

Hydrodynamic Noise and Bjorken Expansion

M. Stephanov

U. of Illinois at Chicago

with J. Kapusta and B. Müller

arXiv:1112.6405 [nucl-th] (PRC)

Main point

- There are two sources of fluctuations in hydrodynamics:
 - Initial conditions;
 - Statistical noise.


Why noise?

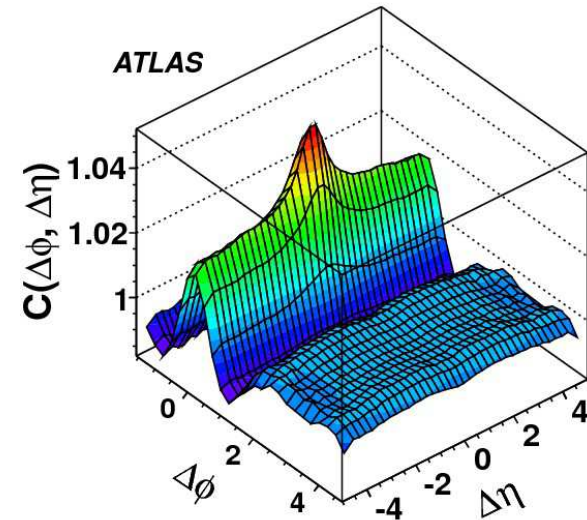
Local thermal equilibrium is a statistical concept. The “state” is an ensemble.

E.g., equation of state $P = P(\varepsilon)$ only gives the average value of pressure.

If there were no statistical fluctuations, the correlation functions, such as, e.g., $\langle T^{\mu\nu} T^{\alpha\beta} \rangle$ (which gives viscosity via Kubo) would vanish.

Fluctuations and viscosity

- Idea: magnitude of fluctuations (hydrodynamic noise) is proportional to dissipation, i.e., viscosity.
- The magnitude of fluctuations (correlations) can be measured. 
- Can the magnitude of these fluctuations tell us about viscosity?



What is Hydrodynamics?

Fluid left alone tends to equilibrium.

There are two time scales:

- 1) local thermodynamic equilibration – fast;
- 2) achieving same conditions throughout – slow.

Hydrodynamics describes that slower process.



It is an effective theory – only operates with degrees of freedom that matter – densities of energy, momentum, charge. They are slow to change on large scales because they carry conserved quantities.

The remaining, faster degrees of freedom are the “noise”.

Relativistic Hydrodynamics

Variables: conserved quantities characterizing local thermal equilibrium, i.e., energy and momentum densities.

Or: ϵ, u^μ – defined by $T^{\mu\nu} u_\nu = \epsilon u^\mu$ – which fixes 4 components of $T^{\mu\nu}$.

The remaining 6 components (stress) must be also expressed in terms of ϵ and u^μ :

$$T^{\mu\nu} = T_{\text{eq}}^{\mu\nu} + \Delta T^{\mu\nu}$$

where the value in equilibrium (a homogeneous/isotropic state) is

$$T_{\text{eq}}^{\mu\nu} = \epsilon u^\mu u^\nu + P(\epsilon) \Delta^{\mu\nu} \quad [\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu]$$

and the deviations from equilibrium due to gradients:

$$\Delta T^{\mu\nu} = -\eta \Delta_\lambda^\mu \left[\nabla^\lambda u^\nu + \nabla^\nu u^\lambda - \frac{2}{3} g^{\lambda\nu} (\nabla \cdot u) \right] - \zeta \Delta^{\mu\nu} (\nabla \cdot u)$$

Equations: $\nabla_\mu T^{\mu\nu} = 0$.

Fluctuations and Noise

- $T^{\mu\nu} = T_{\text{eq}}^{\mu\nu} + \Delta T_{\text{visc}}^{\mu\nu}$ is only true on average. Both sides fluctuate and

$$T^{\mu\nu} = T_{\text{eq}}^{\mu\nu} + \Delta T_{\text{visc}}^{\mu\nu} + S^{\mu\nu}.$$

- The discrepancy comes from the “fast” modes. Thus the “noise” is local:

$$\langle S^{\mu\nu}(x) S^{\alpha\beta}(y) \rangle \sim \delta^4(x - y).$$

- The magnitude is determined by the condition that the equilibrium distribution is given by $e^{\text{Entropy}(\epsilon)}$ (Einstein). Dissipation

$$\nabla_{\mu}(su^{\mu}) = \frac{\eta}{2T} \left[\Delta^{\mu} u^{\nu} + \Delta^{\nu} u^{\mu} - \frac{2}{3} \Delta^{\mu\nu} (\nabla \cdot u) \right]^2 + \frac{\zeta}{T} (\nabla \cdot u)^2$$

must be matched by noise:

$$\langle S^{\mu\nu}(x) S^{\alpha\beta}(y) \rangle = 2T \left[\eta \left(\Delta^{\mu\alpha} \Delta^{\nu\beta} + \Delta^{\mu\beta} \Delta^{\nu\alpha} \right) + \left(\zeta - \frac{2}{3} \eta \right) \Delta^{\mu\nu} \Delta^{\alpha\beta} \right] \delta^4(x - y)$$

Now $\nabla_{\mu} T^{\mu\nu} = 0$ is a system of stochastic eqs. for ϵ, u^{μ} .

Fluctuations and Bjorken expansion

Although noise is local, the correlations induced are propagated by hydrodynamic modes over macroscopic distances.

Equations

$$\nabla_\nu (T_{\text{eq}}^{\mu\nu} + \Delta T_{\text{visc}}^{\mu\nu} + S^{\mu\nu}) = 0.$$

applied to fluctuations around a static equilibrium solution give well-known equilibrium correlation functions (and, e.g., Kubo equation).

Our goal is to apply this to determine correlations in an expanding fireball.

The simplicity and symmetry of the Bjorken solution allows analytical treatment.

In this work we only consider rapidity dependence (integrate over azimuth).

Correlations

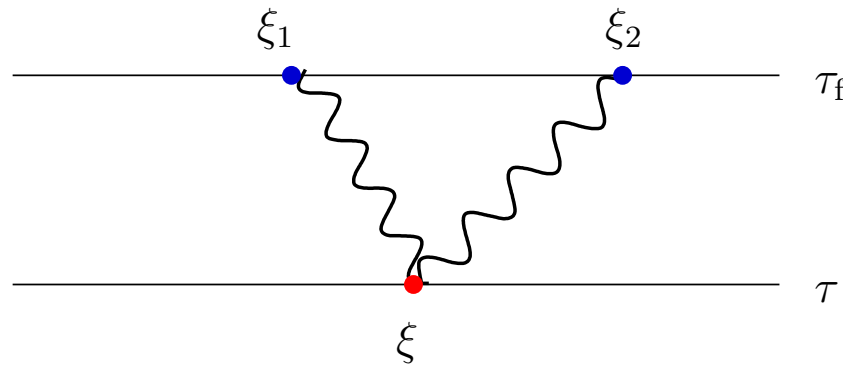
The equal-(proper)time correlation function at freeze-out time τ_f :

$$\langle \rho(\xi_1, \tau_f) \rho(\xi_2, \tau_f) \rangle = \frac{2}{A} \int_{\tau_0}^{\tau_f} \frac{d\tau}{\tau^3} \frac{\nu}{\epsilon + P} \int_{-\infty}^{\infty} d\xi G_\rho(\xi_1 - \xi; \tau_f, \tau) G_\rho(\xi_2 - \xi; \tau_f, \tau).$$

Convenient variable $\rho \equiv \delta s/s$. Convenient notation: $\nu \equiv \frac{4\eta/3 + \zeta}{s}$.

Every point ξ, τ is a source of noise, $\rho = G_\rho \circ f$:

$$[S^{\mu\nu} = \Delta^{\mu\nu} (\epsilon + P) f]$$



Correlations

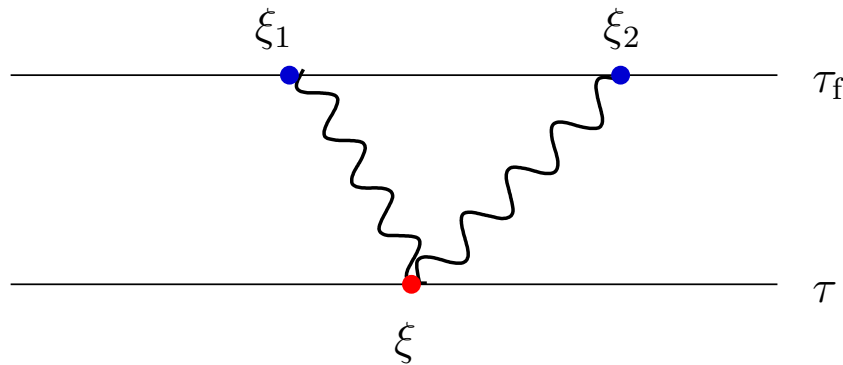
The equal-(proper)time correlation function at freeze-out time τ_f :

$$\langle \rho(\xi_1, \tau_f) \rho(\xi_2, \tau_f) \rangle = \frac{2}{A} \int_{\tau_0}^{\tau_f} \frac{d\tau}{\tau^3} \frac{\nu}{\epsilon + P} \underbrace{\int_{-\infty}^{\infty} d\xi G_\rho(\xi_1 - \xi; \tau_f, \tau) G_\rho(\xi_2 - \xi; \tau_f, \tau)}_{G_{\rho\rho}(\Delta\xi; \tau_f, \tau)}.$$

Convenient variable $\rho \equiv \delta s/s$. Convenient notation: $\nu \equiv \frac{4\eta/3 + \zeta}{s}$.

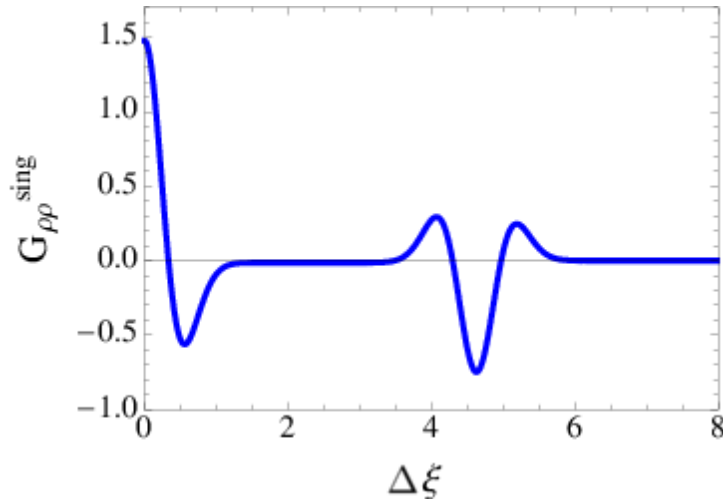
Every point ξ, τ is a source of noise, $\rho = G_\rho \circ f$:

$$[S^{\mu\nu} = \Delta^{\mu\nu} (\epsilon + P) f]$$



Sound horizon and the wake

$$G_{\rho\rho} \equiv G_{\rho\rho}^{\text{sing}} + G_{\rho\rho}^{\text{reg}}$$

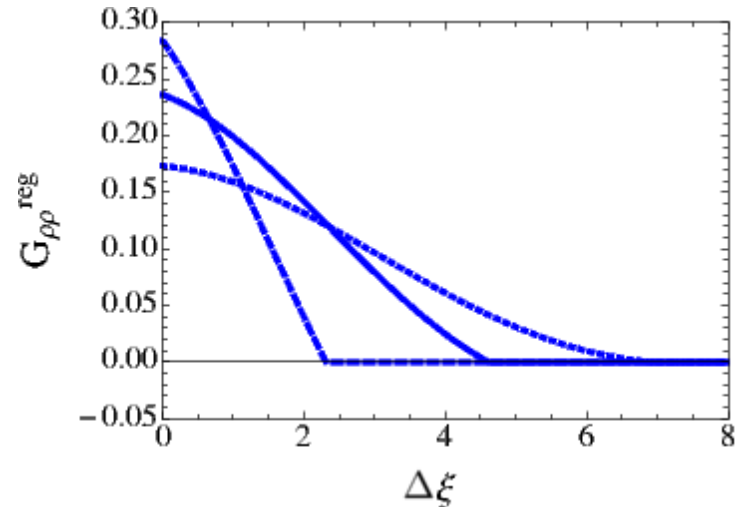


$$v_s^2 = 1/3 \text{ and } \ln(\tau_f/\tau) = 4$$

If the dispersion was linear, there would only be the sound front at $\Delta\xi = 2v_s \ln(\tau_f/\tau)$.

Viscosity smears the sound-front singularity

$$\sigma^2 = \frac{\nu}{\alpha} \left(\frac{1}{\tau T} - \frac{1}{\tau_f T_f} \right)$$



$$\ln(\tau_f/\tau) = 2, 4, 6$$

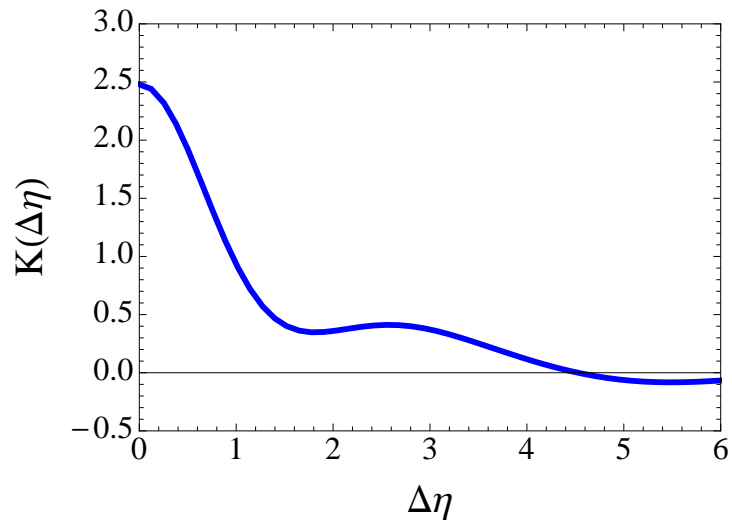
However, $\omega = i\alpha \pm \sqrt{v_s^2 k^2 - \alpha^2}$
 $[\alpha \equiv (1 - v_s^2)/2]$.

Slowest mode $\omega_- \sim -ik^2$, for $k \rightarrow 0$.
 Diffusion-like, but no dissipation.

Freeze-out

Translation from ξ to η using Cooper-Frye leads to additional (thermal) smearing.
Finally

$$\left\langle \delta \frac{dN}{d\eta_1} \delta \frac{dN}{d\eta_2} \right\rangle \left\langle \frac{dN}{d\eta} \right\rangle^{-1} = \frac{45d_s}{4\pi^4 N_{\text{eff}}(T_0)} \frac{\nu}{T_f \tau_f} \left(\frac{T_0^2}{T_f^2} \right)^{v_s^{-2}-2} K(\Delta\eta),$$



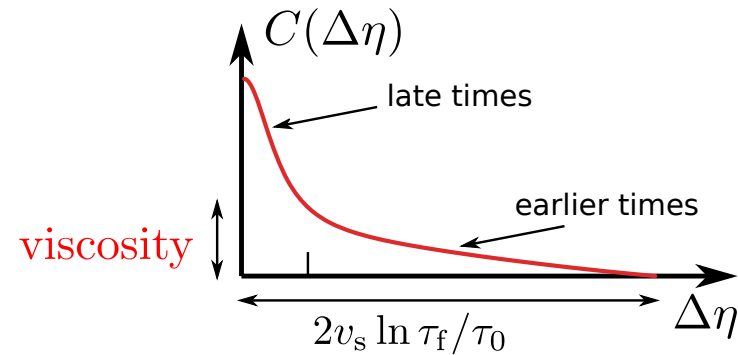
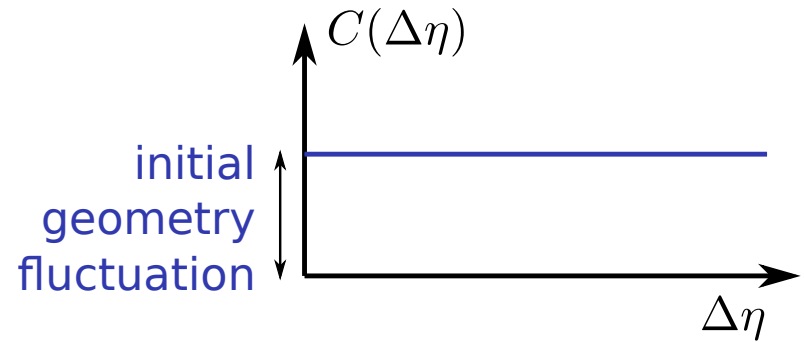
Conclusions and outlook

● Disentangling correlation sources:

Fluctuations of initial conditions produce $\Delta\eta$ -independent correlation.

Hydrodynamic noise has local origin and produces characteristic $\Delta\eta$ dependence:

with magnitude proportional to viscosity.



Conclusions and outlook

- Disentangling correlation sources:

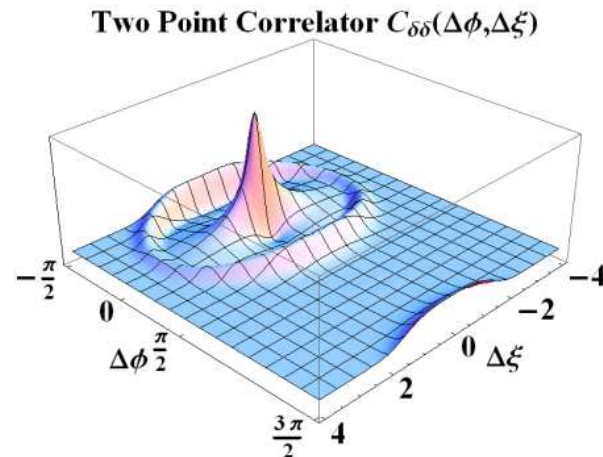
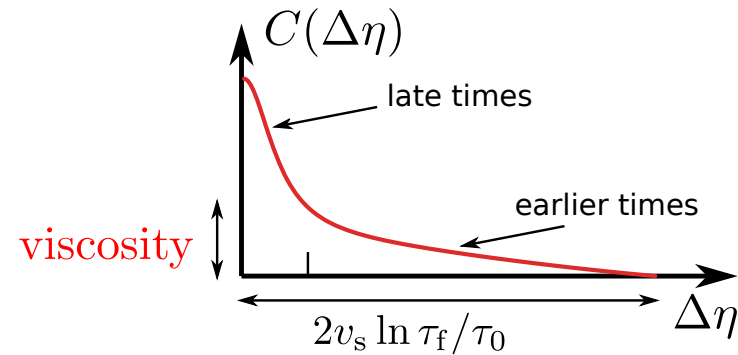
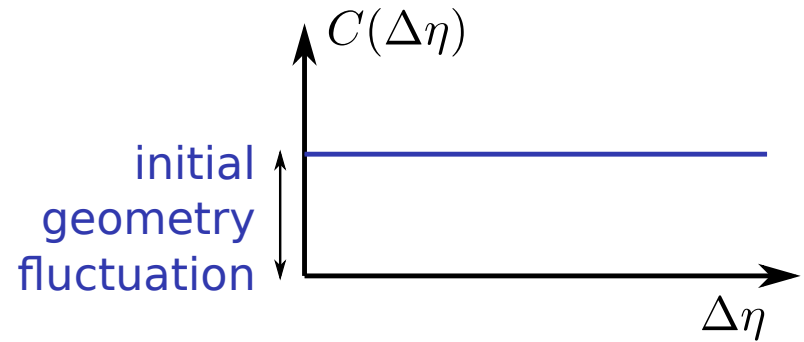
Fluctuations of initial conditions produce $\Delta\eta$ -independent correlation.

Hydrodynamic noise has local origin and produces characteristic $\Delta\eta$ dependence:

with magnitude proportional to viscosity.

- More can be learned from 2D $\Delta\phi, \Delta\eta$ correlations.

Analytical work is in progress (Todd Springer, Saturday).



Conclusions and outlook

- Disentangling correlation sources:

Fluctuations of initial conditions produce $\Delta\eta$ -independent correlation.

Hydrodynamic noise has local origin and produces characteristic $\Delta\eta$ dependence:

with magnitude proportional to viscosity.

- More can be learned from 2D $\Delta\phi, \Delta\eta$ correlations.

Analytical work is in progress (Todd Springer, Saturday).

- What can we learn about QGP from $\Delta\eta$ dependence of v_n ? Viscosity? τ_0 ? T_0 ?

