Perturbative calculation in 1+1dimensional relativistic viscous hydrodynamics

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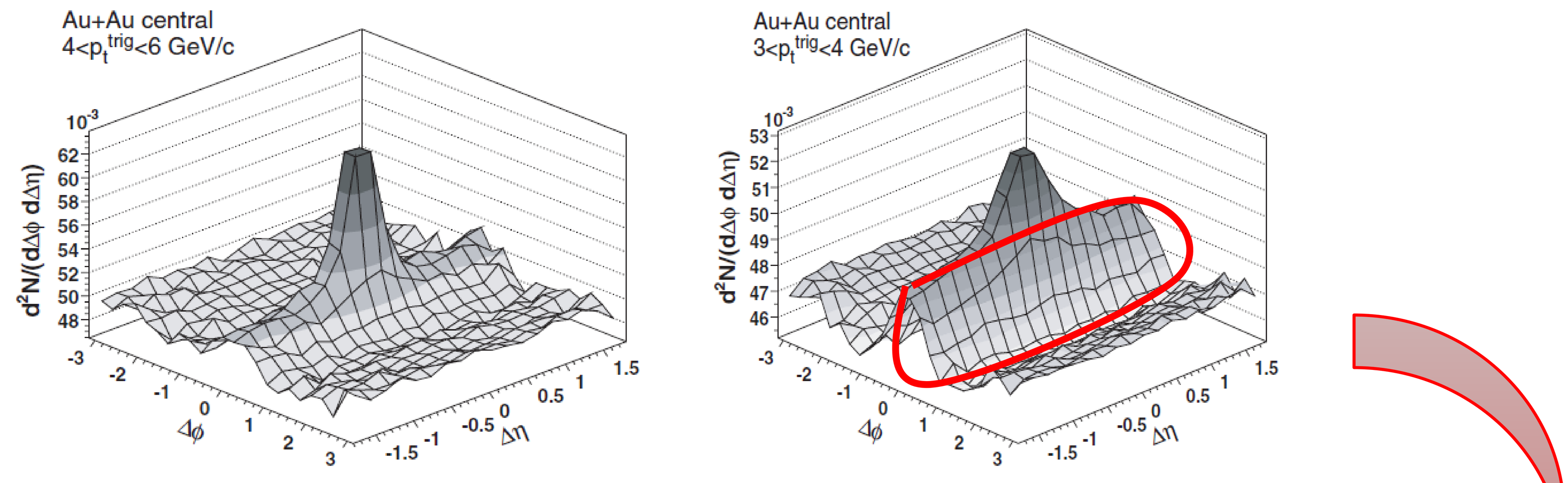
Abstract

We discuss the evolution of the fluctuation in the initial conditions for entropy densities (energy densities) using hydrodynamic calculations. Here we try to explore the origin of the fluctuation. If the local thermal equilibrium is established at an early time of the heavy ion collision and the mean free path of particles produced is sufficiently short, then the evolution of the system may be described by relativistic hydrodynamics. Most of studies have been performed with ideal hydrodynamics; but it is necessary to use viscous hydrodynamics to understand experimental data in detail. In particular, we focus on the dynamics of collision axis and investigate how the viscous effects appear in the framework of perturbation with respect to the bulk and shear viscosities. Assuming that the bulk and shear viscosities are small we calculate the entropy production and the initial condition dependence. For example when the ideal part of solutions is described by Bjorken's solution, we can show the result explicitly.

1. Motivation



Charged di-hadron distribution $\frac{d^2N}{d\Delta\phi\, d\Delta\eta}$ [1]. η:rapidity, ϕ:azimuthal angle.

Ridge
- Long range rapidity correlations up to large rapidity
- Narrow in relative azimuthal angle

We discuss dynamics for the longitudinal direction with hydrodynamic calculation. In particular we investigate whether this structure is formed from the initial condition due to the bulk viscosity.

2. 1+1 dimensional perturbative calculation

$$\partial_\mu T^{\mu\nu} = 0,\ \epsilon = gp$$

variables: (Ref.[2])

$$z^\pm = t \pm z,\quad e^{-\theta} = \frac{T}{T_0},\quad y = \frac{1}{2}\log\frac{u^0+u^1}{u^0-u^1}$$

ϵ:energy density
p:pressure
T:temperature
u^μ:velocity

$$\partial_+ e^{-\theta+y} - \partial_- e^{-\theta-y} + \Delta P = 0$$
$$\partial_+ e^{-g\theta+y} - \partial_- e^{-g\theta-y} + \Delta Q = 0$$

$T_{(\Pi)}{}^{\mu\nu} = \Pi\Delta^{\mu\nu}, (\Delta^{\mu\nu} = u^\mu u^\nu - g^{\mu\nu})$
$\Pi = -\varsigma\partial_\mu u^\mu$

$\Delta P, \Delta Q$: viscous terms

$$\Delta P = \frac{2g^2}{(g+1)^2\epsilon_0} e^{g\theta}\left(-\sinh y\, \partial_\mu T_{(\Pi)}{}^{\mu 0} + \cosh y\, \partial_\mu T_{(\Pi)}{}^{\mu 1}\right)$$
$$\Delta Q = \frac{g}{(g+1)\epsilon_0} e^{\theta}\left(\cosh y\, \partial_\mu T_{(\Pi)}{}^{\mu 0} - \sinh y\, \partial_\mu T_{(\Pi)}{}^{\mu 1}\right)$$

Poincare's lemma: $\exists\Phi$

$$\partial_-\Phi = e^{-\theta+y} + F,\quad \partial_+\Phi = e^{-\theta-y} - F,\quad \left(F(t,z) = \int_0^{2z} dz\, \Delta P(t,z)\right)$$

$$\partial_+\left\{(\partial_-\Phi - F)^{\frac{g+1}{2}}(\partial_+\Phi + F)^{\frac{g-1}{2}}\right\} + \partial_-\left\{(\partial_-\Phi - F)^{\frac{g-1}{2}}(\partial_+\Phi + F)^{\frac{g+1}{2}}\right\} + \Delta Q = 0$$

Thermodynamical quantities:

$$\Phi \to T = T_0(\partial_-\Phi - F)^{\frac{1}{2}}(\partial_+\Phi + F)^{\frac{1}{2}} \longrightarrow \quad \epsilon = \epsilon_0\left(\frac{T}{T_0}\right)^{g+1},\quad p = p_0 g^{-1}\left(\frac{T}{T_0}\right)^{g+1},\quad s = s_0\left(\frac{T}{T_0}\right)^{g}$$

Perturbative

expansion parameter: ζ_0 ($\zeta = \zeta_0\tilde{\zeta}(\tau)$)

$$\Phi = \Phi^{(0)} + \varsigma_0\Phi^{(1)} + \cdots$$

0-th order solution: $\left(\frac{T}{T_0}\right)\Big|^{(0)} = \left(\frac{\tau_0}{\tau}\right)^{1/g}$, $y = \eta$ (Bjorken solution)

1-st order equation:

$$\frac{1}{2\tau_0{}^2} e^{-\left(-\frac{1}{g}+2\right)t}\{g\partial_{tt} - \partial_{xx} - (g-1)\partial_t\}\Phi^{(1)} - \frac{1}{2\tau_0{}^2} e^{-\left(-\frac{1}{g}+2\right)t}\frac{2g}{(g+1)\epsilon_0}\tilde{\zeta} = 0$$

where $t = \log\left(\frac{\tau}{\tau_0}\right), x = \eta$

The homogeneous part of Φ is independent of the bulk viscosity. Namely this form is common in the all orders with respect to ζ_0.

$$\Phi^{(1)} = e^{2\alpha\tilde{t}}\Psi^{(1)},\ \tilde{t} = \frac{t}{\sqrt{g}},\ \alpha = \frac{g-1}{2\sqrt{g}}$$

$$(\partial_{xx} - \partial_{\tilde{t}\tilde{t}} - 2\alpha\partial_{\tilde{t}})\Psi^{(1)} = -\frac{2g}{(g+1)\epsilon_0}\tilde{\zeta} e^{-(2\alpha-\sqrt{g})\tilde{t}}$$

Initial condition: $\Psi^{(1)}\big|_{\tilde{t}=0} = f_1(x), \partial_{\tilde{t}}\Psi^{(1)}\big|_{\tilde{t}=0} = g_1(x)$

$$y^{(1)} = -\frac{1}{8\tau_0} e^{-\left(1-\frac{1}{g}\right)t}\partial_x\Phi^{(1)},\ e^{-\theta}\Big|^{(1)} = \frac{g}{2\tau_0} e^{-t}\partial_t\Phi^{(1)}$$

Solution with the Green's function method

$$\Phi^{(1)} = \Phi^{(1)}_{\rm src} + \Phi^{(1)}_{\rm IF1} + \Phi^{(1)}_{\rm IF2} + \Phi^{(1)}_{\rm IG} + \Phi^{(1)}_{\rm non-int}$$

where

$$\Phi^{(1)}_{\rm src} = \frac{g}{2\alpha g_+\epsilon_0}\left[-\frac{1}{\frac{(1+\beta)\sqrt{g}}{2} - \alpha}\left(e^{2\alpha\tilde{t}} - e^{(1+\beta)\sqrt{g}\tilde{t}}\right) + \frac{2}{(1+\beta)\sqrt{g}}\left(1 - e^{(1+\beta)\sqrt{g}\tilde{t}}\right)\right]$$

$$\Phi^{(1)}_{\rm IF1} = \alpha\tilde{t}\frac{e^{\alpha\tilde{t}}}{2}\int_{-1}^{1} du I_0(\alpha\tilde{t}\sqrt{1-u^2}) f_1(x+\tilde{t}u)$$

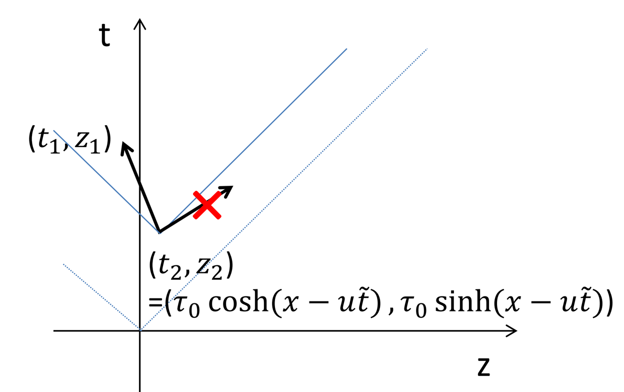
$$\Phi^{(1)}_{\rm IF2} = \alpha\tilde{t}\frac{e^{\alpha\tilde{t}}}{2}\int_{-1}^{1} \frac{du}{\sqrt{1-u^2}} I_1(\alpha\tilde{t}\sqrt{1-u^2}) f_1(x+\tilde{t}u)$$

$$\Phi^{(1)}_{\rm IG} = \alpha\tilde{t}\frac{e^{\alpha\tilde{t}}}{2}\int_{-1}^{1} du I_0(\alpha\tilde{t}\sqrt{1-u^2}) g_1(x+\tilde{t}u)$$

$$\Phi^{(1)}_{\rm non-int} = \frac{e^{\alpha\tilde{t}}}{2}\left(f_1(x+\tilde{t}) + f_1(x-\tilde{t})\right)$$

- modified Bessel function: I_0, I_1
- $\zeta = \zeta_0 e^{2\beta t}$ (β: parameter)

Causality

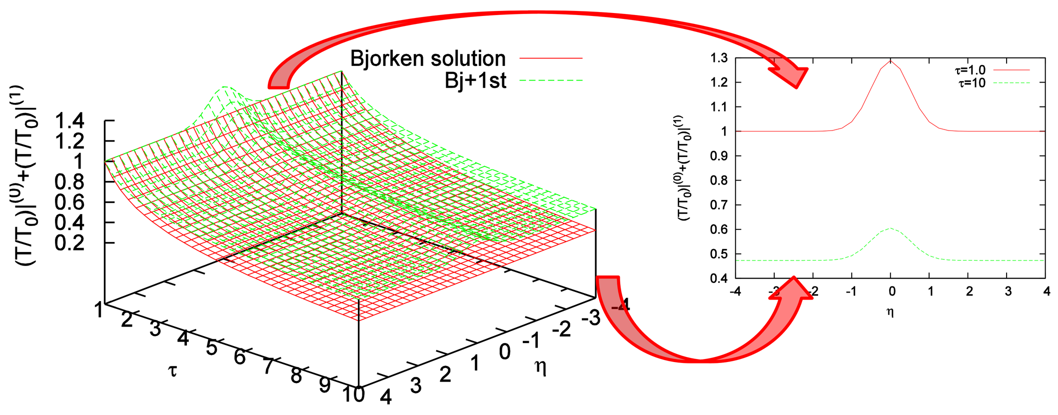


Based on the solution form, we can confirm if the causality is conserve or not. Indeed it is possible to verify that the equations for Φ in terms of all orders ζ satisfy the causality.

Results

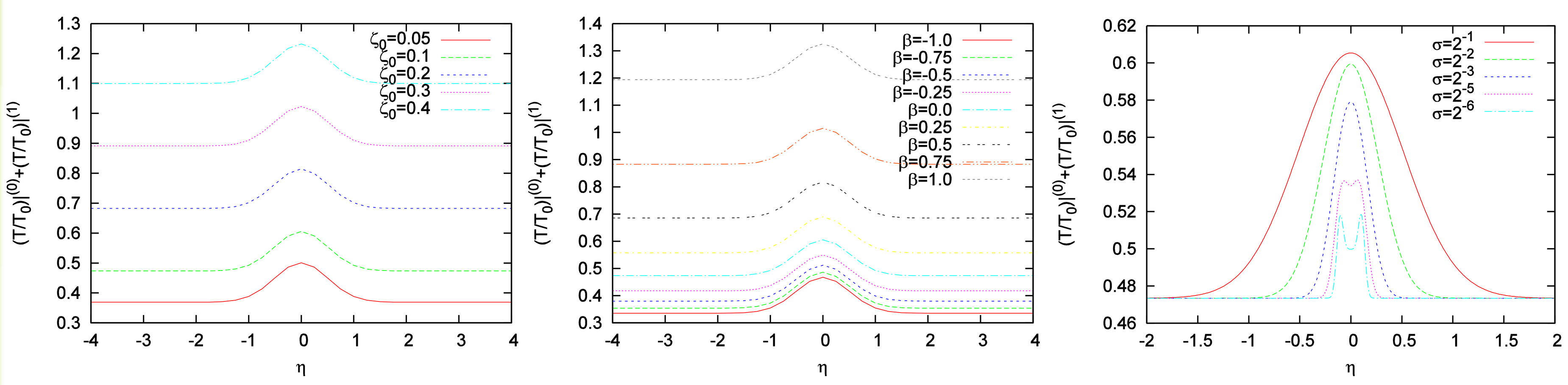
Parameters: $\tau_0 = 1.0$(fm), $\epsilon_0 = 1.0$(GeV^2), $g = 3$, $\zeta_0 = 0.1$, $\beta = 0.0$

Initial conditions: $e^{-\theta}\big|^{(1)}_{\tau=\tau_0} = \left(\frac{T}{T_0}\right)\Big|^{(1)}_{\tau=\tau_0} = 5e^{-2\eta^2}$, $y|^{(1)}_{\tau=\tau_0} = 0$



- shown comparison between Bjorken solution and our result
- Our result finds that the dissipation driven by the bulk viscosity is more effective.

➢ rapidity dependence at $\tau = 10$ (fm/c).



- extrapolation of the expansion parameter ζ_0: $\zeta_0 \gtrsim 0.3 \to T^{(0)} \gtrsim T^{(1)}$
- proper time dependence of the bulk viscosity: $\beta = 0 \leftrightarrow \zeta = $ const, $\beta = -0.5 \leftrightarrow \zeta \propto s$
- initial condition σ dependence: $\left(\frac{T}{T_0}\right)\Big|^{(1)}_{\tau=\tau_0} = 5e^{-\frac{\eta^2}{2\sigma^2}}$

3. Summary

- We formulate the perturbative calculation for 1+1dimensional relativistic viscous hydrodynamics.
- We show the space-time evolution of the temperature T.
- The causality is not violated in all orders.
- We confirm the dissipation due to the effect of the bulk viscosity and find the structure around the mid-rapidity when the width η at the initial condition is small.

Reference

[1] B. I. Abelev et al. [STAR Collaboration], Phys. Rev. C 80, 064912 (2009) [arXiv:0909.0191 [nucl-ex]].
[2] R. Peschanski and E. N. Saridakis, Nucl. Phys. A **849**, 147 (2011) [arXiv:1006.1603 [hep-th]].