Baryon number conservation and limited acceptance vs. cumulants of net proton distribution

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AB, V. Koch, V. Skokov, arXiv:1203.4529

AB, V. Koch, arXiv:1206.4286

Outline

- Short introduction
- Baryon number conservation
  - calculation
  - new observable
- Limited acceptance
  - required vs. actual acceptance
  - results, problems and hopes
- Conclusions
- Backup with equations

Introduction

To make a long story short we hope to see a minimum <u>and</u> a maximum of net baryon/proton or charge cumulant ratios as a function of energy

$$c_1 = \langle N_B - N_{\bar{B}} \rangle$$

$$c_2 = \langle (N_B - N_{\bar{B}})^2 \rangle - \langle N_B - N_{\bar{B}} \rangle^2$$

$c_3, c_4, c_5, c_6, \ldots$ or $B \rightarrow Q$

see talk by V. Skokov

Baryon number conservation

AB, V. Koch, V. Skokov, arXiv:1203.4529

Calculation

$$P(n_B, n_{\bar{B}}) = P(n_B)P(n_{\bar{B}})$$

↓

$$P_\Delta(n_B - n_{\bar{B}})$$ Skellam distribution

$$P_B(n_B, n_{\bar{B}}) \sim \sum P(N_B)P(N_{\bar{B}})\delta_{N_B - N_{\bar{B}} - B} \times$$

$$\times B(N_B, n_B; p_B)B(N_{\bar{B}}, n_{\bar{B}}, p_{\bar{B}})$$

$P(x)$ – Poisson dist., $B(\dots)$ – Binomial dist.

N_B – total # of baryons, n_B – measured # of baryons

Binomial parameter

$$p = \frac{\#\ of\ measured\ protons/baryons}{total\ \#\ of\ baryons}$$

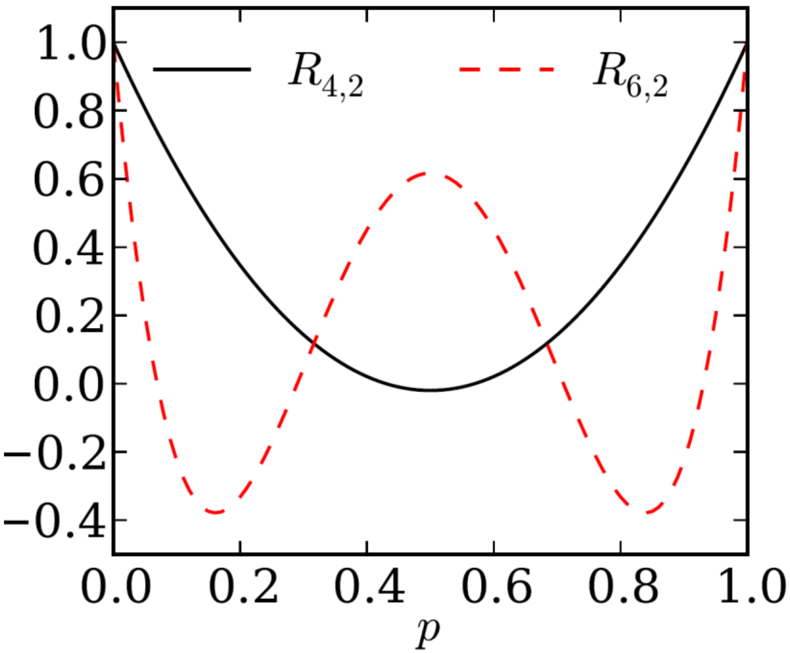
$p_{max} \approx \frac{1}{2}$ if only protons are measured

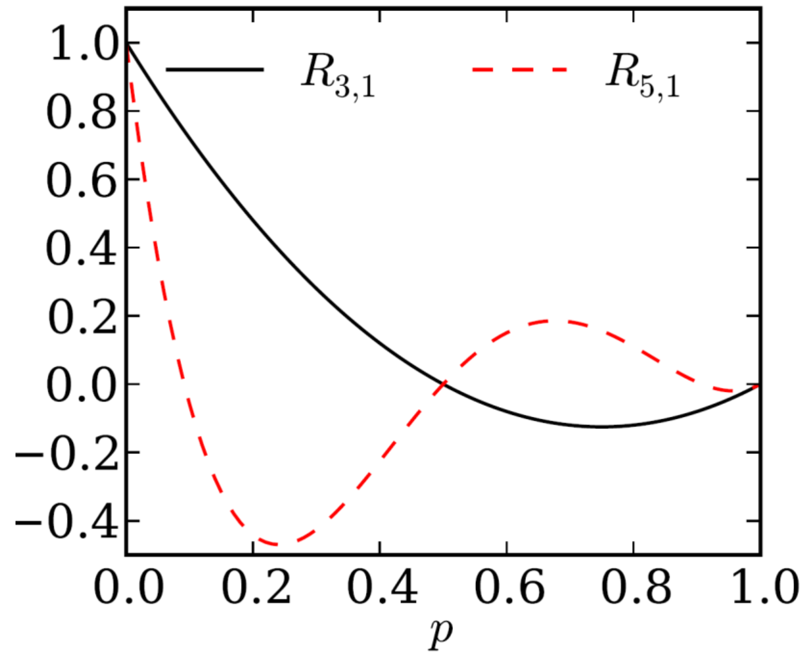
$p_{max} = 1$ if baryons are measured

… of course $p_{min} = 0$

Everything can be calculated analytically (see backup)

Results for $\langle N_B \rangle = 400,\ \langle N_{\bar{B}} \rangle = 100$

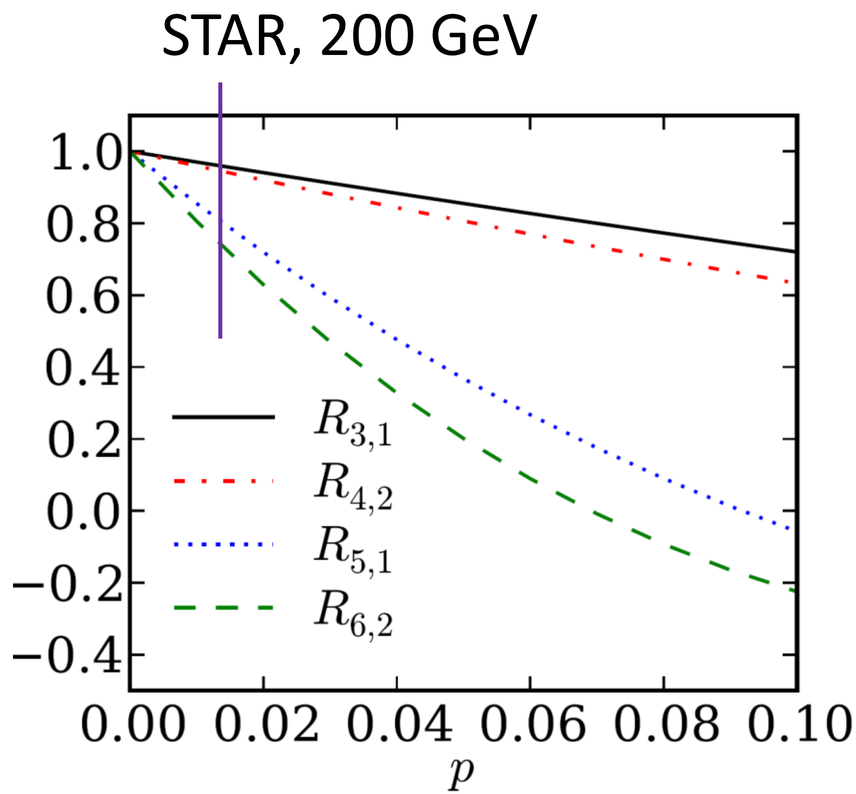




$$R_{n,m} = \frac{c_n}{c_m}$$

$$p = \frac{\#\ of\ measured\ protons/baryons}{total\ \#\ of\ baryons}$$

… for small *p*



We obtain:

200 GeV: $R_{4,2} \approx 0.95$, $R_{6,2} \approx 0.77$

5 GeV: $R_{4,2} \approx 0.85$, $R_{6,2} \approx 0.32$

New observable

$$D = R_{5,1} - R_{3,1}\left[1 - \frac{3}{4}(1+\gamma)(3-\gamma)\right]$$

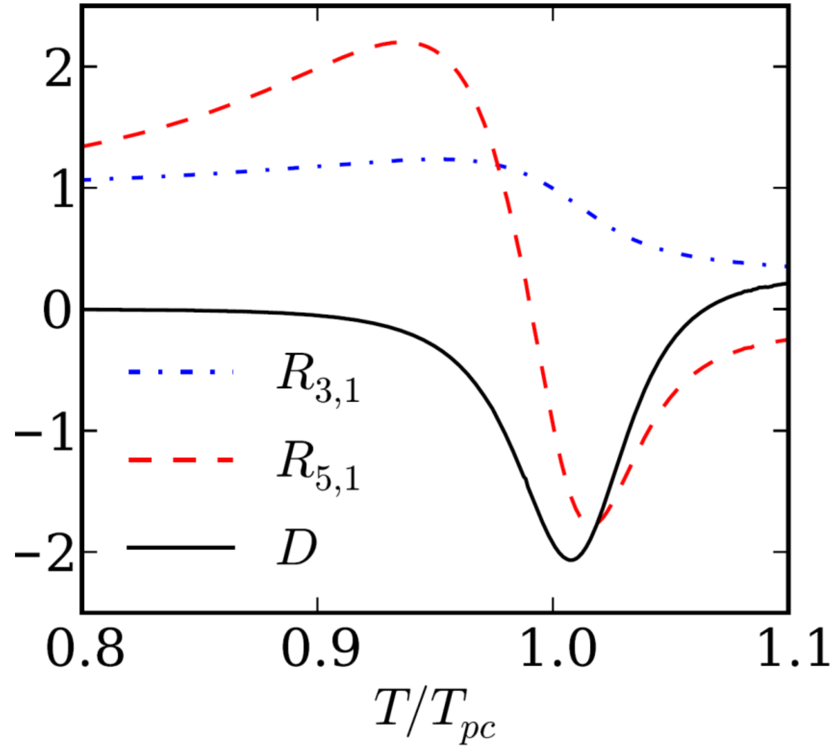
$$\gamma = \sqrt{1 + 8R_{3,1}}$$

$D = 0$ for a system with only baryon conservation

PQM calculation

$\mu_B/T = 0.5$

T_{pc} – crossover temperature

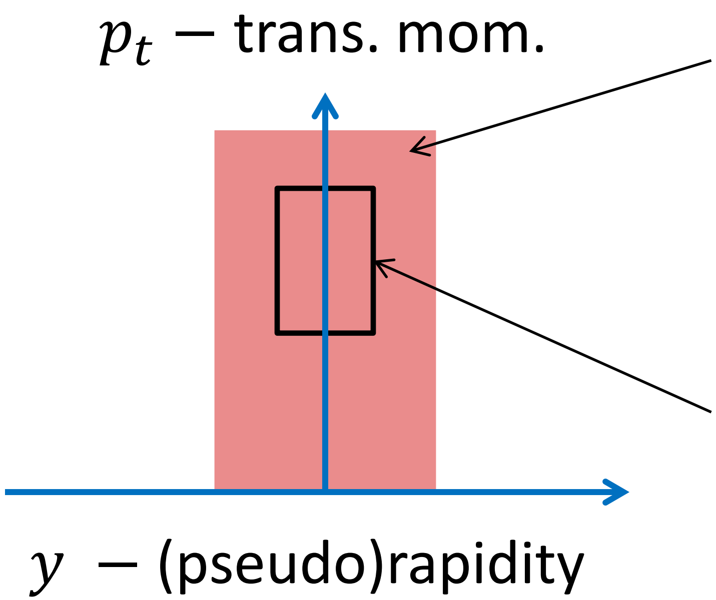


Limited acceptance

M. Kitazawa, M. Asakawa, arXiv:1205.3292; Phys. Rev. C85 (2012) 021901

AB, V. Koch, arXiv:1206.4286

Definitions



Required acceptance.
If we measure all relevant particles in this acceptance we will capture the desired physics.

Actual acceptance. In addition we usually cannot measure all relevant particles, e.g., neutrons.

$$p = 1:\ c_n = K_n$$

K_n – cumulants in the required acceptance

c_n – cumulants in the actual acceptance

Calculation

$$p(n_1, n_2) = \sum P(N_1, N_2)\, B(N_1, n_1; p_1) B(N_2, n_2, p_2)$$

↓ c_n — what we measure

↓ $K_n, F_{i,k}$ — what we would like to measure

$$p_1 = p_2 = 1\text{:}\ c_n = K_n$$

factorial moments $F_{i,k} = \left\langle \frac{N_1! N_2!}{(N_1 - i)!(N_2 - k)!} \right\rangle$

$B(\ldots) -$ binomial dist.

Binomial parameter

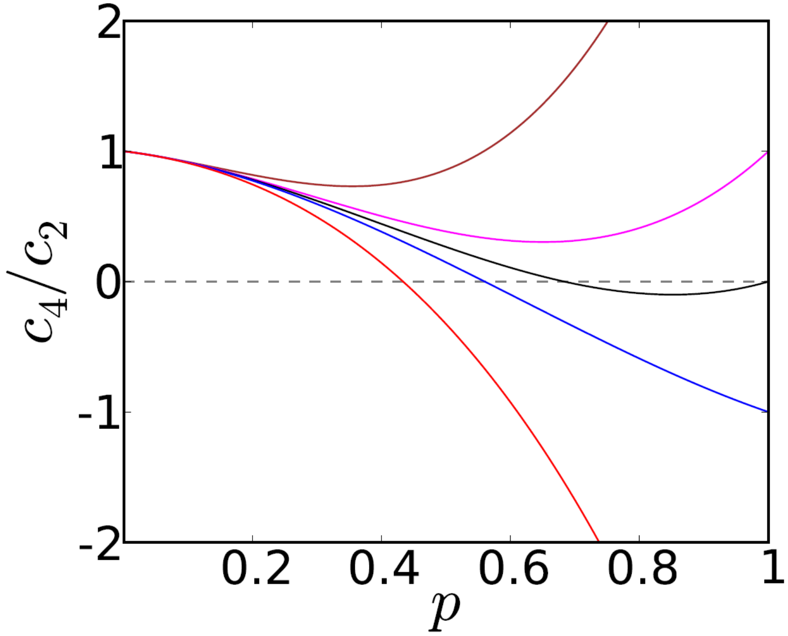
$$p = \frac{\#\ of\ measured\ particles}{total\ \#\ of\ particles\ that\ should\ be\ measured}$$

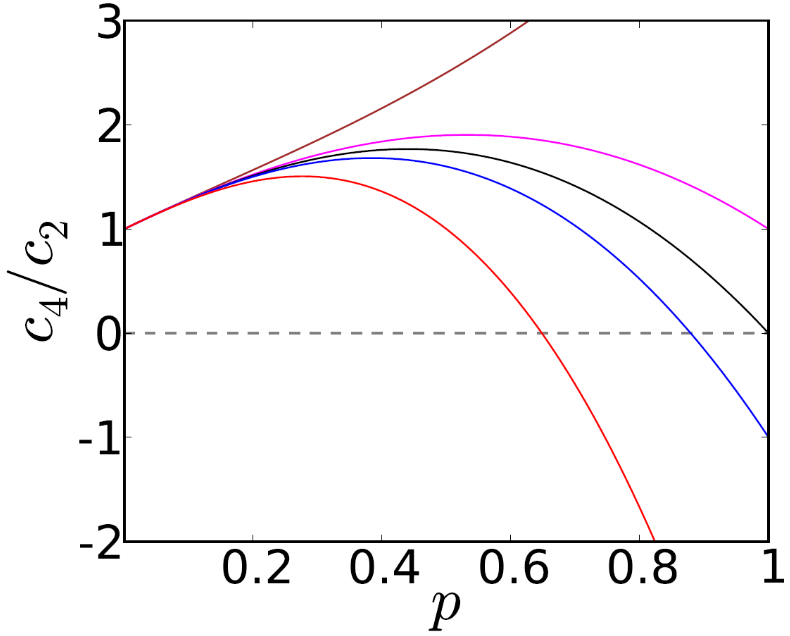
this *p* is different than *p* in baryon conservation

If we want to study net-baryon cumulants but we measure net-proton cumulants then $p_{max} \approx 1/2$.
Detector efficiency, cuts in rapidity or transverse momentum lead to $p < 1/2$.

STAR: $p < 1/2$ and probably $p \approx 1/5$ (nobody really knows)

Results





Experimental efforts may be futile

for more details see arXiv:1206.4286

Can we do something? Yes

We can extract K_n, e.g.,

$$pK_1 = c_1$$

$$p^2 K_2 = c_2 - n(1-p)$$

n – measured number of protons and anti-protons

See our paper for $K_{3,4,5,6}$. However, the smaller *p* the better precision of measurement is needed.

What about net charge? There is no problem to have $p > 1/2$

Conclusions

- Baryon number conservation results in a comparable signal as the experimental data for net proton cumulants

- Limited acceptance, especially inability to measure neutrons, is the most serious problem that makes the interpretation of net proton cumulants very challenging. Net charge is more promising.

Backup

Modified (baryon conservation) Skellam distribution:

$$P_B(n) = \left(\frac{p_B}{p_{\bar{B}}}\right)^{n/2} \left(\frac{1-p_B}{1-p_{\bar{B}}}\right)^{(B-n)/2} \times \frac{I_n\left(2z\sqrt{p_B p_{\bar{B}}}\right) I_{B-n}\left(2z\sqrt{(1-p_B)(1-p_{\bar{B}})}\right)}{I_B(2z)}$$

$n = n_B - n_{\bar{B}}$ (net baryon)
or $n = n_p - n_{\bar{p}}$ (net proton)

$$\langle N_{B,\bar{B}} \rangle_C = z\frac{I_{B\mp 1}(2z)}{I_B(2z)}$$

Cumulants ($p_B = p_{\bar{B}} = p$; $q = 1 - p$):

$$c_1 = pB,$$
$$c_3 = c_1(1-p)(1-2p),$$
$$c_5 = c_3\left(1 - 12p(1-p)\right).$$

$$c_2 = p(1-p)\langle N \rangle_C,$$
$$c_4 = c_2 + 3(p^2q^2B^2 - c_2^2) + 6pq(2z^2pq - c_2),$$
$$c_6 = c_4 + 4(c_4 - c_2) - 10(2pq + c_2)(c_4 - c_2) - 30pq(p^2q^2B^2 + c_2^2),$$

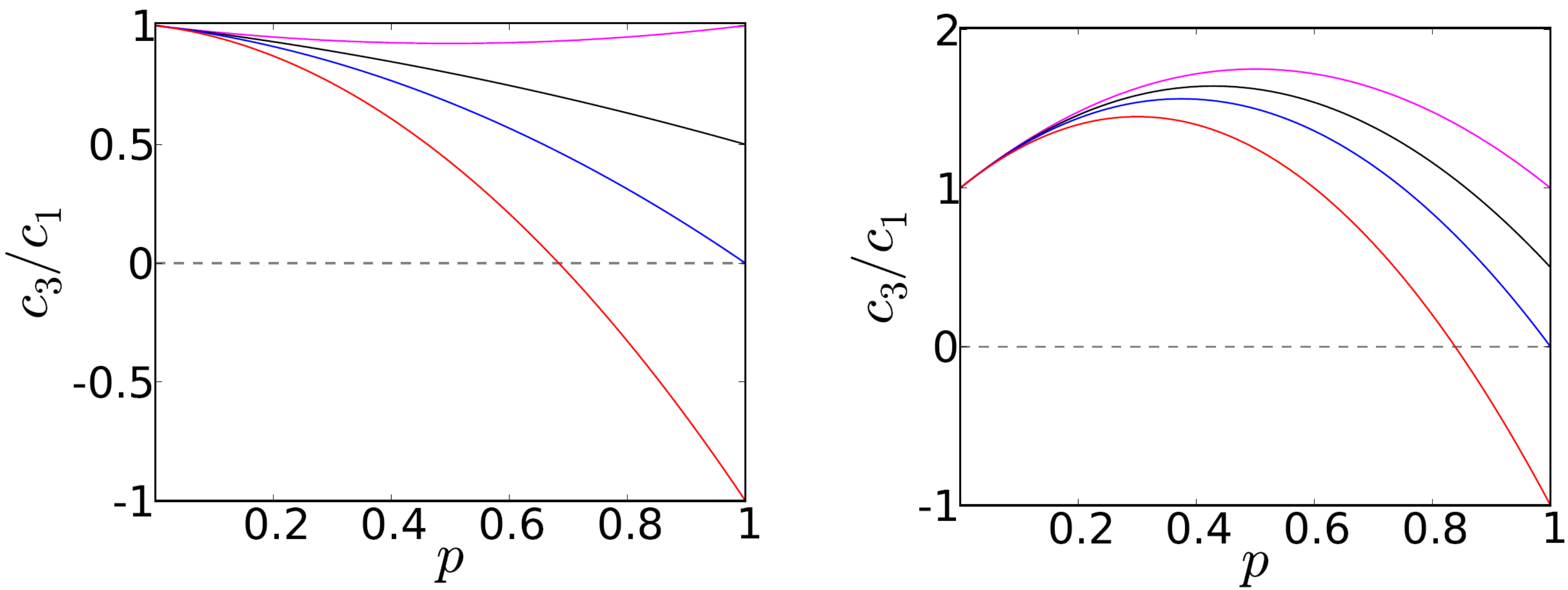
Relations between c_n and K_n (actual vs. required acceptance).
Here $p_1 = p_2 = p$.

$$c_1 = pK_1,$$
$$c_2 = p\,(1-p)\,N + p^2 K_2,$$
$$c_3 = p(1-p^2)K_1 + 3p^2(1-p)\,(F_{20} - F_{02} - NK_1) + p^3 K_3,$$

$$\begin{aligned} c_4 \;=\; & Np(1-p) - 3N^2p^2(1-p)^2 + 6p^2(1-p)(F_{02}+F_{20}) - 12K_1p^3(1-p)(F_{20}-F_{02}) \\ & +6Np^3(1-p)(K_1^2 - K_2) + p^2(1-p^2)(K_2 - 3K_1^2) \\ & +6p^3(1-p)(F_{03} - F_{12} + F_{02} + F_{20} - F_{21} + F_{30}) + p^4 K_4. \end{aligned}$$

$F_{i,k}$ − factorial moments in the required acceptance

c_3/c_1 as a function of binomial parameter *p*



Relations between K_n and c_n (required vs. actual acceptance).
Here $p_1 = p_2 = p$.

$$
\begin{aligned}
pK_1 &= c_1, \\
p^2 K_2 &= c_2 - n(1-p), \\
p^3 K_3 &= c_3 - c_1(1-p^2) - 3(1-p)(f_{20} - f_{02} - nc_1),
\end{aligned}
$$

$$
\begin{aligned}
p^4 K_4 \;=\; & c_4 - np^2(1-p) - 3n^2(1-p)^2 - 6p(1-p)(f_{20} + f_{02}) + 12c_1(1-p)(f_{20} - f_{02}) \\
& -(1-p^2)(c_2 - 3c_1^2) - 6n(1-p)(c_1^2 - c_2) \\
& -6(1-p)(f_{03} - f_{12} + f_{02} + f_{20} - f_{21} + f_{30}).
\end{aligned}
$$

$f_{i,k}$ – measured factorial moments

General case $p_1 \neq p_2$, see arXiv:1206.4286