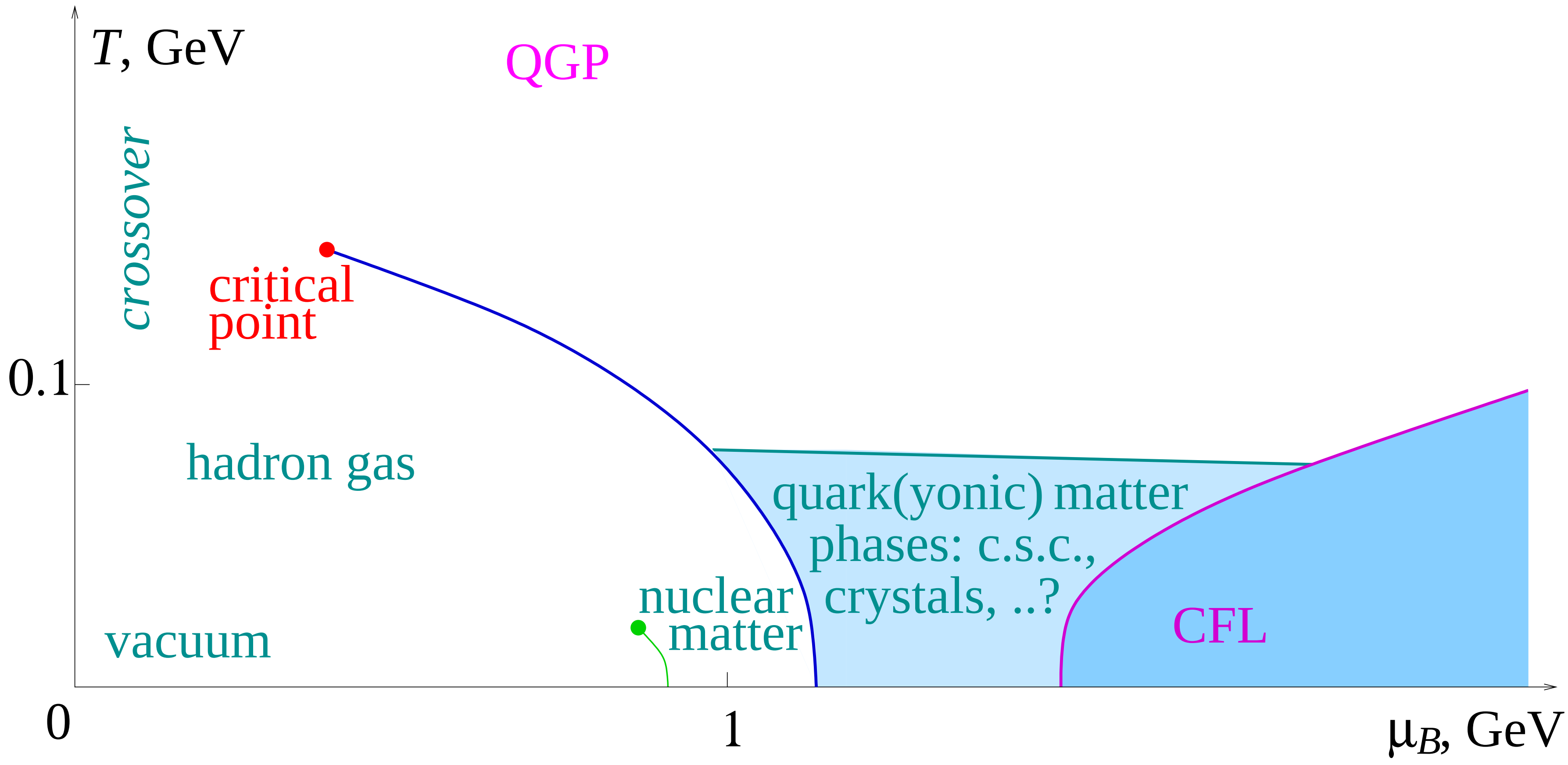
QCD phase diagram

the search for the critical point

M. Stephanov

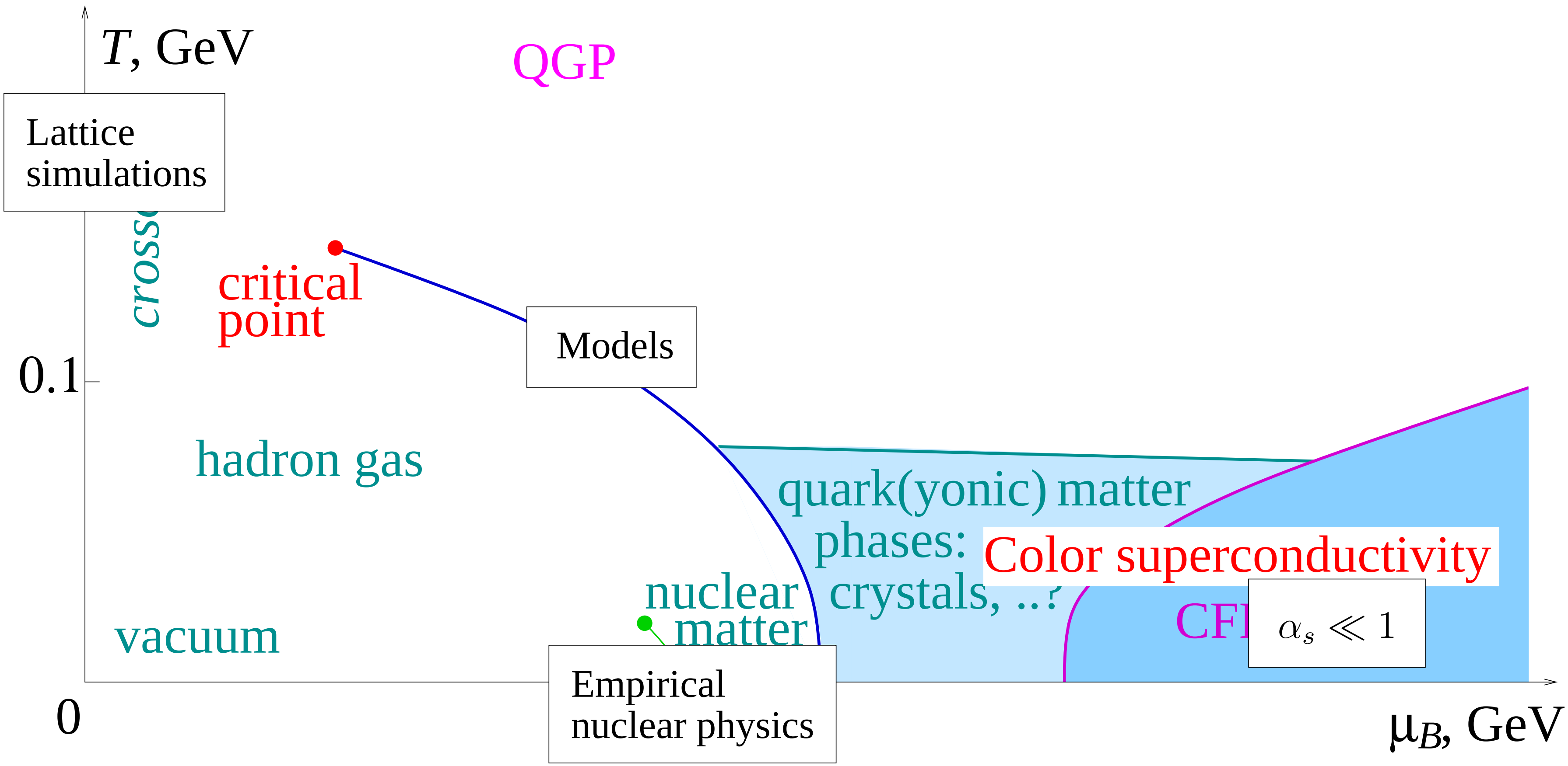
U. of Illinois at Chicago

The map of QCD phases



- There are many phases and transitions we can expect, but we do not always know their location, or if they actually do occur.
- Models (and lattice) suggest the transition becomes 1st order at some μ_B.
- Can we observe the critical point in heavy ion collisions, and how?

The map of QCD phases



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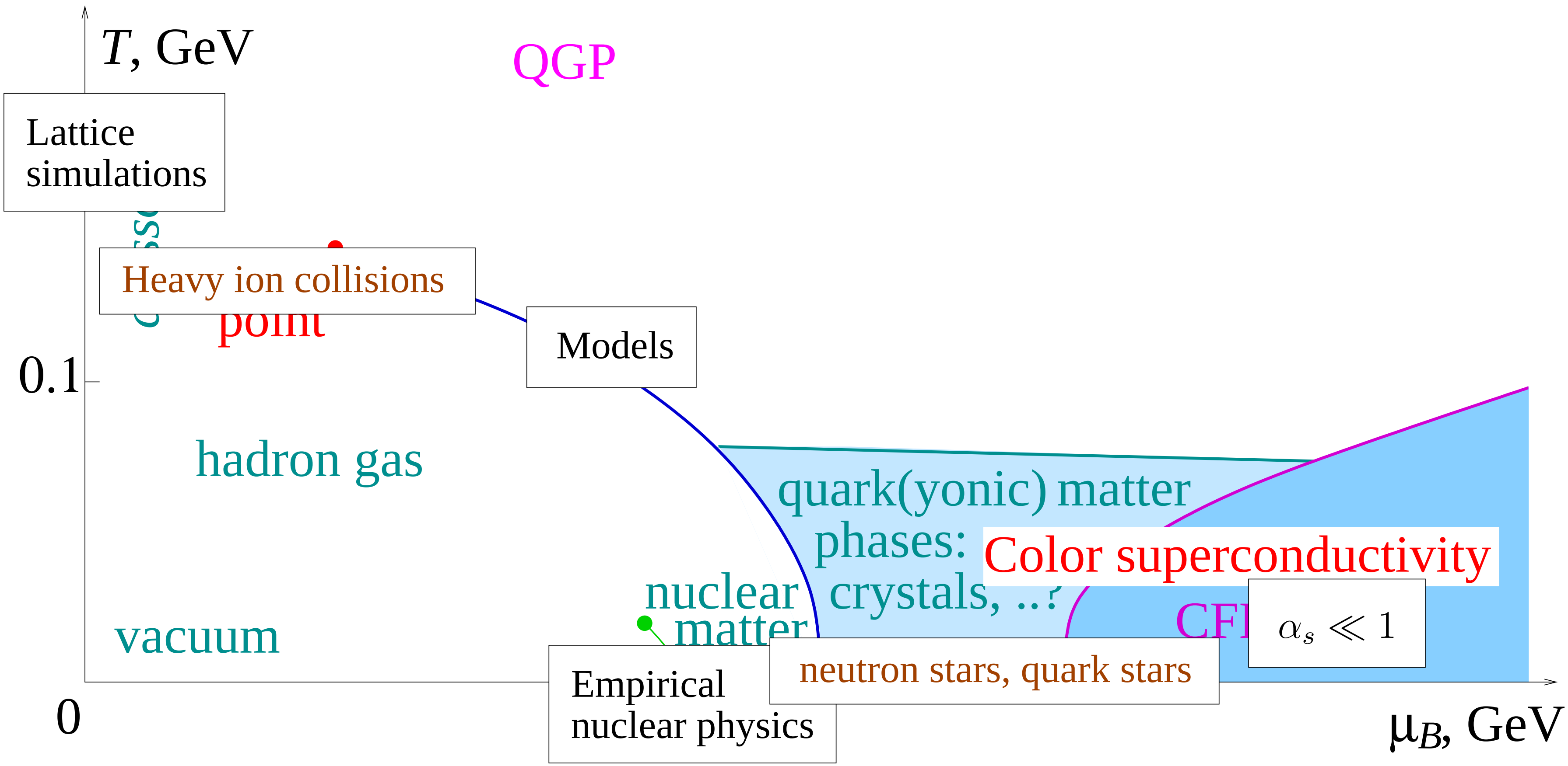

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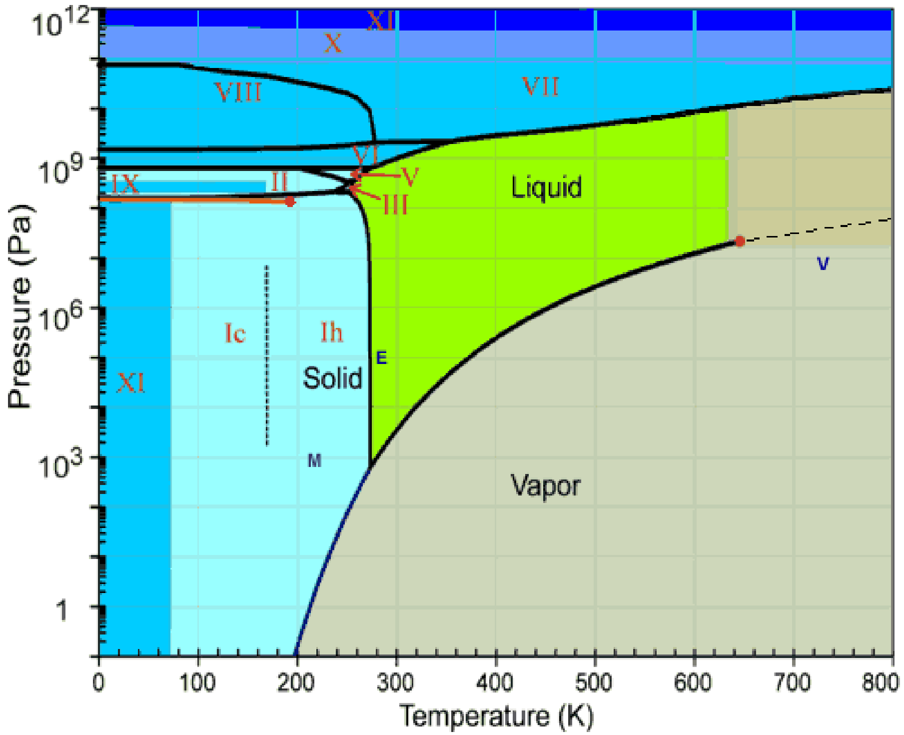
Critical points in known liquids

Critical point ∃ in many liquids (critical opalescence).

Water:



The transition

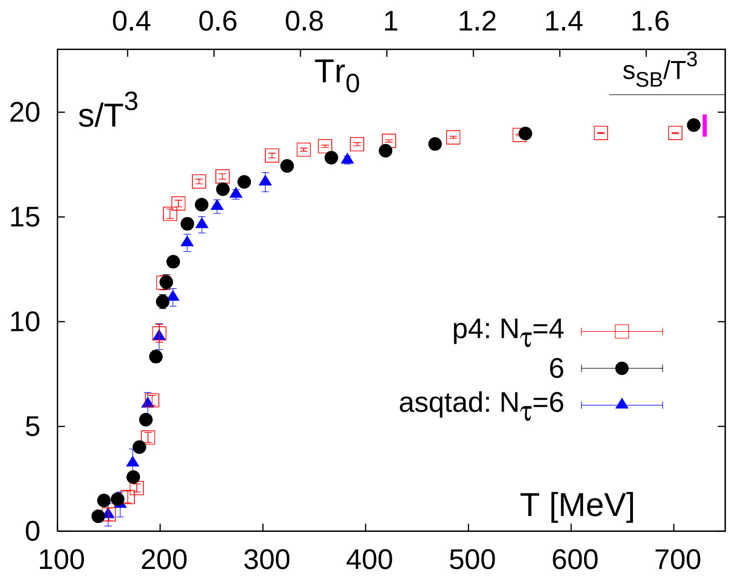
- Deconfinement? Confinement is difficult to define for theories with quarks.
  - Polyakov's definition, $\langle P \rangle = 0$, does not work, because $\langle P \rangle \neq 0$. The Z_3 symmetry is out once quarks are in.
  - Confining string between two color sources is *not* infinite — it snaps:

$$Q - - - - - - \bar{Q} \quad \Rightarrow \quad Q - - \bar{q} \quad + \quad q - - \bar{Q}$$

  - "No colored states"? This is true by *definition* of the theory. Not a dynamical property. There is no *de*confinement in this definition of confinement.
- In the limit of massless quarks there is a well-defined T_c. But this is chiral symmetry restoration.
- Our world is not ideal: neither chiral symmetry ($m_q = 0$) nor confinement ($m_q = \infty$) is well-defined. And neither is the distinction between the two phases.

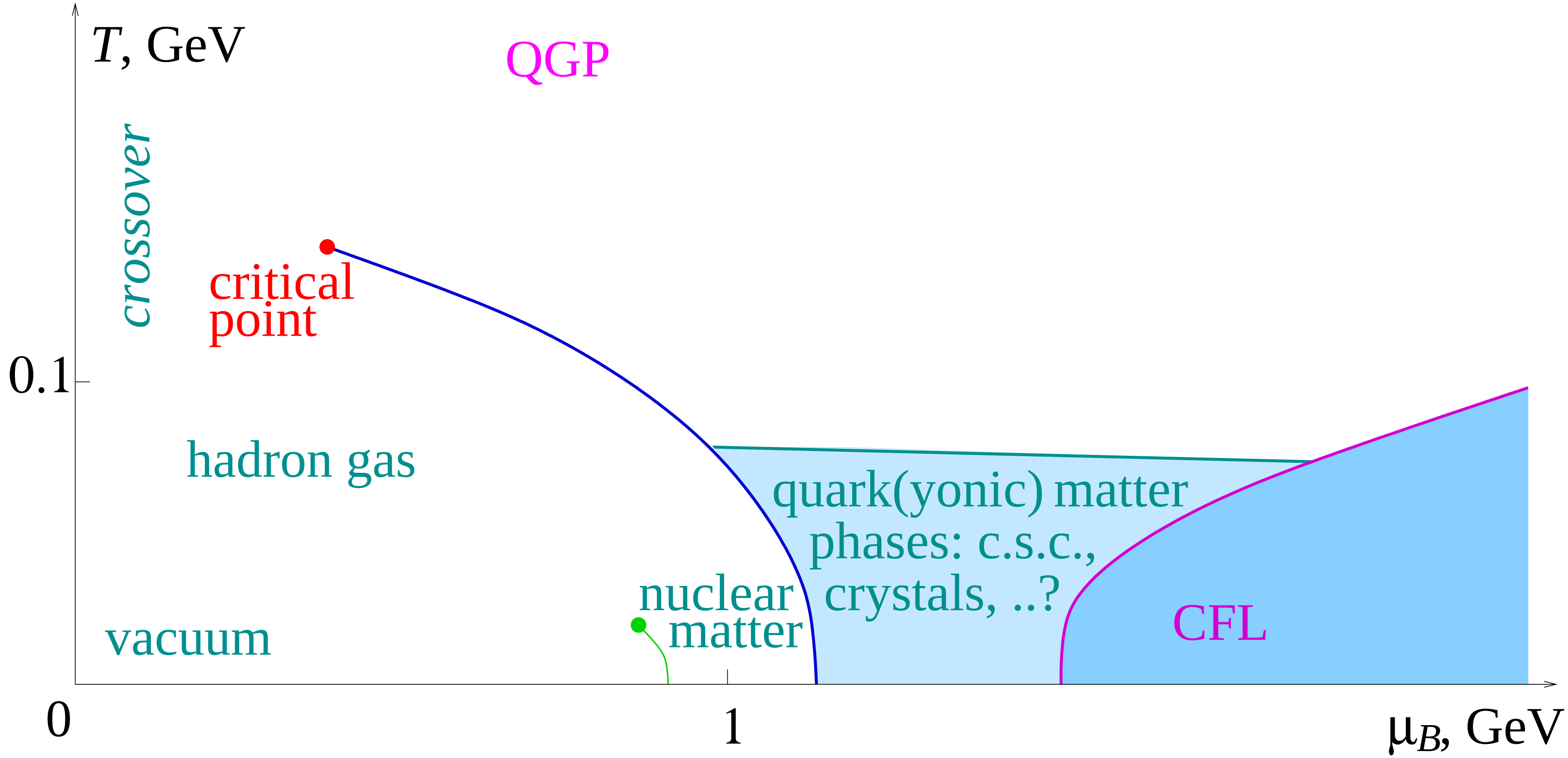
Deconfinement transition in QCD

- But there is a sense in which deconfinement does happen in QCD:

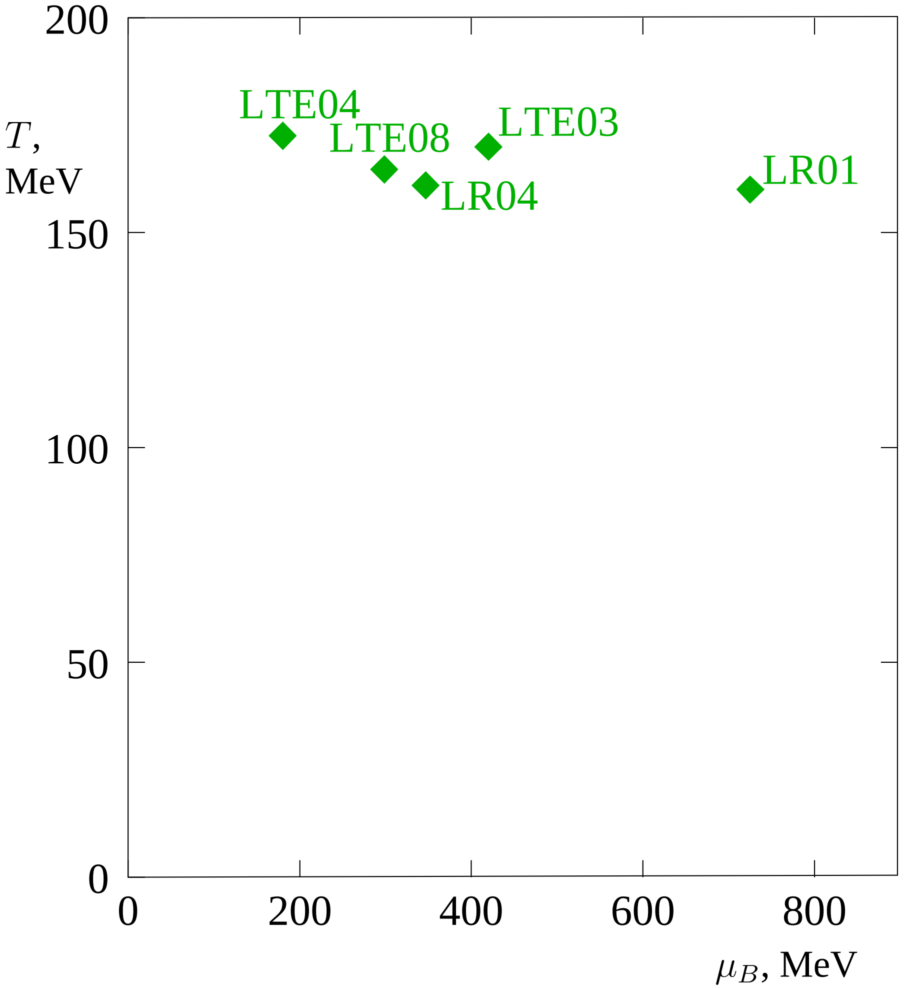


- s/T^3 – a measure of the number of (massless) particle “species”.
- gluons and quarks act as (count as) unconfined (“free”) above T_c!
- NB: “free” as far as d.o.f. counting (s), but not necessarily as far as hydro-dynamics (η).
- NB: even as $T \to \infty$ interaction energies are actually large ($\alpha_s T$), but the kinetic energies are larger still (T).

Where exactly is the critical point?



Location of the critical point from the Lattice



- Rough estimates based on various assumptions.
- Systematic errors are not shown/known.

Sign Problem

- Thermodynamics is encoded in the partition function

$$Z = \sum_{\text{quantum states}} \exp\{-\beta(\mathcal{E} - \mu N)\} = \int \mathcal{D}(\text{paths}) \, \exp\{-S_E\}$$

  - S_E - action on a path in imaginary time τ from 0 to β.

- Usually, S_E - real. So $\int \mathcal{D}(\text{paths})\, e^{-S_E}$ - itself is a partition function for *classical* statistical system in $3+1$ dimensions. Monte Carlo methods work.

- Not so for $\mu \neq 0$.

$$e^{-S_E} = e^{-S_{\text{gluons}}} \det D_{\text{quarks}}.$$

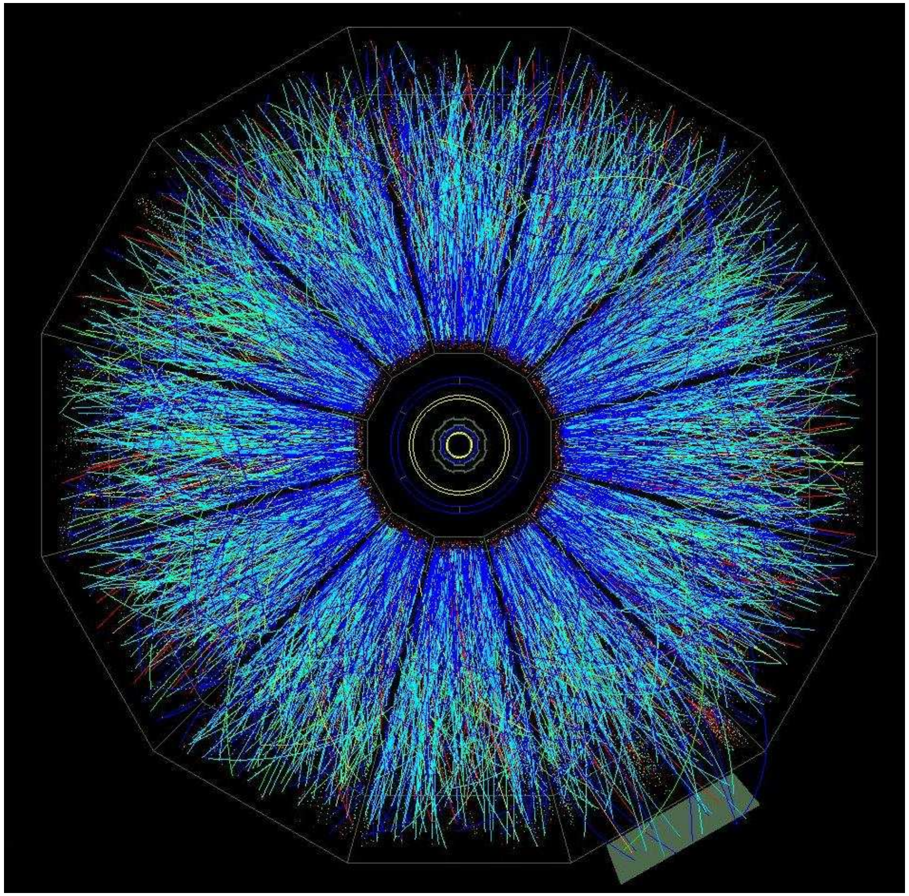
and $\det D_{\text{quarks}}$ - complex for $\mu \neq 0$.

Monte Carlo translates weight e^{-S_E} into probability and fails if S_E is not real.

- Recent progress based on various techniques of circumventing the problem:
  - Reweighting (use weight at $\mu = 0$);
  - Taylor expansion;
  - Imaginary μ;
  - ...

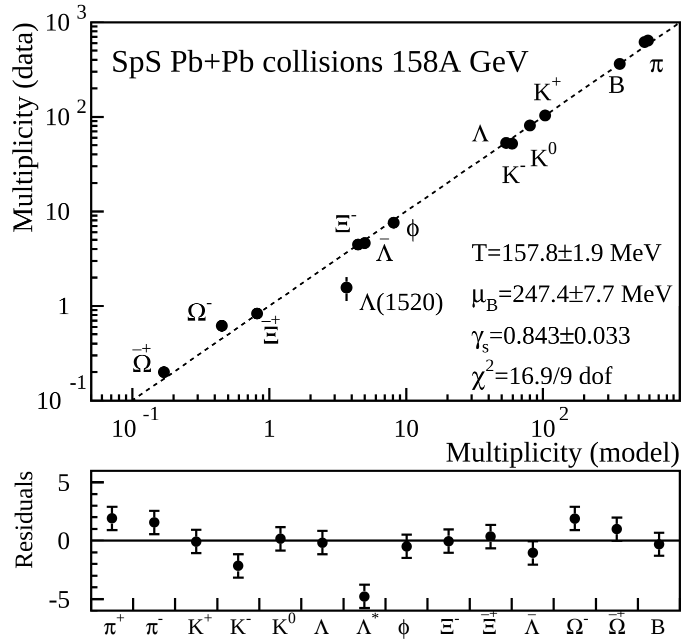
Heavy-ion collisions and the phase diagram

STAR@RHIC



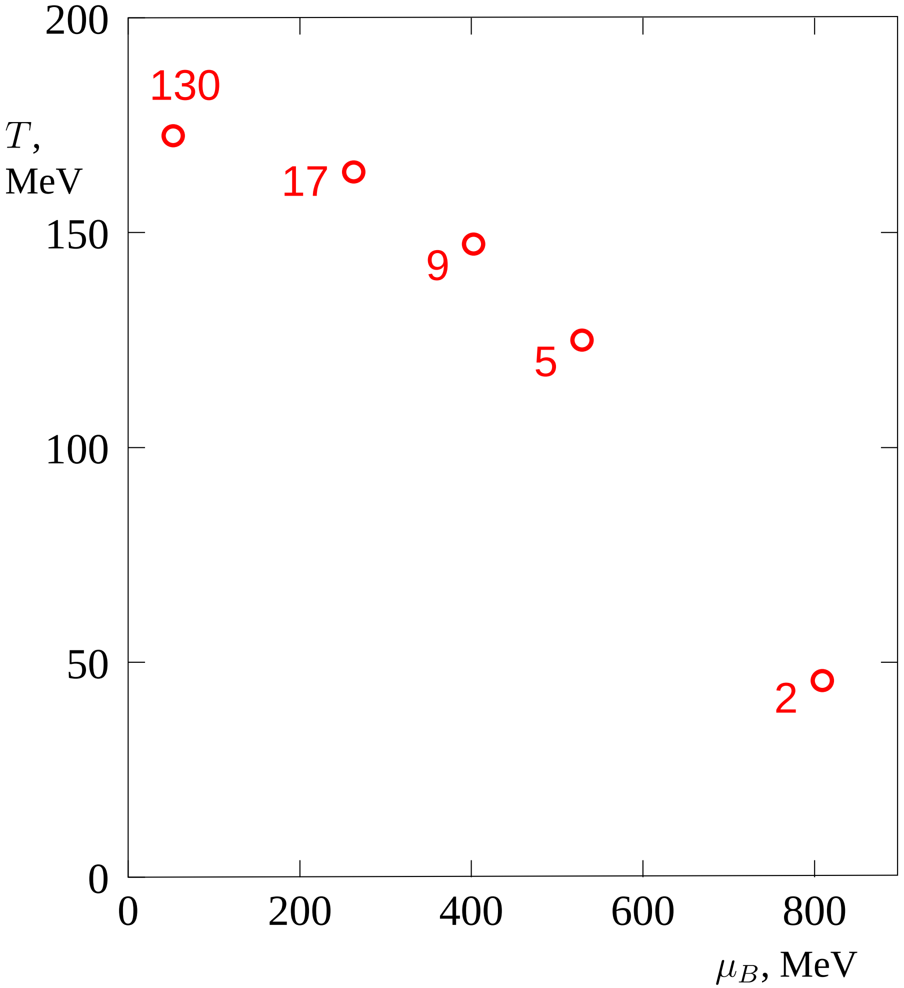
an event
“Little Bang”

Final state is thermal



(from Becattini et al)

Location of the critical point vs freeze-out



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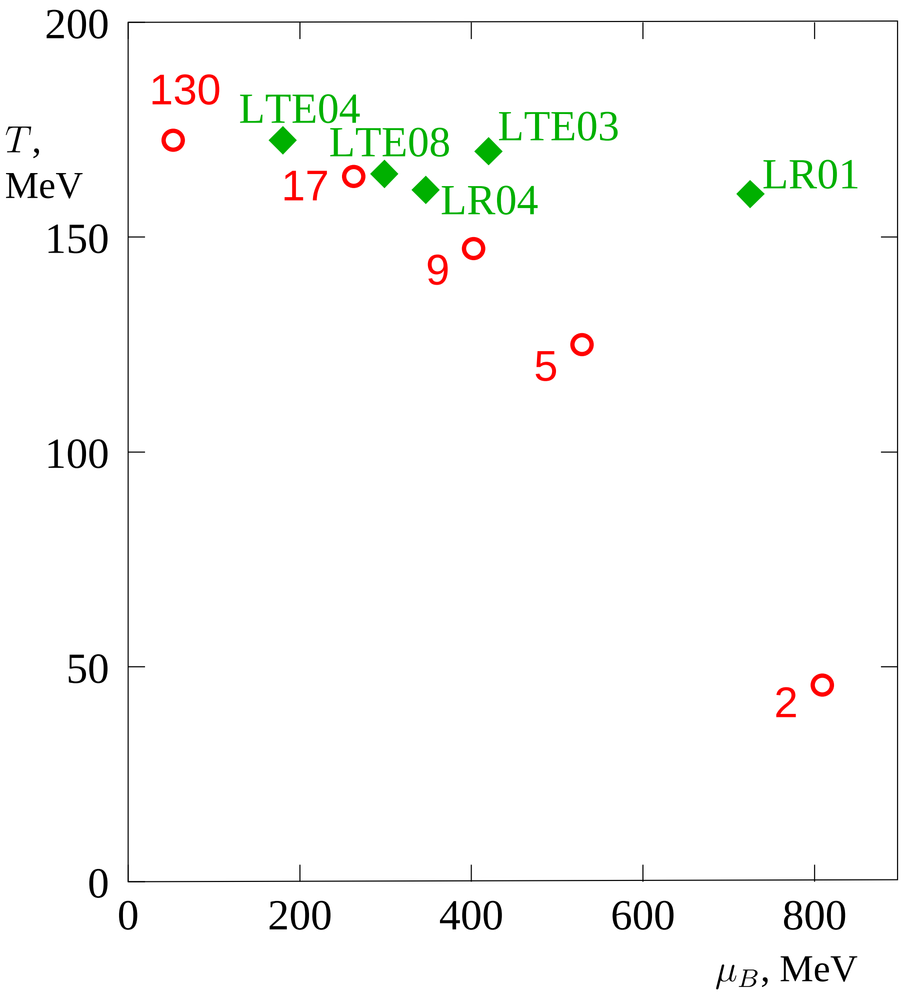

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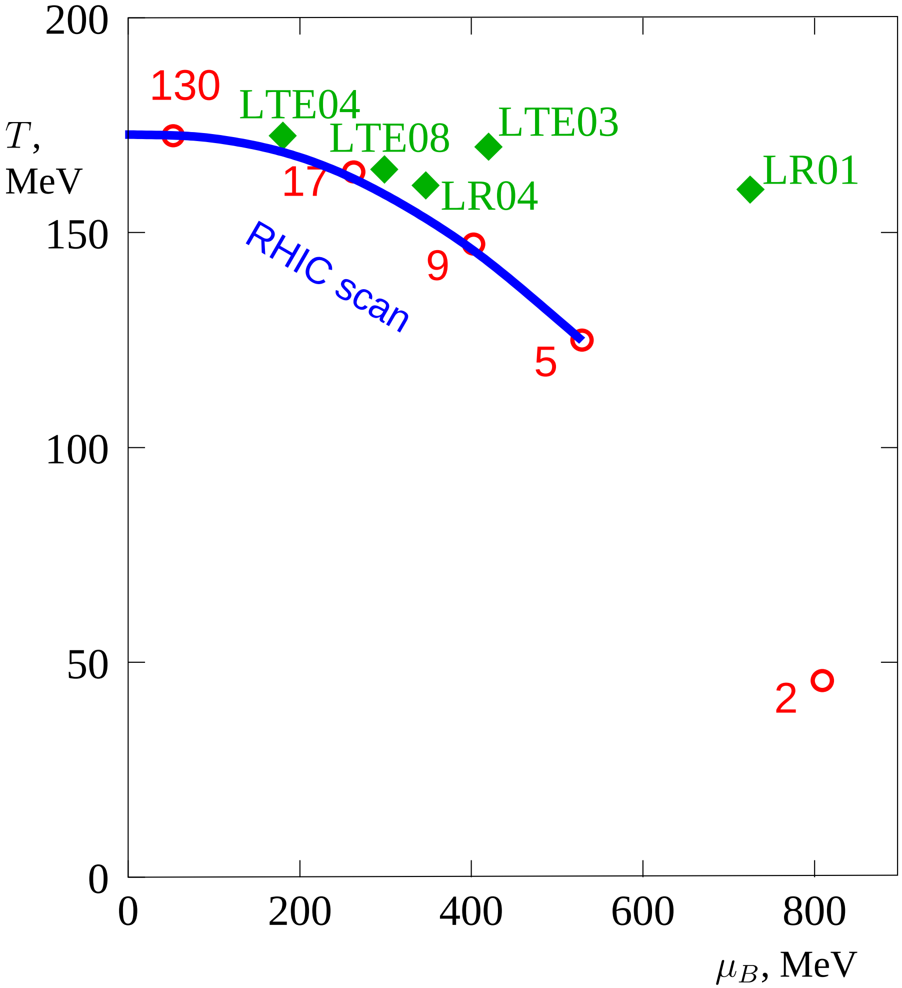
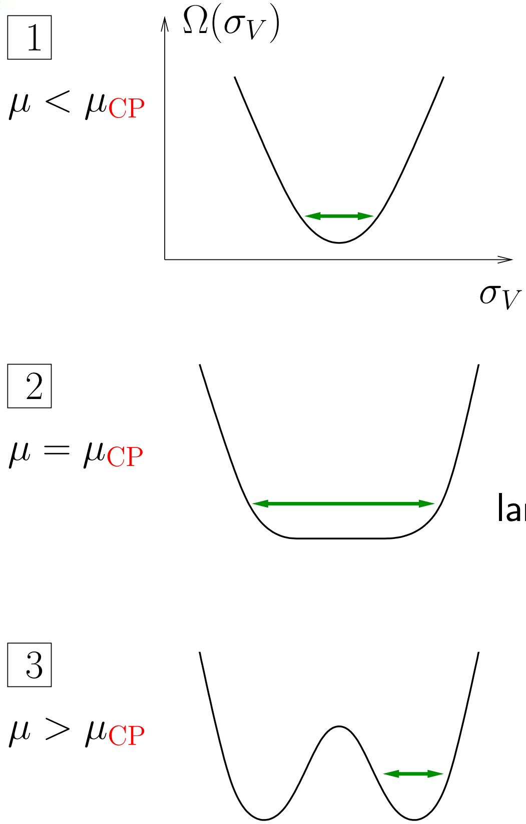


To do:

- Experiments:
  - RHIC,
  - NA61(SHINE) @ SPS,
  - CBM @ FAIR/GSI
  - NICA @ JINR
- Improve lattice predictions, understand systematic errors.
- Find most sensitive/optimal signatures and understand the effects of the dynamics of a h.i.c. on them.

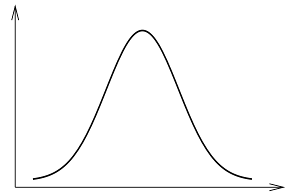
Critical mode and fluctuations

- 

  Consider a quantity (order param.) such as, e.g., $\sigma_V = \int_V \sigma$, where $\sigma \sim \bar{\psi}\psi$.

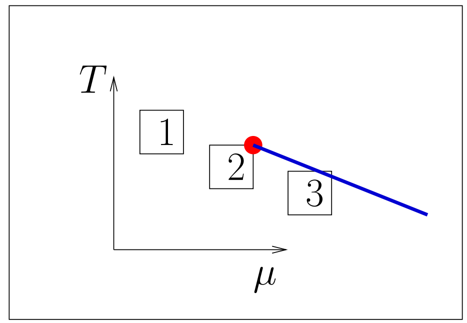
  $\langle \sigma_V^2 \rangle \sim (\Omega'')^{-1}$

  Einstein, 1910:
  $P(\sigma_V) \sim$ number of states with that σ_V i.e., e^S, or $e^{-\Omega/T}$

  $(\Omega'')^{-1} \to \infty$

  large equilibrium fluctuations

- Magnitude of fluctuation and correlation length:

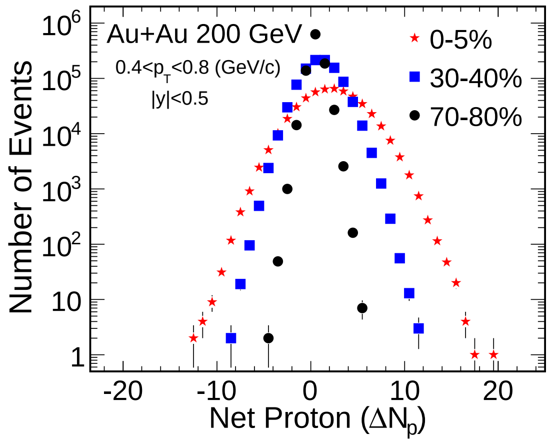
$$\langle \sigma(\boldsymbol{x})\sigma(\mathbf{0})\rangle \sim \begin{cases} e^{-|\boldsymbol{x}|/\xi} & \text{for} \quad |\boldsymbol{x}| \gg \xi \\ 1/|\boldsymbol{x}|^{1+\eta} & \text{for} \quad |\boldsymbol{x}| \ll \xi \end{cases}$$

$$\langle \sigma_{\mathbf{0}}^2 \rangle = \int d^3\boldsymbol{x} \langle \sigma(\boldsymbol{x})\sigma(\mathbf{0})\rangle \sim \xi^{2-\eta}$$

critical singularity is a *collective* phenomenon

- σ or n_B or T^{00}? Because they mix, only *one* linear combination is critical.

Fluctuation signatures



- Experiments give for each event: multiplicities N_π, N_p, ..., set of momenta p, etc.
  These quantities fluctuate event-by-event.
  - Measure – sq. var., e.g., $\langle(\delta N)^2\rangle$,$\langle(\delta p_T)^2\rangle$.
- What is the magnitude of these fluctuations near the QCD C.P.? (Rajagopal-Shuryak-MS, 1998)
- *Universality* tells us how it grows at the critical point: $\langle(\delta N)^2\rangle \sim \xi^2$.
  Correlation length is a universal measure of the "distance" from the c.p.
  It diverges as $\xi \sim (\Delta\mu \text{ or } \Delta T)^{-2/5}$ as the c.p. is approached.
- Magnitude of ξ is limited $< \mathcal{O}($2–3 fm$)$ (Berdnikov-Rajagopal).

- "Shape" of the fluctuations can be measured: non-Gaussian moments.
  As $\xi \to \infty$ fluctuations become less Gaussian ($\xi \to \infty$ vs $N \to \infty$).
- Higher cumulants show even stronger dependence on ξ (PRL 102:032301,2009):

$$\langle(\delta N)^3\rangle \sim \xi^{4.5}, \qquad \langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2 \sim \xi^{7}$$

which makes them more sensitive signatures of the critical point.

Fluctuations of order parameter and ξ

- Consider probability distribution for the order-parameter field:

$$P[\sigma] \sim \exp\left\{-\Omega[\sigma]/T\right\},$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\boldsymbol{\nabla}\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \ldots\right]. \qquad \Rightarrow \qquad \xi = m_\sigma^{-1}$$

- Moments (connected) of $\boldsymbol{q} = 0$ mode $\sigma_V \equiv \int d^3x\, \sigma(x)$:

$$\langle \sigma_V^2 \rangle = VT\,\xi^2\,; \qquad \langle \sigma_V^3 \rangle = 2VT^2\,\lambda_3\,\xi^6\,;$$

$$\langle \sigma_V^4 \rangle_c \equiv \langle \sigma_V^4 \rangle - 3\langle \sigma_V^2 \rangle^2 = 6VT^3\,[\,2(\lambda_3\xi)^2 - \lambda_4\,]\,\xi^8\,.$$

- Tree graphs. Each propagator gives ξ^2.

- Scaling requires “running”: $\lambda_3 = \tilde\lambda_3 T(T\xi)^{-3/2}$ and $\lambda_4 = \tilde\lambda_4(T\xi)^{-1}$, i.e.,

$$\langle \sigma_V^3 \rangle = 2VT^{3/2}\,\tilde\lambda_3\,\xi^{4.5}\,; \quad 6VT^2\,[\,2(\tilde\lambda_3)^2 - \tilde\lambda_4\,]\,\xi^7\,.$$

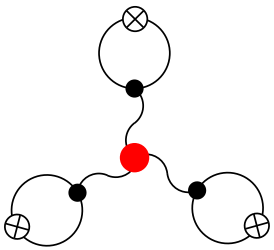
Experment: fluctuations of observables

- Example:

$$\delta N = \sum_{\boldsymbol{p}} \delta n_{\boldsymbol{p}}.$$

- $n_{\boldsymbol{p}}$ fluctuates around $\bar{n}_{\boldsymbol{p}}(m)$, which also fluctuates.
  Because $\delta m = g\delta\sigma$, where σ – order parameter field.

$$\delta n_{\boldsymbol{p}} = \underbrace{\delta n^0_{\boldsymbol{p}}}_{\text{statistical}} + \underbrace{\frac{\partial \bar{n}_{\boldsymbol{p}}}{\partial m}\, g\, \delta\sigma}_{\text{critical}} .$$

- 

$$\langle (\delta N)^3 \rangle = \sum_{\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3} \langle \delta n_{\boldsymbol{p}_1} \delta n_{\boldsymbol{p}_2} \delta n_{\boldsymbol{p}_3} \rangle$$

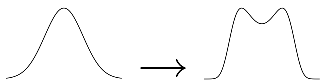
$$= (\text{Statistical}) - \langle \sigma_V^3 \rangle \left(\frac{g}{T}\right)^3 \frac{v^2_{\boldsymbol{p}_1}}{\gamma_{\boldsymbol{p}_1}} \frac{v^2_{\boldsymbol{p}_2}}{\gamma_{\boldsymbol{p}_2}} \frac{v^2_{\boldsymbol{p}_3}}{\gamma_{\boldsymbol{p}_3}}$$

$$v^2_{\boldsymbol{p}} = f_{\boldsymbol{p}}(1 \pm f_{\boldsymbol{p}}), \quad \gamma_{\boldsymbol{p}} = (dE_{\boldsymbol{p}}/dm)^{-1}$$

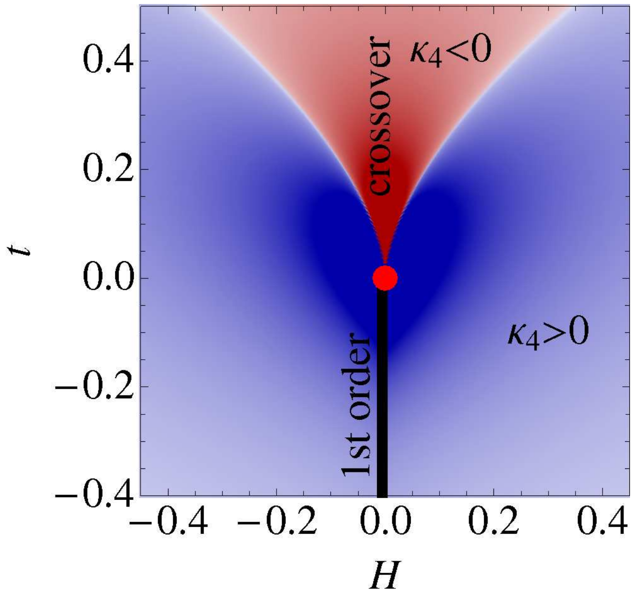
- $\langle \sigma_V^3 \rangle$ – a cumulant of the order parameter field – *universal*.

$$\langle \sigma_V^2 \rangle = VT\,\xi^2\,; \quad \langle \sigma_V^3 \rangle = 2VT^{3/2}\,\tilde{\lambda}_3\,\xi^{4.5}\,; \quad \langle \sigma_V^4 \rangle = 6VT^2\,[\,2(\tilde{\lambda}_3)^2 - \tilde{\lambda}_4\,]\,\xi^7\,.$$

Negative kurtosis

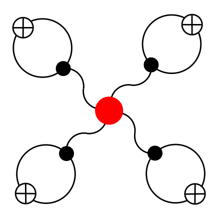
- Not only kurtosis becomes large, but it also changes sign (PRL 107:052301,2011)
- $P(\sigma_V)$: 

Thus $\langle \sigma_V^4 \rangle_c < 0$ on the crossover line ($\lambda_3 = 0$). And around it.



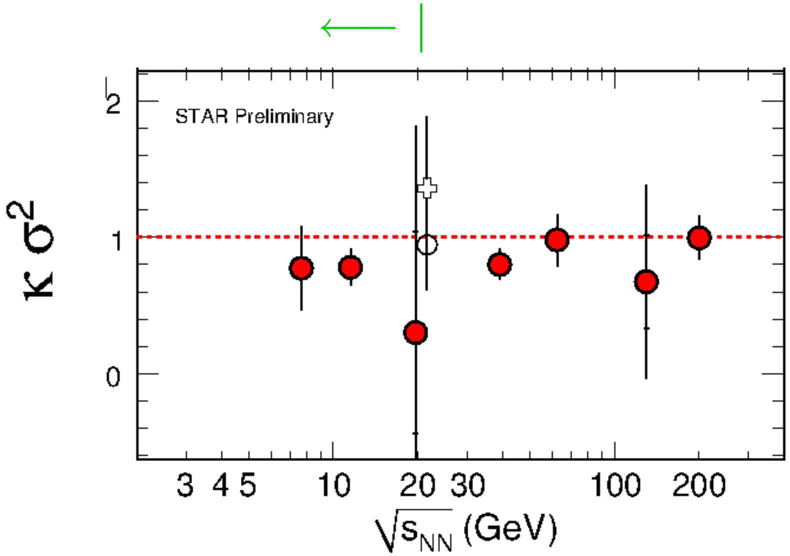
- Universal Ising eq. of state $M(H)$:
  $M = R^\beta \theta, \qquad t = R(1-\theta^2), \qquad H = R^{\beta\delta} h(\theta)$
  - here κ_4 is $\kappa_4(M) \equiv \langle M^4 \rangle_c$
  - in QCD $M \to \sigma_V$, and $(t, H) \to (\mu - \mu_{\rm CP}, T - T_{\rm CP})$

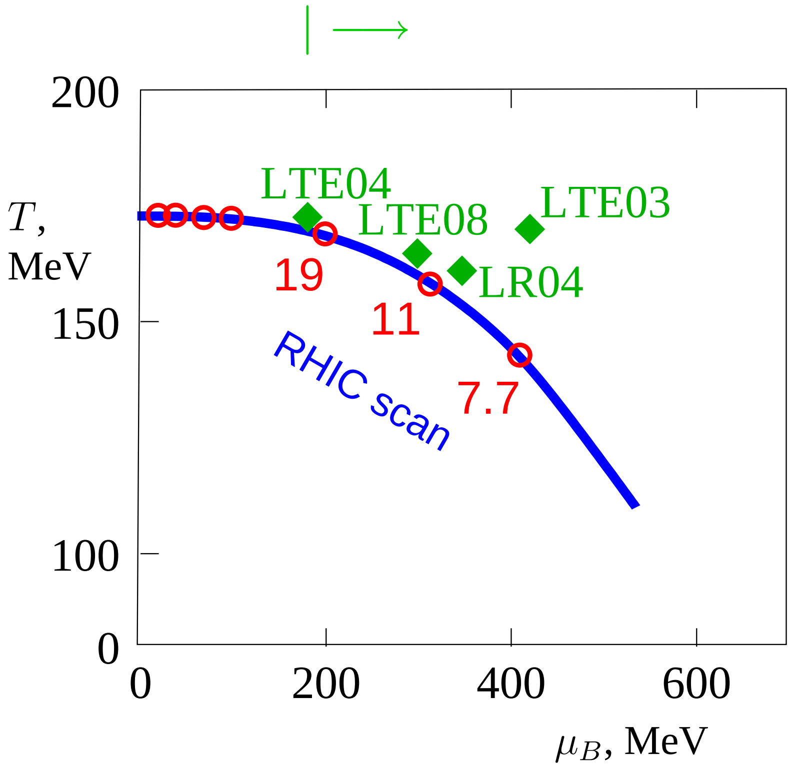
$$\langle (\delta N)^4 \rangle_c = \langle N \rangle + \langle \sigma_V^4 \rangle_c \left(\frac{g}{T} \int_{\boldsymbol{p}} \frac{v_{\boldsymbol{p}}^2}{\gamma_{\boldsymbol{p}}} \right)^4 + \ldots,$$



$\langle \sigma_V^4 \rangle_c < 0$ means $\quad \omega_4(N) \equiv \langle (\delta N)^4 \rangle_c / \langle N \rangle < 1$

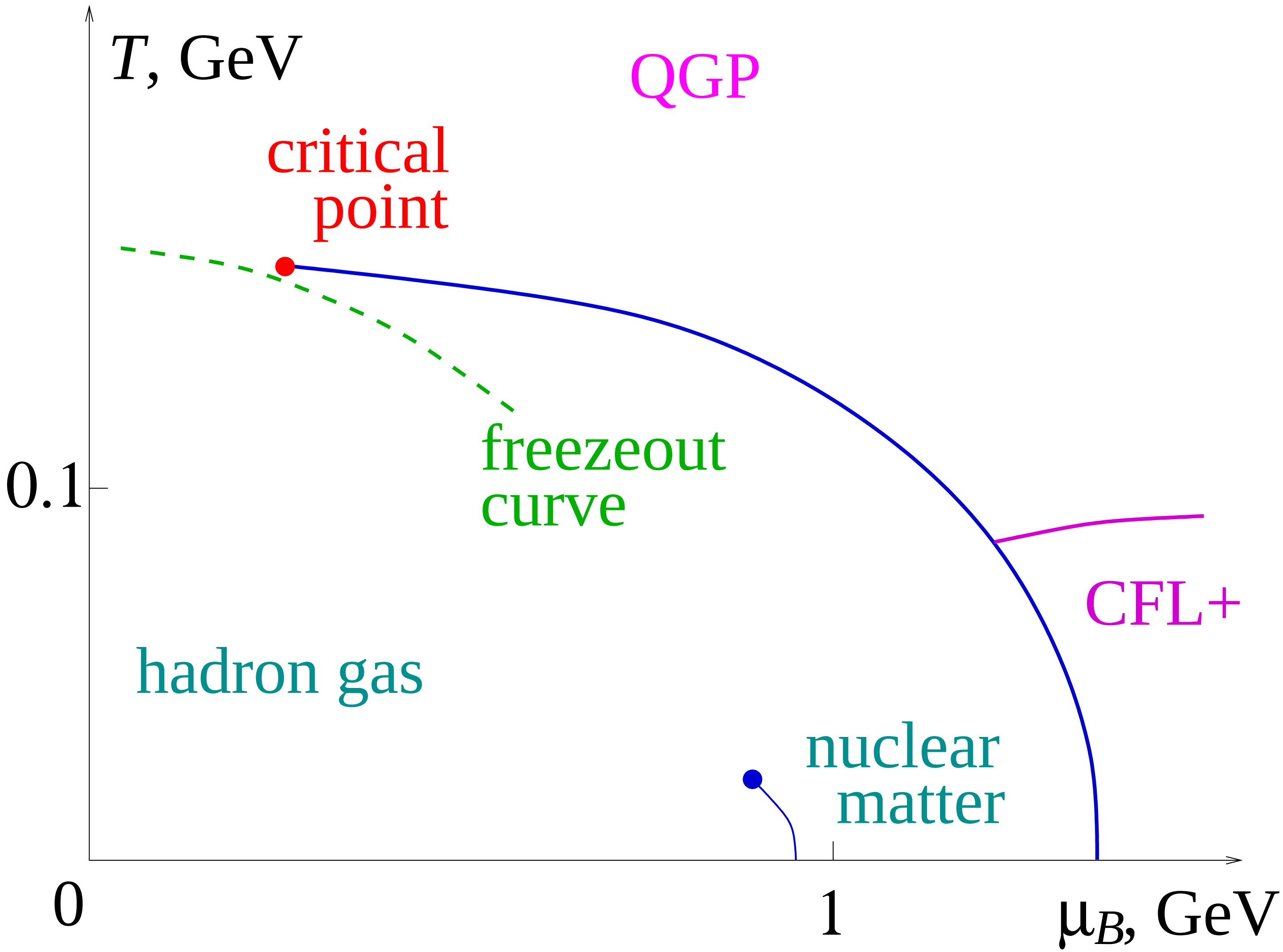
Early data from RHIC energy scan



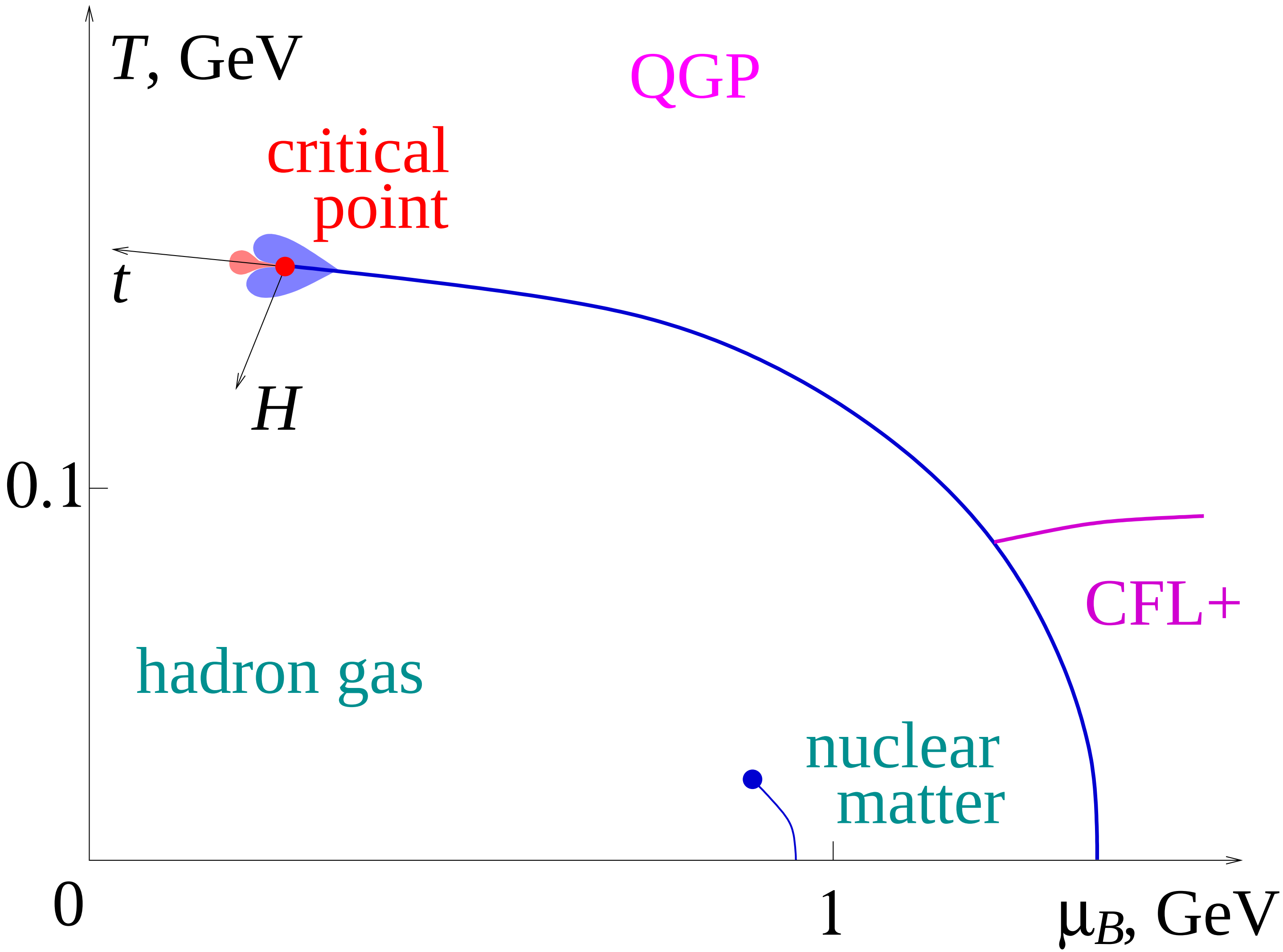


- 3 points at $\mu_B > 200$ MeV.

A scenario


T, GeV
QGP
critical point
0.1
freezeout curve
CFL+
hadron gas
nuclear matter
0
1
μB, GeV

A scenario



- Critical region $\Delta\mu_B \sim \mathcal{O}(100 - 150)$ MeV.

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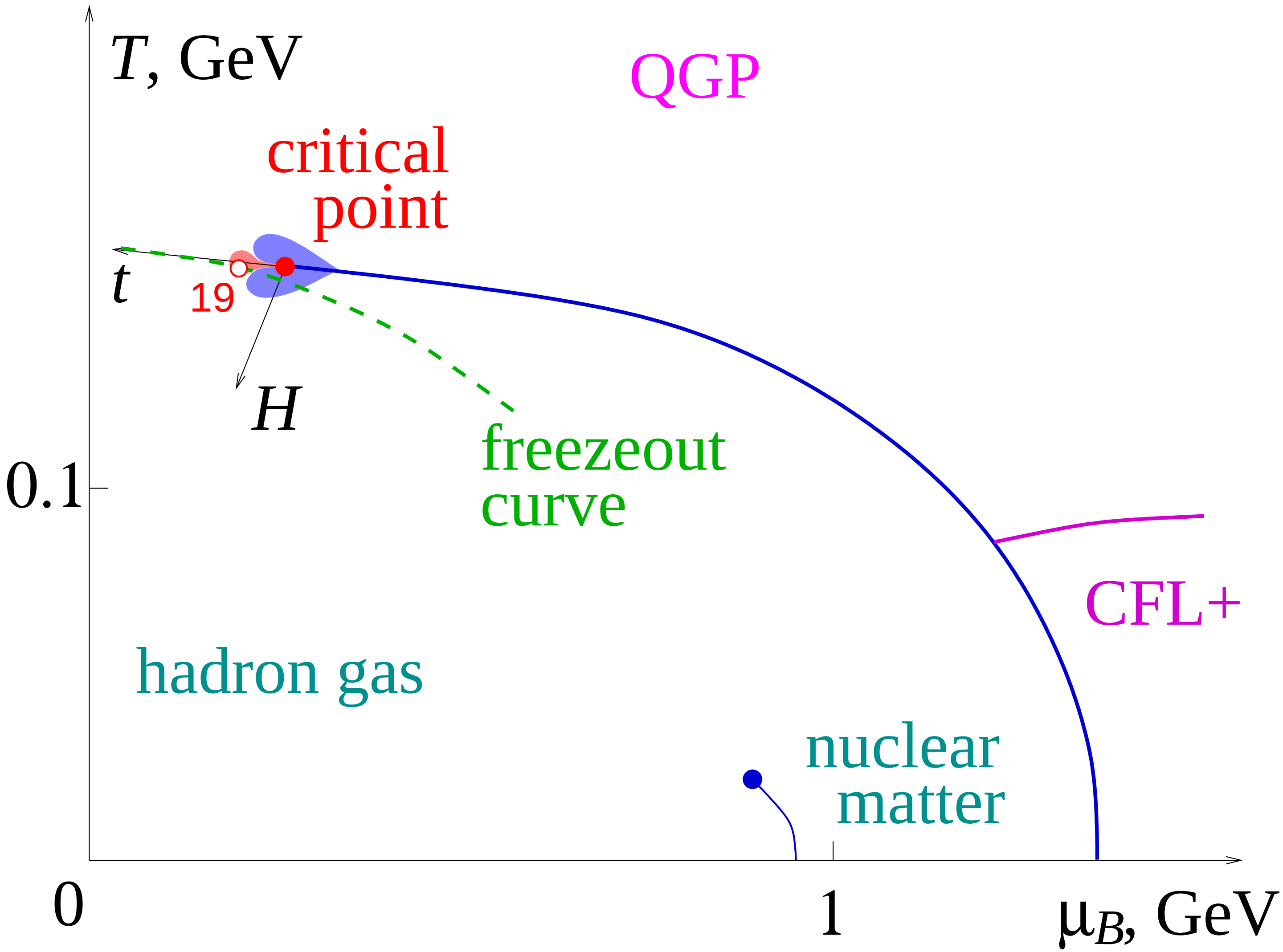

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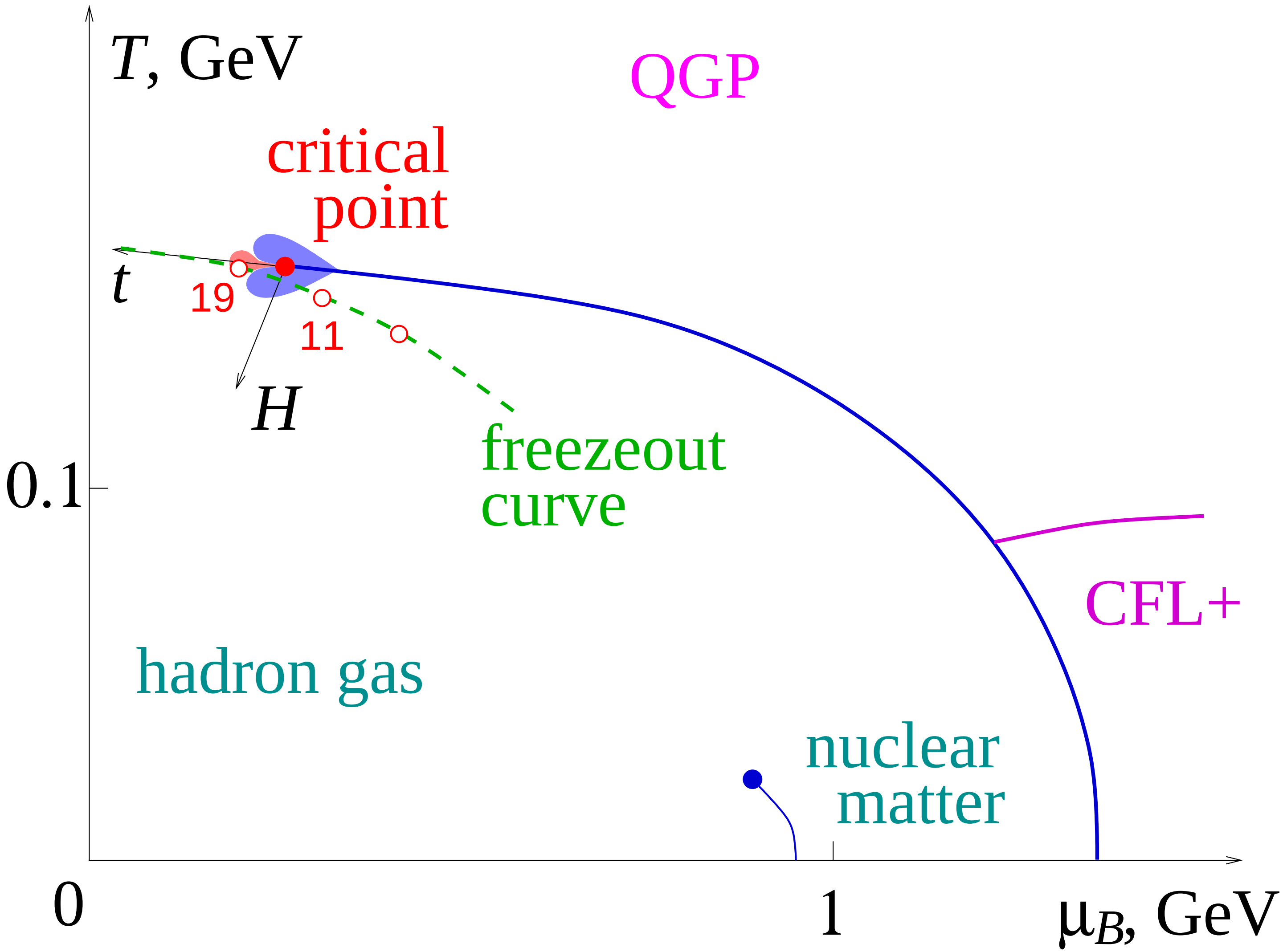

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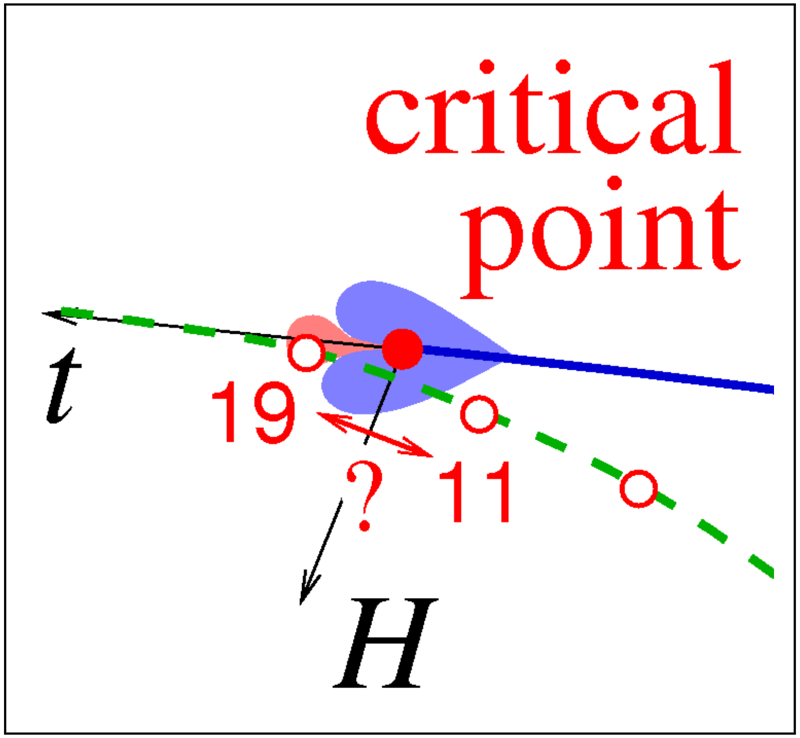
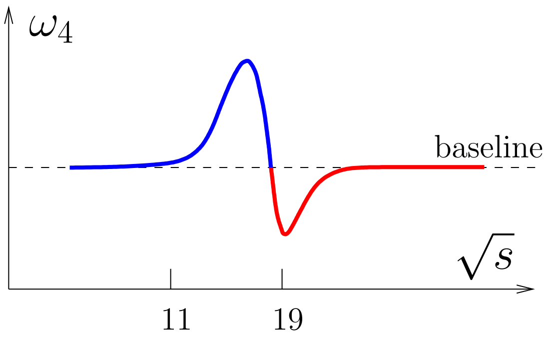
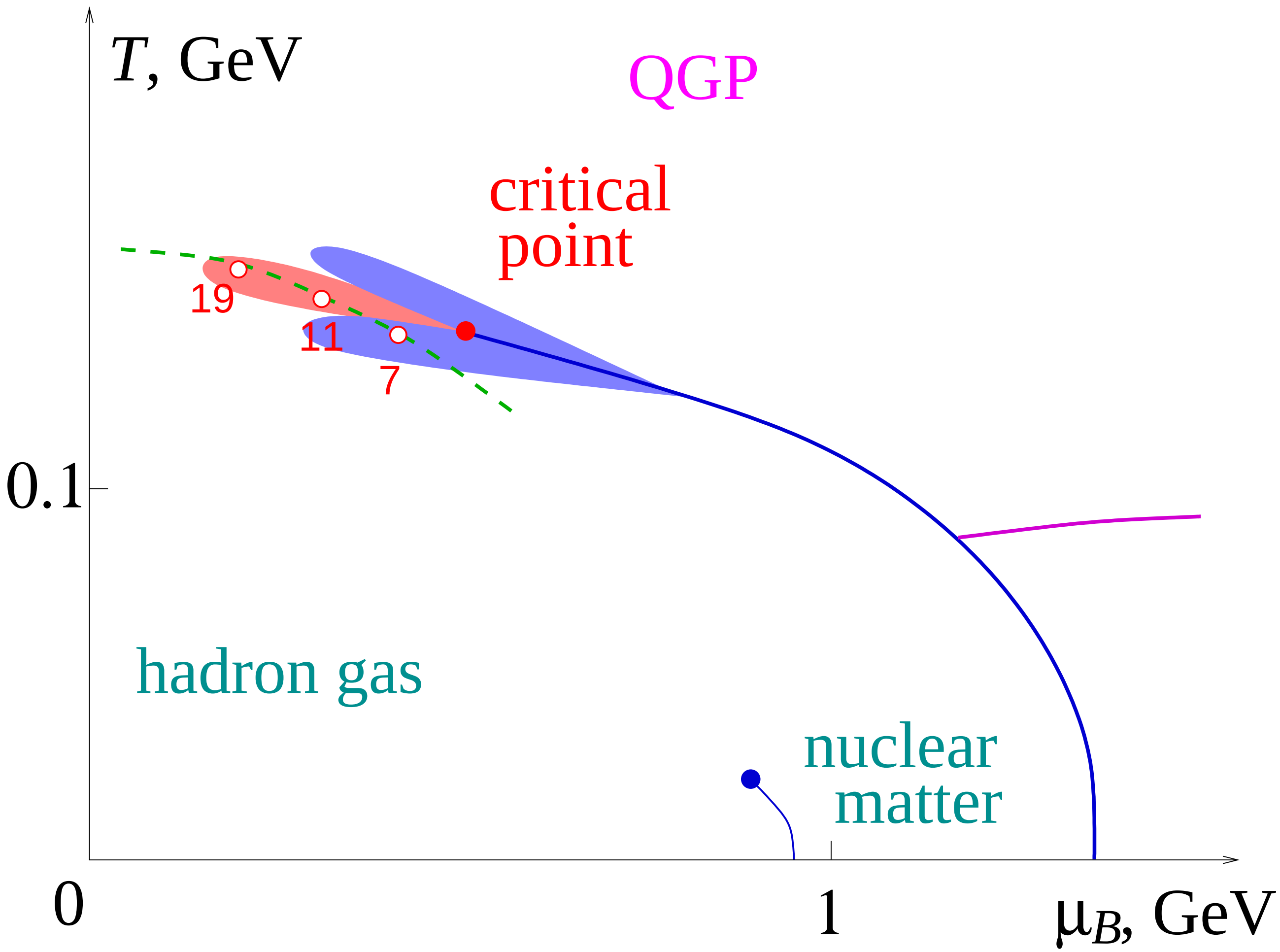


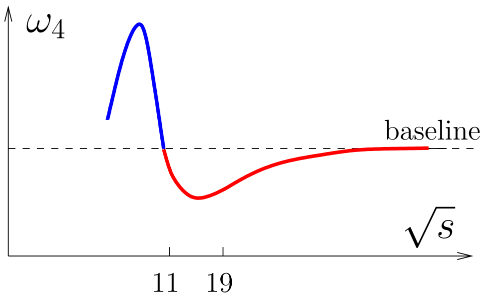


- Critical region $\Delta\mu_B \sim \mathcal{O}(100 - 150)$ MeV.

Another scenario



- Wider critical region.
  The region is stretched due to the proximity of the O(4) critical line, chiral limit.
  Seen in models (e.g., Skokov et al).



Conclusions/Outlook

- Critical point is a special singular point on the phase diagram, with unique signatures. This makes its experimental discovery possible.

- Locating the point is still a challenge for theory.
  Continued progress in lattice calculations: towards infinite volume, continuum limit and even tackling the sign problem.
  Inconclusive so far (lower bound $\mu_B \sim 200$ MeV).

- New sensitive signatures of the critical point based on higher moments are under study: the effects of the time evolution, conservation laws, finite acceptance.

- The search for the critical point is on. New RHIC results for 2 points with $\mu_B > 200$ MeV ($\sqrt{s} = 11$ and 7.7 GeV) were presented at QM11.

  19 and 27 GeV at QM12?

- More measurements at $\sqrt{s}$ values below 19 GeV are needed to map QCD phase diagram.