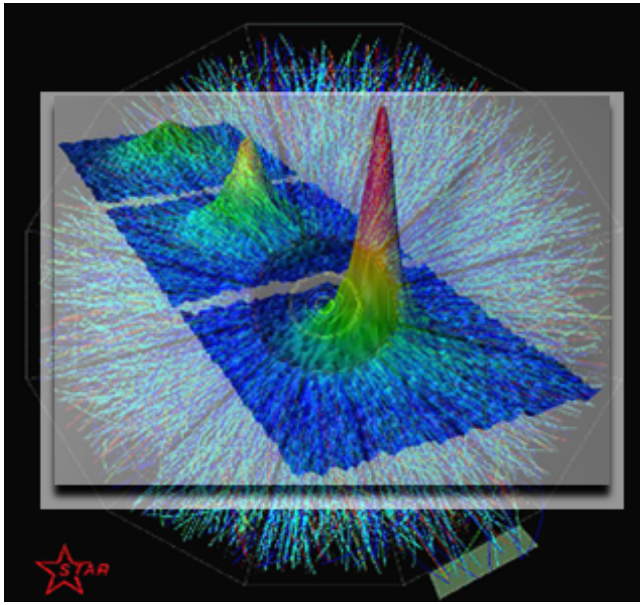
QM2012 @ Washington D.C.    AUG.17th, 2012

# Thermalization and Possible Bose-Einstein Condensation in Over-populated Glasma

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# Outline

- The Pre-Equilibrium Matter in Heavy Ion Collisions
- Important Feature: High Overpopulation
- A Kinetic Approach: Dynamical BEC & Separation of Scales
- Numeric Study of the BEC Onset
- Discussions

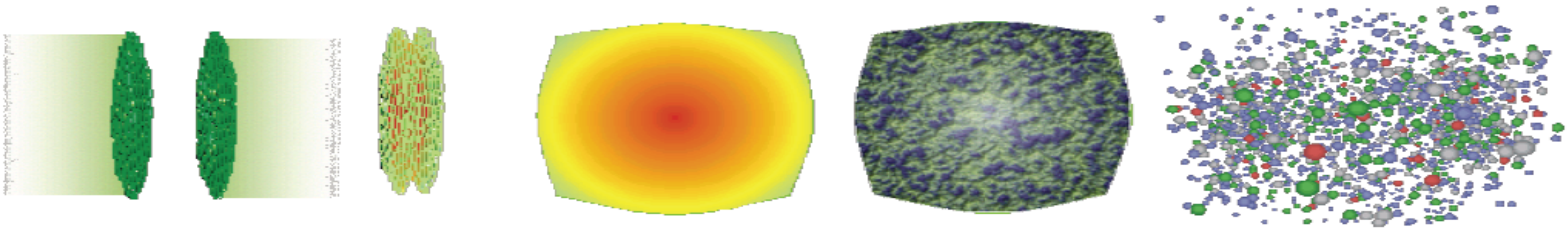
*Collaborators and References:*

*Blaizot, Gelis, JL, McLerran, Venugopalan, Nucl. Phys. A873, 68 (2012);*

*Blaizot, JL, McLerran, to appear;*

*Chiu, Hemmick, Khachatryan, Leonidov, JL, McLerran, arXiv:1202.3679 [nucl-th].*

# Just After the "Little Bang"



Color Glass Condensates — Initial Singularity — Glasma — sQGP - perfect fluid — Hadron Gas → t

Strong constraint from the initial condition: **saturation**

$$\frac{xG(x,Q^2)}{\pi R^2 Q_s^2} \sim \frac{1}{\alpha_s}$$

**Pre-Equilibrium Matter (We focus on this!)**

Strong constraint from Hydro modeling and empirical data: **fast "thermalization" ~ fm/c (to the extent of justifying hydro)**

**Scale in PEM high! Qs~GeV Asyp. Free. Is weak coupling description viable with "fast thermalization"? YES → More is different!**

# A High Density Gluon System

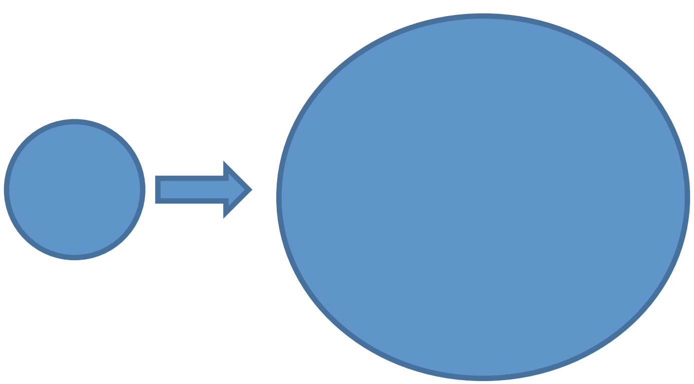
We consider a high density gluon system, starting at a time $\tau_0 \sim \frac{1}{Q_s}$

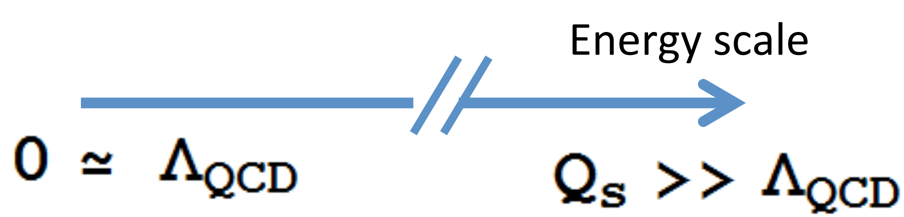
$$f \sim \frac{1}{\alpha_s} \;, \quad p < Q_s$$

$$f \sim 0 \;, \quad p > Q_s$$

***Our starting point***

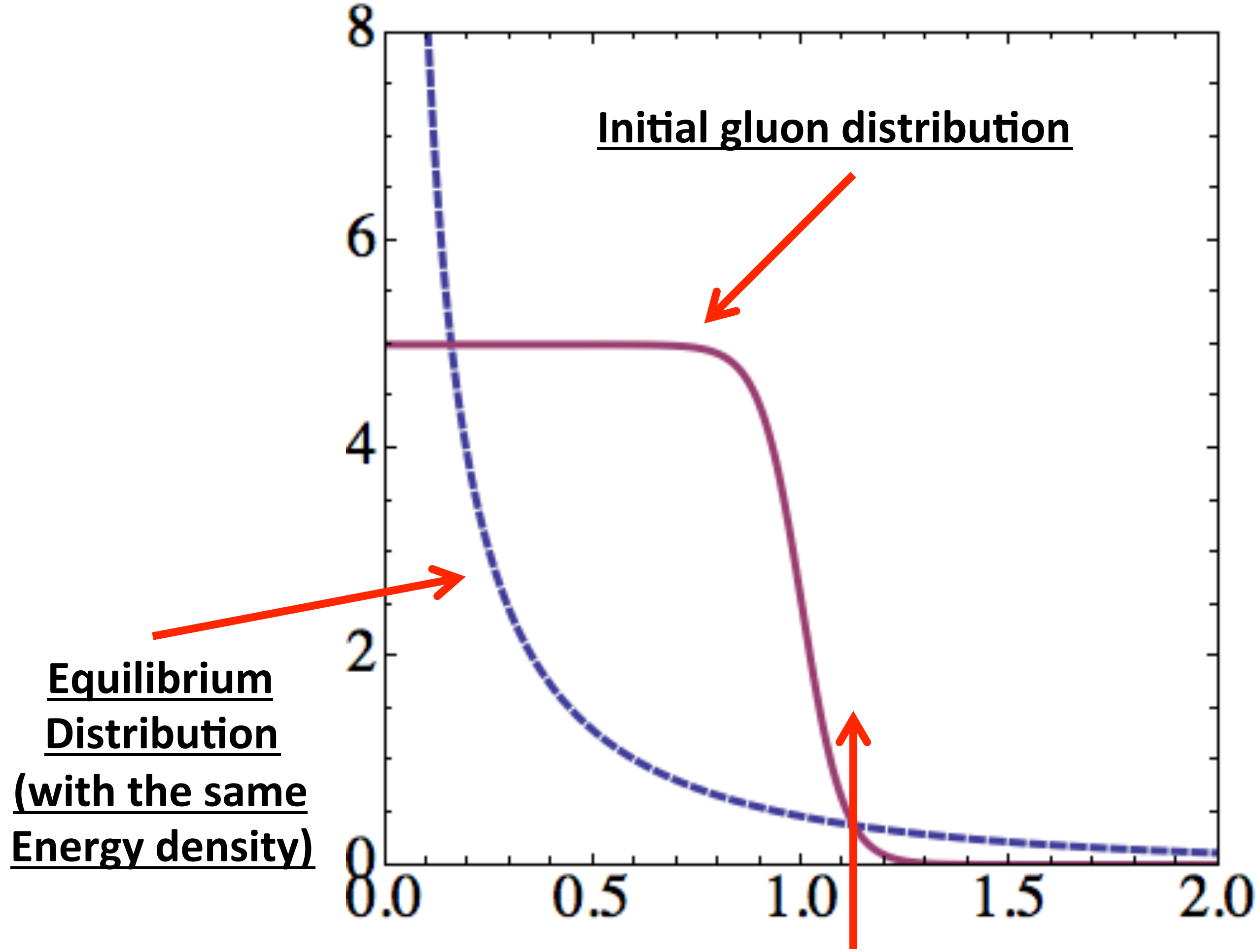
**Some idealization concerning real heavy ion collisions:**
**very large transversely; very high energy , i.e. large Qs and small coupling**





# Far from Equilibrium...

**The initial gluon system is far from equilibrium ! (Plotted here: f(p) )**




*Saturation Scale Qs ~ 1 GeV or larger, weakly coupled*

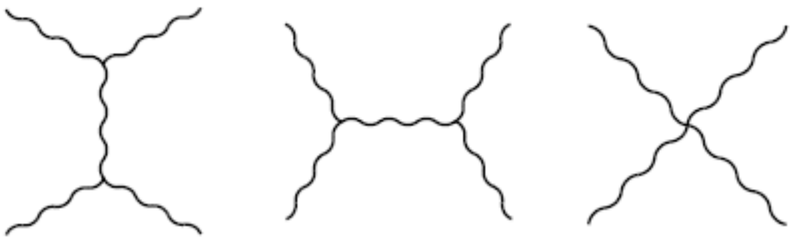
- **Very high phase space density f ~ 1/alpha_s**
- **Strong overpopulation**
- **Only one scale that characterizes the distribution**

**Thermalization: how the initial distribution evolves toward the equilibrium?**

# Amplified Scattering

**The initial gluon system is** ***highly occupied → change the power-counting***

***e.g. for the collision integral in kinetic evolution***



$$f * f * \alpha_s^2 \sim O(1)$$

$$f(p) \sim \frac{1}{\alpha_s}$$

**Coherent amplification of scattering →**
**Fast thermalization from overpopulation?!**

# OVERPOPULATION

**We consider a high density gluon system, starting at a time** $\tau_0 \sim \frac{1}{Q_s}$

$$f \sim \frac{1}{\alpha_s} \quad , \quad p < Q_s \qquad\qquad f \sim 0 \quad , \quad p > Q_s$$

$$\epsilon_0 = \epsilon(\tau = Q_s^{-1}) \sim \frac{Q_s^4}{\alpha_s} \qquad\qquad n_0 = n(\tau = Q_s^{-1}) \sim \frac{Q_s^3}{\alpha_s}$$

**Overpopulation parameter:** $n_0\, \epsilon_0^{-3/4}$

***For our initial gluon system:*** $n_0\, \epsilon_0^{-3/4} \sim 1/\alpha_s^{1/4}$ ***Much larger than unity at weak coupling!***

***In contrast, for equilibrated QGP:***

$$\epsilon_{eq} \sim T^4 \qquad\qquad n_{eq} \sim T^3 \qquad\qquad n_{eq}\, \epsilon_{eq}^{-3/4} \sim 1$$

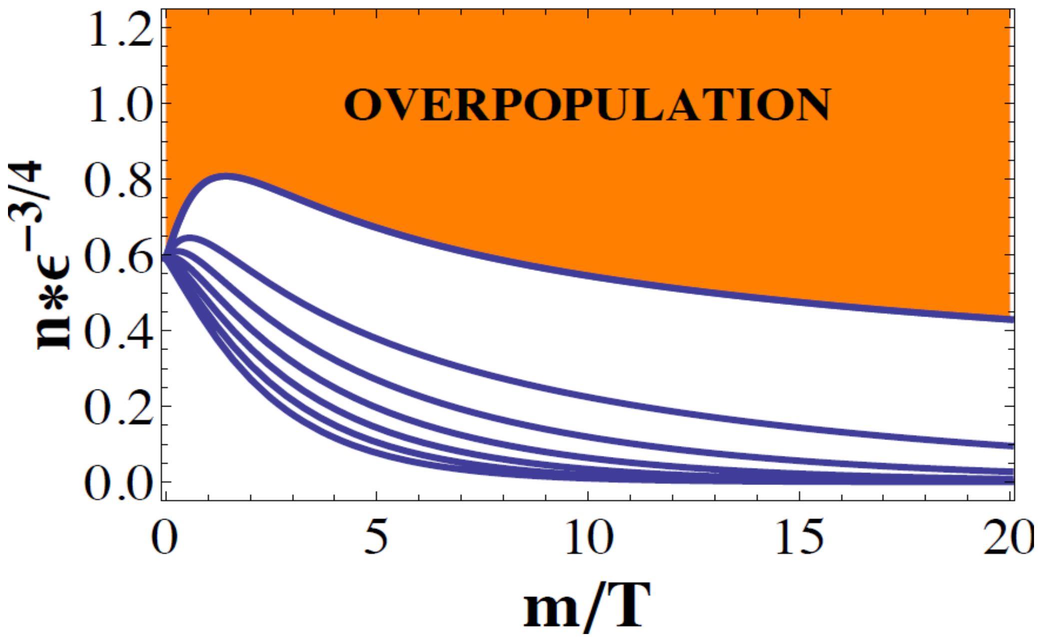
**For the given amount of energy, there are initially way too many gluons !**

# OVERPOPULATION

**Does building up a chemical potential help? NO!**
**The maximum gluons that can be accommodated by a Bose-Einstein distribution with \mu:**

$$f_{\rm eq}(\boldsymbol{k}) \equiv \frac{1}{e^{\beta(\omega_{\boldsymbol{k}}-\mu)}-1}$$

$$\mu \le \omega_{p=0} = m \neq 0$$




***For the given amount of initial energy, our gluon system has too many gluons***

***→Actually a "cold" dense system of gluons initially***

***→ can NOT equilibrate to a BE distribution by default (assuming elastic dominance)***

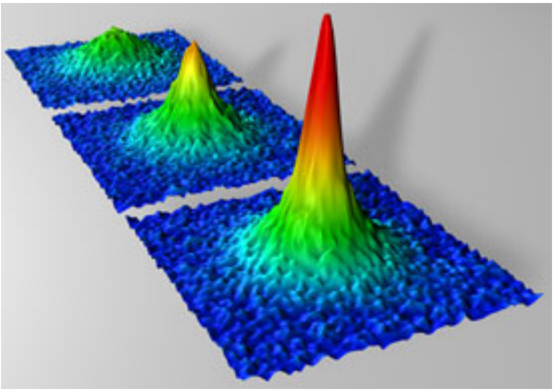
***→Rather:***

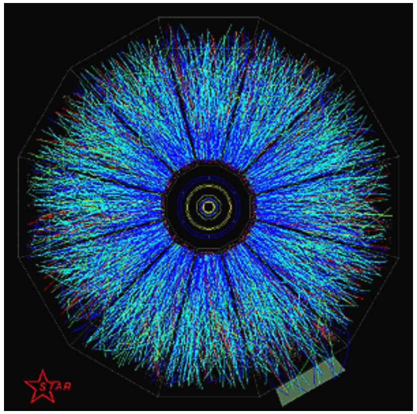
$$f_{\rm eq}(\boldsymbol{k}) = n_{\rm c}\delta(\boldsymbol{k}) + \frac{1}{e^{\beta(\omega_{\boldsymbol{k}}-m_0)}-1}$$

# Bose-Einstein Condensation from Overpopulation

**Initial overpopulation + Energy & Number conservation → BEC !!**

$$n_0 \, \epsilon_0^{-3/4}$$





**Atomic BEC: for given number of atoms, reduce energy (cooling) toward overpopulation**

**Dynamical formation of BEC at the early stage after heavy ion collisions:**
**born to have too many gluons for the available energy , really fascinating !**

$$T \sim Q_s / \alpha_s^{1/4} \qquad n_c = n - n_g \qquad n_c \sim \frac{Q_s^3}{\alpha_s}\left(1 - \alpha_s^{1/4}\right)$$

# Initially Uni-Scale

**There is ONLY ONE SCALE initially, i.e. the Qs**

$$f \sim \frac{1}{\alpha_s}\ ,\quad p < Q_s \qquad\qquad f \sim 0\ ,\quad p > Q_s$$

**This distribution is highly un-desired thermodynamically, i.e. by examining the entropy:**

$$s \sim \int_p [\,(1+f) * Ln\,(1+f) - f * Ln\,(f)\,]$$

$$[\,] \sim O\,(1 + Ln\,(f))\ ,\quad f >> 1$$

$$[\,] \sim O\,(1)\ ,\quad f \sim 1$$

$$[\,] \sim O\,(f * Ln\,(1/f))\ ,\quad f << 1$$

Thermalization → maximization of entropy →

More beneficial to distribute the gluons

to wider region in p-space with f ~ 1

# Separation of Scales

**We introduce two scales --- momentum cutoff scale & the saturated scale**

$$\Lambda : \; f << 1 \text{ for } p > \Lambda \qquad \Lambda_s : \; f \sim \frac{1}{\alpha_s}$$

**There is ONLY ONE SCALE initially,** $\Lambda \sim \Lambda_s \sim Q_s$

$$f \sim \frac{1}{\alpha_s} \;, \; p < Q_s \qquad f \sim 0 \;, \; p > Q_s$$

**Toward thermalization, the two scales must be separated!**

***By how much?***

**Again, for a very weakly coupled equilibrated QGP**

$$\Lambda \sim T \qquad \Lambda_s \sim \alpha_s * T$$

**Thermalization must be accompanied by specific separation of the two scales:**

$$\frac{\Lambda_s}{\Lambda} \sim \alpha_s$$

# Recap of the P.E.M.

**Bose–Einstein condensation and thermalization of the quark–gluon plasma**

Jean-Paul Blaizot [a], François Gelis [a], Jinfeng Liao [b,*], Larry McLerran [b,c], Raju Venugopalan [b]

*NPA873(2012)68-80 [1107.5296]*

**The Pre-Equilibrium Matter as**

**a highly overpopulated gluon system evolving toward equilibrium:**

**High occupancy**

*→coherent amplification of scattering, ~O(1) effect despite coupling*

**Strong overpopulation** *→ dynamical BEC formation*

**Initial uni-scale** *→ separation of the two scales by coupling \alpha_s*

**Allowing a kinetic approach based on weak coupling**

*Related discussions: Kurkela & Moore, 1107.5050, 1108.4684, 1207.1663; Schlichting, 1207.1450*

# Developing a Kinetic Approach

**Two-stage problem:**

- **The evolution from overpopulation toward onset of condensation --- we will come to this later with numerical studies.**
- **The co-evolution of condensate + regular distribution ---- we will show here *approximate scaling solution of the regular distribution toward thermalization***

$$\mathcal{D}_t f(\vec{p}) = \xi \left(\Lambda_s^2 \Lambda\right) \vec{\nabla} \cdot \left[\vec{\nabla} f(\vec{p}) + \frac{\vec{p}}{p}\left(\frac{\alpha_S}{\Lambda_s}\right) f(\vec{p})[1+f(\vec{p})]\right]$$

$$\Lambda\left(\frac{\Lambda_s}{\alpha_S}\right)^2 \equiv (2\pi^2)\int \frac{d^3p}{(2\pi)^3} f(\vec{p})\,[1+f(\vec{p})]$$

$$\Lambda\frac{\Lambda_s}{\alpha_S} \equiv (2\pi^2)\,2\int \frac{d^3p}{(2\pi)^3}\frac{f(\vec{p})}{p}$$

*Blaizot, JL, McLerran, to appear.*

# The "Static Box" Problem

**A schematic scaling distribution characterized by the two evolving scales:**

$$f(p) \sim \frac{1}{\alpha_s} \text{ for } p < \Lambda_s, \qquad f(p) \sim \frac{1}{\alpha_s}\frac{\Lambda_s}{\omega_p} \text{ for } \Lambda_s < p < \Lambda, \qquad f(p) \sim 0 \text{ for } \Lambda < p$$

$$n_g \sim \frac{1}{\alpha_s}\Lambda^2\Lambda_s \qquad \epsilon_g \sim \frac{1}{\alpha_s}\Lambda_s\Lambda^3 \qquad n = n_c + n_g$$

**Time evolution of the distribution, i.e. of the two scales → need two conditions**

$$\text{At } \tau_0 \sim 1 / Q_s \; : \quad \Lambda \sim \Lambda_s \sim Q_s$$

Essential points here:

- Energy is conserved, but not gluon number → absorption into condensate
- Collision time scale (from transport equation) ~ \tau for scaling solutions

$$t_{scat} = \frac{\Lambda}{\Lambda_s^2}$$

# Thermalization in The "Static Box"

Two conditions fixing the time evolution:

$$\Lambda_s \Lambda^3 \sim \text{constant} \qquad t_{\text{scat}} \sim \frac{\Lambda}{\Lambda_s^2} \sim t$$

The scaling solution:

$$\Lambda_s \sim Q_s \left(\frac{t_0}{t}\right)^{\frac{3}{7}} \qquad \Lambda \sim Q_s \left(\frac{t}{t_0}\right)^{\frac{1}{7}}$$

Upon thermalization: separation of scales; entropy production; eliminating over-pop.

$$\Lambda_s \sim \alpha_s \Lambda \quad \Rightarrow \quad \boxed{t_{\text{th}} \sim \frac{1}{Q_s}\left(\frac{1}{\alpha_s}\right)^{7/4}}$$

$$T \sim \Lambda \sim Q_s / \alpha_s^{1/4} \qquad s \sim \Lambda^3 \sim Q_s^3 / \alpha_s^{3/4} >> s_0 \sim Q_s^3$$

$$n * \epsilon^{-3/4} \sim \left(\frac{\Lambda_s}{\alpha_s * \Lambda}\right)^{1/4} \rightarrow O(1)$$

# Condensate in The "Static Box"

Condensate dominates the number density:

$$n_c \sim n_0[1 - (t_0/t)^{1/7}]$$

$$n_g \sim n_0 \left(\frac{t_o}{t}\right)^{1/7}$$

While gluons dominate the energy density

$$\epsilon_c \sim n_c m \sim n_c \sqrt{\Lambda \Lambda_s}$$

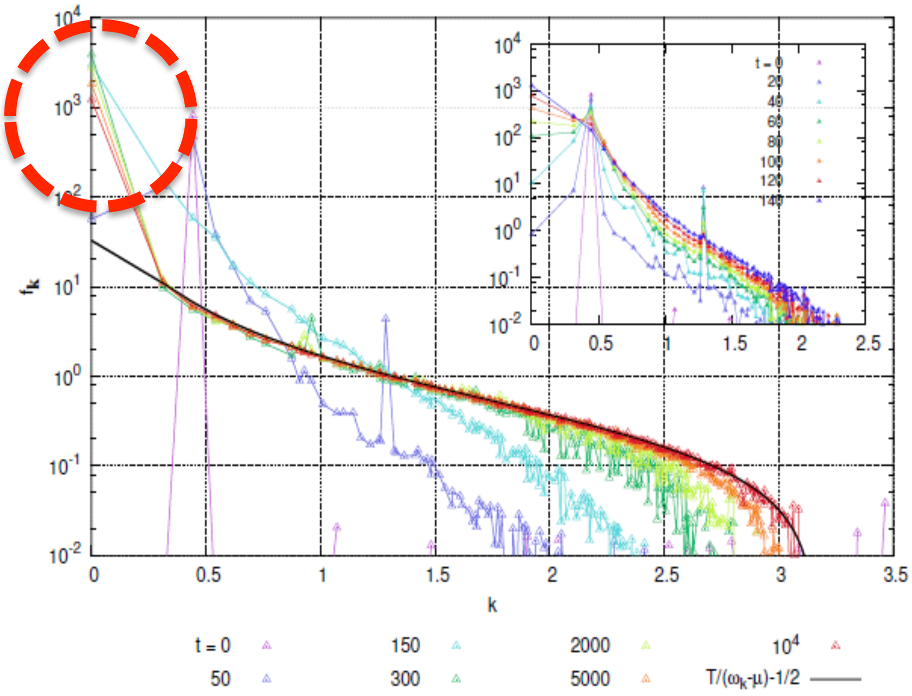
$$m^2 \sim \alpha_s \int dp\, p^2 \frac{df(p)}{d\omega_p} \sim \Lambda \Lambda_s$$

$$\frac{\epsilon_c}{\epsilon_g} \sim \left(\frac{t_0}{t}\right)^{1/7}$$

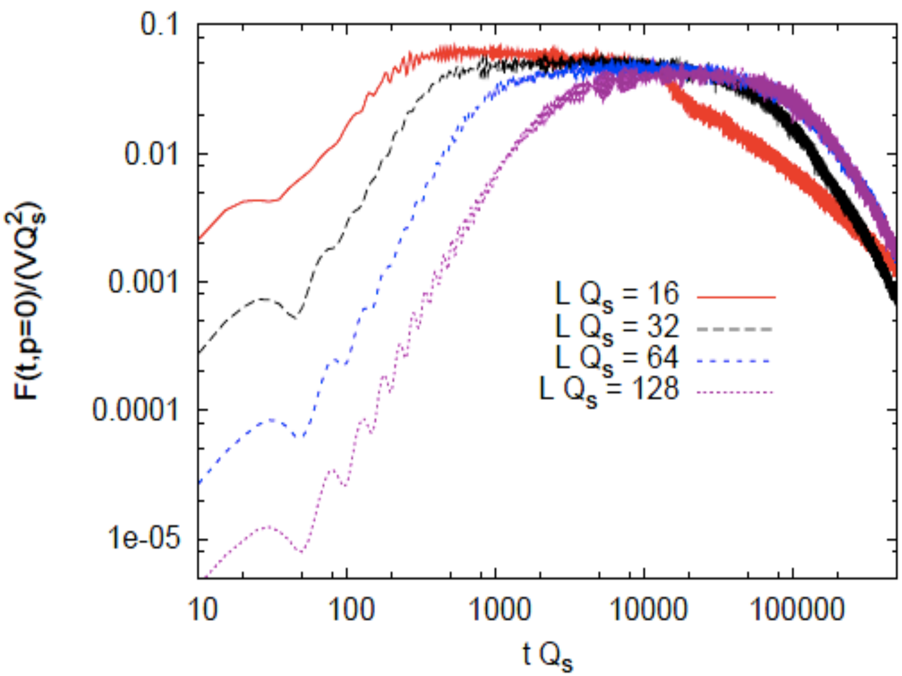
**A robust, though transient,, Bose-Einstein condensate can be dynamically formed and maintained during the thermalization!**

# Evidence of BEC from Scalar Field Theory Computation

*From: Epelbaum & Gelis 1107.0668*




*From: Berges & Sexty 1201.0687*



# Effect of Longitudinal Expansion

**Matter created in heavy ion collision undergoes rapid longitudinal expansion**
**→ Dilution as well as driving anisotropy**

**The scattering competes with the expansion to restore the isotropy.**

**The analysis of scaling solution is similar, except that evolution of energy density is related to the anisotropy now**

$$P_L = \delta \epsilon \quad \Rightarrow \quad \epsilon_g(t) \sim \epsilon(t_0) \left( \frac{t_0}{t} \right)^{1+\delta}$$

$\delta = 0$ : free streaming $\qquad \delta = 1/3$ : isotropic

$$\Lambda_s \sim Q_s \left( \frac{t_0}{t} \right)^{(4+\delta)/7}, \qquad \Lambda \sim Q_s \left( \frac{t_0}{t} \right)^{(1+2\delta)/7}.$$

$$\left( \frac{t_{th}}{t_0} \right) \sim \left( \frac{1}{\alpha_s} \right)^{7/(3-\delta)}$$

$$n_c \sim \frac{Q_s^3}{\alpha_s} \left( \frac{t_0}{t} \right) \left[ 1 - \left( \frac{t_0}{t} \right)^{(-1+5\delta)/7} \right]$$

# Toward Onset of BEC

$$\mathcal{D}_t f(\vec{p}) = \xi \left(\Lambda_s^2 \Lambda\right) \vec{\nabla} \cdot \left[ \vec{\nabla} f(\vec{p}) + \frac{\vec{p}}{p} \left(\frac{\alpha_S}{\Lambda_s}\right) f(\vec{p})[1 + f(\vec{p})] \right]$$

**We numerically solve the kinetic equation to study:**

**How the the system evolves from overpopulation toward onset of BEC?**

**How robust this phenomenon is against:**
***different initial overpopulation?***
***different initial distribution shape?***
***different initial anisotropy?***
***and the longitudinal expansion?***

*Blaizot, JL, McLerran, to appear.*

# Small Momentum Behavior

$$\mathcal{D}_t f(\vec{p}) = \xi\left(\Lambda_s^2 \Lambda\right) \vec{\nabla} \cdot \left[\vec{\nabla} f(\vec{p}) + \frac{\vec{p}}{p}\left(\frac{\alpha_S}{\Lambda_s}\right) f(\vec{p})[1 + f(\vec{p})]\right]$$

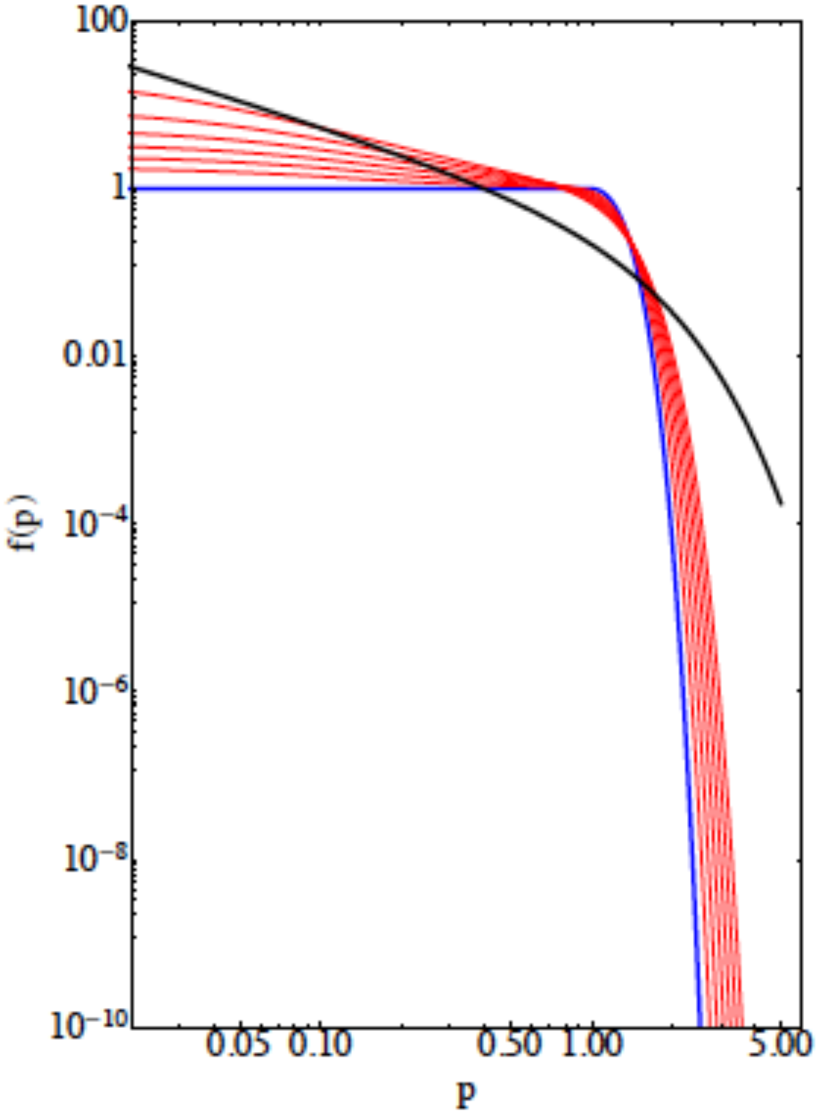
**What happens at small p:**

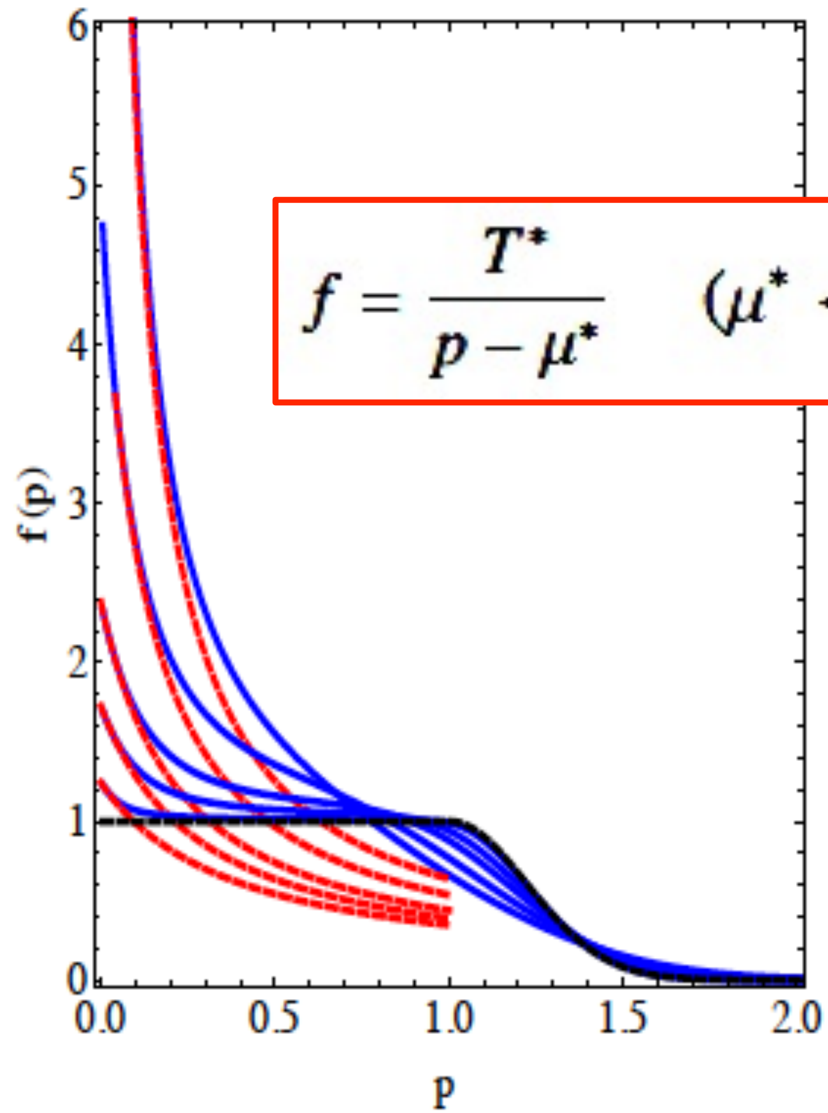
1. **strong particle flux in momentum space toward IR**
2. **almost "instantaneous" local equilibration toward the classical thermal shape**
3. **the local effective "chemical potential" approaches zero → ONSET!**

$$f = \frac{T^*}{p - \mu^*} \qquad (\mu^* < 0)$$

*Blaizot, JL, McLerran, to appear.*

# Numerical Solutions



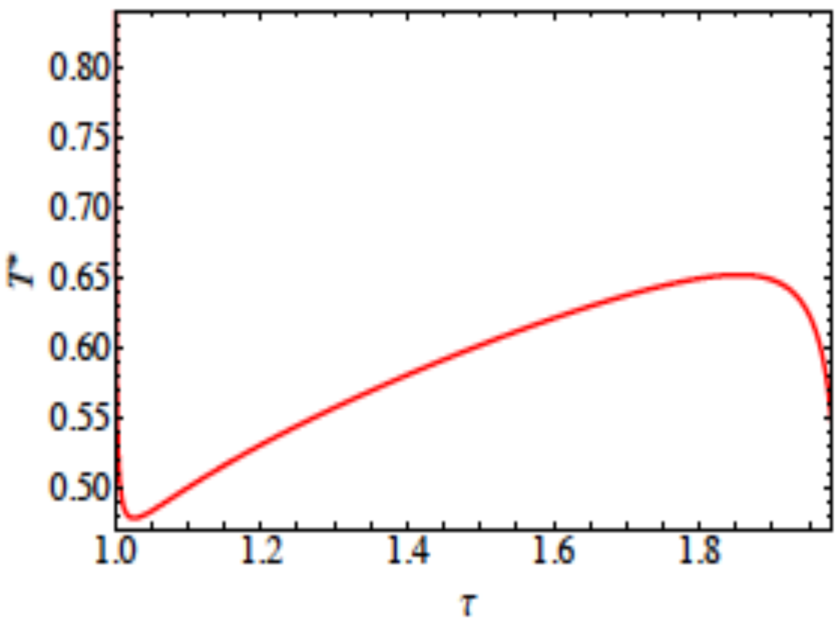

$$f = \frac{T^*}{p - \mu^*} \quad (\mu^* < 0)$$

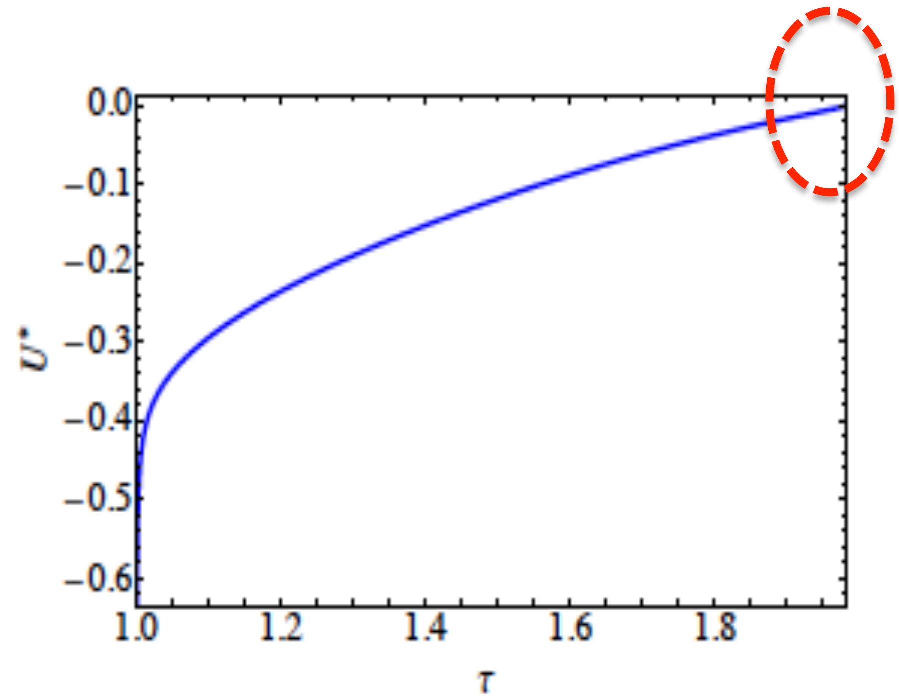
$$f(p) = f_0\,\theta(Q_s - p)$$

$$f_0^c \approx 0.154$$

***Onset seen for all f0 greater than critical***

# BEC Onset via Vanishing \Mu

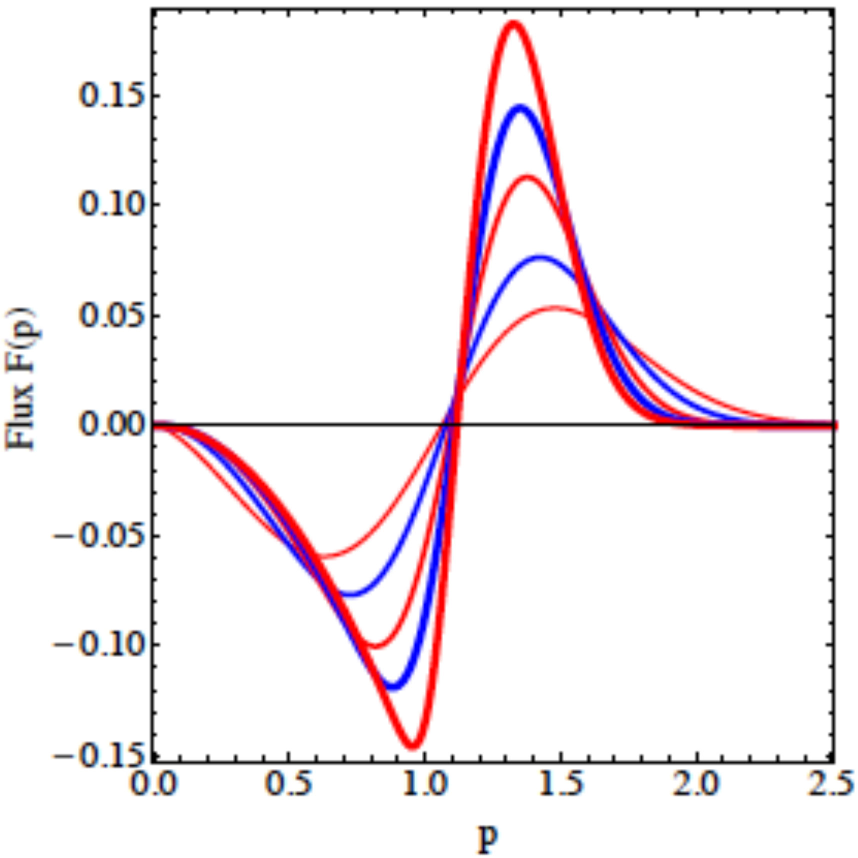


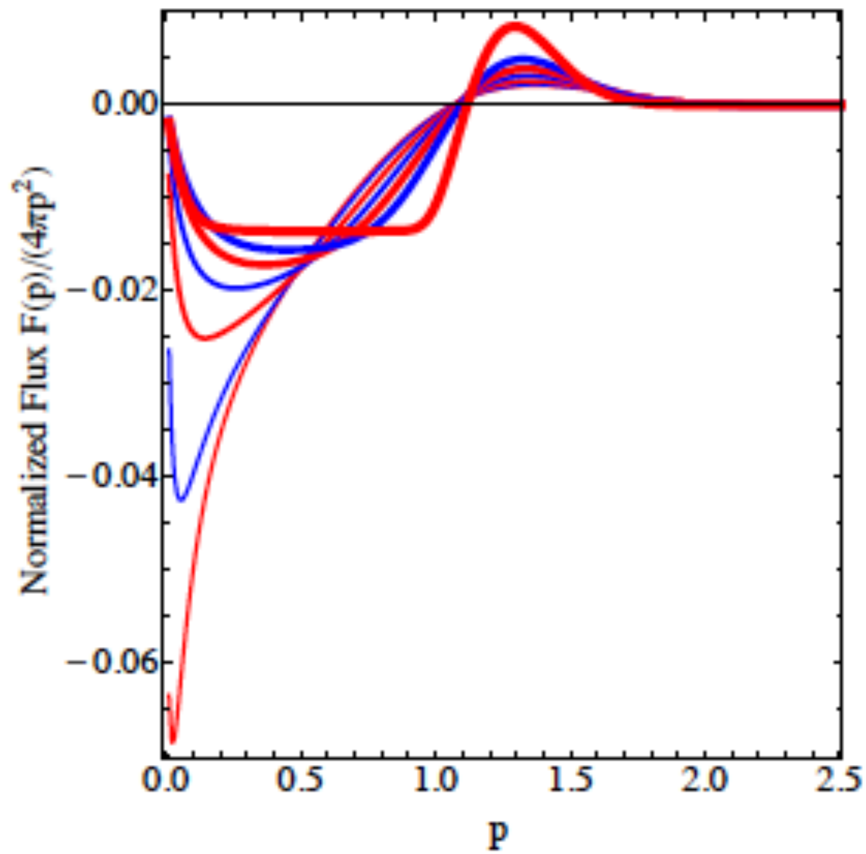




$$f = \frac{T^*}{p - \mu^*} \quad (\mu^* < 0)$$

**“local thermal” shape with local T and \mu**

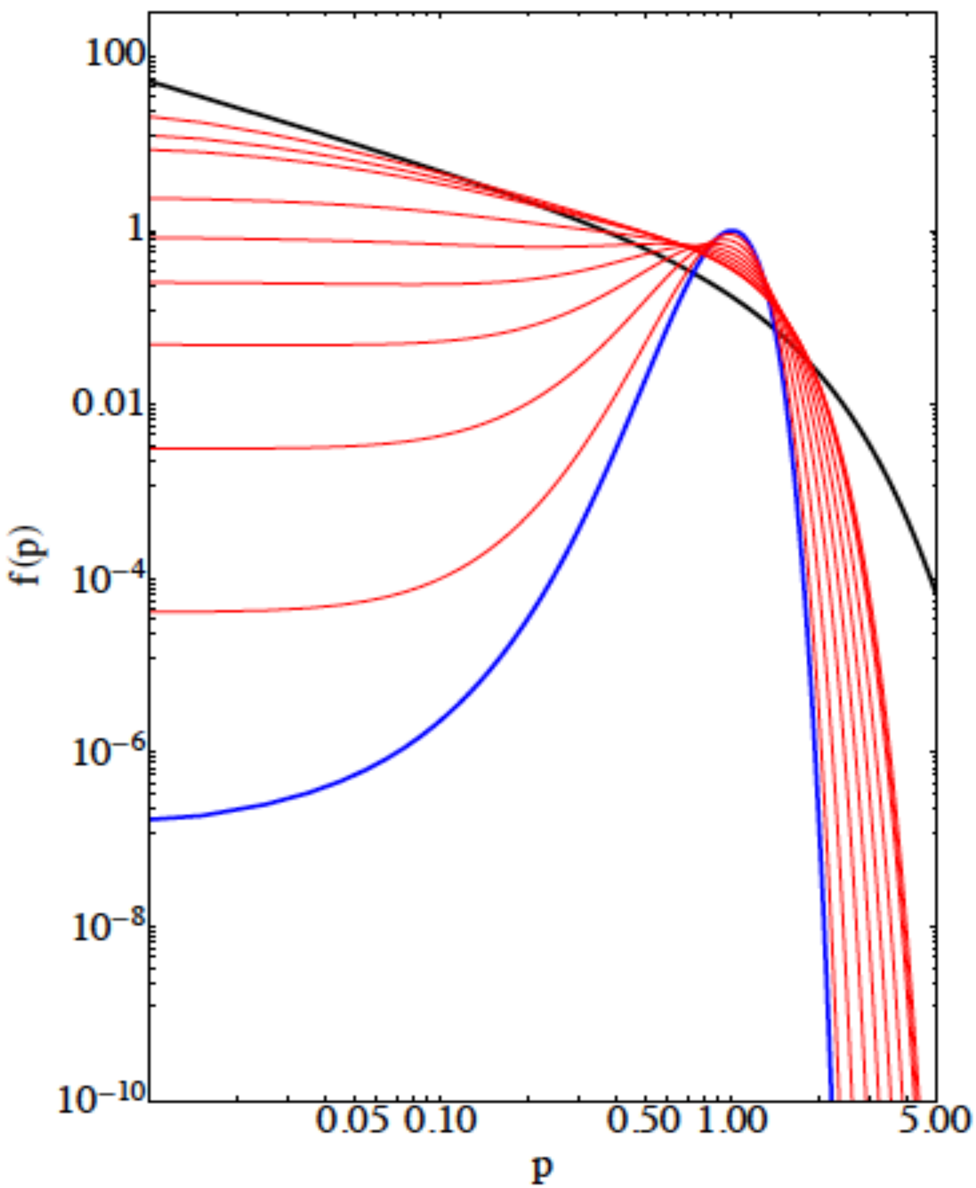
# IR and UV Cascades



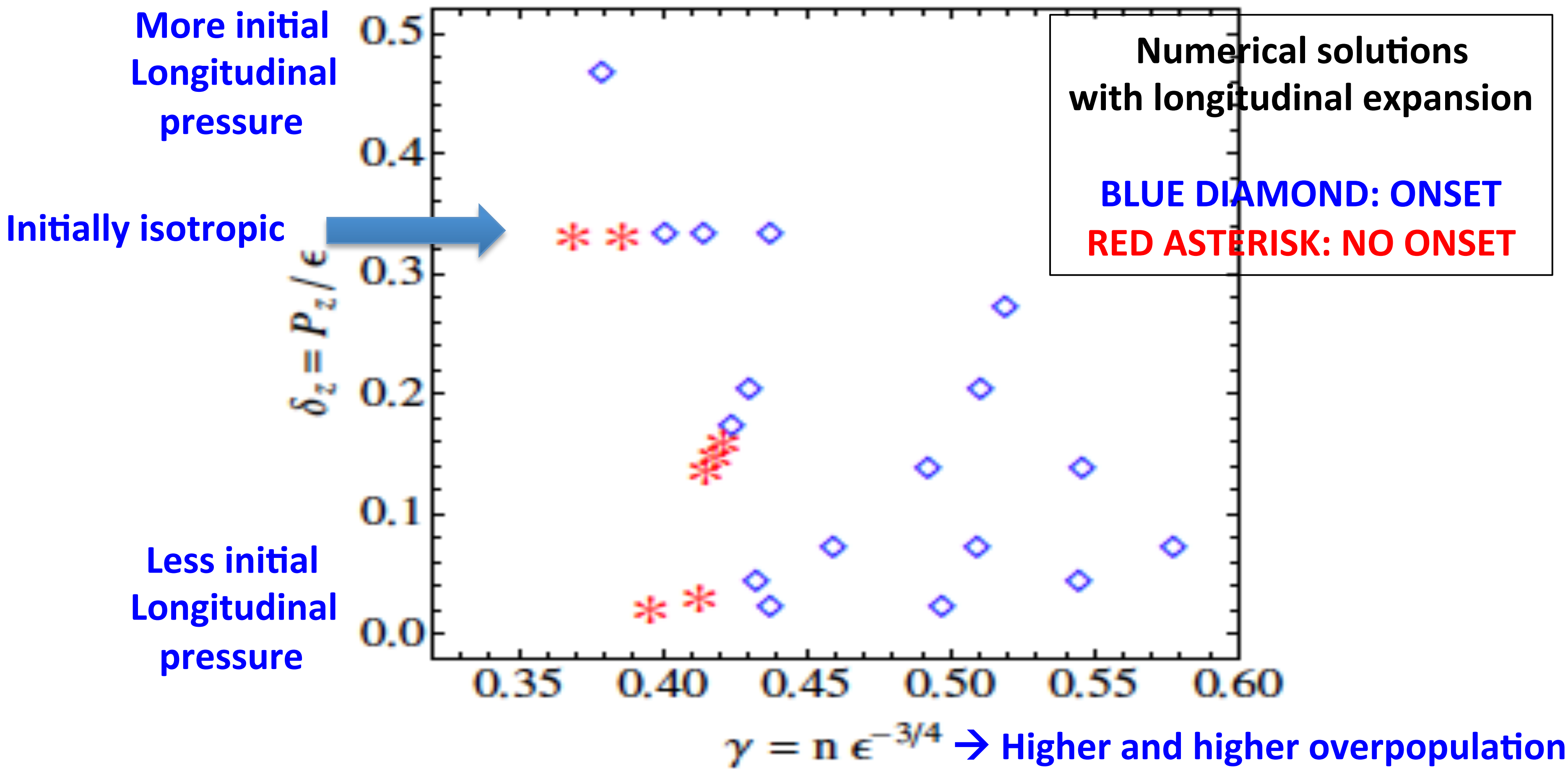

**Flux in momentum space:**
**Particle cascade toward IR**
**Energy cascade toward UV**
**(similar behaviors observed on lattice scalar field simulations)**

# Robust against Initial Shape




**Starting with Gaussian type of initial condition, we see the same onset of BEC driven by overpopulation**

# Robust against Anisotropy & Expansion



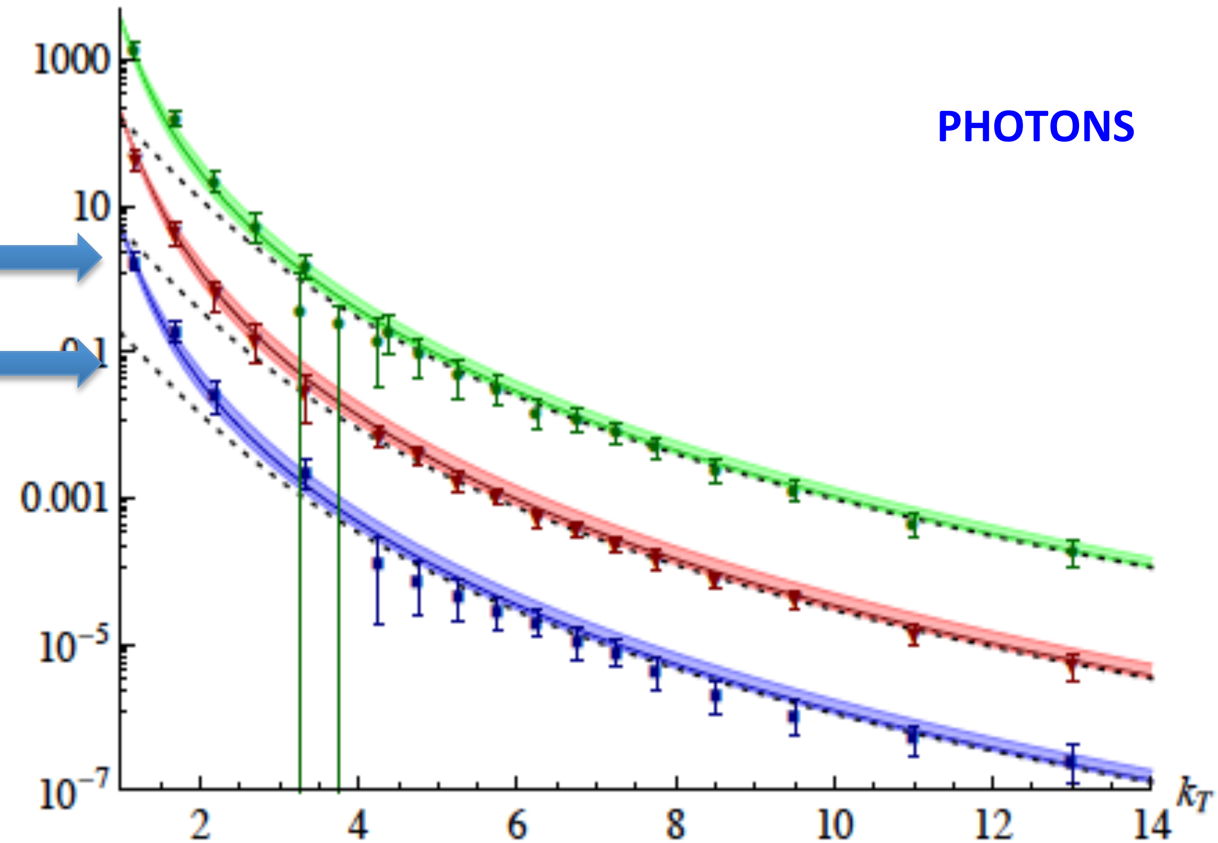

**High enough initial occupation f_0~1 is able to compete with anisotropy and expansion to reach BEC onset!**

# Phenomenology: EM Production in the Pre-Equilibrium Matter

*There should be additional EM emission from the pre-equilibrium stage*
*We derived fitting formula motivated by our thermalization scenario.*

Conventional sources + **our PEM contribution**

Conventional sources total ("cocktail")




**Important contributions to EM production from the pre-equilibrium matter !**

*Chiu, Hemmick, Khachatryan, Leonidov, JL, McLerran, arXiv:1202.3679 [nucl-th].*

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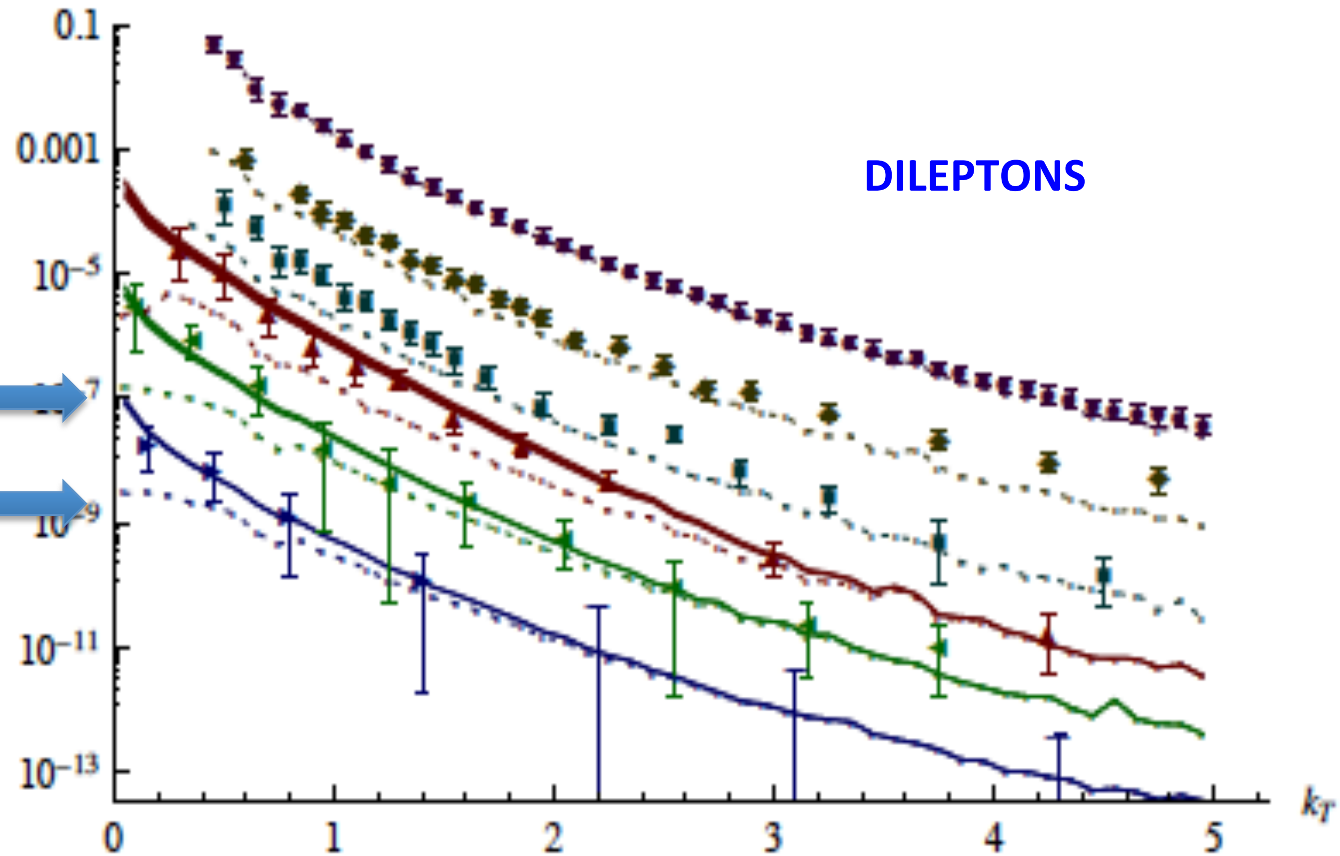


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*Chiu, Hemmick, Khachatryan, Leonidov, JL, McLerran, arXiv:1202.3679 [nucl-th].*

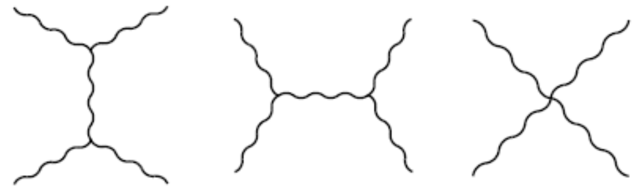
# Summary: a Scenario for the First-Fermi/c after Little Bang

- The pre-equilibrium matter starts with **high gluon density and one intrinsic scale.**
- **Elastic scattering is coherently enhanced** despite the small coupling, therefore leading to strongly interacting matter even at early time and driving the thermalization.
- Strong **overpopulation** enforces the system **toward BEC**.
- The onset of BEC is robust against initial shape, anisotropy, as well as the longitudinal expansion.
- Important phenomenological consequences, e.g. EM emission.
- Many more to be studied and reported in the future.

# Backup Slides

# Kinetic Evolution

With 2→2 gluon scattering
& small-angle approximation
(Blaizot, JL, McLerran, to appear)



$$\mathcal{D}_t f(p) = \xi \frac{\Lambda_s^2 \Lambda}{p^2} \partial_p \left\{ p^2 \left[ f'(p) + \left( \frac{\alpha_S}{\Lambda_s} \right) f(p)[1 + f(p)] \right] \right\}$$

*(BE as fixed point)*

$$\Lambda \left( \frac{\Lambda_s}{\alpha_S} \right)^2 \equiv \int_0^\infty dp\, p^2 f(p) [1 + f(p))]$$

$$\Lambda \frac{\Lambda_s}{\alpha_S} \equiv - \int_0^\infty dp\, p^2 \frac{d}{dp} [f(p))] = 2 \int_0^\infty dp\, p^2 \frac{f(p)}{p}$$

**For highly occupied initial condition:**
***coupling constant disappears*** **in the scales!**

$$f \sim \frac{1}{\alpha_s} \qquad \Lambda_s^2 * \Lambda \sim O(1)$$

**In contrast, for thermalized BE distribution:**

$$\Lambda \sim T \qquad \Lambda_s \sim \alpha_s * T \qquad \Lambda_s^2 * \Lambda \sim O(\alpha_s^2)$$

# Effect of Longitudinal Expansion

Two conditions fixing the time evolution:

$$\epsilon_g(t) \sim \epsilon(t_0)\left(\frac{t_0}{t}\right)^{1+\delta} \qquad t_{scat} \sim \frac{\Lambda}{\Lambda_s^2} \sim t$$

The scaling solution:

$$\Lambda_s \sim Q_s\left(\frac{t_0}{t}\right)^{(4+\delta)/7}, \qquad \Lambda \sim Q_s\left(\frac{t_0}{t}\right)^{(1+2\delta)/7}.$$

Upon thermalization: separation of scales; entropy production; eliminating over-pop.

$$\Lambda_s \sim \alpha_s \Lambda \quad \Rightarrow \quad \boxed{\left(\frac{t_{th}}{t_0}\right) \sim \left(\frac{1}{\alpha_s}\right)^{7/(3-\delta)}}$$

$$T \sim Q_s * \alpha_s^{\frac{1+2\delta}{3-\delta}} \qquad s \sim T^3 \sim Q_s^3 * \alpha_s^{\frac{3*(1+2\delta)}{3-\delta}}$$

Strong scattering competes
With expansion → fixed asymmetry

$$\boxed{\zeta = \frac{16+21\kappa}{36}\left[1-\sqrt{1+\frac{1008}{(16+21\kappa)^2}}\right]}$$

# Condensate in Expanding Case

NOTE: anisotropic expansion reduces overpopulation

Condensate still can exist and dominate density: for $\delta > 1/5$

$$n_c \sim \frac{Q_s^3}{\alpha_s}\left(\frac{t_0}{t}\right)\left[1-\left(\frac{t_0}{t}\right)^{(-1+5\delta)/7}\right]$$

Energy carried by condensate is subleading:

$$\frac{\epsilon_c}{\epsilon_g} \sim \left(\frac{t_0}{t}\right)^{(5-11\delta)/14}$$

***The more isotropic the expansion is, the stronger the condensation will be.***

***Would be great to test in scalar/gauge field simulations aand kinetic computation.***

# Small Momentum Behavior

At small momentum $(f \gg 1)$

$$\mathcal{D}_\tau f(p) = I_a \frac{\partial^2 f}{\partial p^2} + \frac{2 I_b}{p} f^2 + \left[\frac{2 I_a}{p} + 2 I_b f\right] \frac{\partial f}{\partial p}$$

'instantaneous' equilibrium distribution function

$$f = \frac{T^*}{p - \mu^*} \quad (\mu^* < 0)$$

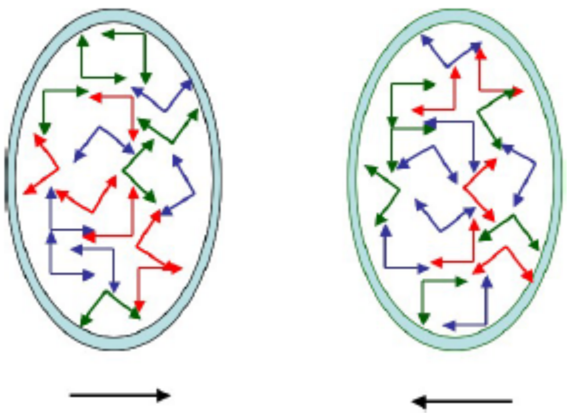
$$\mathcal{D}_\tau f \approx \frac{2T^*}{(p-\mu^*)^3}(I_a - T^* I_b) - \frac{2T^*}{p(p-\mu^*)^2}(I_a - T^* I_b)$$

$$I_a = \int \frac{d^3p}{(2\pi)^3} f(\boldsymbol{p})(1 + f(\boldsymbol{p})) \qquad I_b = \int \frac{d^3p}{(2\pi)^3} \frac{2 f(\boldsymbol{p})}{p}$$

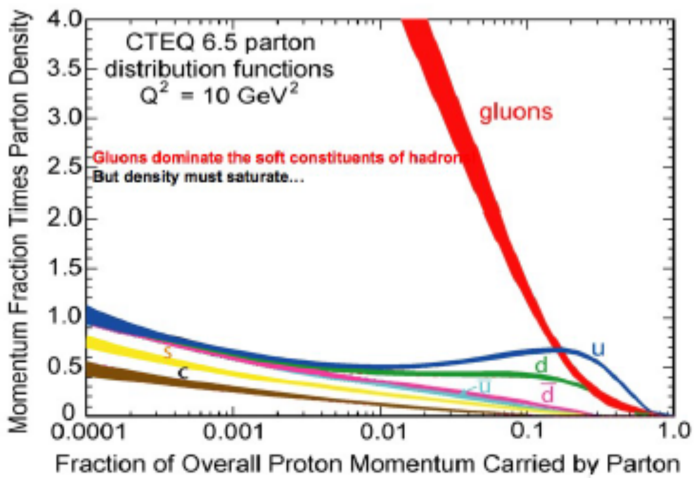
Blaizot, JL, McLerran, to appear.

# The Pre-Pre-Equilibrium

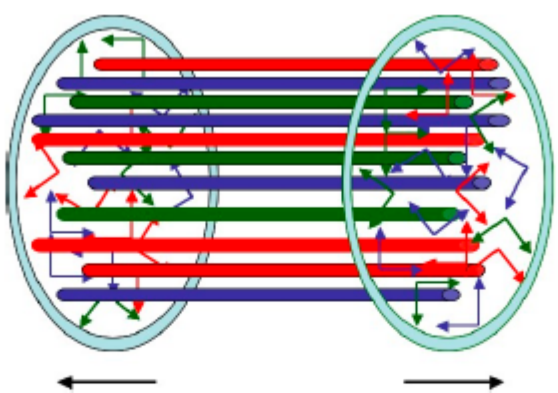
**Before the collision: saturation**



$$\frac{xG(x, Q^2)}{\pi R^2 Q_s^2} \sim \frac{1}{\alpha_s}$$




**Right after the collision**



Strong longitudinal expansion → anisotropy;

Instabilities play an important role:

Isotropization of energy-momentum tensor;

rapid growth toward large occupation of soft modes;

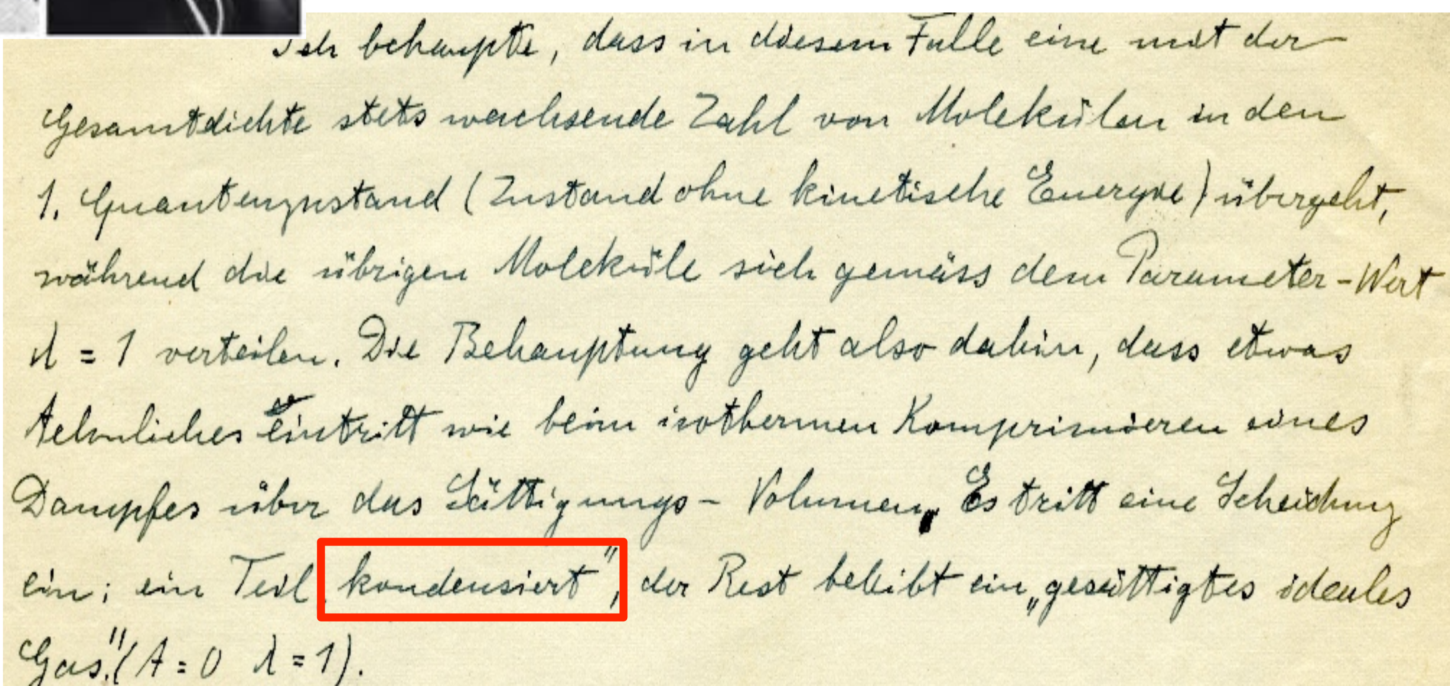
on the time scale ~ 1/Qs .

# OVERPOPULATION

***Overpopulation → Bose-Einstein condensation for equilibrium state***

***All the extra gluons get absorbed into zero momentum mode.***

$$f_{\mathrm{eq}}(\boldsymbol{k}) = n_{\mathrm{c}}\delta(\boldsymbol{k}) + \frac{1}{e^{\beta(\omega_{\boldsymbol{k}} - m_0)} - 1}$$



Ich behaupte, dass in diesem Falle eine mit der Gesamtdichte stets wachsende Zahl von Molekülen in den 1. Quantenzustand (Zustand ohne kinetische Energie) übergeht, während die übrigen Moleküle sich gemäss dem Parameter-Wert $\lambda = 1$ verteilen. Die Behauptung geht also dahin, dass etwas Aehnliches eintritt wie beim isothermen Komprimieren eines Dampfes über das Sättigungs-Volumen. Es tritt eine Scheidung ein; ein Teil „kondensiert", der Rest bleibt ein „gesättigtes ideales Gas." ($A = 0$ $\lambda = 1$).

# From Phenomenology to Understanding?

**Phenomenology:**
**Start far-from equilibrium, expand as a fluid shortly after ~ 1 fm/c**

**Understanding is lacking:**
**Fast local equilibration → strong scattering right from the start**
**But how? --- single most challenging issue in heavy ion physics**

**Phenomenology:**
**strong soft-soft and soft-hard re-scattering in the matter**

**Understanding:**
***Strongly coupled* constituents or *strongly interacting* constituents?**
**Or both, at different time stages?**