The QCD Equation of State with 2+1 flavors of Highly Improved Staggered Quarks

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Lattice QCD and numerical setup

HISQ/tree – scale setting

Trace anomaly, $N_\tau = 6, 8, 10, 12$ comparison

Trace anomaly at low and high temperature

Conclusion

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Lattice QCD

- Quantum field theory on a discrete (Euclidian) space-time lattice, $N_s^3 \times N_\tau$, $T = 1/(aN_\tau)$
- Evaluate path integrals stochastically (importance sampling)

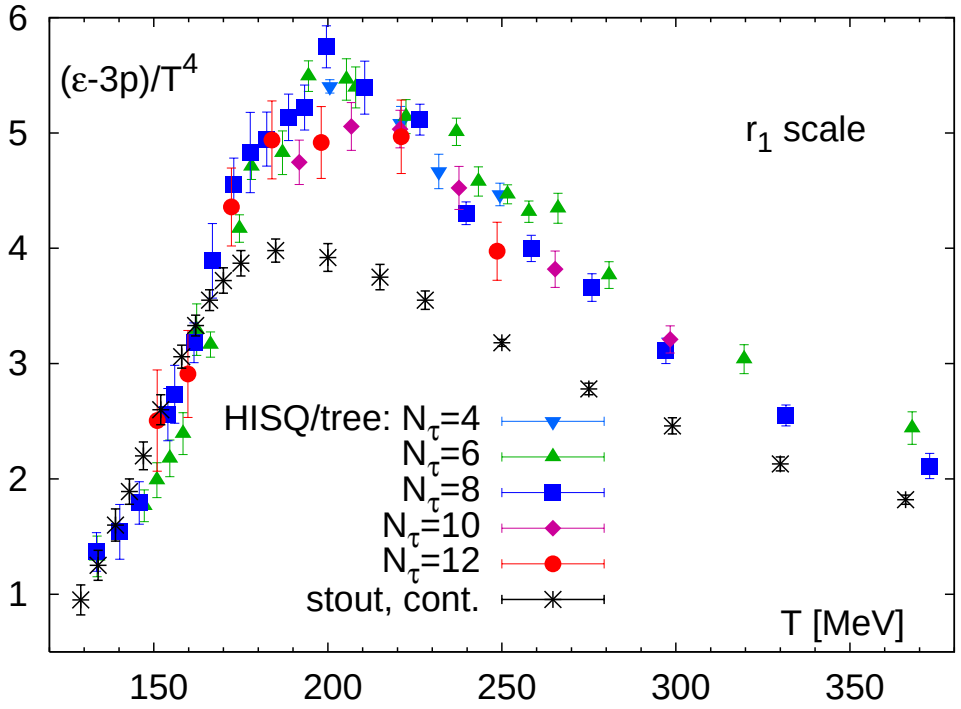
$$\begin{aligned} Z &= \int DUD\bar{\psi}D\psi \exp\{-S\}, \quad S = S_g + S_f \\ \langle \mathcal{O} \rangle &= \frac{1}{Z} \int DUD\bar{\psi}D\psi \mathcal{O} \exp\{-S\} \end{aligned}$$

- We use Highly Improved Staggered Quarks (HISQ)[1] and the tree-level Symanzik-improved gauge action, hence HISQ/tree
- The physics is recovered in the continuum limit ($a \to 0$, or $N_\tau \to \infty$ in the finite-temperature geometry)
- The trace anomaly

$$\varepsilon - 3p = -\frac{T}{V}\frac{d \ln Z}{d \ln a} \quad \Rightarrow \quad \frac{p}{T^4} - \frac{p}{T_0^4} = \int_{T_0}^{T} dT' \frac{\varepsilon - 3p}{T'^5}$$

[1] Follana et al. (HPQCD), PRD75, 054502 (2007)

Trace anomaly – current status



- HISQ/tree[2] – at fixed $N_\tau = 6$, 8, 10 and 12 with r_1 scale
- Stout[3] – continuum estimate obtained as mean of $N_\tau = 8$ and 10 data after applying a tree-level correction (i.e., the results of the simulation are divided by a ratio of the pressure in the Stefan-Boltzmann limit at given N_τ and in the continuum)
- Can the apparent discrepancy in the peak region be due to the tree-level corrections applied to the stout data?

[2] HotQCD, PoS(Lat2011), 182 (2011) [arxiv:1201.5345 [hep-lat]] and work in progress
[3] Borsanyi et al. (BW), JHEP1011, 077 (2010)

HISQ/tree – numerical setup

- Calculation of the trace anomaly requires subtraction of UV divergencies (take difference of zero- and finite-temperature quantities evaluated at the same values of the gauge coupling):

$$\begin{aligned}\frac{\varepsilon - 3p}{T^4} &= R_\beta[\langle S_g\rangle_0 - \langle S_g\rangle_T] \\ &- R_\beta R_m[2m_l(\langle \bar{l}l\rangle_0 - \langle \bar{l}l\rangle_T) + m_s(\langle \bar{s}s\rangle_0 - \langle \bar{s}s\rangle_T)]\end{aligned}$$

$$R_\beta(\beta) = -a\frac{d\beta}{da}, \quad R_m(\beta) = \frac{1}{m}\frac{dm}{d\beta}, \quad \beta = \frac{10}{g^2}$$

- Line of constant physics $m_l = m_s/20$ (physical $m_l = m_s/27$)
- Statistics (in molecular dynamics time units):

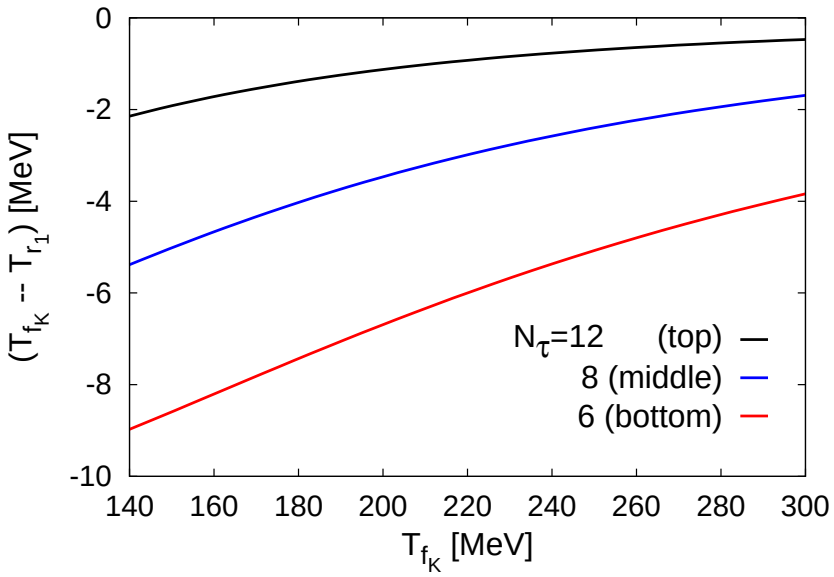
$T > 0$		$T = 0$	
$16^3 \times 4$	4K	$24^3 \times 32$	3-4K
$24^3 \times 6$	5-30K	32^4, $32^3 \times 64$	2-5K
$32^3 \times 8$	12-30K	48^4	4-6K
$40^3 \times 10$	10-20K	$48^3 \times 64$	5-6K
$48^3 \times 12$	30-40K		

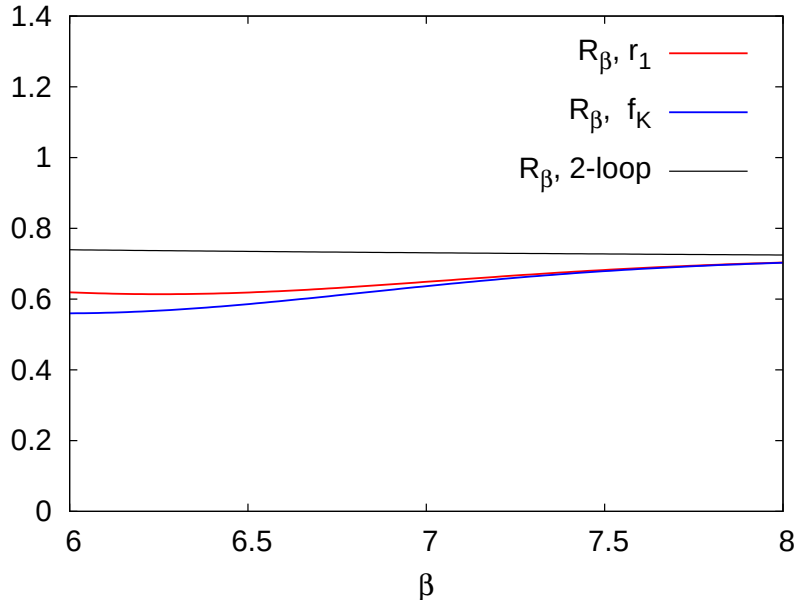
HISQ/tree - scale setting

- Sommer scale, $r_1 = 0.31$ fm (derived requiring that f_π is at the experimental value[4])

$$r^2 \frac{dV_{\bar{q}q}}{dr}\bigg|_{r=r_1} = 1$$

- Direct hadronic scale, f_K





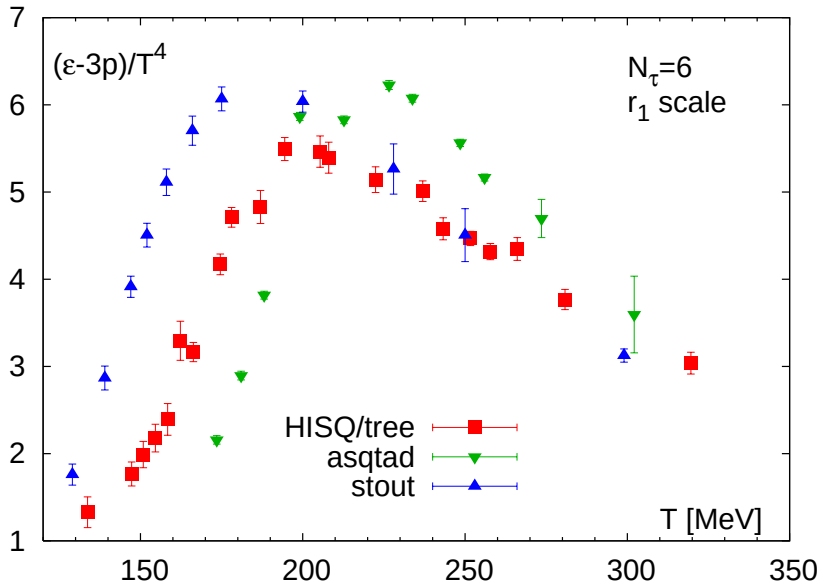
- Left: difference in temperature with lattice spacing set using r_1 or f_K
- Right: the β-function in r_1 and f_K scheme in the region of the inverse gauge coupling $10/g^2$ used in simulations

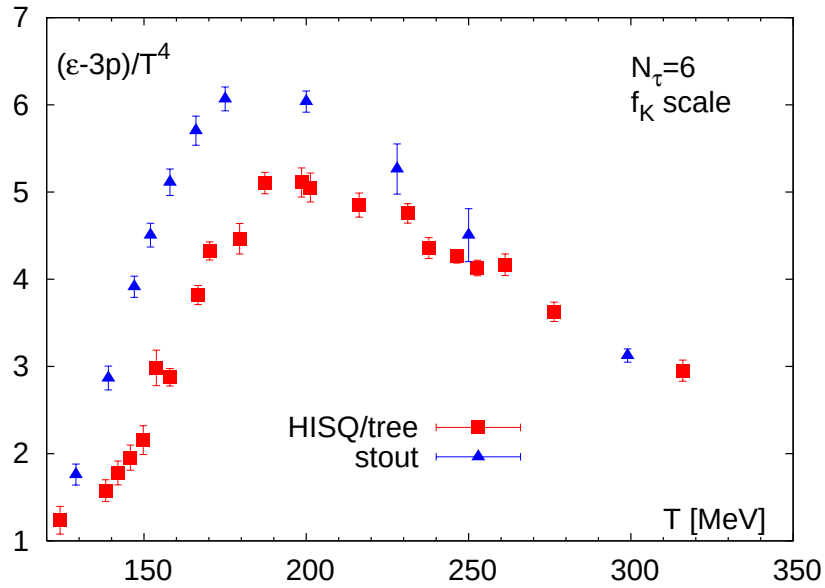
[4] MILC, PoS(Lat2010), 074 (2010) [arxiv:1012.0868 [hep-lat]]

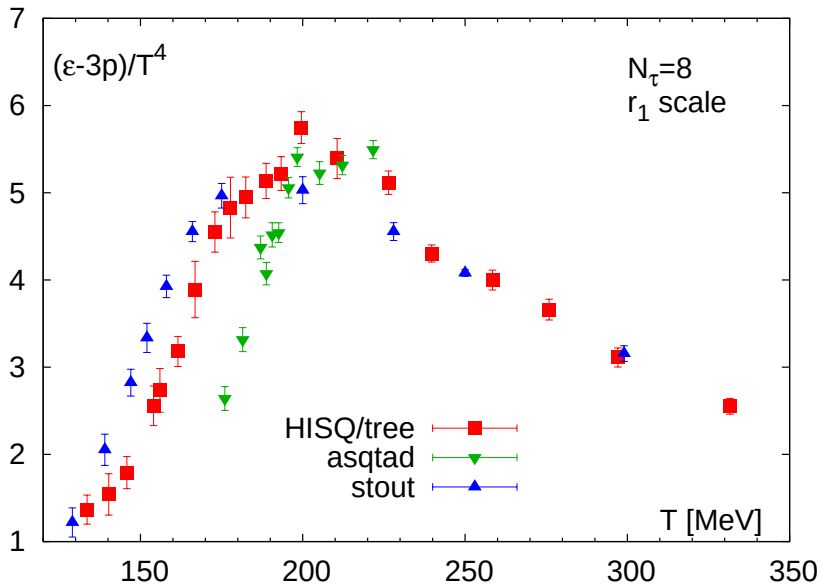
Trace anomaly

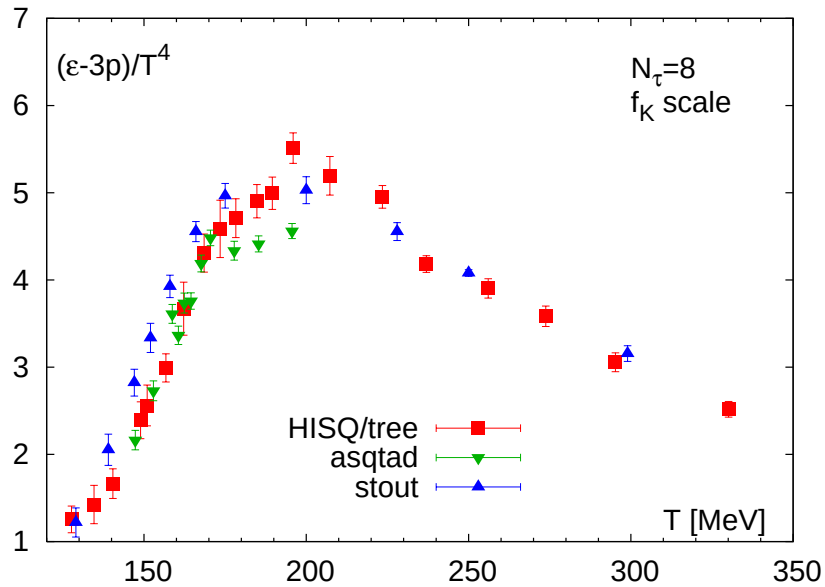
- Compare the trace anomaly for HISQ/tree, asqtad and stout actions
- Compare with two scales, r_1 (left) and f_K (right), stout with f_K only
- Stout plotted without the tree-level correction
- At fixed temporal extent $N_\tau = 6$, 8, 10 and 12

Trace anomaly, $N_\tau = 6$ and 8

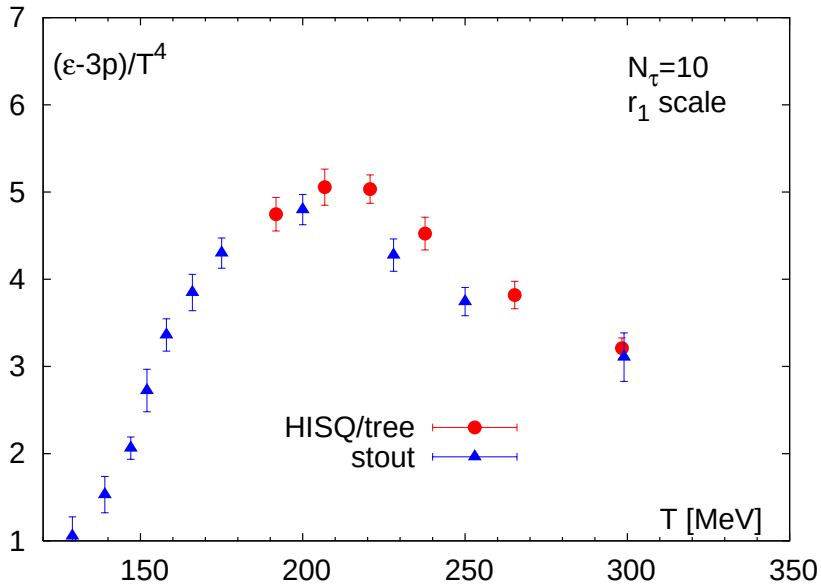


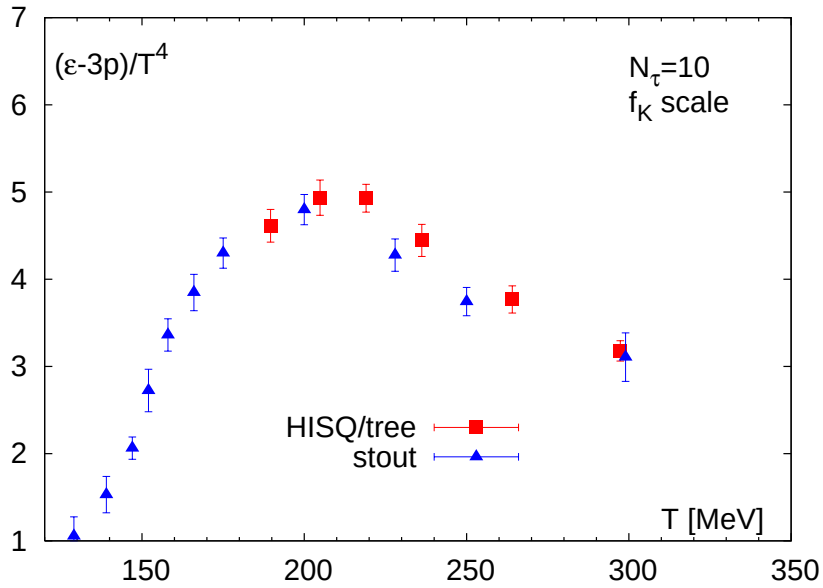


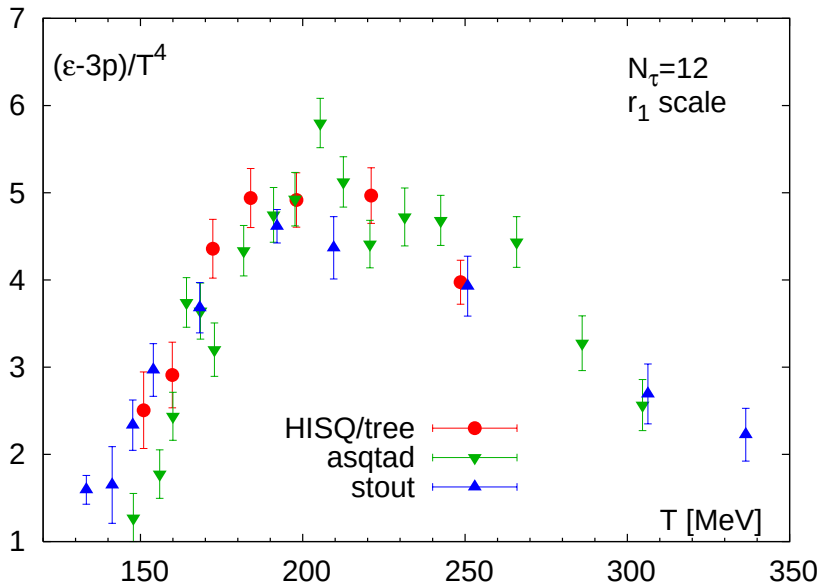


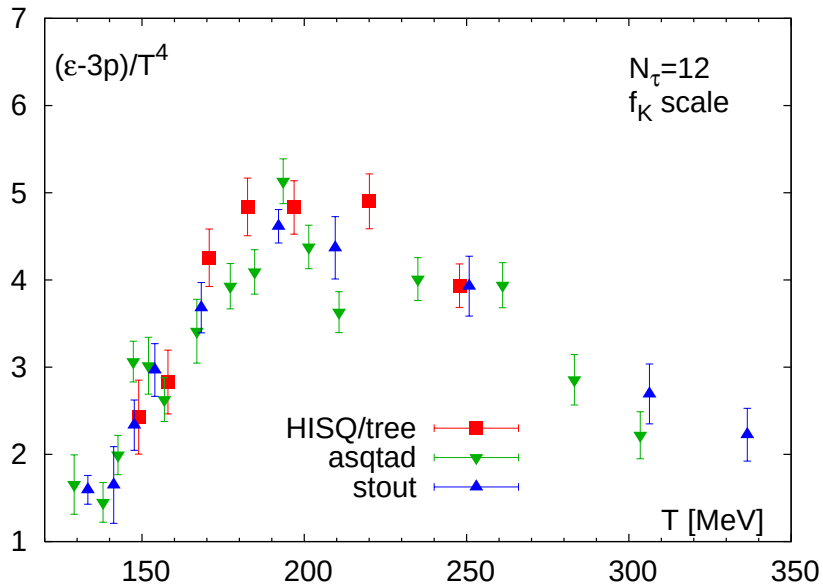


Trace anomaly, $N_\tau = 10$ and 12





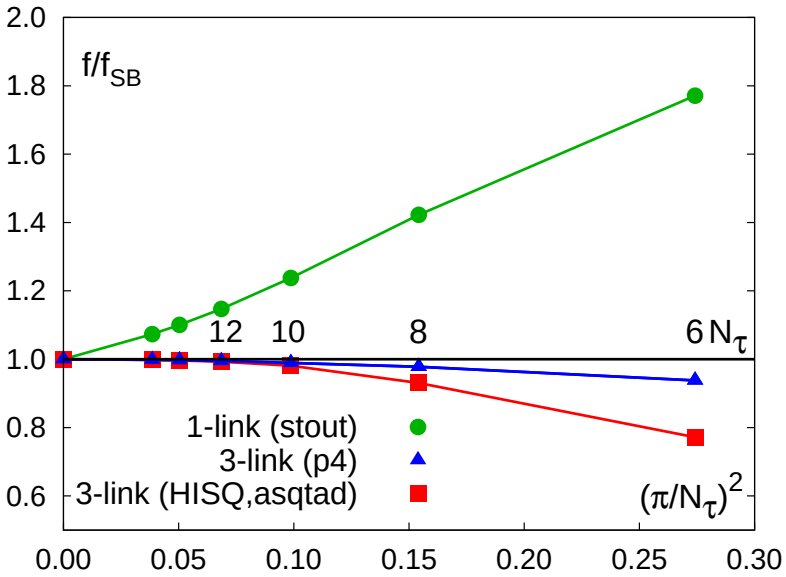


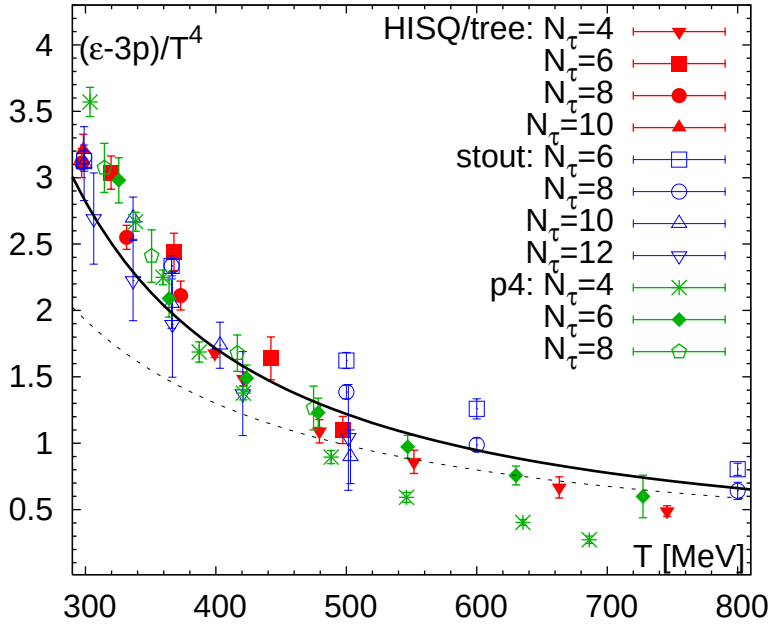


Agreement of trace anomaly $(\varepsilon - 3p)/T^4$ data

- On $N_\tau = 6$, 8 lattices the agreement between data from different actions is improved by using the f_K scale
- On $N_\tau = 10$, 12 lattices the difference between HISQ/tree data using r_1 and f_K scale is insignificant
- On $N_\tau = 10$, 12 lattices the HISQ/tree, asqtad and stout data agree within statistics if the tree-level correction is removed from the stout data

Trace anomaly at high temperature

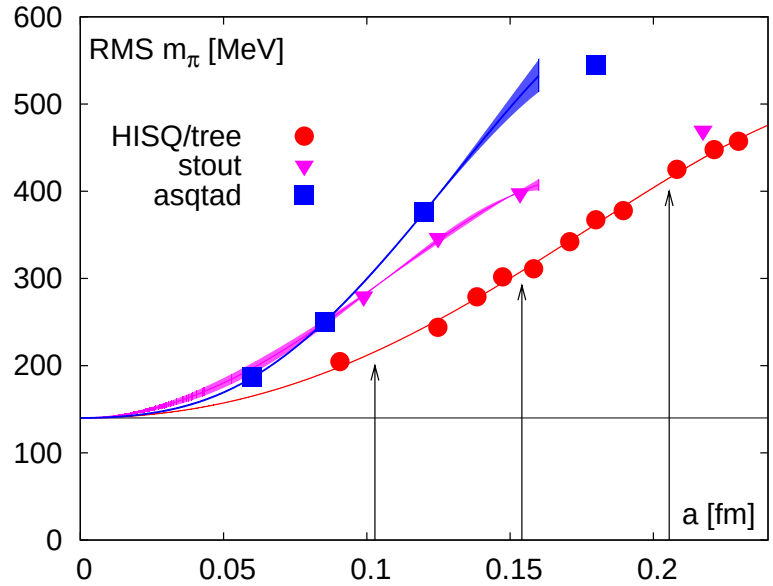


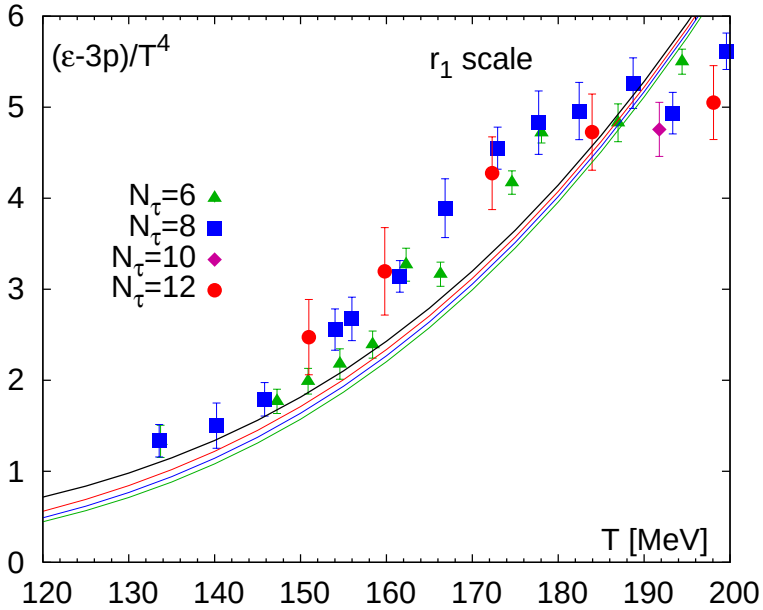


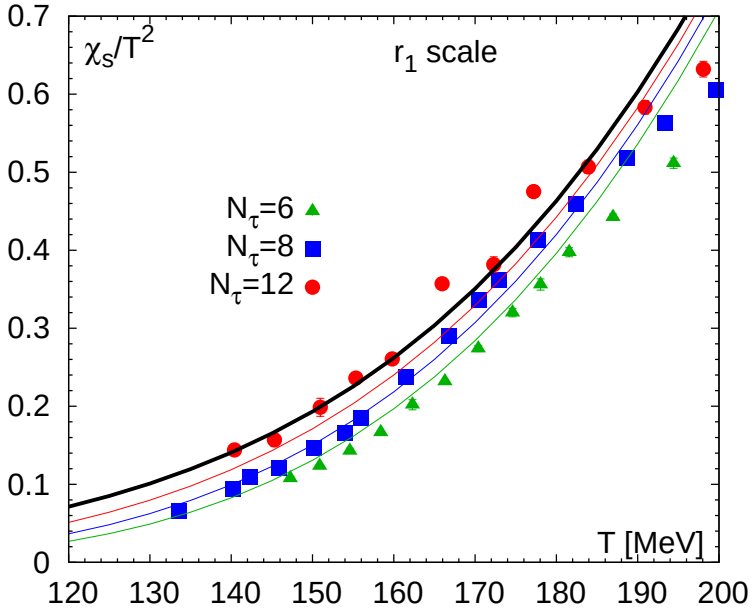
- The free energy of an ideal quark gas at different N_τ (left)
- The trace anomaly at high temperature (right)
- Reasonable agreement with resummed perturbative calculations (solid line) using 2-loop running coupling. Dashed line is the 1-loop result.

Trace anomaly at low temperature

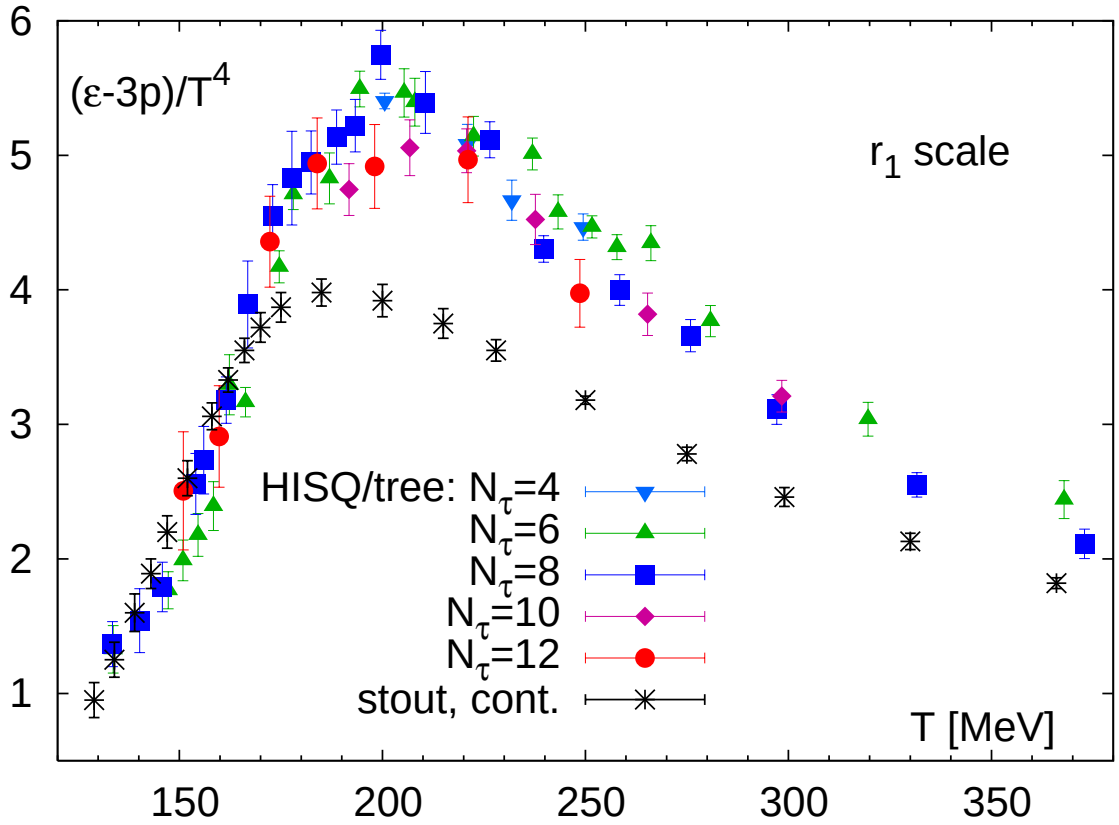
- Taste symmetry breaking makes the average pion mass heavier. Problem becomes worse on coarser lattices
- These effects are less pronounced in the trace anomaly. Note mild N_τ-dependence compared to χ_s (right panel)







Trace anomaly in the peak region



- Noticeable cut-off dependence in the peak region
- The continuum limit is approached from above
- We are increasing statistics on $N_\tau = 10$, 12 ensembles

Conclusion

- The HISQ/tree discretization scheme has better taste symmetry compared to other staggered actions (asqtad and stout). This improves the scaling behavior at low temperature.
- The HISQ/tree action also includes correction to the quark dispersion relation which is relevant at high temperatures.
- The scale setting procedure can make a difference at finite a, however, for HISQ/tree data at $N_\tau = 10$ and 12 the effect is negligible.
- The trace anomaly with HISQ/tree agrees with stout on $N_\tau = 10$ and 12 lattices once the tree-level correction is removed from the stout data.
- The low-temperature behavior of the trace anomaly qualitatively follows the Hadron Resonance Gas model, but disagrees quantitatively.
- Need higher statistics to control the continuum limit.