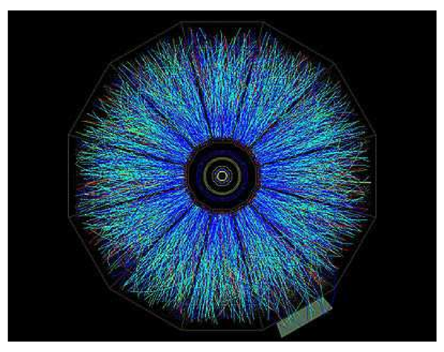

Quarkyonic percolation

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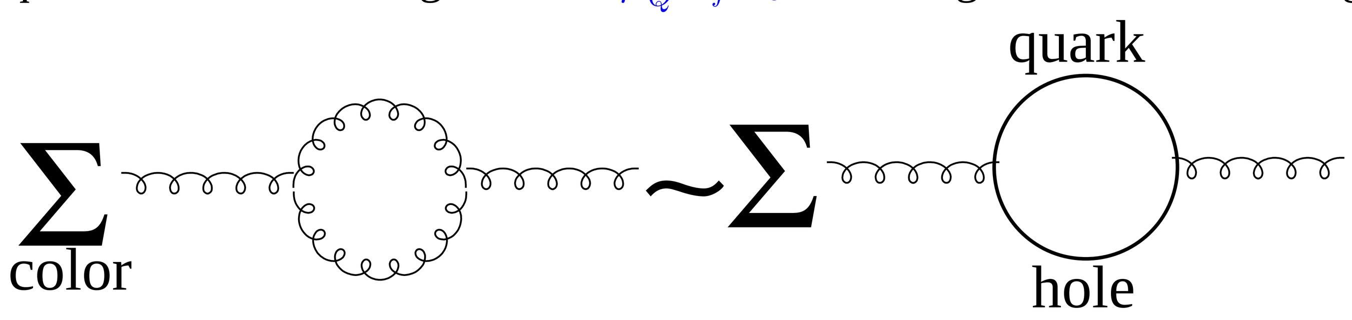
Introduction: quarkyonic matter

Let us think about QCD at moderate temperature and chemical potential ($T \leq T_c, \mu_Q \sim \Lambda_{QCD}$) at the large $N_c \gg N_f \gg 1$ limit

In momentum space , infrared slavery should still be maintained between $\mu_Q \sim \Lambda_{QCD}$ and $\mu_Q \sim \sqrt{N_c/N_f}\Lambda_{QCD}$. This is because quark-hole screening $\sim \mu_Q^2 N_f N_c$ and gluon antiscreening $\sim N_c^2$.

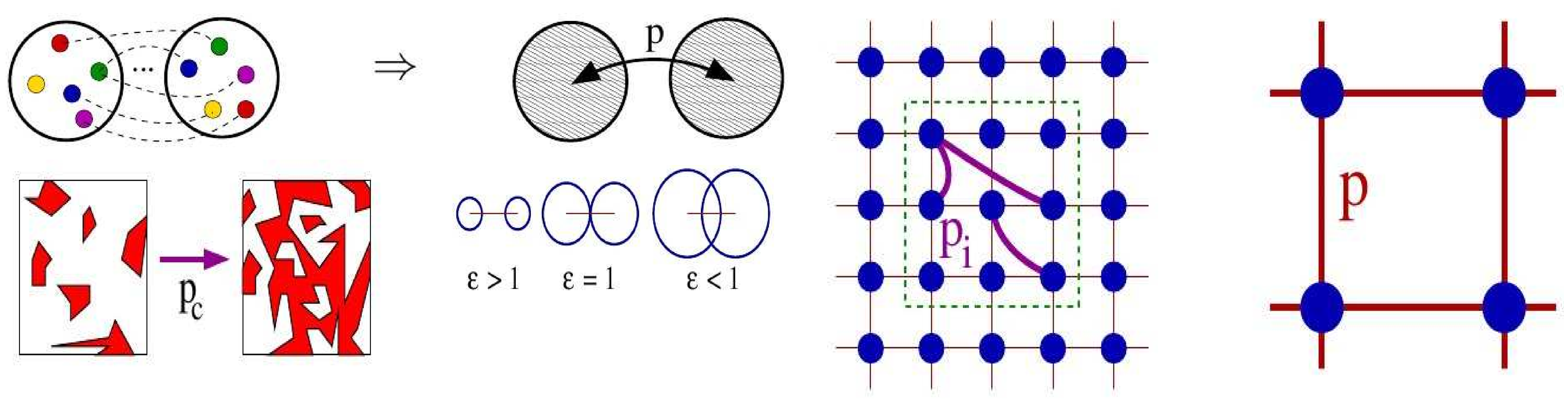


higher order diagrams will not change this power

In configuration space , baryons should "touch"/"overlap" with each other, since N_c quarks in a baryon, quarks of neighboring baryons at $\sim N_c^{-1/3}\Lambda_{QCD} \to 0$ apart

So matter seems to be both confining and asymptotically free. potentially very interesting. [1] suggested the "quarkyonic" matter $\Lambda_{QCD} \geq \mu_Q \leq \sqrt{N_c/N_f}\Lambda_{QCD}$ has quark degrees of freedom below the fermi surface and baryonic degrees of freedom at the Fermi surface. This would imply pressure and entropy $\sim N_c$ even in confining matter.
Interesting, but... there is no controlled approximation in this regime: lattice has sign problem, EFTs diverge if $T, \mu_Q \sim \Lambda_{QCD}$, and $N_c \to \infty$ for semiclassical Gauge/string.

Some insight from percolation [3,4]



Look for a percolation type transition: integrate out quark degrees of freedom into baryon-baryon interactions

$$p = 1 - \left(q_{(1),ij}\right)^{(N_c)^\alpha} \qquad q_{(1),ij} = \int f_A(x_i)\mathrm{d}x_i \int f_B(x_j)\mathrm{d}x_j \left(1 - F(|x_i - x_j|)\right)$$

Then decimate a lattice into a super-lattice,and look for a fixed point
We assume [3] a "hard-sphere" distribution for $f_{A,B}(x)$, and a propagator respecting the scaling expected from QCD, a coupling constant λ/N_c and a range $r_T \sim \Lambda_{QCD} \sim N_c^0$ Propagator choices $g(y) = (\lambda N_c^{-1}) F_{K,S,T}(y)$ include popular functions found in the literature [2]

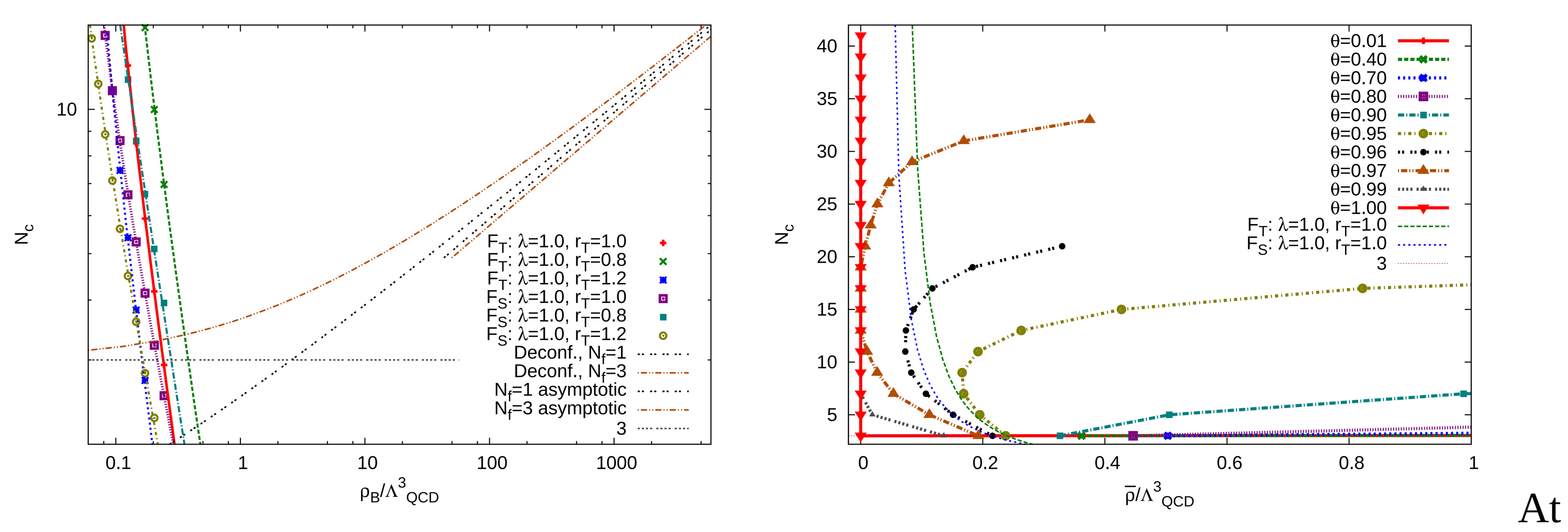
$$F_T(y) = \Theta\left(1 - \frac{y}{r_T/\Lambda_{QCD}}\right), F_K(y) = \frac{2r_T^2}{\pi y^2}\sin^2\left(\frac{y}{r_T/\Lambda_{QCD}}\right), F_S(y) = F_K(y)e^{-M|y|}$$

The first is a Θ -function in configuration space, the second a Θ -function in momentum space and the third a Debye-screened Θ -function in momentum space.
Not deconfinement ($\mu_q^{deconf} \sim N_c\Lambda_{QCD}$) . but percolation, deconfinement cross@N_c^{crit}
What is N_c^{crit} ? Relationship to $\left(N_c^{crit}\right)_{confinement}$?

Percolation and deconfinement

We model $T \ll \Lambda_{QCD}, \mu_Q \sim \Lambda_{QCD}$ matter by an ideal gas EoS (interactions, while not weak, impact pressure rather than density). For $\mu_Q \sim N_c\Lambda_{QCD}$, the baryonic density is

$$\rho_B^{\mathrm{conf}}(T=0) = \Lambda_{\mathrm{QCD}}^3 \frac{1}{6\pi^2} g_f g_s \frac{N_c^3}{N_f^{3/2}} (N_c - N_f)^{3/2} \sim \frac{g_f g_s}{6\pi^2} \Lambda_{\mathrm{QCD}}^3 \frac{N_c^{9/2}}{N_f^{3/2}}$$



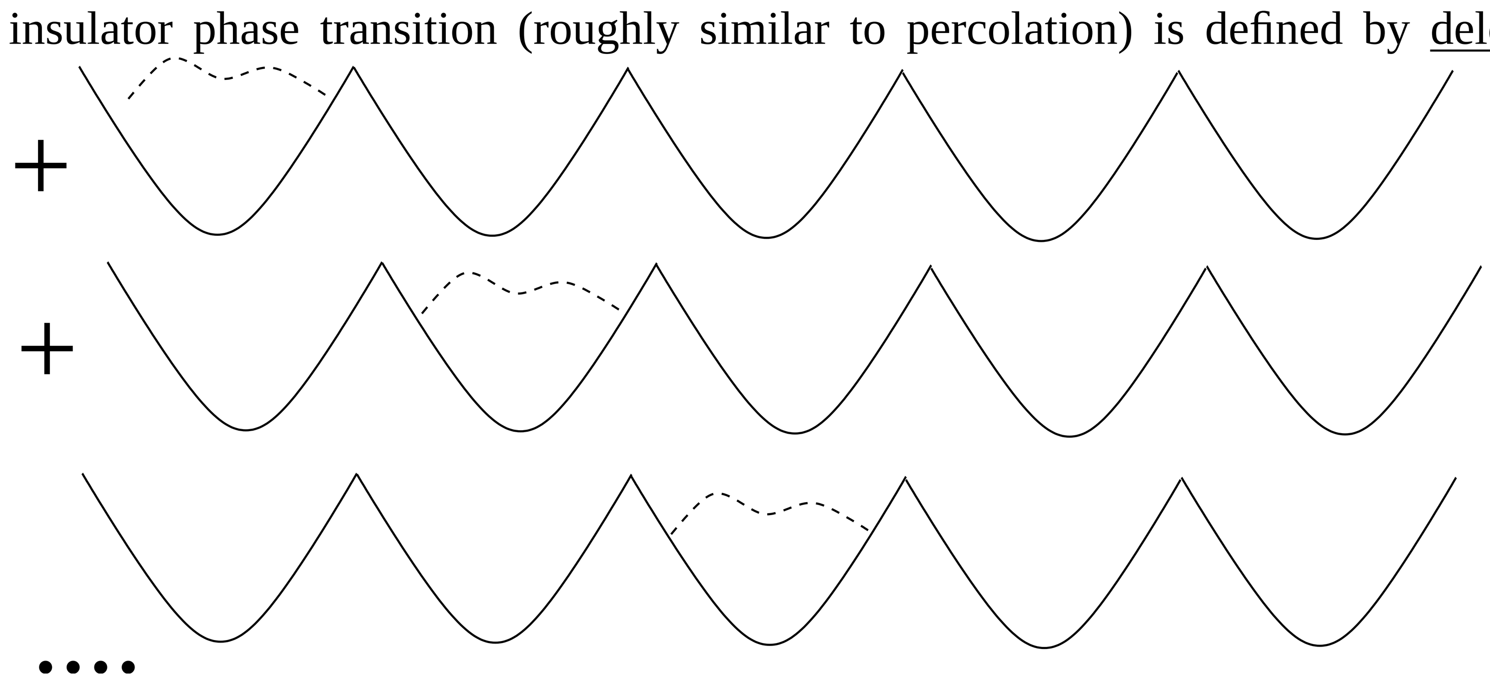
At $T \sim \mu_Q \sim \Lambda_{QCD}$ confinement $\sim N_c^0$. Account for curvature in $T - \mu_Q$ space by parametrizing phase diagram $1 - \theta^2 = \left(\frac{\mu_B^{\mathrm{conf}}}{\mu_0}\right)^2$, $\theta = \frac{T}{T_c} \simeq \frac{3}{2}\frac{T}{\Lambda_{QCD}}$ finite density ρ_B is

$$\rho_B^{\mathrm{conf}} = \sum_{\eta=0,1...}^{Q} (2\eta+2)\left\{ \int \frac{\alpha^2 \mathrm{d}\alpha}{1 + \exp\left[\frac{3}{2}\frac{N_c}{\sqrt{N_f}}\frac{1}{\theta}\left(\sqrt{\alpha^2 + N_f + \eta\frac{\sqrt{N_f}}{N_c^2}} - \sqrt{N_c}\sqrt{1-\theta^2}\right)\right]} - \int \frac{\alpha^2 \mathrm{d}\alpha}{1 + \exp\left[\frac{3}{2}\frac{N_c}{\sqrt{N_f}}\frac{1}{\theta}\left(\sqrt{\alpha^2 + N_f + \eta\frac{\sqrt{N_f}}{N_c^2}} + \sqrt{N_c}\sqrt{1-\theta^2}\right)\right]}\right\} \frac{4\pi g_f}{(2\pi)^3}\frac{N_c^3}{N_f^{3/2}}\Lambda_{\mathrm{QCD}}^3$$

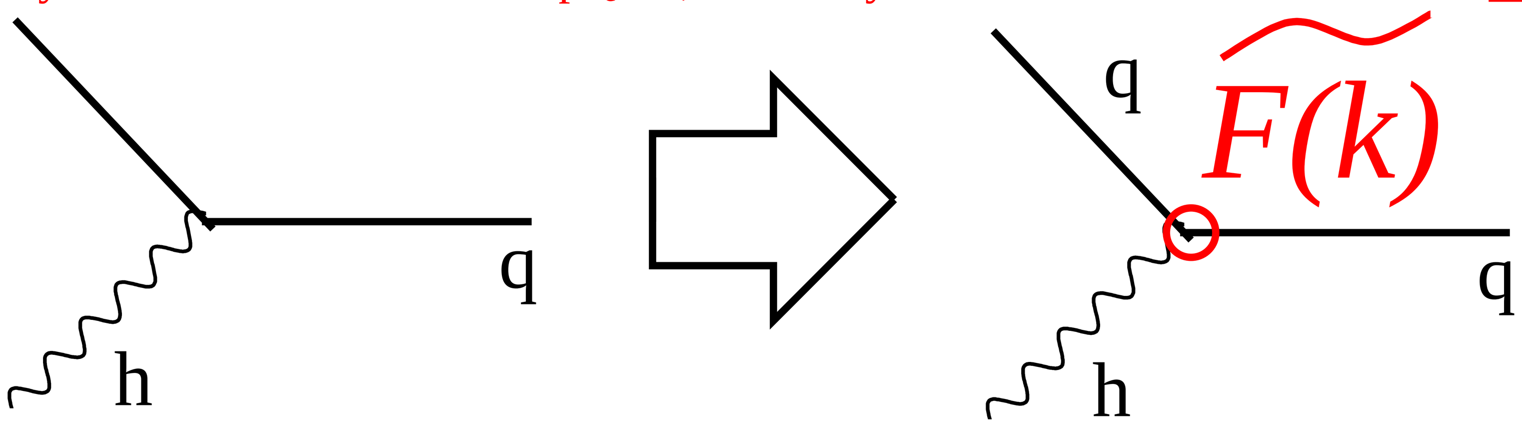
It looks like space for a sliver of confining but percolating quark matter remains@$N_c = 3, \Lambda_{QCD}\mu_Q < \mathcal{O}(2)\,\Lambda_{QCD}$

Theory of quarkyonic percolation

We will try to outline what an EFT of quarkyonic matter looks like from the point of view of large N_c and percolation. A ready analogy is the electron gas model of the metal, where atoms are semi-classical potential wells (analogous to large N_c baryons) and electrons are quantum particles moving in them (analogous to quarks). The conductor-insulator phase transition (roughly similar to percolation) is defined by delocalization.



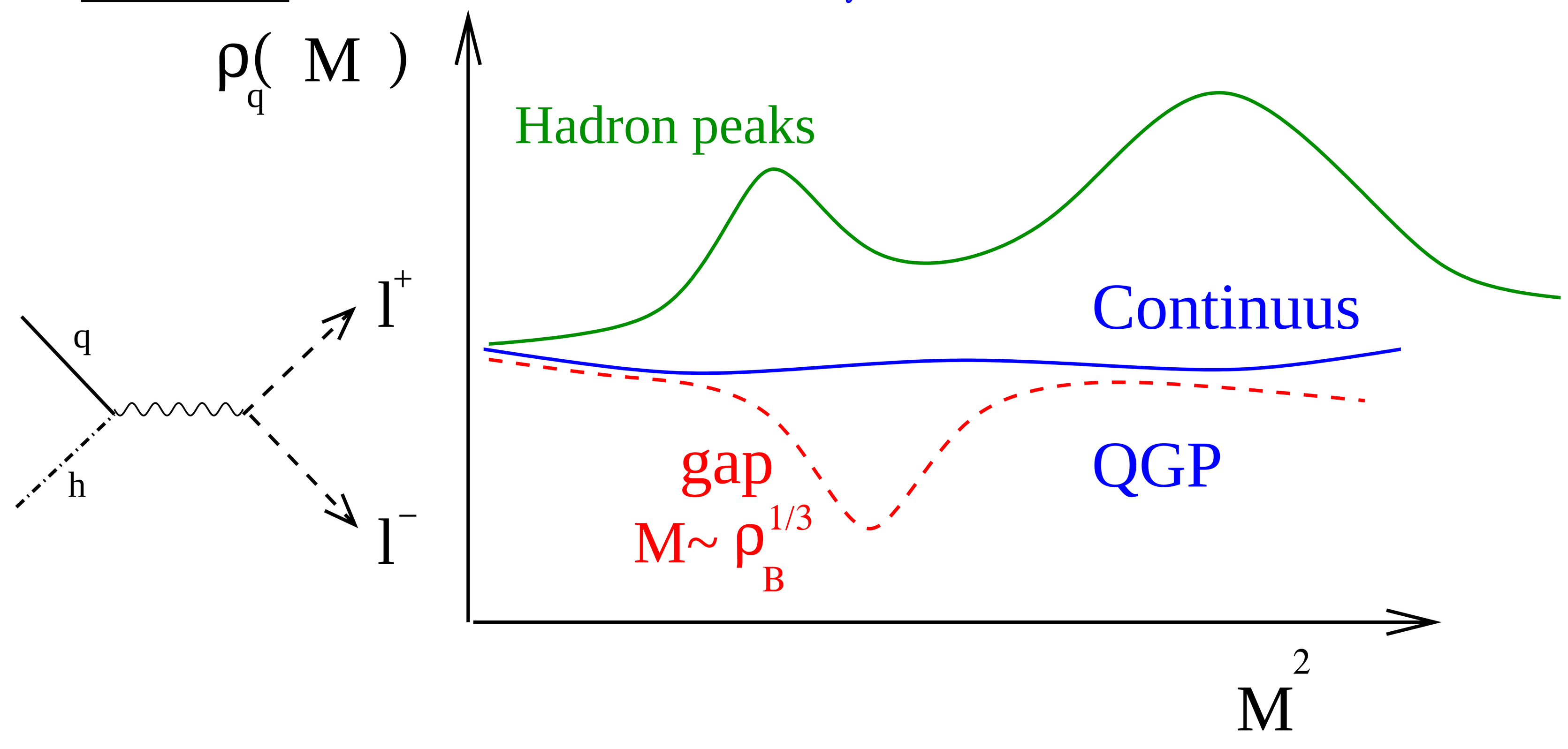
In the percolating phase, quark wavefunctions are delocalized w.r.t. baryons. Color is local on scales $\geq \Lambda_{QCD}^{-1}$ but, because of delocalization, this does not mean quark wavefunctions at large distances are necessarily correlated (models like WZW can be modified to explicitly eliminate such long-range correlations).
Dynamics could look like pQCD, but baryon mean fields remain as a form factor



So, confinement is encoded in the baryonic form-factors, but baryons the partition function is still pQCD like, a la [1]. In particular, p,s $\sim N_c$. In baryon matter, $e \sim N_c$ but p,s $\sim N_c^0$, so these two phases should be distinguisheable.

Phenomenology of quarkyonic percolation

FAIR/RHIC/NICA the logical place to see the form factor is in dilepton spectral functions, $q\bar{q} \to ll$. Note that periodicity of form factor will lead to suppression of signal at $M^2 \sim \Lambda_{QCD}^2$ (usually signal is enhancement)



neutron stars $P \sim N_c$ means a "quasi-QGP" is formed at $\mu_B \sim m_B$, but without mixed phase (percolation is second order. $P \sim N_c \sim \mathcal{O}(3)$ in proto-neutron stars means proto-neutron star much stiffer than "normal" nuclear proto-neutron star. Could quarkyonic matter make stars explode ?

Conclusions, discussion and outlook

- A percolation-type "quarkyonic" transition, distinct from deconfinement, appears natural in $T - \rho - N_c$ space
- At $\mu_Q \sim \Lambda_{QCD}$ the critical $N_c \sim \mathcal{O}(10)$, so nuclear density not percolation. however, percolation could happen at $\Lambda_{QCD} < \mu_Q < \sqrt{N_c/N_f}\Lambda_{QCD}$
- A natural analogy for constructing an EFT of percolating matter is the con ductor-insulator transition, baryons fulfil the role of "atomic potential wells", quarks of the "free electron gas"
- This picture has phenomenological consequences: A dip in the spectral function accessible at FAIR, and the effect of "electron gas pressure" in neutron and proto-neutron stars

References

[1] L. McLerran and R. D. Pisarski, Nucl. Phys. A **796**, 83 (2007)
[2] T. Kojo, Y. Hidaka, L. McLerran and R. D. Pisarski, Nucl. Phys. A **843**, 37 (2010)
[3] S. Lottini and G. Torrieri, Phys. Rev. Lett. **107**, 152301 (2011)
[4] S. Lottini and G. Torrieri, arXiv:1204.3272 [nucl-th].