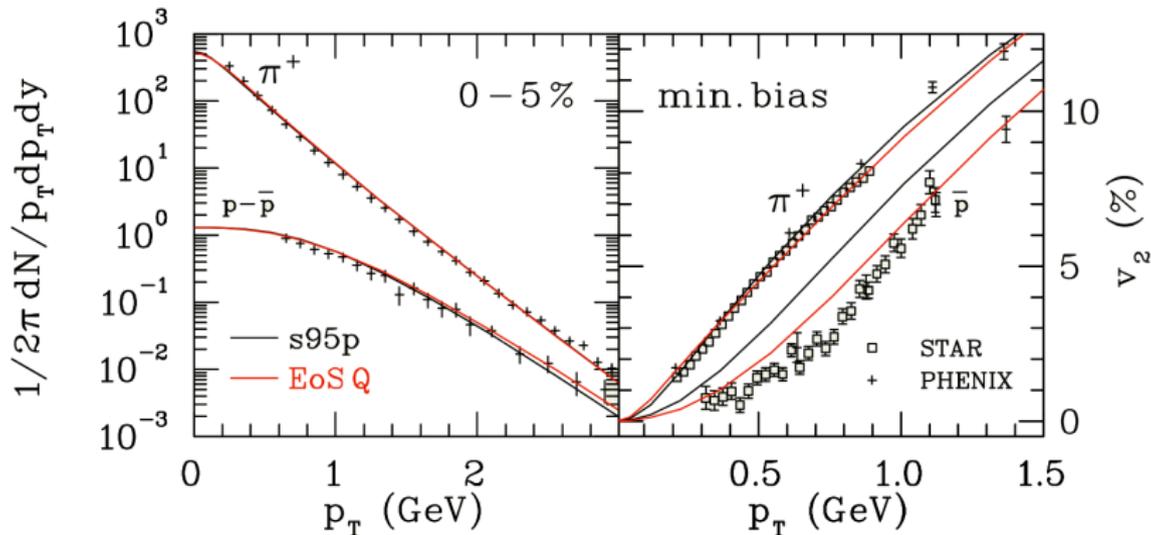
Au+Au, minimum bias

No perfect agreement but EoS with plasma phase favored

P. Huovinen (2001)

Lattice equation of state

ideal hydro, Au+Au at $\sqrt{s} = 200 A$ GeV
 chemical equilibrium



s95p: $T_{FO} = 140$ MeV

EoS Q: first order phase transition at $T_c = 170$ MeV, $T_{FO} = 125$ MeV

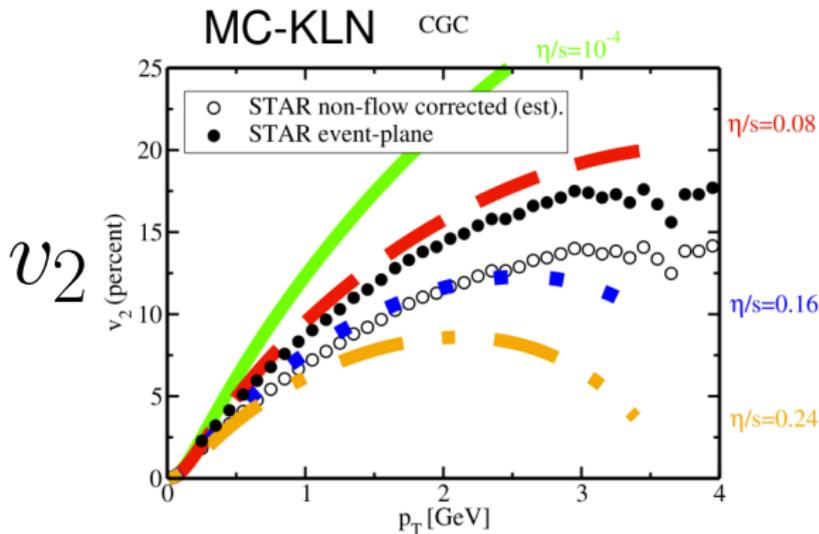
Chemical freeze-out (when you see "PCE")

Hadronic phase: ideal gas of massive hadrons and resonances assumed to be in chemical equilibrium.

- Thermal model fits to particle ratios indicate a chemical freeze-out temperature of $T_{\text{ch}} \approx 160$ MeV.
- We evolve down to $T_{FO} \approx 120$ MeV
 \Rightarrow **particle ratios will come out wrong in hydro**
- Solution:
include (partial) chemical non-equilibrium (PCE)
fixing particles ratios at $T_{\text{ch}} > T_{FO}$.
Number of pions, Kaons, etc. are conserved quantities below T_{ch} .
- This modifies the EoS, in particular $T(\varepsilon, n_b)$.

Extracting transport properties of the QGP

Viscous hydrodynamics differs from ideal hydro, especially for v_n :
Early work (smooth initial conditions):



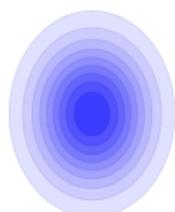
M. Luzum and P. Romatschke, Phys.Rev. C78, 034915 (2008)

Experimental data: STAR, Phys.Rev.C77, 054901 (2008)

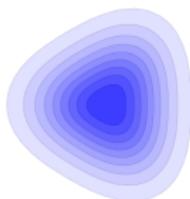
Higher harmonic flow

$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left(1 + \sum_n (2v_n \cos[n(\phi - \psi_n)]) \right)$$

When including fluctuations, all moments appear:



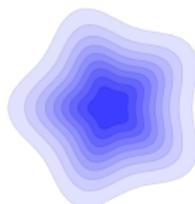
$n = 2$



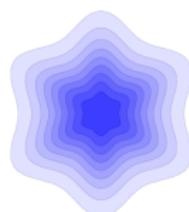
$n = 3$



$n = 4$



$n = 5$



$n = 6$

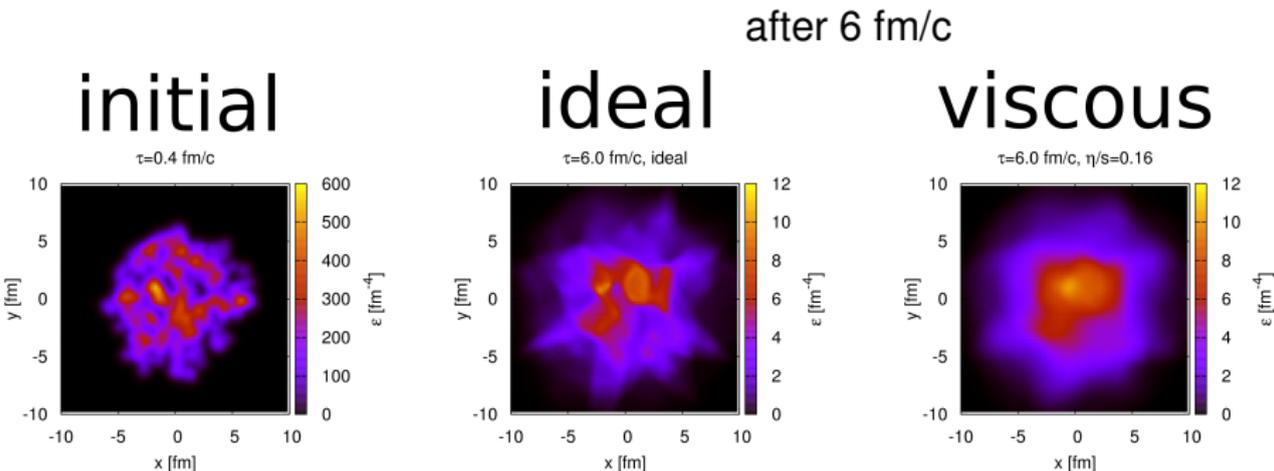
also v_1 and $n > 6$

Compute $v_n = \langle \cos[n(\phi - \psi_n)] \rangle$

with the event-plane angle $\psi_n = \frac{1}{n} \arctan \frac{\langle \sin(n\phi) \rangle}{\langle \cos(n\phi) \rangle}$

Fluctuations and viscosity

We can see the effect of viscosity on higher harmonics in event-by-event simulations!



energy density in the transverse plane

B. Schenke, S. Jeon, and C. Gale, Phys.Rev.Lett.106, 042301 (2011)

Viscosity in a single event

B. Schenke, S. Jeon, and C. Gale, PRL 106, 042301 (2011)

ideal

$$\eta/s = 0.16$$

energy density (scale adjusted with time)

Viscosity in a single event

B. Schenke, S. Jeon, and C. Gale, PRL 106, 042301 (2011)

ideal

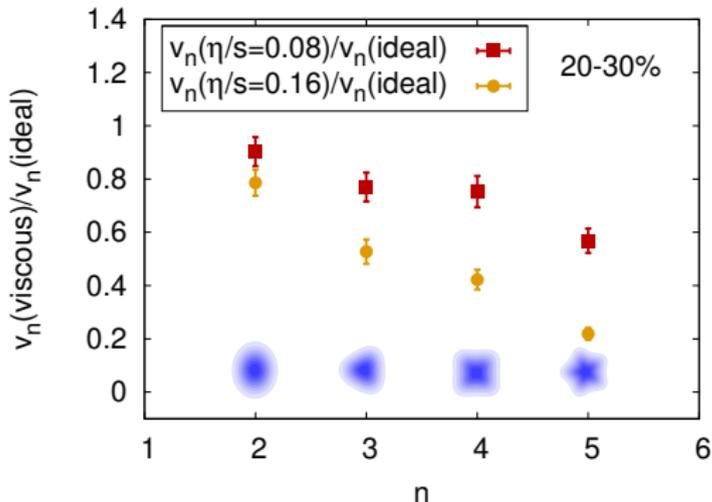
$$\eta/s = 0.16$$

energy density (scale adjusted with time)

More quantitatively: Sensitivity to η/s

B. Schenke, S. Jeon, C. Gale, Phys.Rev. C85, 024901 (2012)

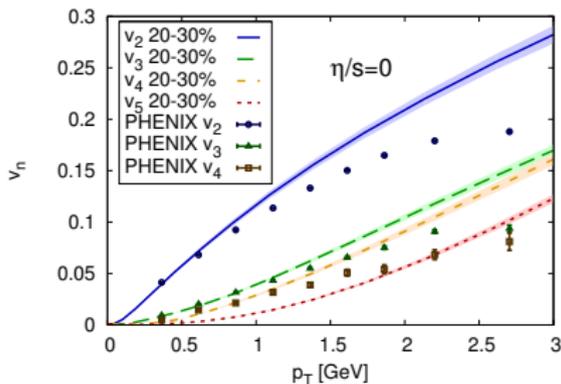
Higher Fourier coefficients are suppressed more by viscosity.



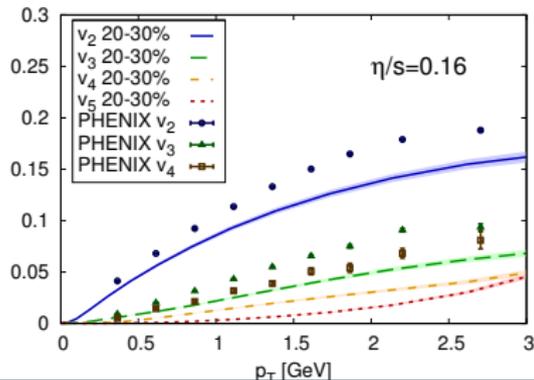
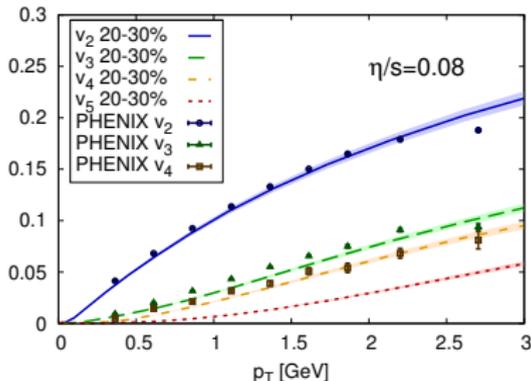
Using higher harmonics to determine η/s

B. Schenke, S. Jeon, C. Gale, arXiv:1109.6289

Data is from event-plane method. Calculations are $\sqrt{\langle v_n^2 \rangle}$.



MC-Glauber initial conditions



This is promising.

Need systematic study of all v_n as function of initial conditions, granularity, η/s , ...

Experimental data: PHENIX, arXiv:1105.3928

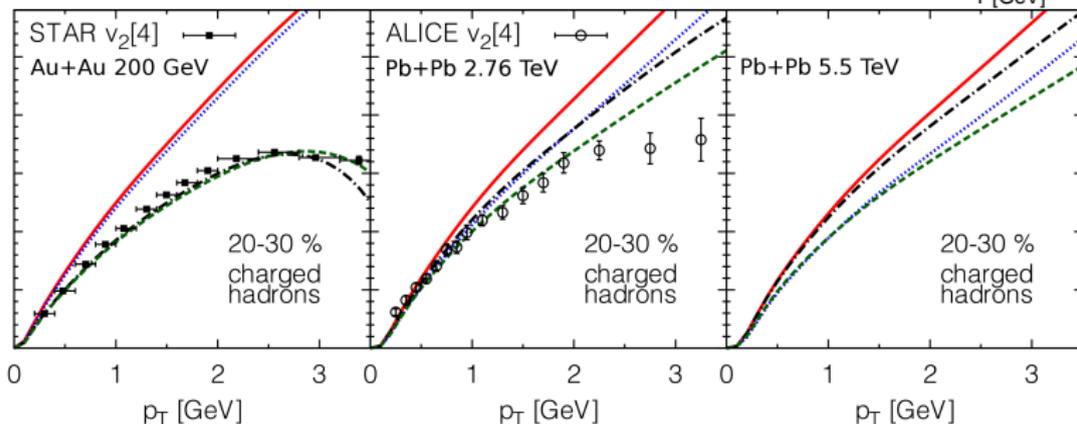
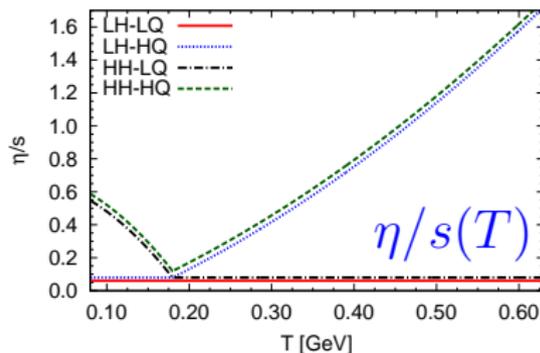
Beyond constant η/s

Determine dependence of v_2 on modeled $\eta/s(T)$.

L="low", H="high"

H=hadronic phase, Q=QGP

H. Niemi et al, Phys.Rev.Lett. 106 (2011) 212302



Weak dependence on QGP $\eta/s(T)$ at RHIC. Dependent on minimum.
Different at LHC energies (longer QGP phase, smaller gradients in the hadronic phase)

Do we really need local thermal equilibrium?

Rapid expansion \rightarrow large momentum space anisotropies
 \rightarrow shear \geq isotropic pressure.

Breakdown of expansion in terms of shear corrections

Try this: Hydrodynamics for highly anisotropic systems

Hydrodynamic expansion around anisotropic mom. dist. function

\rightarrow New evolution equations

- by requiring energy-momentum conservation and using an ansatz for an entropy source

W. Florkowski, R. Ryblewski, Phys.Rev. C83 034907 (2011) and arXiv:1103.1260

- by taking moments of the Boltzmann equation

M. Martinez and M. Strickland, Nucl.Phys. A848 183-197 (2010) and Nucl.Phys. A856 68-87 (2011)

One formalism for anisotropic early time dynamics and late time near-equilibrium dynamics (right limits)

Other recent developments and different approaches

- **Thermal fluct. in sound and shear waves contribute to viscosity**

Important when microscopic $\eta/s = 0.08$ (breakdown of 2nd order visc. hydro)
negligible when 0.16 P. Kovtun, G.D. Moore, P. Romatschke, Phys.Rev. D84 (2011) 025006

- **Hydrodynamic fluctuations**

in addition to initial state fluctuations, should be there and can be important
J.I. Kapusta, B. Müller, M. Stephanov, Phys.Rev. C85 (2012) 054906

- **Shear viscosity from parton cascade with 2 \leftrightarrow 3 processes:
 $\eta/s = 0.13 - 0.16$, also elliptic flow computation**

F. Reining, I. Bouras, A. El, C. Wesp, Z. Xu, C. Greiner, Phys.Rev. E85 (2012) 026302
Z. Xu, C. Greiner, Phys.Rev.Lett.100 172301 (2008)
A. El, A. Muronga, Z. Xu, C. Greiner, Phys.Rev. C79, 044914 (2009)

- **Flow in multiphase transport model (AMPT)**

J. Xu, C.M. Ko, arXiv:1103.5187, Phys.Rev. C83, 021903 (2011)

- **Flux-tube model with split into hydrodynamic and high-momentum part, EPOS - describes wide momentum range.**

K. Werner, Iu. Karpenko, M. Bleicher, T. Pierog, S. Porteboeuf-Houssais, Phys.Rev. C85 (2012) 064907

- Hydrodynamics is a powerful tool to describe the low momentum, bulk properties of heavy-ion collisions
- QCD enters through the equation of state (EoS) and (sometimes) through the initial state
- Many 2+1D and 3+1D simulations are on the market. 2 of them viscous 3+1D event-by-event
- Some simulations are coupled to hadronic afterburners (like UrQMD) to describe hadronic stage better
- Different models for the initial state exist. Some are mainly geometric (Glauber), some based on QCD (MC-KLN, IP-Glasma), some are extracted from cascade or string models
- Particle spectra and anisotropic flow are well described by e-b-e viscous hydrodynamics
- Using hydrodynamics we can learn about transport properties of the QGP and test models for the initial state

- Thermalization. How does the system become isotropic or even thermal in the first $\sim 0.5 \text{ fm}/c$?
- Does the system actually become isotropic and thermal in the first $\sim 0.5 \text{ fm}/c$?
- What exactly happens in the first $\sim 0.5 \text{ fm}/c$? Is there flow built up?
- Freeze-out. Is Cooper-Frye at constant T good enough? What about different species?
- Viscous corrections. Do they become too large? At least for photons they do...
- Need more studies on bulk viscosity
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