A No-Go Theorem for Critical Phenomena in QCD at finite temperature and density

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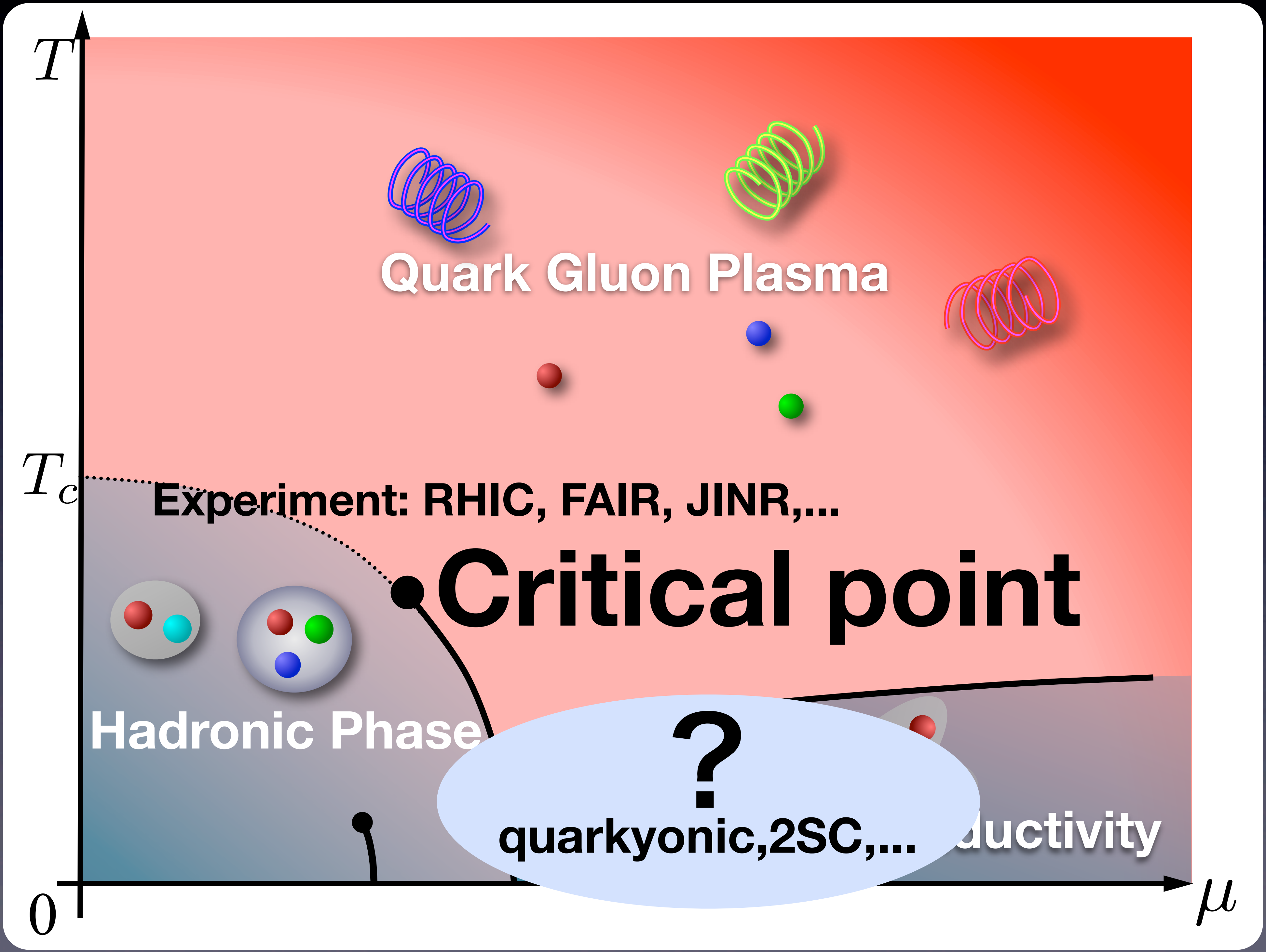
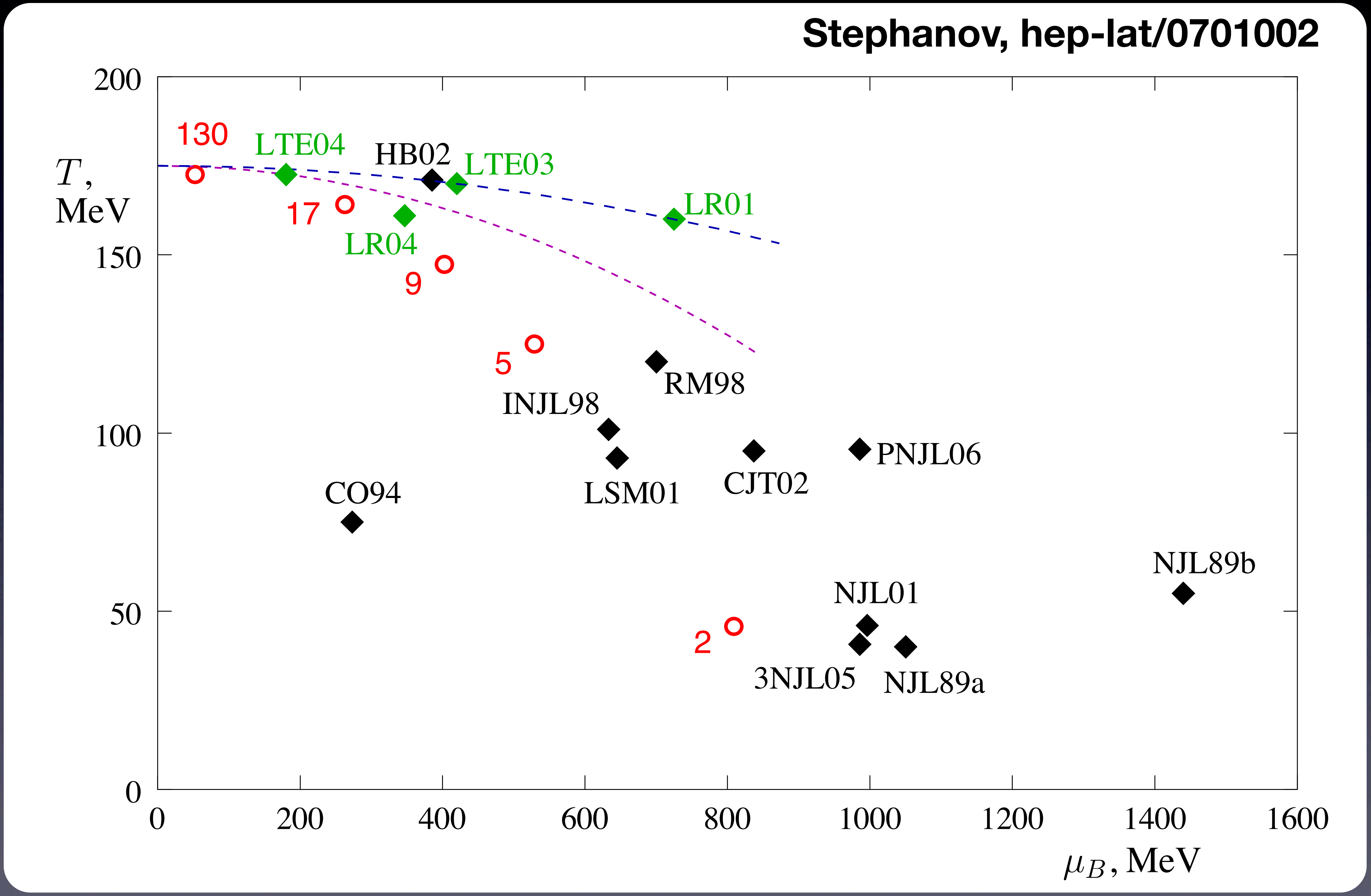
Where is the QCD critical point?

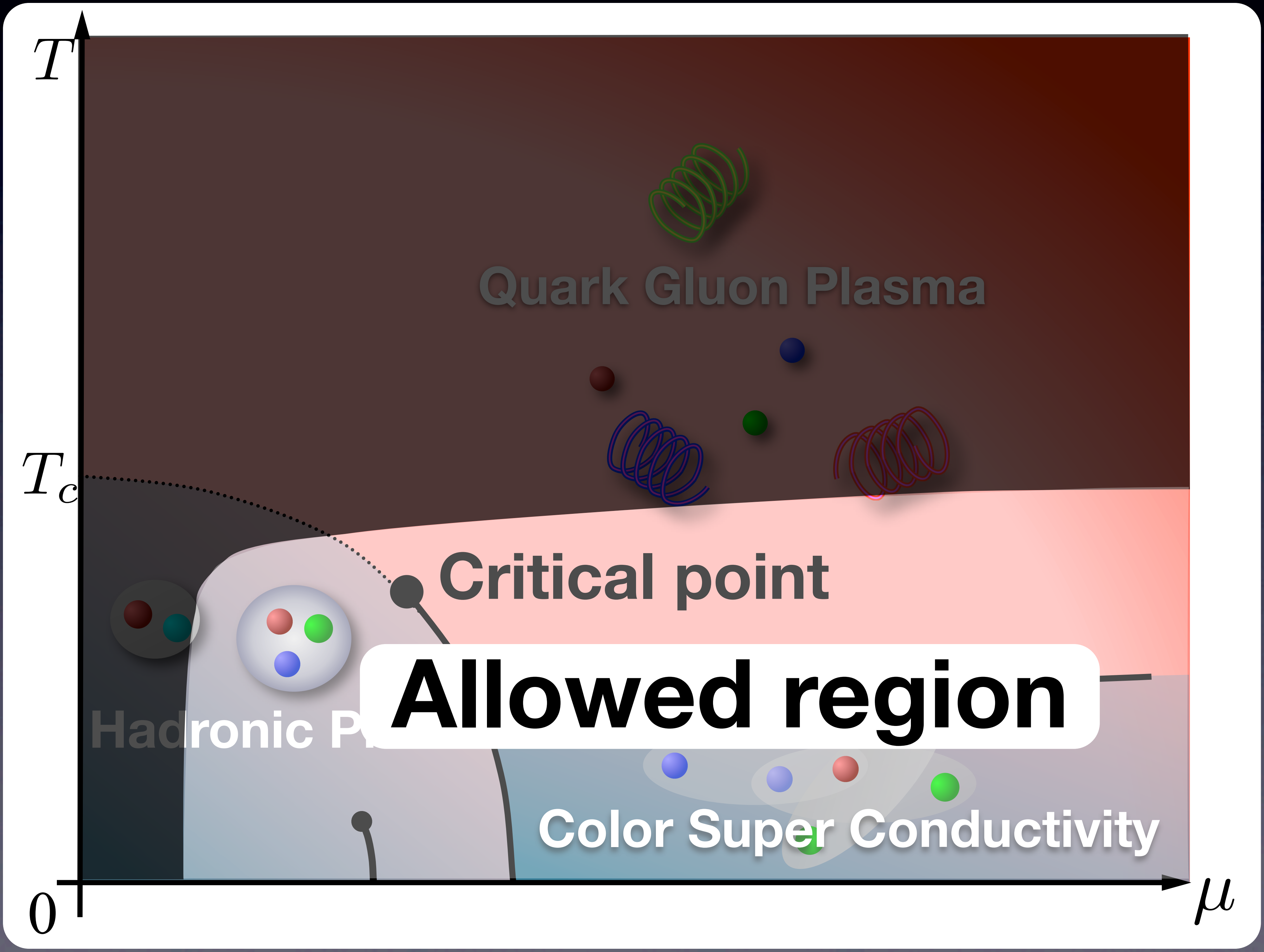
Phase diagram of QCD



QCD critical point



We want to determine the allowed region.



Fluctuation of the order parameter

$$\langle \delta\sigma(\boldsymbol{x})\delta\sigma(0)\rangle \sim \exp(-|\boldsymbol{x}|m_\sigma)$$

At the critical point

$$m_\sigma \to 0$$

QCD inequality

+ some approximations

- Neglecting disconnected diagrams
- Neglecting quark loops mixing flavors

Large-N_c QCD satisfies both approximations!

Several model such as NJL with mean field approximation, random matrix also satisfy.

QCD Inequality

Weingarten ('83), Witten ('83), Nussinov ('84),
Espriu, Gross, Wheater ('84)

At *T=0, μ=0*

For flavor nonsinglet channel

$$m_\Gamma \geq m_\pi$$

No SSB of isospin and baryon symm.

Vafa-Witten ('84)

QCD Inequality

Weingarten ('83), Witten ('83), Nussinov ('84),
Espriu, Gross, Wheater ('84)

Dirac operator $D = \gamma_\mu(\partial_\mu + igA_\mu)$

Anti-Hermite $D^\dagger = -D$

Chiral symmetry $\gamma_5 D \gamma_5 = -D$

$$\Rightarrow \quad \mathcal{D} = D + m$$

$$\det \mathcal{D} \geq 0$$

For isospin chemical potential

$$\tau_1 \gamma_5 \mathcal{D} \gamma_5 \tau_1 = \mathcal{D}^\dagger \qquad \mathcal{D}(\mu_I) = D + \frac{\mu_I}{2}\gamma_0 \tau_3 + m$$

Alford, Kapustin and Wilczek ('99)

QCD Inequality

Weingarten ('83), Witten ('83), Nussinov ('84), Espriu, Gross, Wheater ('84)

Flavor nonsinglet operator: $M_\Gamma(x) = \bar{\psi}\Gamma\psi$

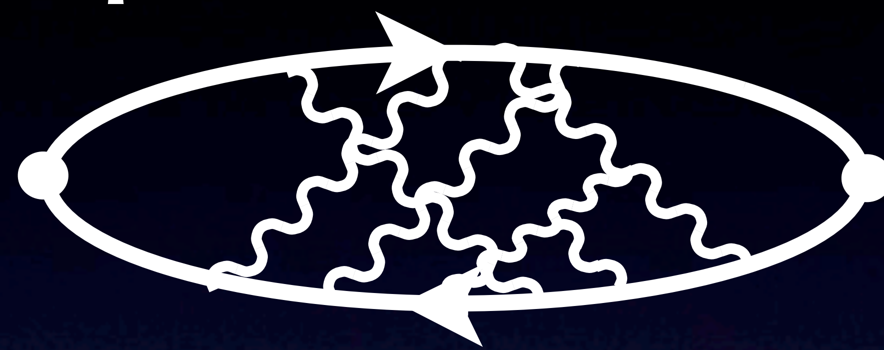
$$\langle M_\Gamma(x) M_\Gamma^\dagger(y)\rangle_{\psi,A} = -\langle \mathrm{tr}[S_A(x,y)\Gamma S_A(y,x)\bar{\Gamma}]\rangle_A$$

Cauchy–Schwarz inequality

$$\langle \mathcal{O}_1 \mathcal{O}_2\rangle \le \sqrt{\langle \mathcal{O}_1 \mathcal{O}_1^\dagger\rangle\langle \mathcal{O}_2 \mathcal{O}_2^\dagger\rangle}$$

QCD Inequality

Flavor nonsinglet operator

$$\langle M_\Gamma(x) M_\Gamma^\dagger(y)\rangle_{\psi,A} =$$ 

$$= -\langle \mathrm{tr}[S_A(x,y)\Gamma S_A(y,x)\bar{\Gamma}]\rangle_A$$

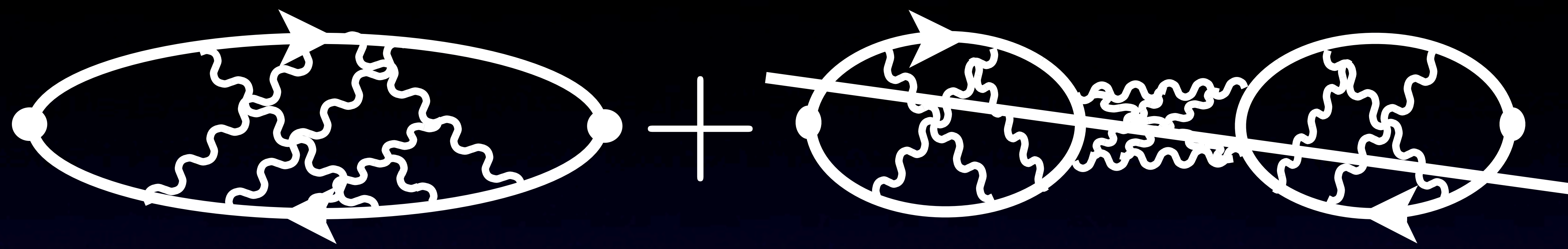
$$\leq \langle \mathrm{tr}[S_A(x,y) S_A^\dagger(y,x)]\rangle$$

$$= \langle M_\pi(x) M_\pi^\dagger(y)\rangle_{\psi,A}$$

$$\Rightarrow \quad m_\Gamma \geq m_\pi$$

$$\langle M_\Gamma(x) M_\Gamma^\dagger(y)\rangle_{\psi,A} \sim \exp(-m_\Gamma |x-y|)$$

Flavor singlet operator



$$\langle M_\Gamma(x) M_\Gamma^\dagger(y)\rangle_{\psi,A} = -\langle \mathrm{tr}[S_A(x,y)\Gamma S_A(y,x)\bar{\Gamma}]\rangle_A$$

$$+\langle \mathrm{tr}[S_A(x,x)\Gamma]\mathrm{tr}[S_A(y,y)\bar{\Gamma}]\rangle_A$$

If disconnected diagram is neglected, $m_\sigma \geq m_\pi$

No second order phase transition as long as $m_\pi \neq 0$

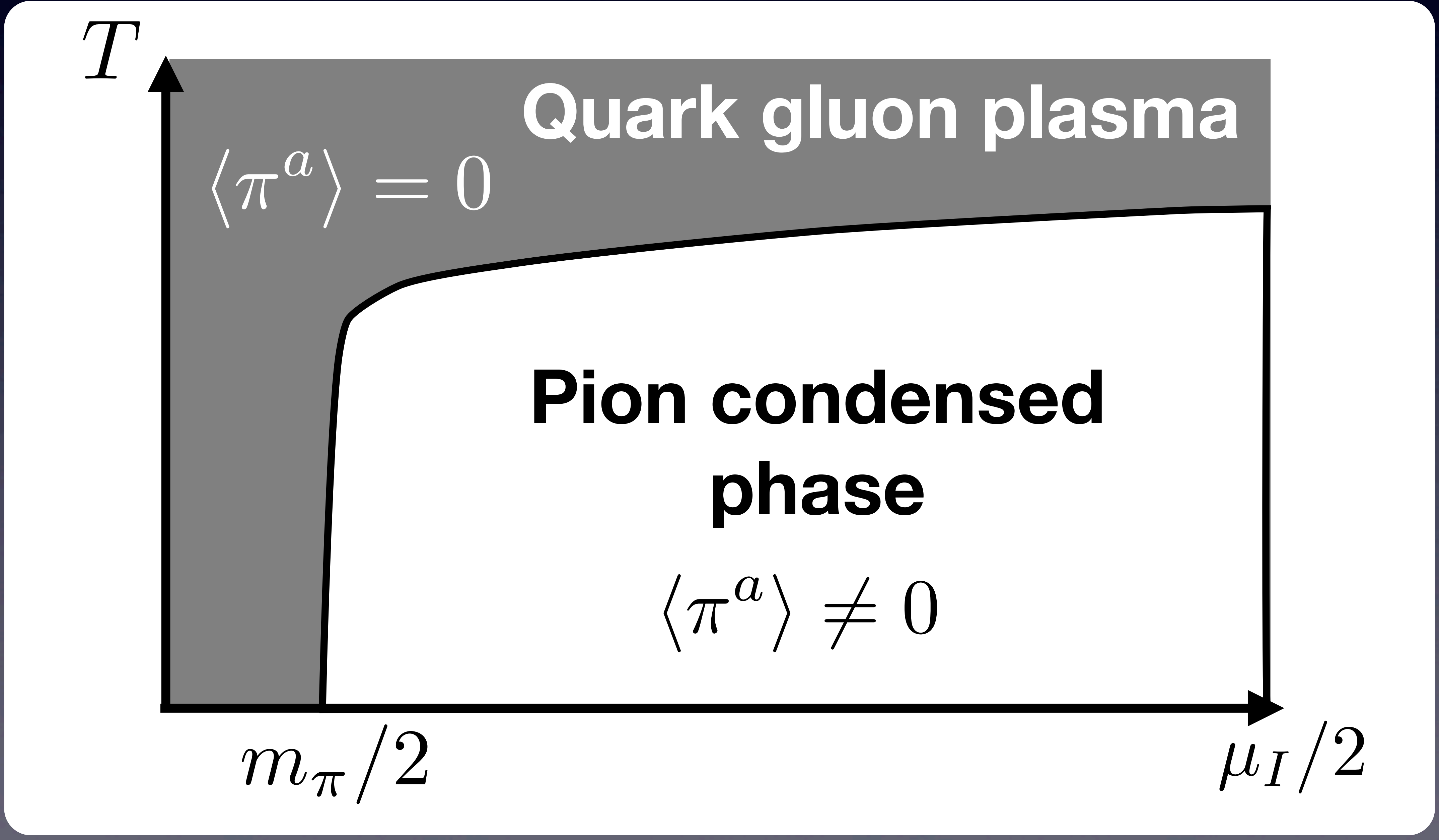
QCD inequality also works at finite *T* and μ_I.

Son and Stephanov ('01)

QCD phase diagram at μ_I

No critical point out side of the pion condensed phase

YH, Yamamoto('11)



QCD phase diagram at *μ*

QCD inequality does not work at *μ*...

If some quark loops mixing flavors are negligible,

i.e., complex phase is negligible,

$$P = p(\mu_u^2, \mu_d^2) + \cancel{p_{\mathrm{mix}}(\mu_u\mu_d, \mu_u^2, \mu_d^2)}$$

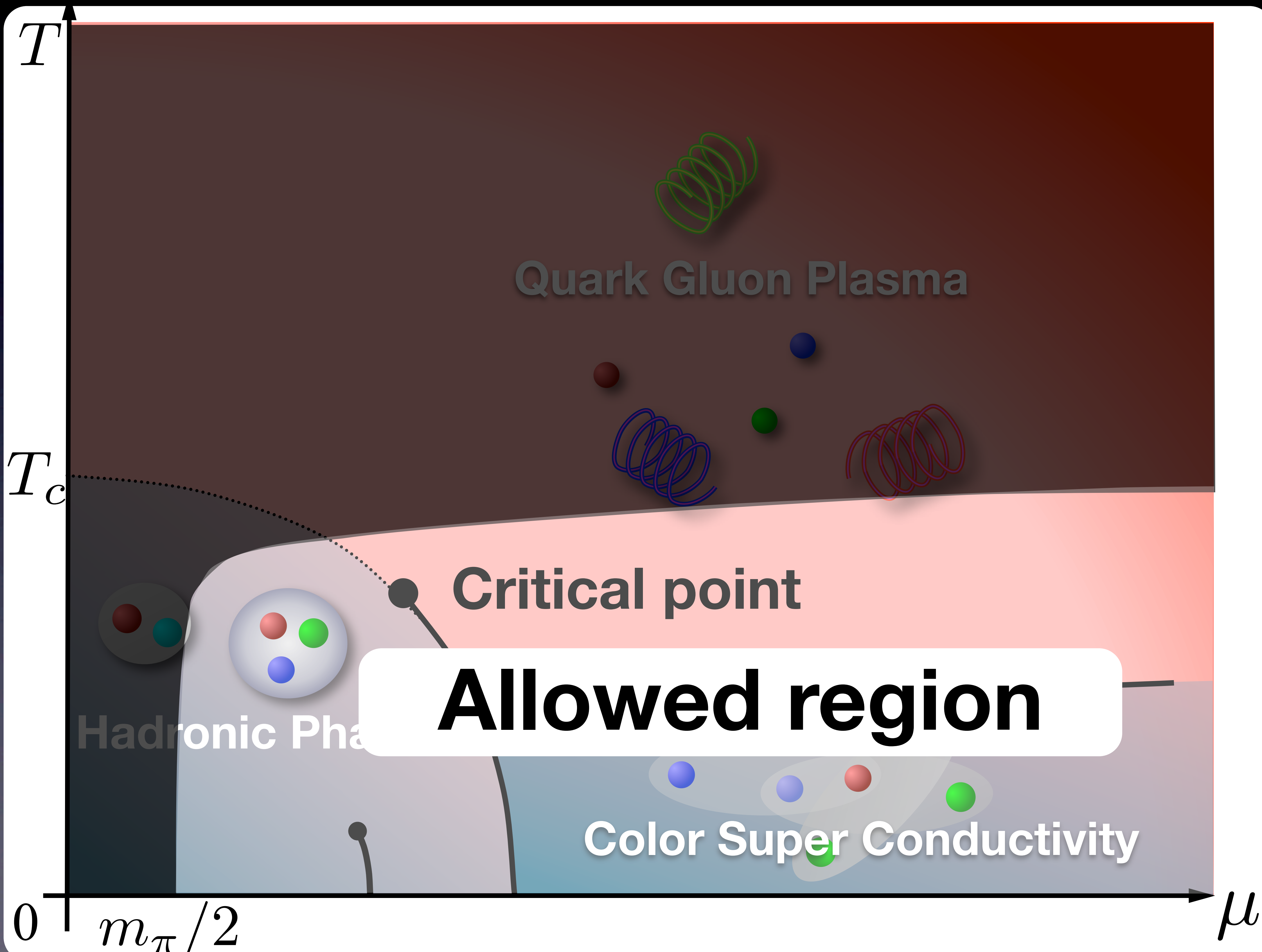
OK, at large-N_c, outside of pion condensed phase

The phase structure at finite *μ* $\approx$ **The phase structure at finite μ_I**

At large N_c, Hanada and Yamamoto ('11)

Phase diagram of QCD

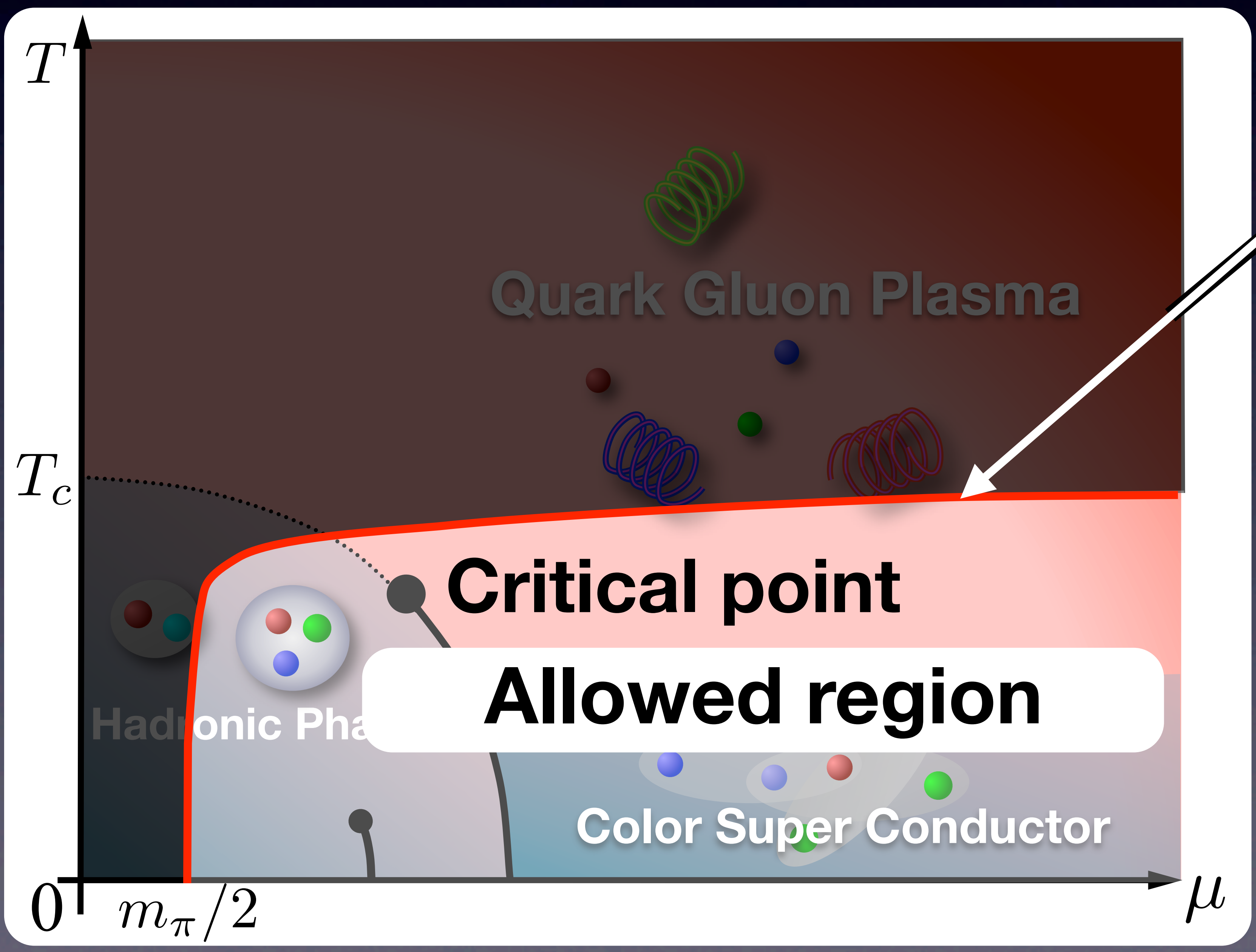
YH, Yamamoto ('11)



Summary

No critical point out side of the pion condensed phase.

(if quark loops and disconnected diagram are suppressed.)



Large N_c QCD, OK.

Lattice QCD can determine the boundary.

cf. Kogut, Sinclair ('04), ('06), ('07),
de Forcrand, Kratochvila ('06)
de Forcrand, Stephanov, Wenger ('07)
Detmold, Orginos, Shi ('12)

Lattice simulation is difficult in the pion condensed phase.

We need to estimate contributions of disconnected diagrams.