Heavy quark potential at non-zero temperature and quarkonium spectral function

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- Motivation : the study and interpretation of static meson correlators is much more easier than the analysis of heavy meson correlators

  static correlators => potential model => spectral functions

- Free energy and singlet free energy of static quark anti-quark pair in 2+1 flavor QCD with Highly Improved Staggered Quark (HISQ) action => controlled discretization effects

- Wilson loops and potential at $T>0$

- Implications for quarkonium spectral functions

in collaboration with Alexei Bazavov

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Static quark anti-quark pair in T>0 QCD

Consider correlation functions of static meson operators

$$G_1(t,x,y,T) = \langle O(x,y;t)O(x,y;0)\rangle, \quad G_8(t,x,y,T) = \langle O^\alpha(x,y;t)O^\alpha(x,y;0)\rangle$$

for color singlet and adjoint represenation at time $t = 1/T$ with

$$O(x,y;t) = \bar{\psi}(x,t)U(x,y;t)\psi(y,t)$$

$$O^\alpha(x,y;t) = \bar{\psi}(x,t)U(x,x_0;t)T^\alpha U(x_0,y;t)\psi(y,t)$$

After integration out the static quarks we get

$$G_1(r,T) = \frac{1}{N}\langle \mathrm{Tr} L^\dagger(x)U(x,y;0)L(y)U^\dagger(x,y,1/T)\rangle,$$

$$G_8(r,T) = \frac{1}{N^2-1}\langle \mathrm{Tr} L^\dagger(x)\mathrm{Tr} L(y)\rangle$$

$$-\frac{1}{N(N^2-1)}\langle \mathrm{Tr} L^\dagger(x)U(x,y;0)L(y)U^\dagger(x,y,1/T)\rangle, r = |x-y|.$$

$L(x)$ is the temporal Wilson line; on the lattice $L(x) = \prod_{\tau=0}^{N_\tau-1} U_0(x,\tau)$

Alternative choice of $O(x,y;t)$: fix the Coulomb gauge and omit $U(x,y;t)$

For $t \to \infty$ there the choice of meson operators is not important but $t \leq 1/T$

Static quark anti-quark pair in T>0 QCD (cont'd)

The color averaged correlator gives the ratio of the partition of partition function of QCD at $T > 0$ with static $Q\bar{Q}$ to partition function without static sources:

$$G(r,T) = \frac{1}{N^2}\langle \mathrm{Tr}L(r)\mathrm{Tr}L^\dagger(0)\rangle = \frac{Z_{QQ}(r,T)}{Z(T)} = e^{-F(r,T)/T}$$

McLerran, Svetitsky 1981

$\rightarrow -T \ln G(r,T) \equiv F(r,T)$ is the excess free energy due to static sources.

$$G(r,T) = \frac{1}{N^2}G_1(r,T) + \frac{N^2-1}{N^2}G_a(r,T) \equiv \frac{1}{N^2}e^{-F_1(r,T)/T} + \frac{N^2-1}{N^2}e^{-F_8(r,T)/T}$$

The spectral represenation of singlet and averaged correlators $(T < T_c)$:

$$G_1(r,T) = \sum_{n=1}^{\infty} c_n(r) e^{-E_n(r)/T}, \qquad G(r,T) = \frac{1}{N^2}\sum_{n=1}^{\infty} e^{-E_n(r)/T}$$

Jahn, Philipsen, 2004

Perturbation theory $(T \gg T_c)$:

$$F_1(r,T) = -\frac{N^2-1}{2N}\frac{\alpha_s}{r}e^{-m_D r} - \frac{(N^2-1)\alpha_s m_D}{2N},$$

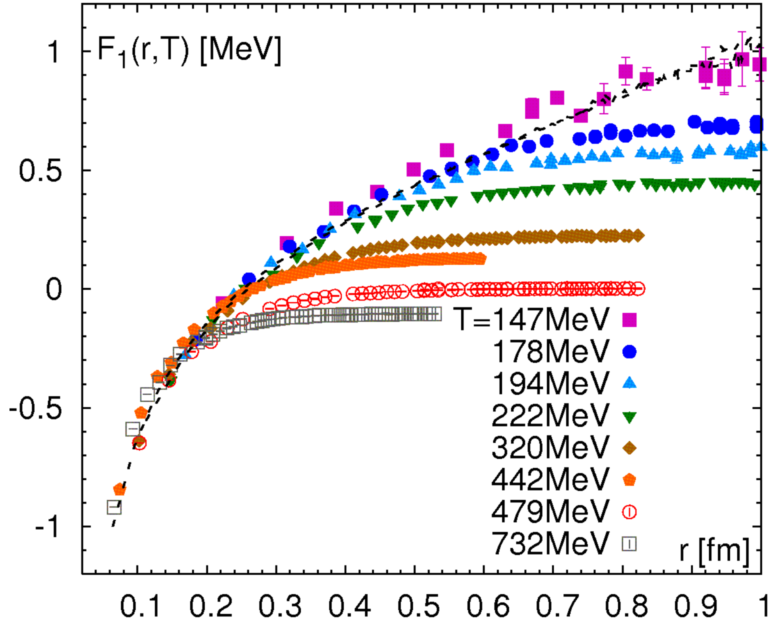
$$F_8(r,T) = +\frac{1}{2N}\frac{\alpha_s}{r}e^{-m_D r} - \frac{(N^2-1)\alpha_s m_D}{2N},$$

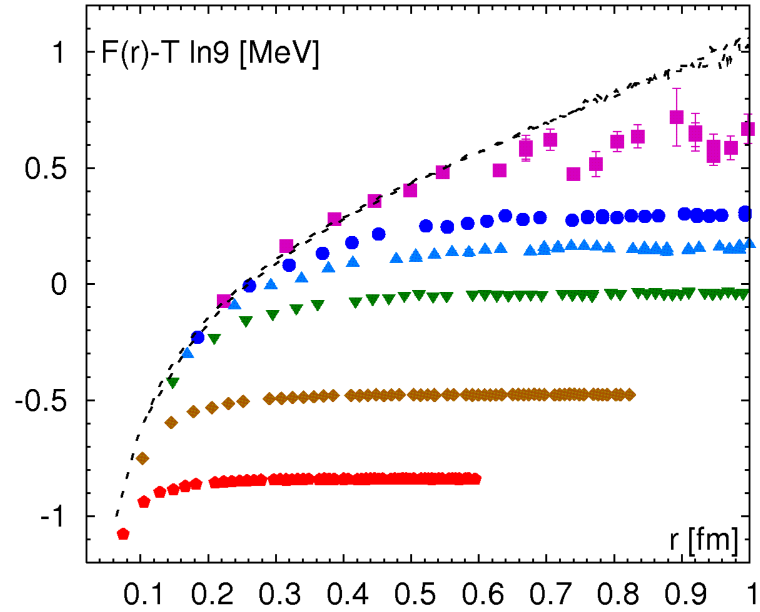
decomposition in terms of F_1 and F_8 can be extended to any order for $rT \ll 1$

Brambilla, Ghiglieri, PP, Vairo 2010

Static quark anti-quark free energy in 2+1f QCD

HISQ action, $24^3 \times 6$, $16^3 \times 4$ (high T) lattices, $m_\pi \simeq 160$ MeV

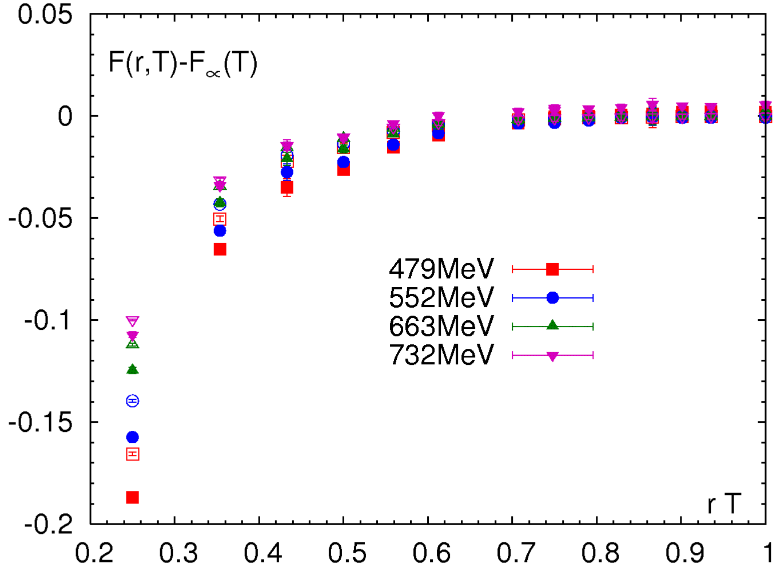




- The strong *T*-dependence for *T<200* MeV is not necessarily related to color screening

- The free energy has much stronger *T*-dependence than the singlet free energy due to the octet contribution

- At high T the temperature dependence of the free energy can be entirely understood in terms of F_1 and Casimir scaling $F_1=-8\ F_8$



Static quark anti-quark free energy in 2+1f QCD (cont'd)

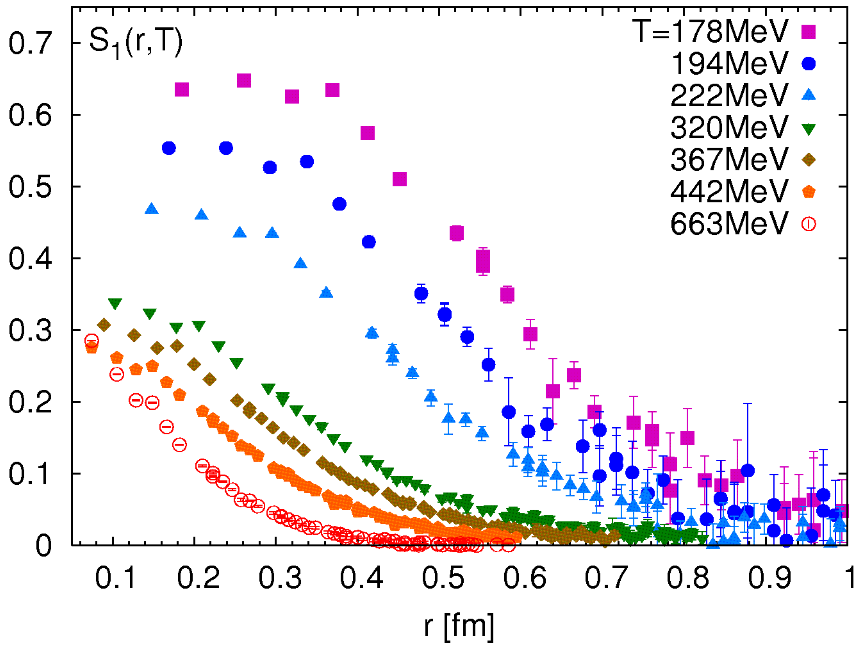
Leading order weak coupling :

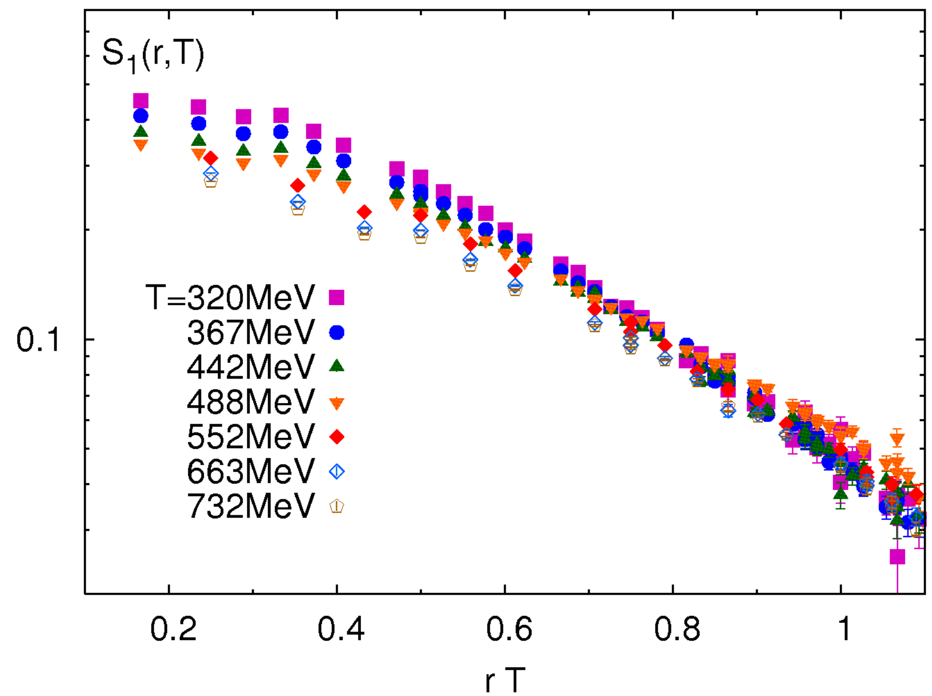
$$F_1(r,T) = C_F \frac{\alpha_s}{r} \exp(-m_D r) + F_\infty(T)$$

Screening function:

$$S_1(r,T) = (F_\infty(T) - F_1(r,T) \cdot r$$

$$rT \ll 1, \quad S_1(r,T) \sim \alpha_s \qquad\qquad rT \gg 1, \quad S_1(r,T) \sim \exp(-m_D r)$$





Wilson loops and potential at T>0

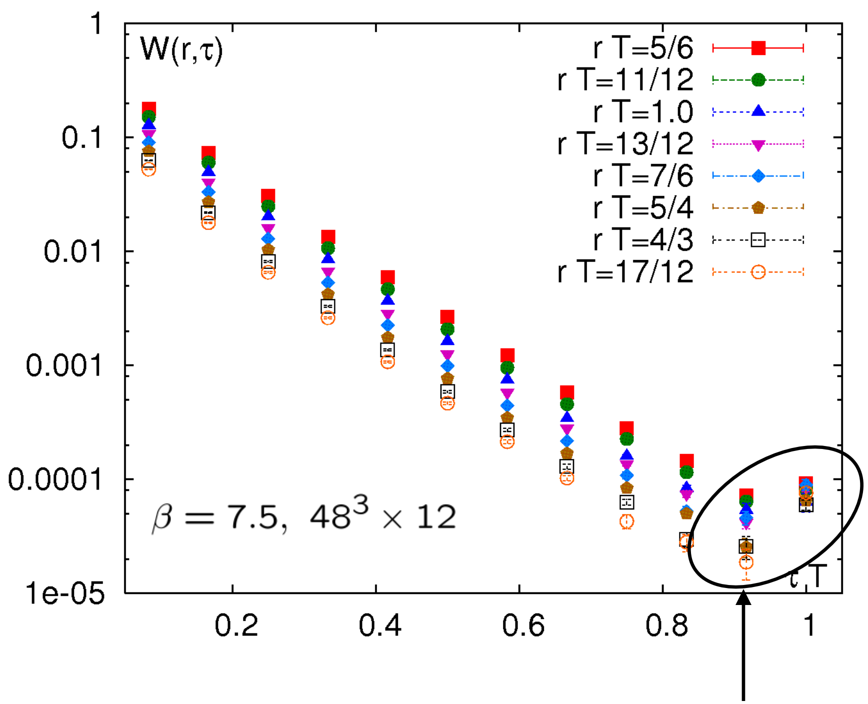
HISQ action, $48^3 \times 16$, $48^3 \times 12$, $24^3 \times 6$ lattices, $m_\pi \simeq 160$ MeV

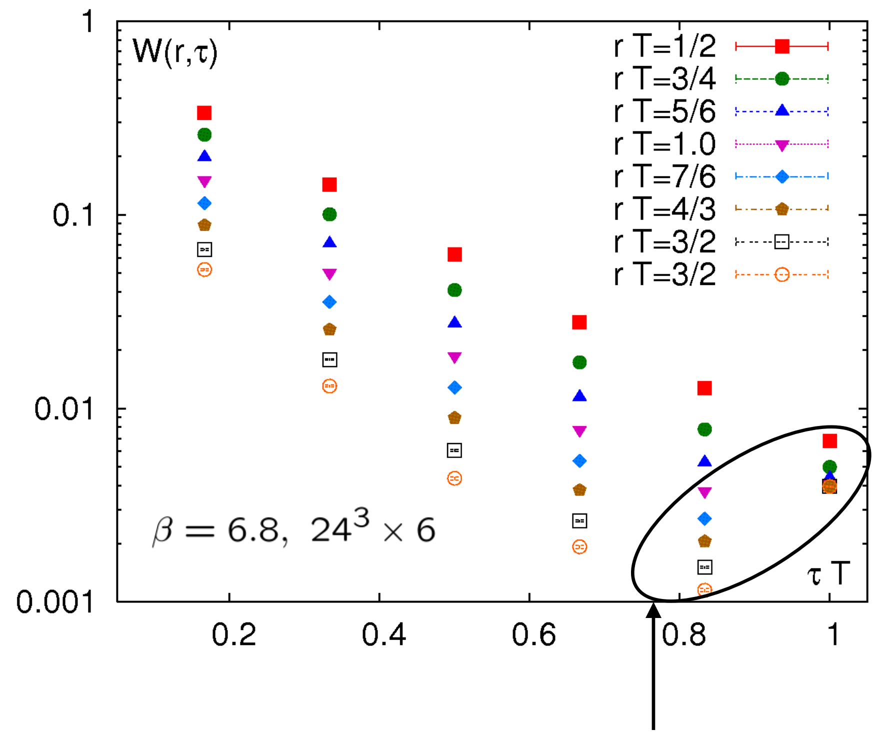
$$W(r,\tau) = \int_{-\infty}^{\infty} d\omega \sigma(\omega, T) e^{-\omega \tau}, \tau < 1/T$$

Rothkopf 2009, Hatsuda, Rothkopf, Hatsuda 2011

not related to the free energy !

assume single state dominance $\sigma(\omega, T) \sim \delta(\omega - V(r,T))$





Backward propagating state $\sim \exp(E_1(T)(1/T - \tau))$ due to gluon interaction across the periodic boundary

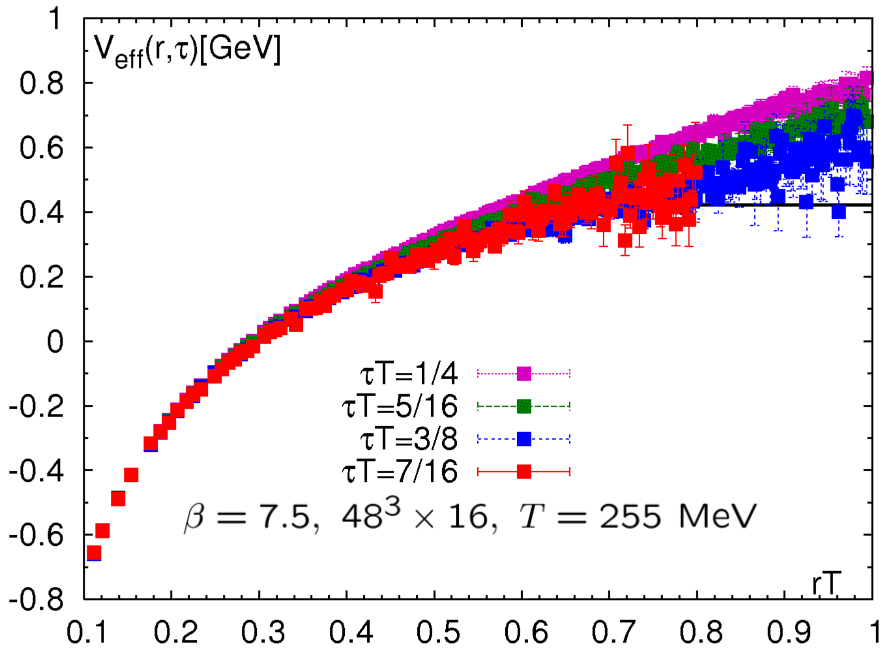
Rothkopf 2009, Allton 2012

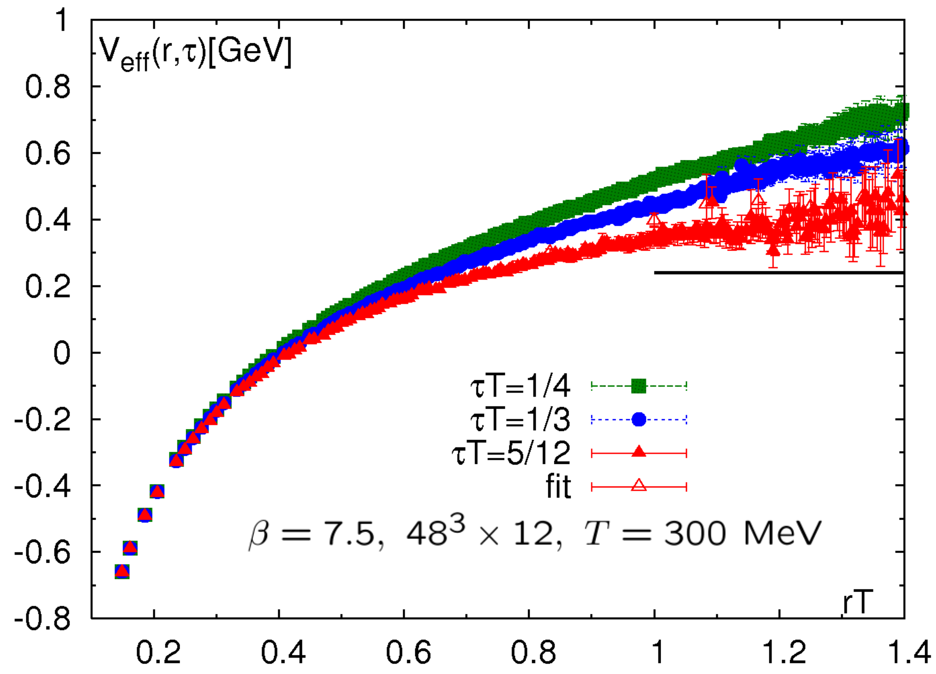
Extracting the potential at T>0

- Consider the ratio of the Wilson loops as function of time

$$V_{eff}(r,\tau) = \ln(W(r,\tau)/W(r,\tau+1))$$

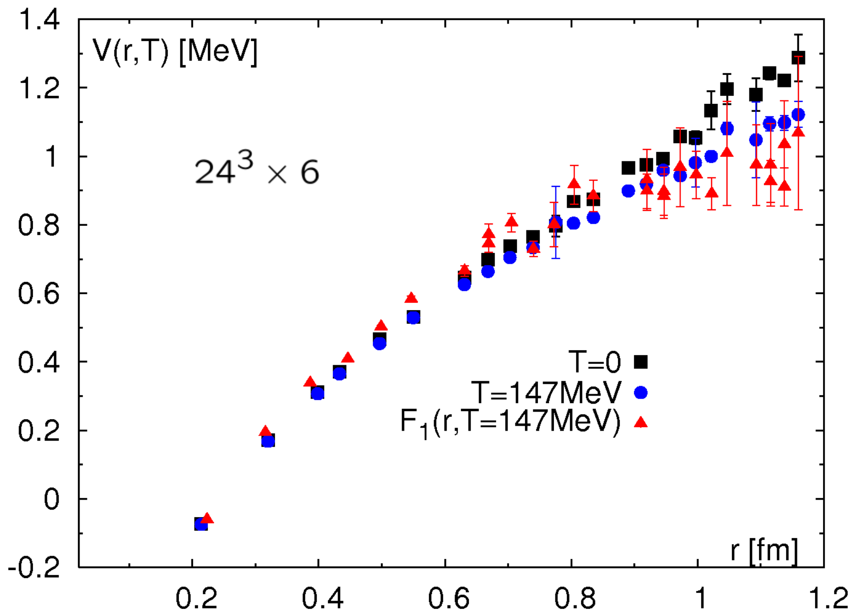
- Fit the Wilson loop with exponential for *τ T < 1/2*
- Remove the backward propagating contribution and fit with exponential

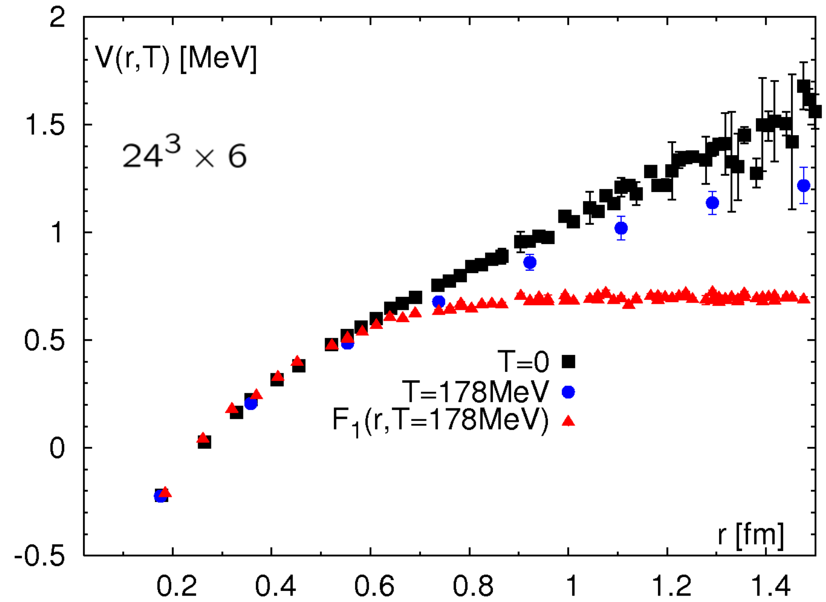




the potential is larger the singlet free energy at distances around *1/T*
to explore larger distances smaller N_τ should be considered

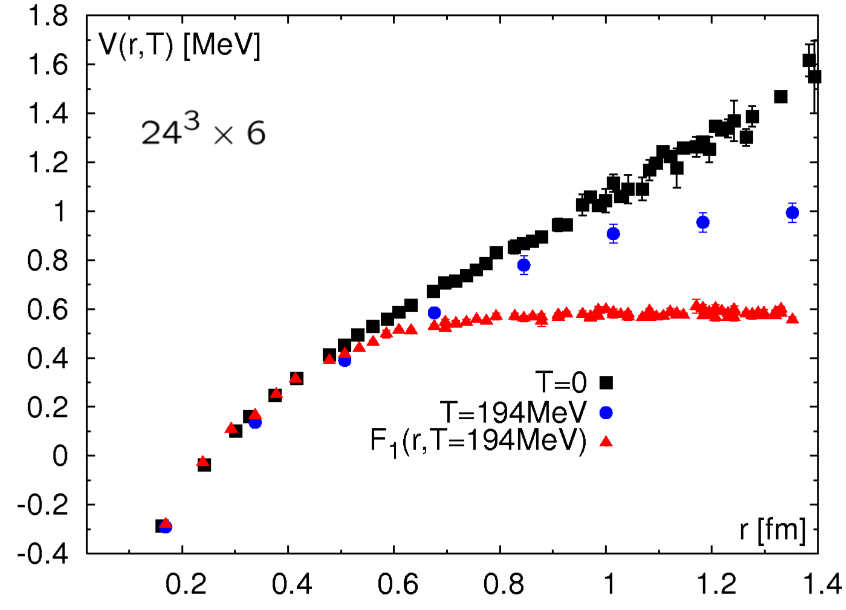
Extracting the potential at T>0 (cont'd)



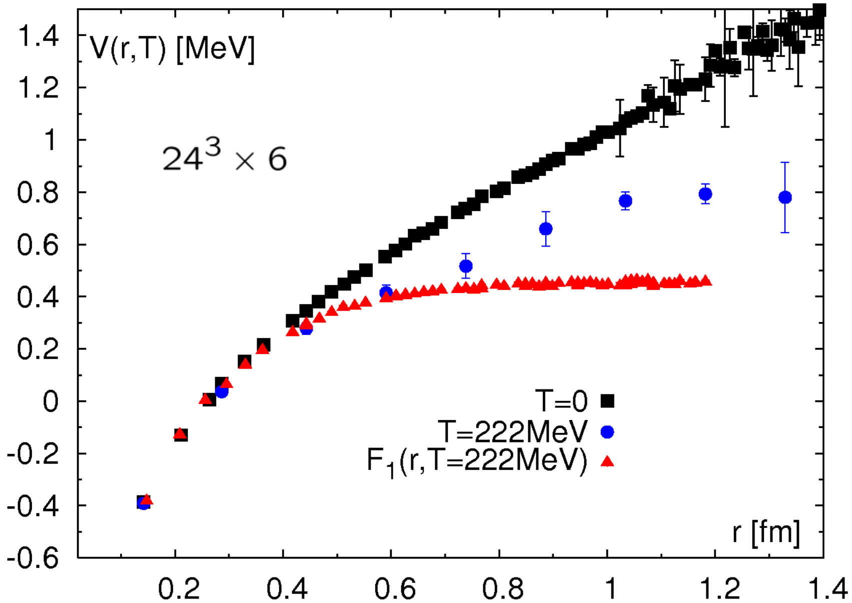


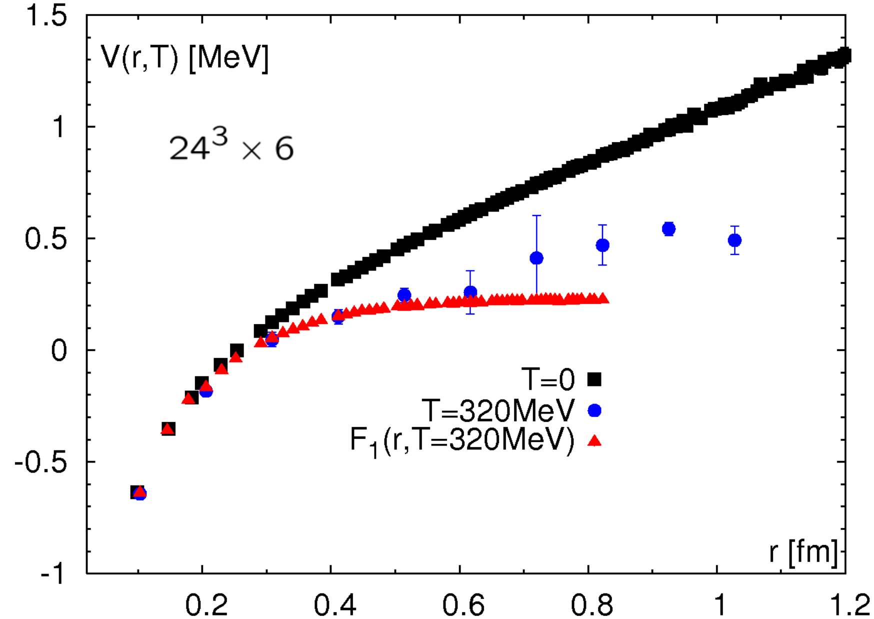
- for the lowest temperature the potential is the same as at *T=0* and agrees with the singlet free energy

- for *150* MeV< *T<200* MeV the potential only slightly differs from the *T=0* potential but much larger than the singlet free energy



Extracting the potential at T>0 (cont'd)

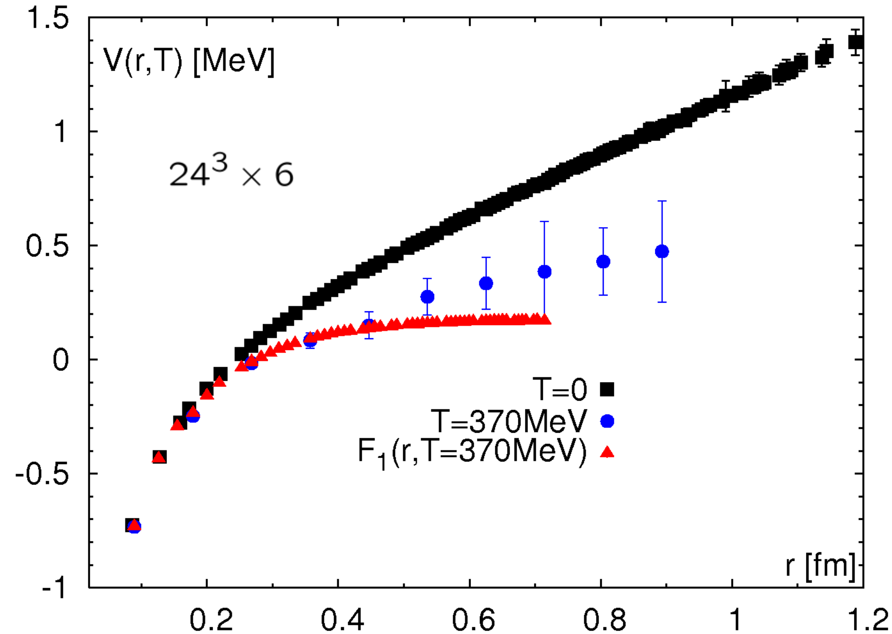




- for *T>200* MeV the potential is significantly modified compared to the *T=0* result

- the potential is larger than the singlet free energy but approaches it from above as the temperature increases

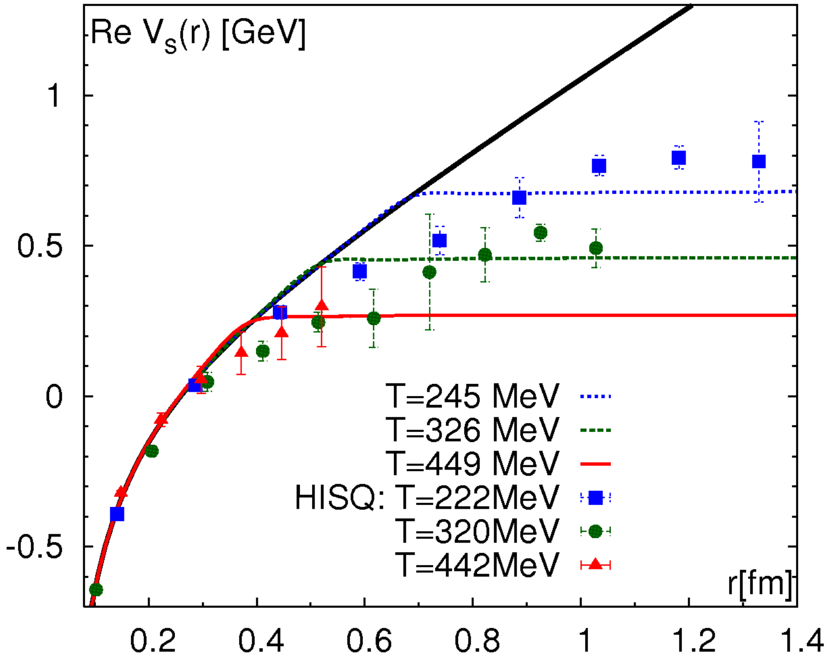
similar behavior in quenched QCD
Burnier, Rothkopf 2012

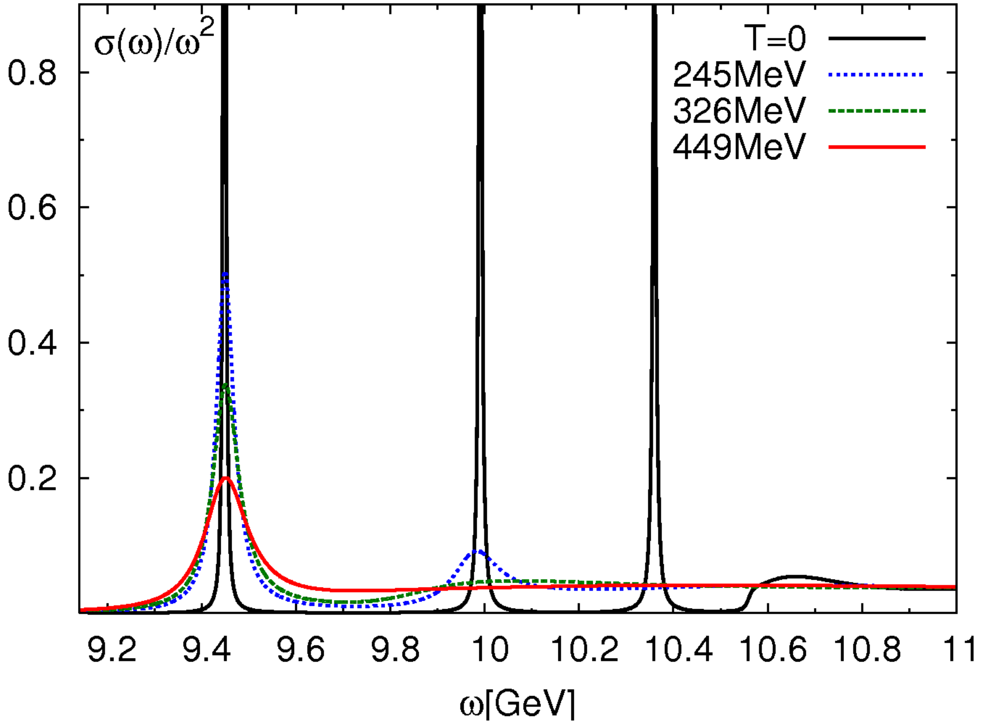


Implications for potential models

• Comparison of lattice results with phenomenological "maximally binding" potential
Mocsy, PP 2007

• Supplement the real part of the potential with perturbative imaginary part and calculate the spectral function
Miao, Mocsy, PP 2010





charmonium and *2S* bottomonium states dissolves for *T>245* MeV,
1S bottomonium states dissolves for *T>450* MeV

Summary

• The free energy and singlet free energy of static quark anti-quark pair calculated with HISQ action show strong screening effects for $T>200$ MeV and the results are in agreement with earlier calculations perfromed with p4 action

• It is possible to extract the potential at $T>0$ from the Wilson loops using exponential fits and the potential shows strong screening effects at $T>200$MeV and is larger than the singlet free energy

• The potential extracted from the lattice is very similar to the phenomenological "maximally binding" potential used in potential model calculations
$\Rightarrow$ confirmation of the earlier conclusions of melting of charmonia and *2S* bottomonia states at $T>245$ MeV and melting of *1S* bottomonium states $T>450$ MeV