Phase diagram and fluctuations from lattice QCD

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Outline

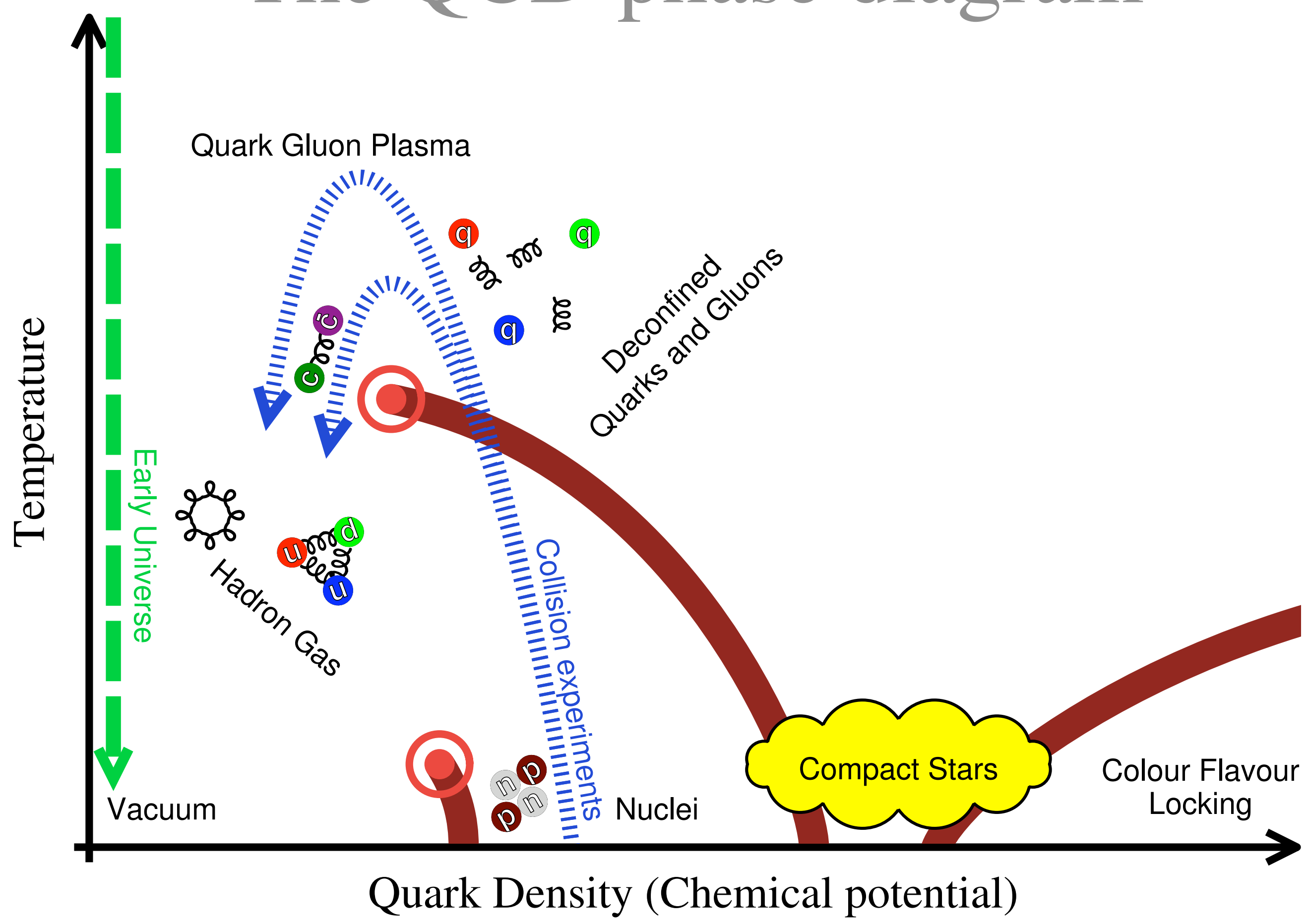
1) Lattice actions

2) Transition temperature:
   - a) zero μ
   - b) O(4) scaling
   - c) finite B
   - d) finite μ

3) Equation of state
   - a) zero μ
   - b) effect of the charm
   - c) finite μ

4) Fluctuations of conserved charges
   - a) second order cumulants at zero μ
   - b) freeze-out conditions
   - c) higher order cumulants

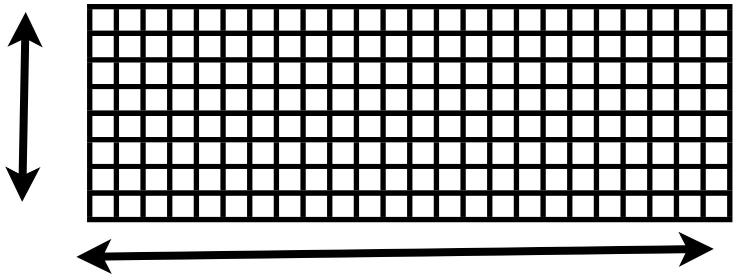
The QCD phase diagram



Lattice QCD

Lattice action is the formula used to discretize QCD on a space-time grid.

$$N_t = \frac{1}{aT}$$



$$N_s = L/a$$

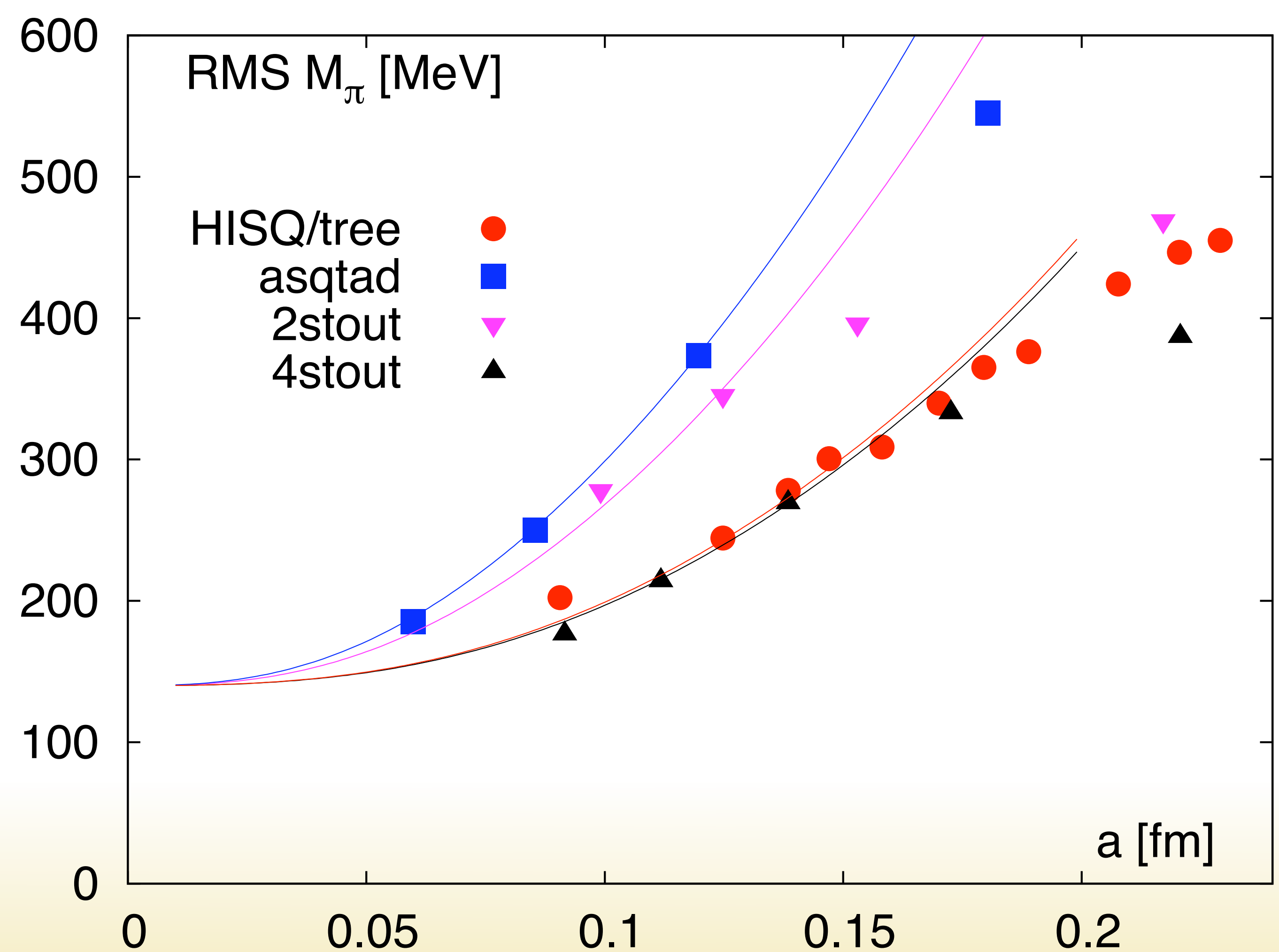
Typical lattice spcaings (T=150 MeV)

N_t =	6	8	10	12	16
a =	0.219	0.164	0.132	0.11	0.082

The most important staggered lattice artefact is **taste breaking**:

Gluon fields feel pions heavier than the lowest energy state, the pseudo-Goldstone boson.

This artefact is characterized by the "average" pion mass, which is restored to physical in the continuum limit.



Good action: small RMS pion mass.

Transition temperature

The transition is a crossover. [Aoki et al Nature 443 (2006) 675-678]

chiral Tc	Budapest-Wuppertal	BNL-Bielefeld	MILC
2004			169(14)(4)
2006	151(3)(3)	192(4)(7)	
2009	146(2)(3) - 157(3)(3)		
2010	147(2)(3) - 155(3)(3)		
2012		154(8)(1)	

BW: hep-lat/0609068
0903.4155
1005.3508

MILC: hep-lat/0608013

BNL-Bielefeld: hep-lat/0405029

HotQCD: 1111.1710

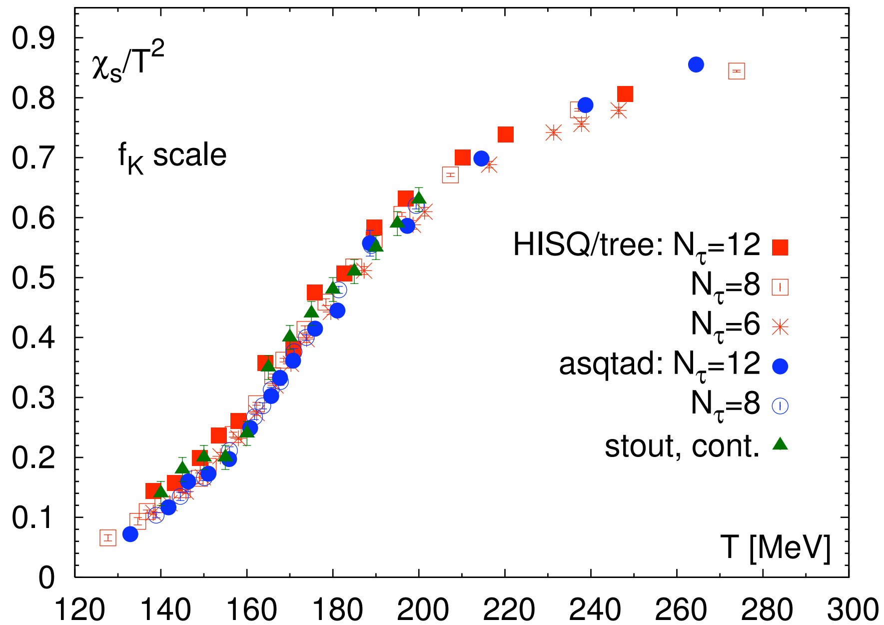
Two different strategies:

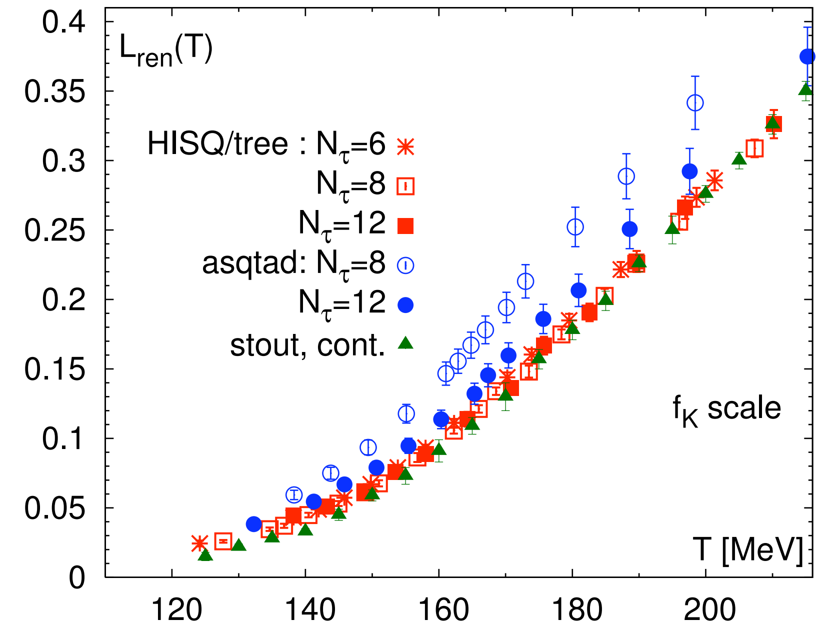
HotQCD has addressed the problem of taste breaking with the introduction of the **HISQ action** and adopted the **scale setting** f_K, also used by Wupperal-Budapest.
The non-physical quarks they used to systematically extra/inter-polate to the physical point using the **O(4) scaling**.

Wuppertal-Budapest is using the cheaper stout action,
working in the physical point with **finer lattices**: N_t=16.

Deconfinement aspects

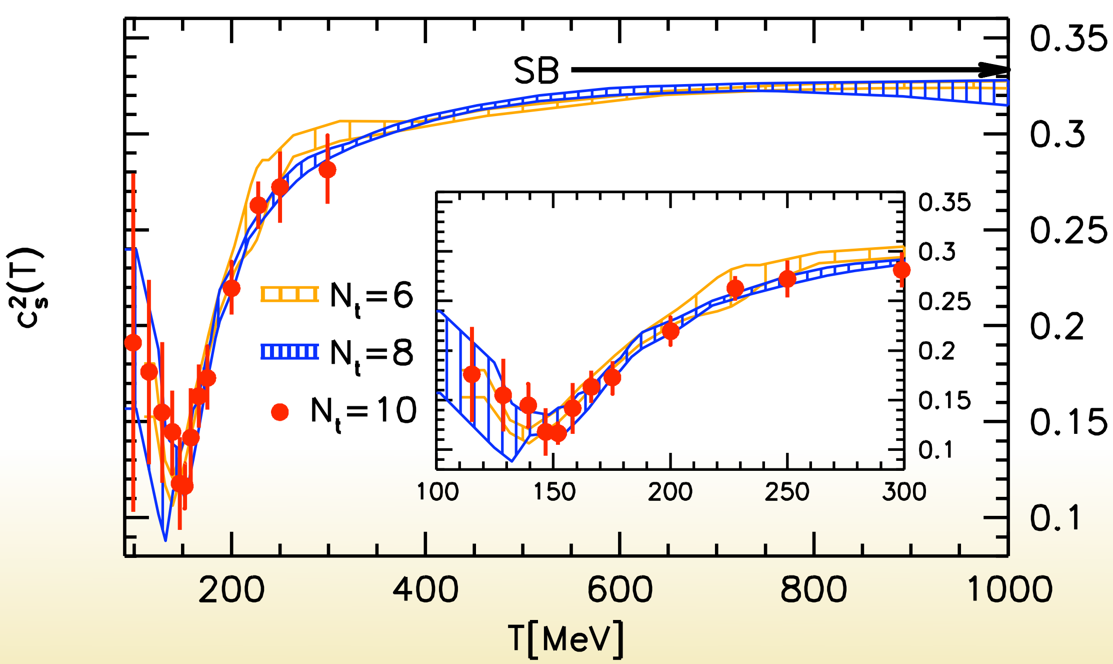
It is difficult to identify a deconfinement transition temperature.





The temperature dependences are in agreement, nevertheless.

Other possibilities: from the equation of state



	characteristic temperature
speed of sound	145(5)
energy density	157(4)(3)
strange susceptibilty	165(5)(3)

T_c in external magnetic field

At non-central collisions very strong magnetic fields arise: $\sqrt{eB} \sim 0.3\mathrm{GeV}$

[Skokov et al 0907.1396]

Linear sima model with Polyakov loop, PNJL predicts increasing Tc with B.

[Mizher et al 1004.2712][Fraga&Mizher 0810.3693][Gatto&Ruggieri 1012.1291][Skokov 1112.5137]

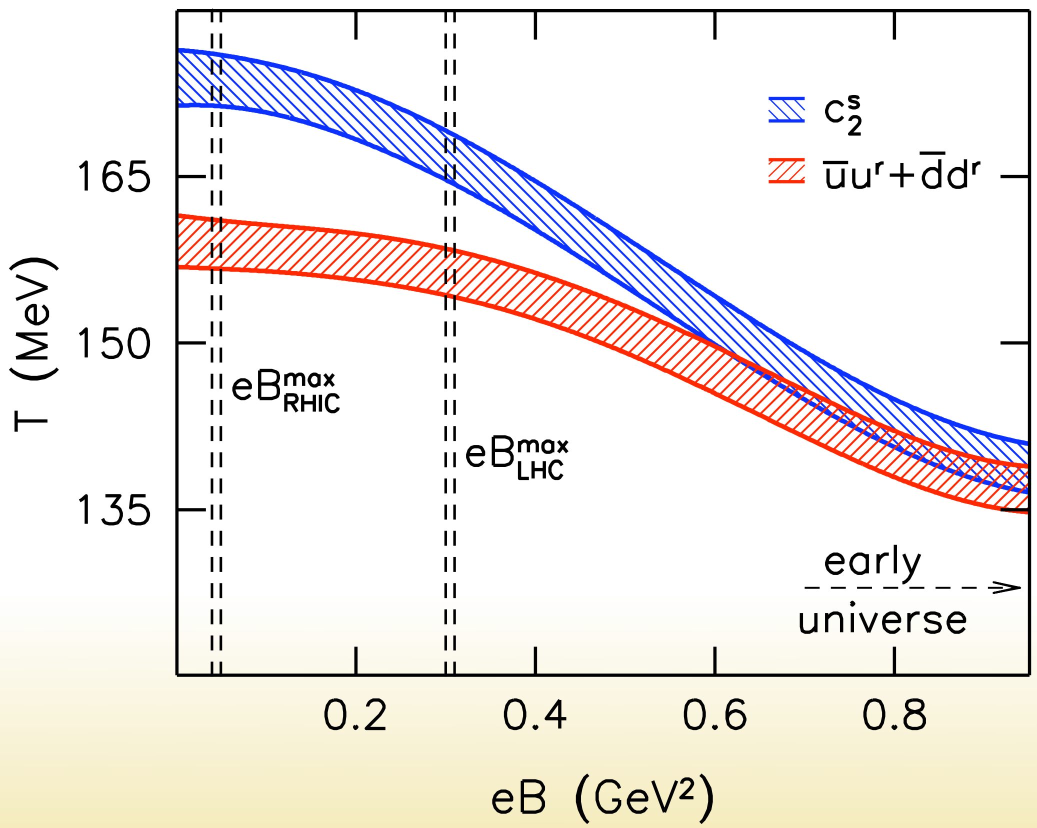
Other models (Nf=2 chiral perturbaton, chromomagnetic fields) predict the opposite.

[Agasian&Fedorov 0803.3156][Cea&Cosmai hep-lat/0204023,05050007][Cea et al 0707.1149]

What does lattice say?

Exploratory studies [D'Elia et al 1005.5365,1103.2080] [Ilgenfritz et al 1203.3360]

Large scale study (physical point, contiuum limit) [Bali et al 1111.4956,1206.4205]



Transition temperature drops with B, and stays a crossover, in nature.

No evidence for the separation of the chiral and deconfinement transitions.

An other set of simulations with heavy pions showed a mild increase of Tc.

Curvature of the transition line

At finite chemical potental direct simulation of QCD is not possible, but indirect methods *(reweighting, extrapolation of imaginary-μ results, Taylor expansion)* may work. see [Gupta QM12]

The curvature transition line can be expressed in terms of **Taylor coefficients** of an order parameter of the transition.

$$\kappa = -T_c \left. \frac{\mathrm{d}T_c(\mu^2)}{\mathrm{d}(\mu^2)} \right|_{\mu=0}$$

(Nt=8, scaling arguments) [Kaczmarek et al 1011.3130]

(Nf=2, isospin/baryon comparison) [Cea et al 1202.5700]

(continuum,physical point) [Endrodi et al 1102.1356]

Endrodi et al (continuum result):

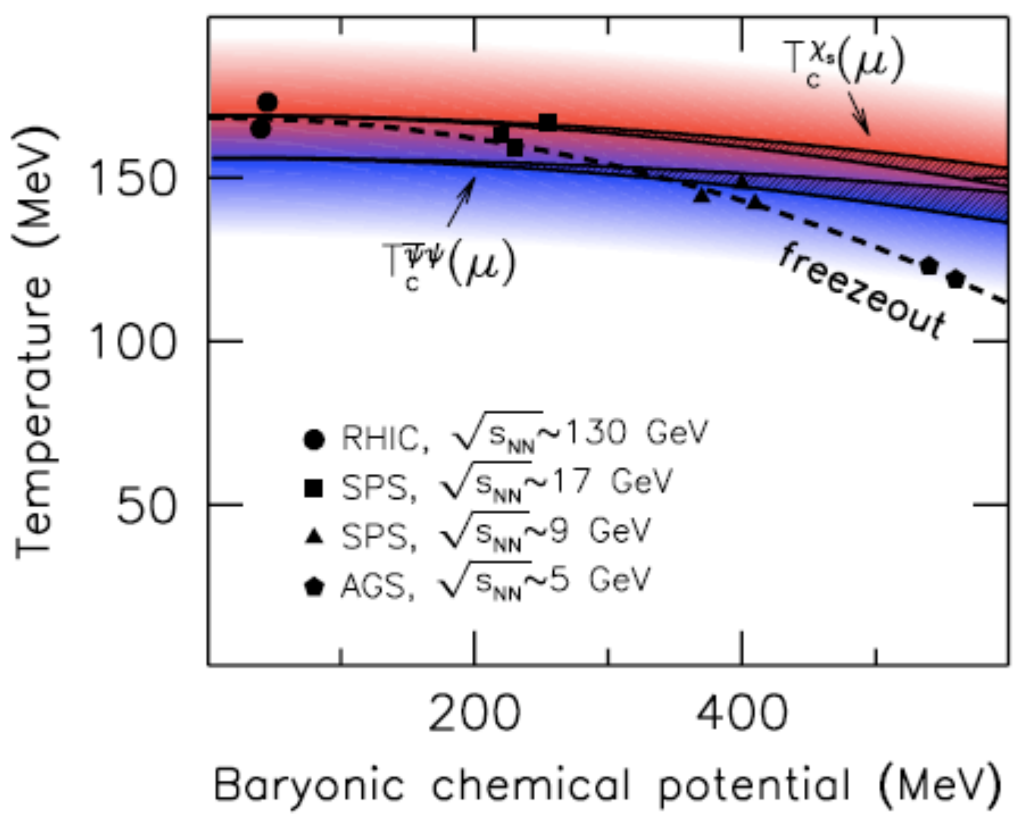
$$\kappa^{(\bar\psi\psi_r)} = 0.0066(20)$$

$$\kappa^{(\chi_s/T^2)} = 0.0089(14),$$

Kaczmarek et al (not continuum, O(4) scaling):

$$\kappa = 0.0066(2)(4)$$

The transition lines do not approach significantly, the transition is not getting stronger: **no sign for a close CEP**.



The freeze-out temperature curve has a stronger curvature. High μ: freeze-out in the hadronic phase

Hadron Resonance Gas

The hadron resonance gas (HRG) model discribes a mixture of free hadrons:
all mesons and baryons and their excited states you find in the particle data book.

[Dashen,Ma,Bernstein 1969]

$$\frac{p^{\mathrm{HRG}}}{T^4} = \frac{1}{VT^3} \sum_{i \in \, mesons} \log \mathcal{Z}^M(T, V, \mu_{X^a}, m_i) + \frac{1}{VT^3} \sum_{i \in \, baryons} \log \mathcal{Z}^B(T, V, \mu_{X^a}, m_i)$$

$$\ln \mathcal{Z}^{M/B}_{M_i} = \mp \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \qquad \varepsilon_i = \sqrt{k^2 + m_i^2}$$

$$= \frac{VT^3}{2\pi^2} d_i \left(\frac{M_i}{T}\right)^2 \sum_{k=1}^{\infty} (\pm 1)^{k+1} \frac{z_i^k}{k^2} K_2(kM_i/T)$$

Degeneracy factor d_i: spin, etc, chemical potentials enter through $z_i = \exp\left((\sum_a X_i^a \mu_{X^a})/T \right)$

This model is a good approximation to QCD in the hadronic phase:

low T: mostly pions, they interact very weakly in QCD and chiPT

[Grasser&Leutwyler 1984] [Greber&Leutwyler 1989]

higher T: interactions are included through the growing number of resonances.

In the strong coupling expansion the partition function reproduces HRG.

[Langelage&Philipsen 1002.1507]

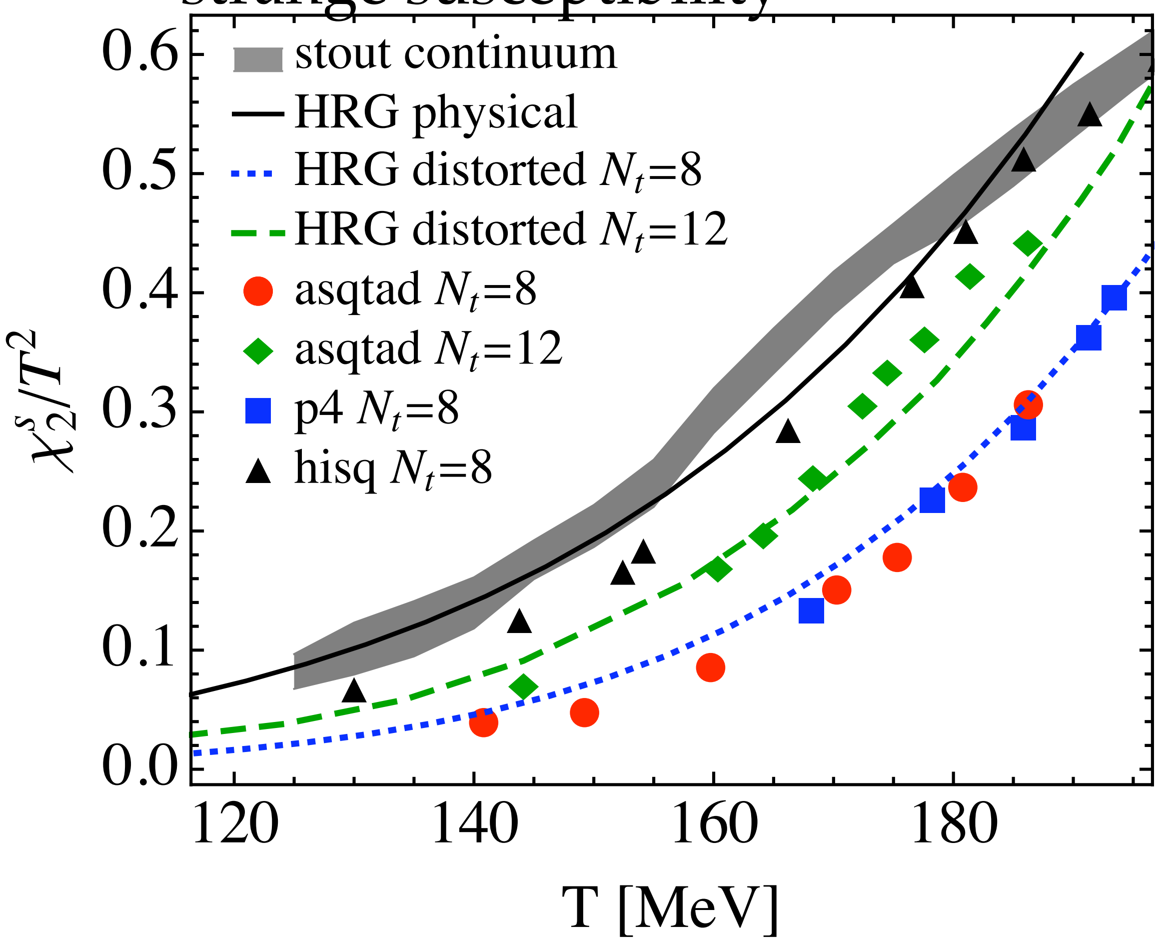
HRG has been tested against lattice in a heavy pion world.

[Karsch et al 0303108] [Petreczky&Huovinen 0912.2541,1005.0324,1106.6227]

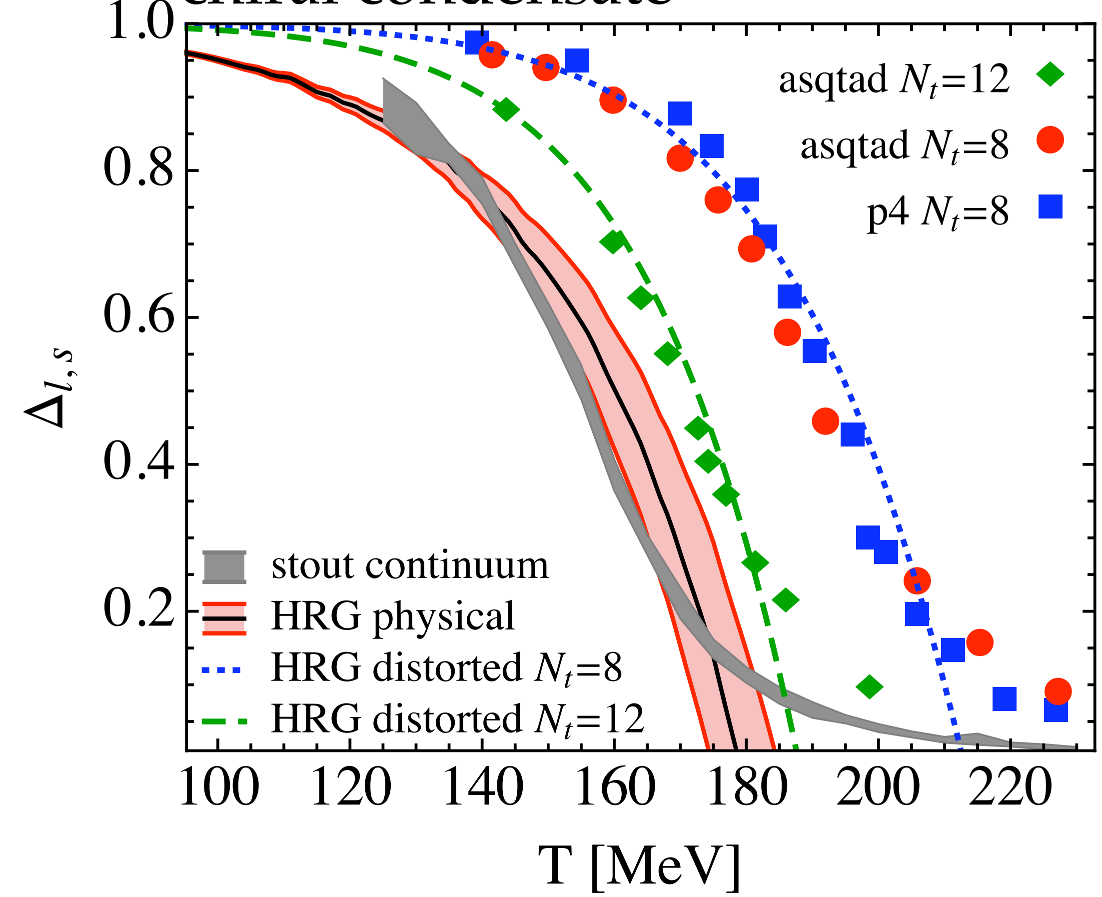
HRG vs lattice continuum limit

We see several observables where the HRG prediction is reproduced by lattice QCD's continuum extrapolation up to ~ 150 MeV.

strange susceptibility



chiral condensate



For HRG's chiral condensate prediction the quark mass dependence of a selection of hadrons were determined with Wilson fermions.

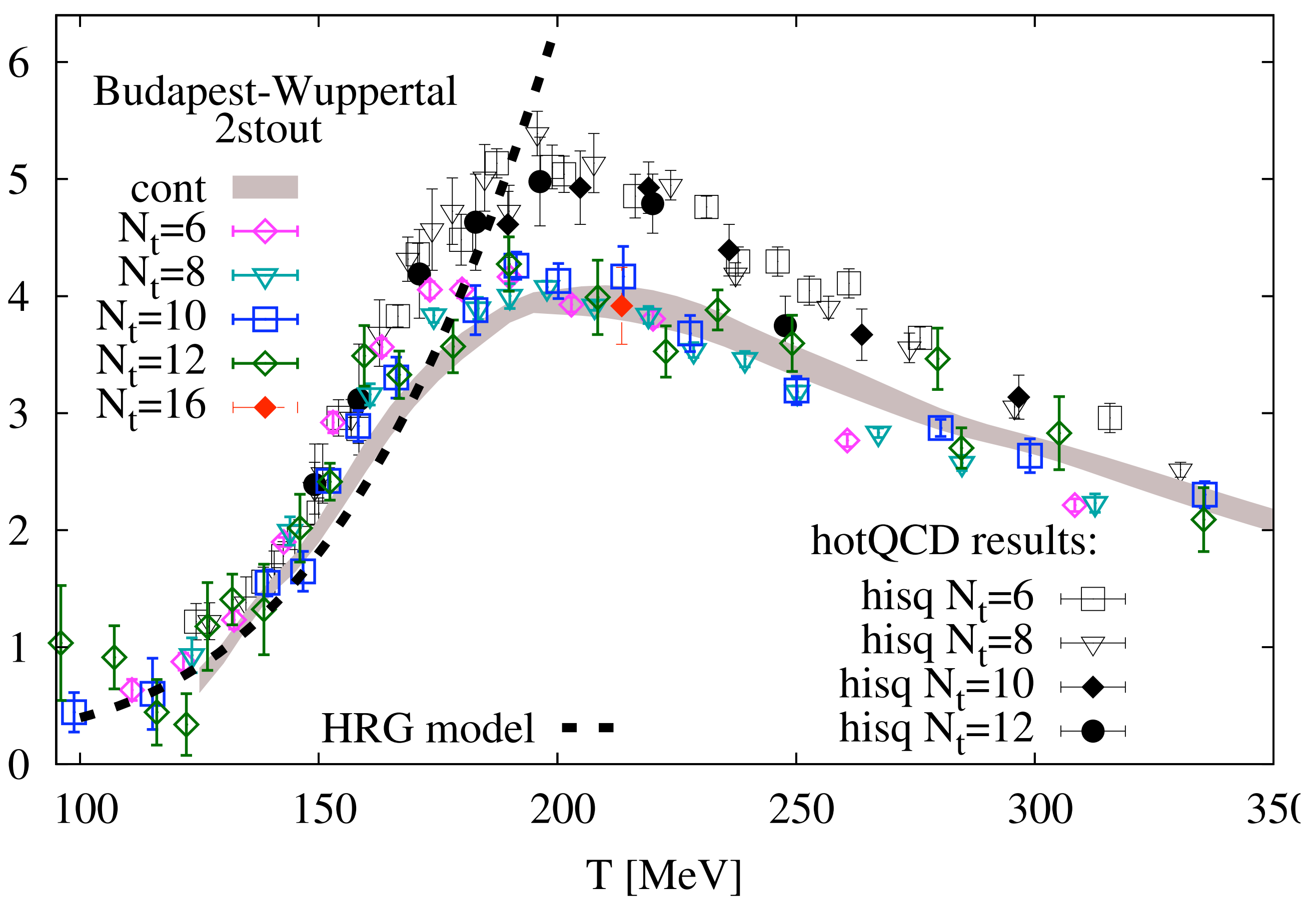
[Wuppertal-Budapest 1007.2580]

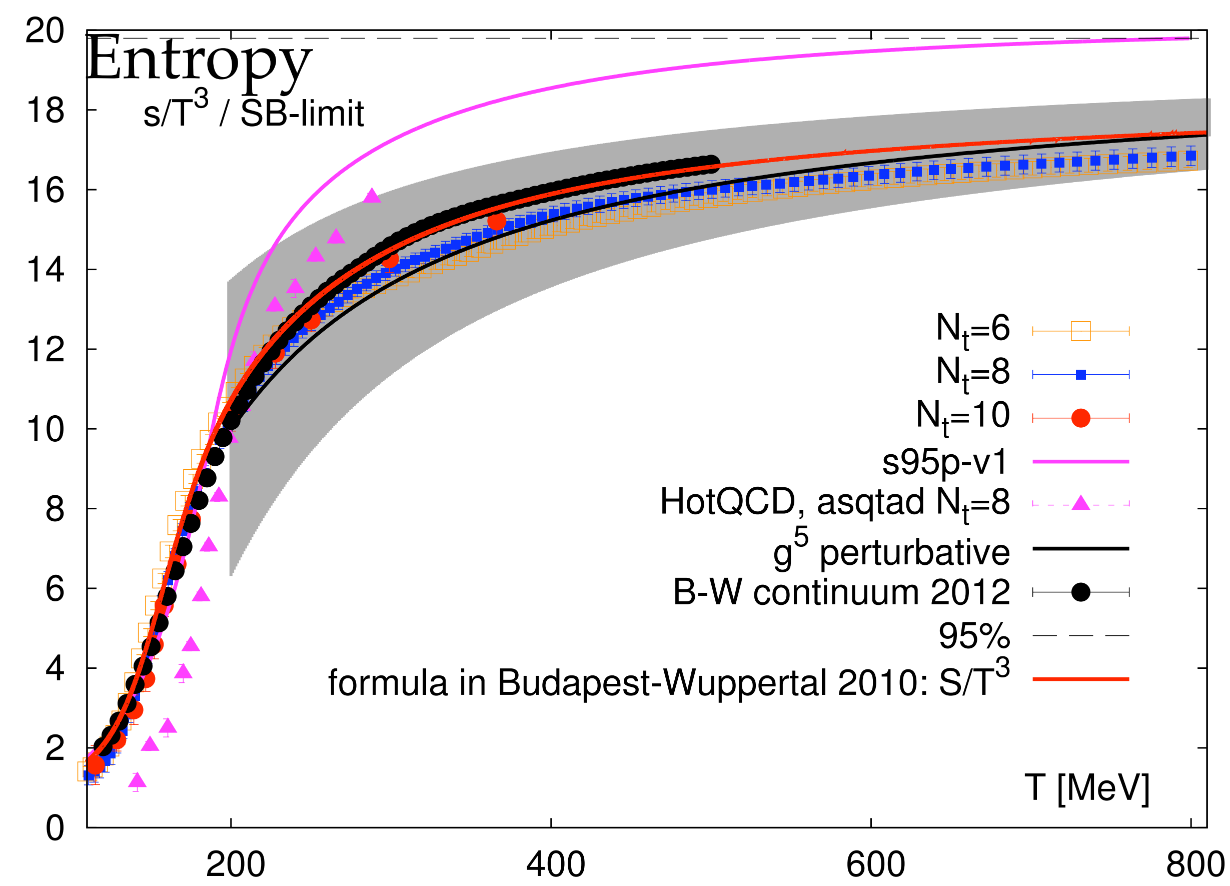
(We also show HotQCD (asqtad) and BNL/Bielefeld (p4) data as of 2010 for comparison.)

Equation of state

In most approaches $\frac{\varepsilon - 3p}{T^4}$ is calculated and the thermodynamic relations are used to calculate the free energy en entropy: $\frac{p}{T^4} = \int_0^T \frac{\varepsilon - 3p}{T'^4} \frac{dT'}{T'}$ and $\frac{s}{T^3} = \frac{\varepsilon + p}{T^4}$

These relations are exact on the lattice in the fixed N_t approach. *(i.e. all the EoS data in this talk)*





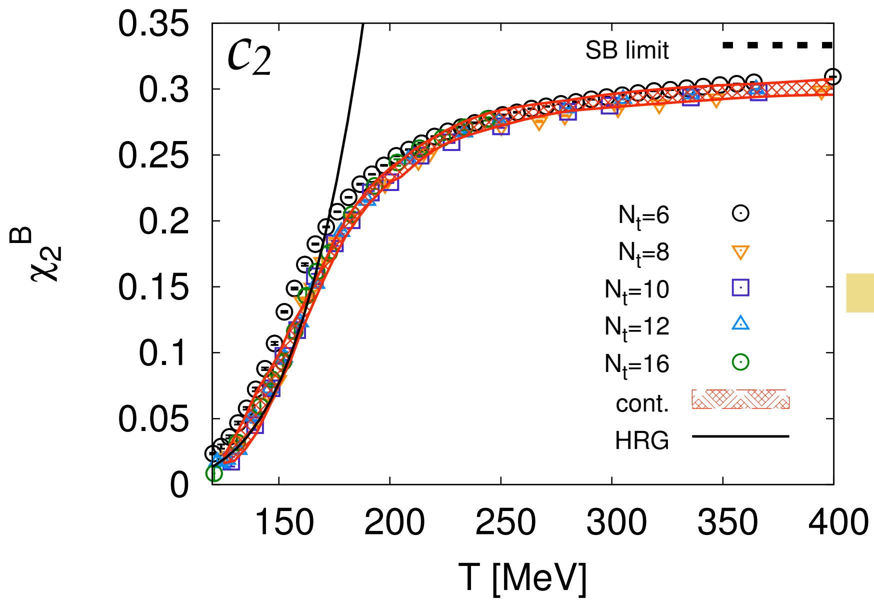
[Wuppertal-Budapest 1007.2580,1204.0995, QM12:Ratti]
[HotQCD 0903.4379(asqtad),1005.1131(hisq),QM12:Bazavov]
s95p: [Petreczky&Huovinen 0912.2541] *HRG + asqtad Nt=8*

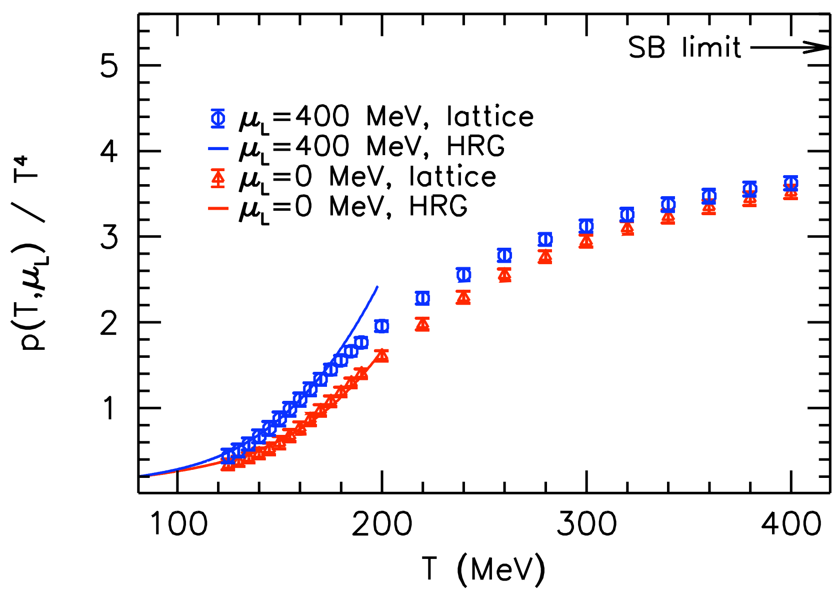
With the HISQ action the former 50-70% discrepancy is down to 20%

EoS at finite chemical potential

Taylor approach:

$$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(T,\mu_q) = \sum_{n=0}^{\infty} c_n(T,m_q) \left(\frac{\mu_q}{T}\right)^n \quad with \quad c_n(T,m_q) = \frac{1}{n!}\frac{1}{VT^3}\frac{\partial^n \ln Z(T,\mu_q)}{\partial(\mu_q/T)^n}\bigg|_{\mu_q=0}$$





Budapest-Wuppertal:

use continuum free energy + continuum 1st derivative in $\left(\frac{\mu}{T}\right)^2$

[Wuppertal-Budapest 1204.6710]

MILC/BNL-Bielefeld:

coarse lattice (Nt=6) + 3rd order expansion in $\left(\frac{\mu}{T}\right)^2$

[De Tar et al 1003.5682] [Ejiri et al hep-lat/0512040]

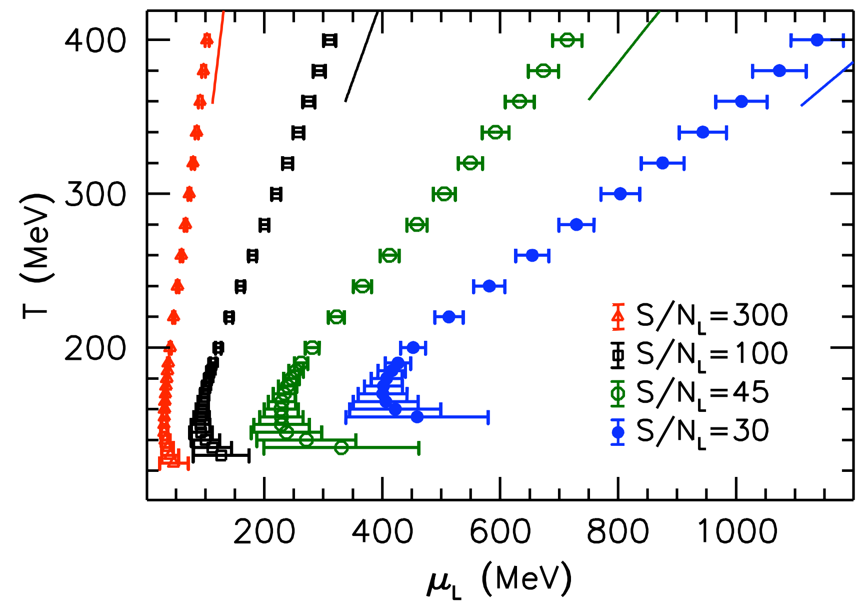
use s95p parametrization (asqtad Nt=8) + c2 (HISQ Nt=8)
+ c4 (p4, Nt=4, shifted) + c6 (p4, Nt=4, shifted)

[Huovinen&Petreczky &Schmidt, 2011]

Isentropic equation of state

Taylor expansion on a constant S/N trajectory.
Exploratory study by Ejiri et al with Nf=2.

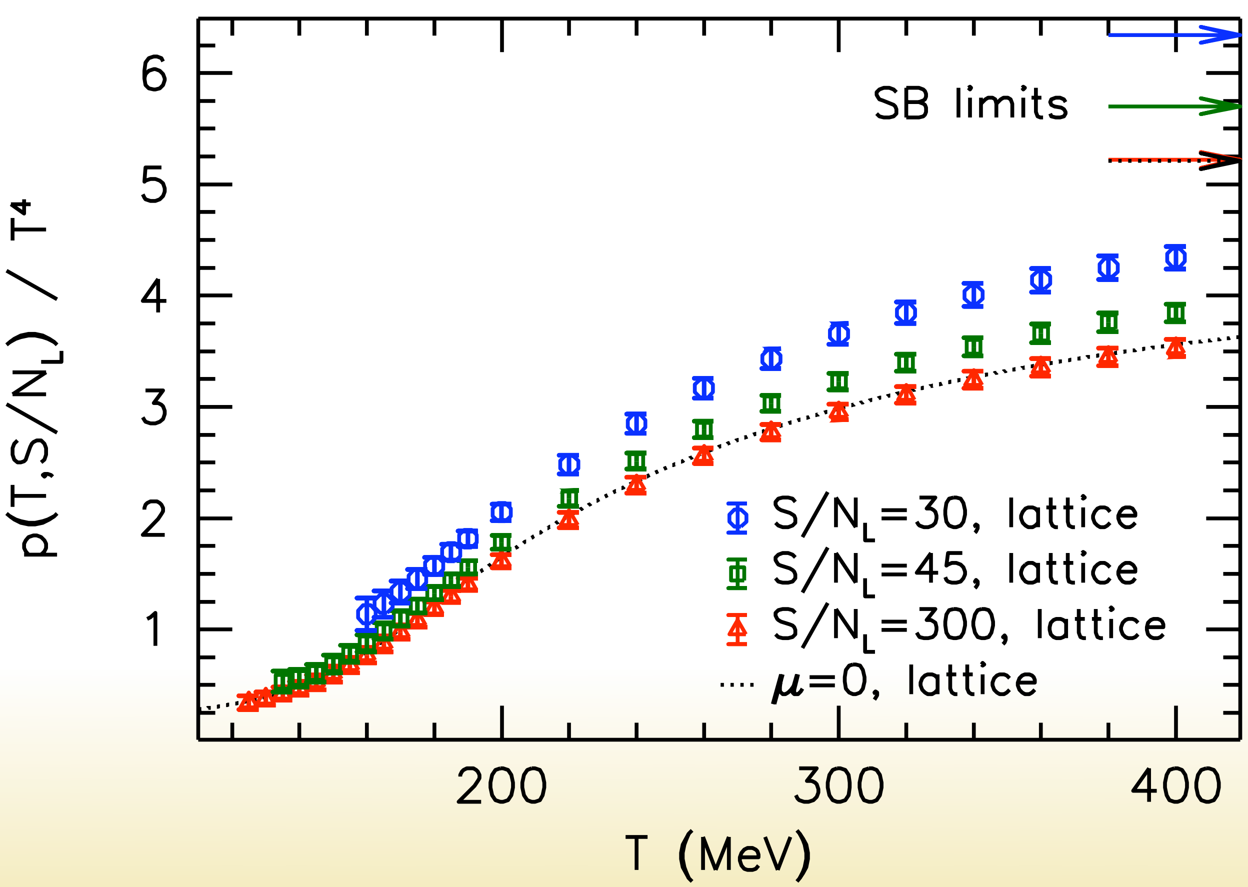
[Ejiri et al Phys. Rev. D 73 (2006) 054506]

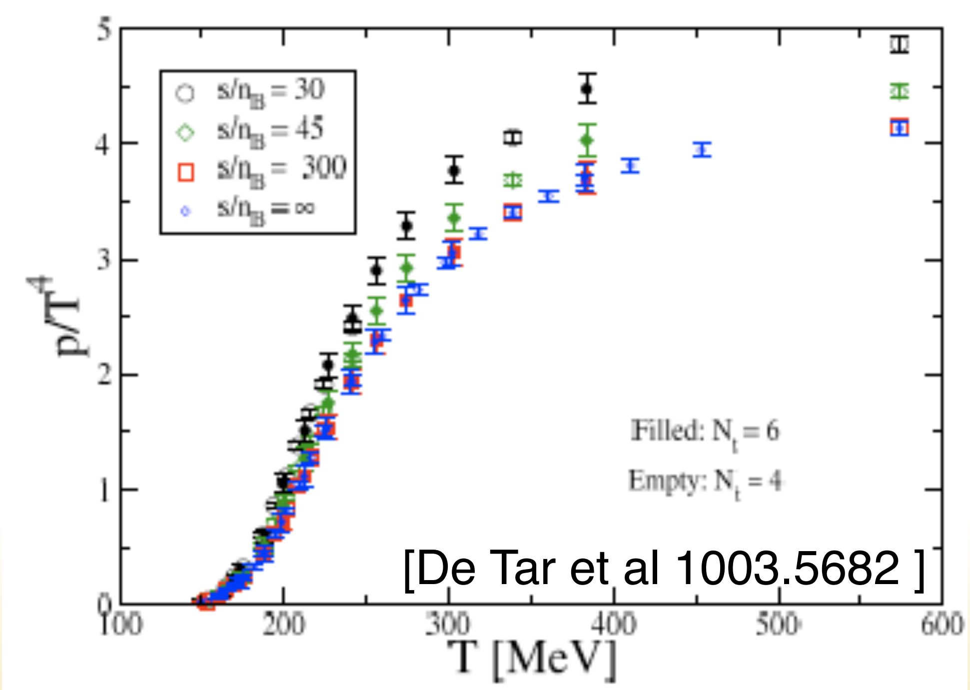


Here: Wupperal-Budapest continuum data.

[Wuppertal-Budapest 1204.6710]
[Ratti QM12]

A parametrization for $\mu > 0$ is also given in the paper.





[De Tar et al 1003.5682]

Fluctuations of conserved charges

For an equilibrium sub-system at freeze-out, the net yields are given by

$$\langle N_X \rangle = -T \frac{\partial \log Z}{\partial \mu_X} \qquad \langle N_X^2 \rangle - \langle N_X \rangle^2 = \frac{\partial^2 \log Z}{(\partial \mu_X / T)^2}$$

On the lattice we calculate these derivatives in a large volume and in equilibrium.

$$\hat{\chi}_2^X = \frac{1}{VT^3} \frac{\partial^2 \log Z}{(\partial \mu_X / T)^2}$$

(typically evaluated at zero chemical potential)

A linear extrapolation to finite baryo-chemical potential is possible through

$$\hat{\chi}_{22}^{XB} = \frac{1}{VT^3} \frac{\partial^4 \log Z}{(\partial \mu_X / T)^2 (\partial \mu_B / T)^2}$$

Non-gaussianity $\hat{\chi}_4^X = \frac{1}{VT^3} \frac{\partial^4 \log Z}{(\partial \mu_X / T)^4}$ is used in a V-independent ratio: $\hat{\chi}_4^X / \hat{\chi}_2^X$

<u>Challenges for lattice:</u>

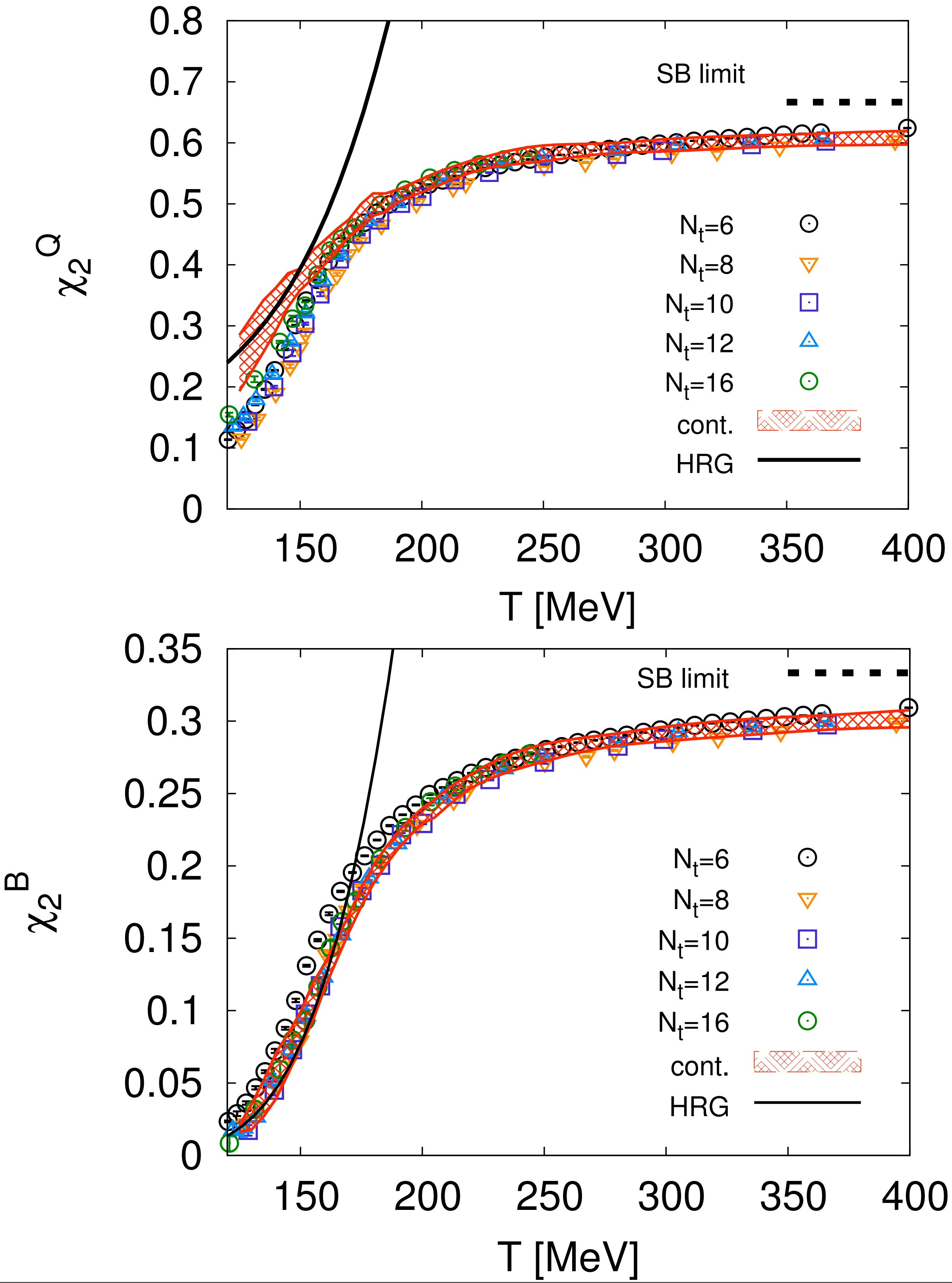
Each additional derivative come with a volume penalty in the statistics (reweighting...)

Electric charge is pion-dominated and require fine lattices **and** taste-improvement.

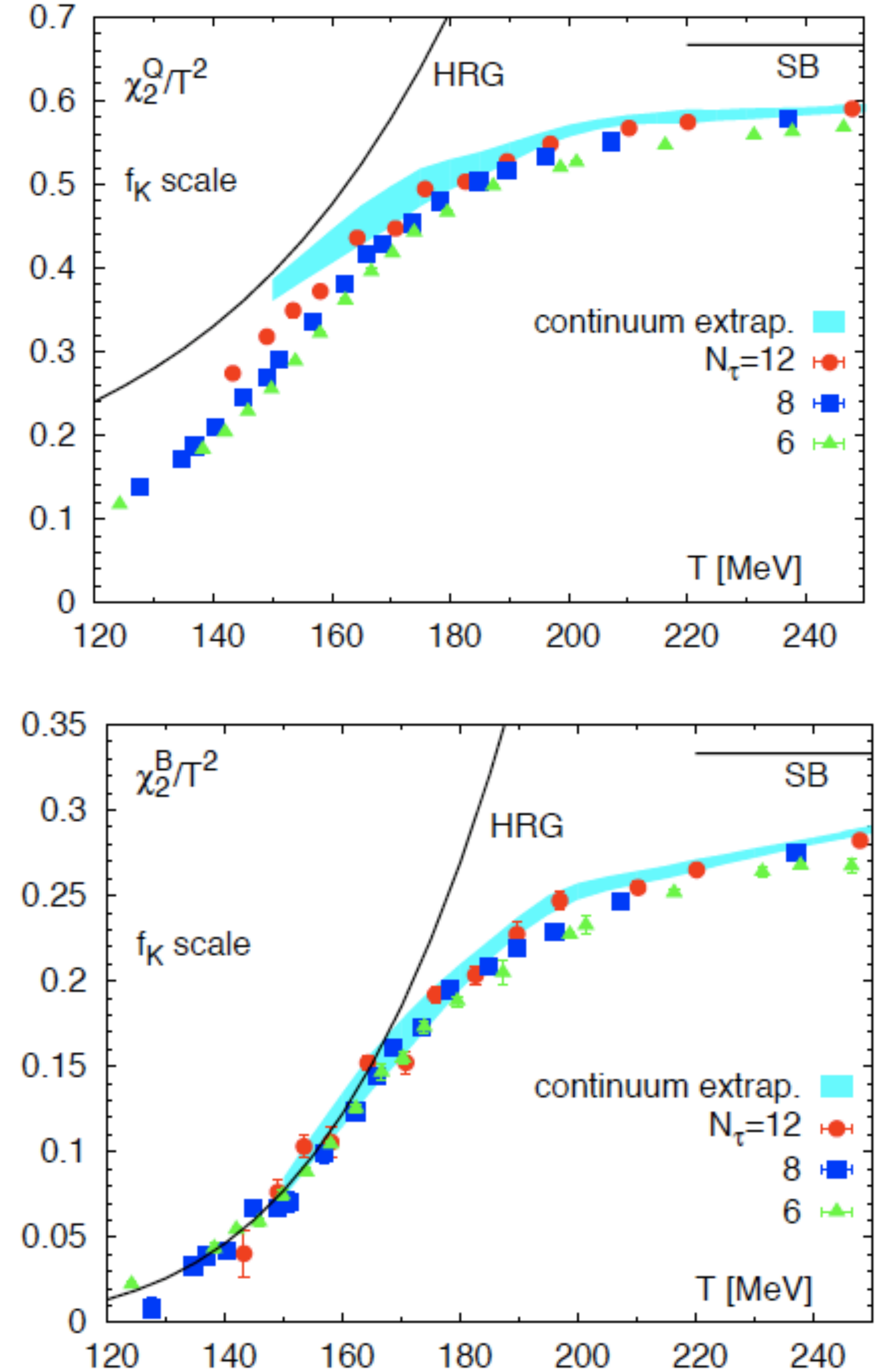
Baryon number requires a large statistics.

Diagonal fluctuations *(continuum)*

[Wuppertal-Budapest: 1112.4416]

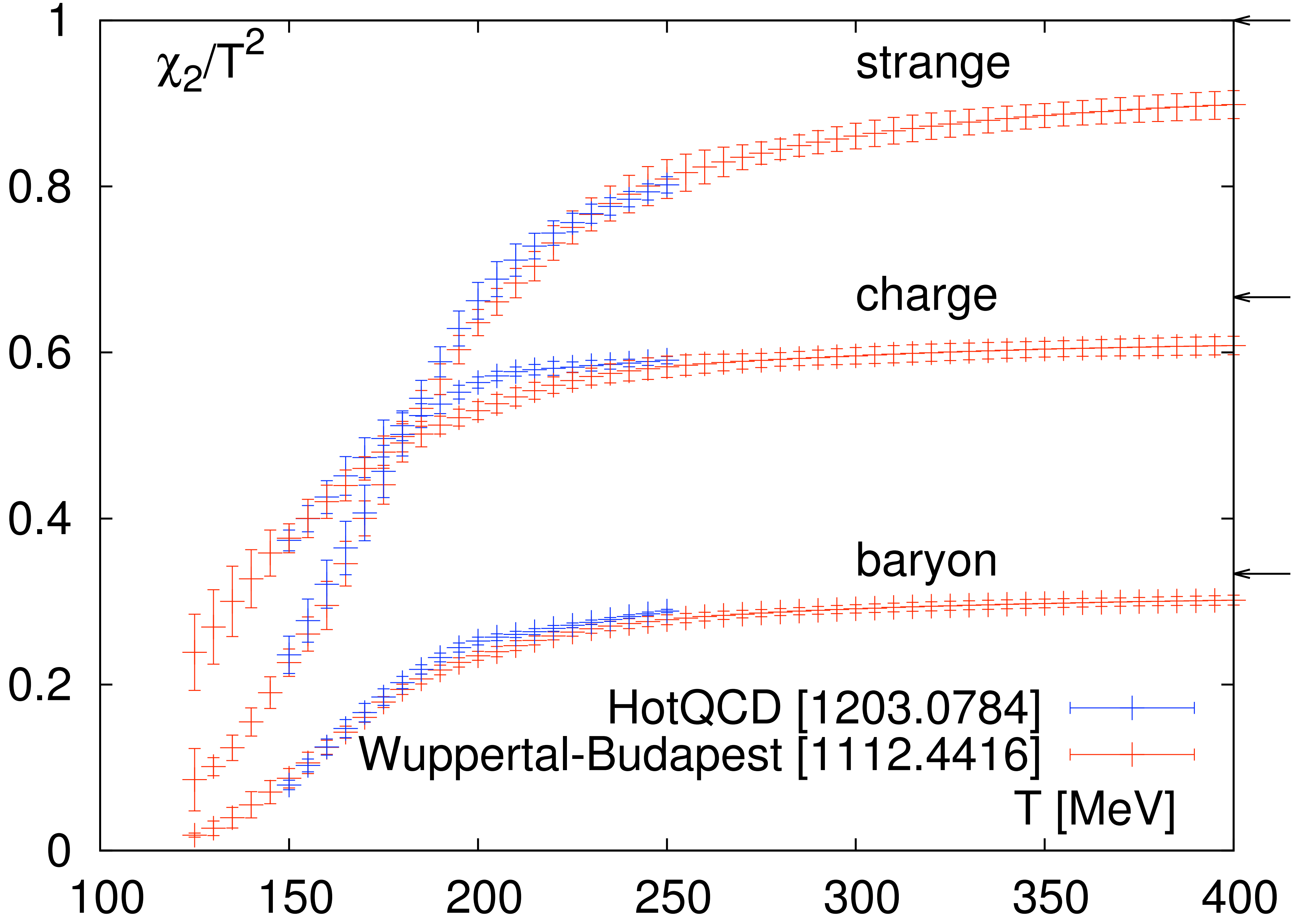


[HotQCD: 1203.0784]



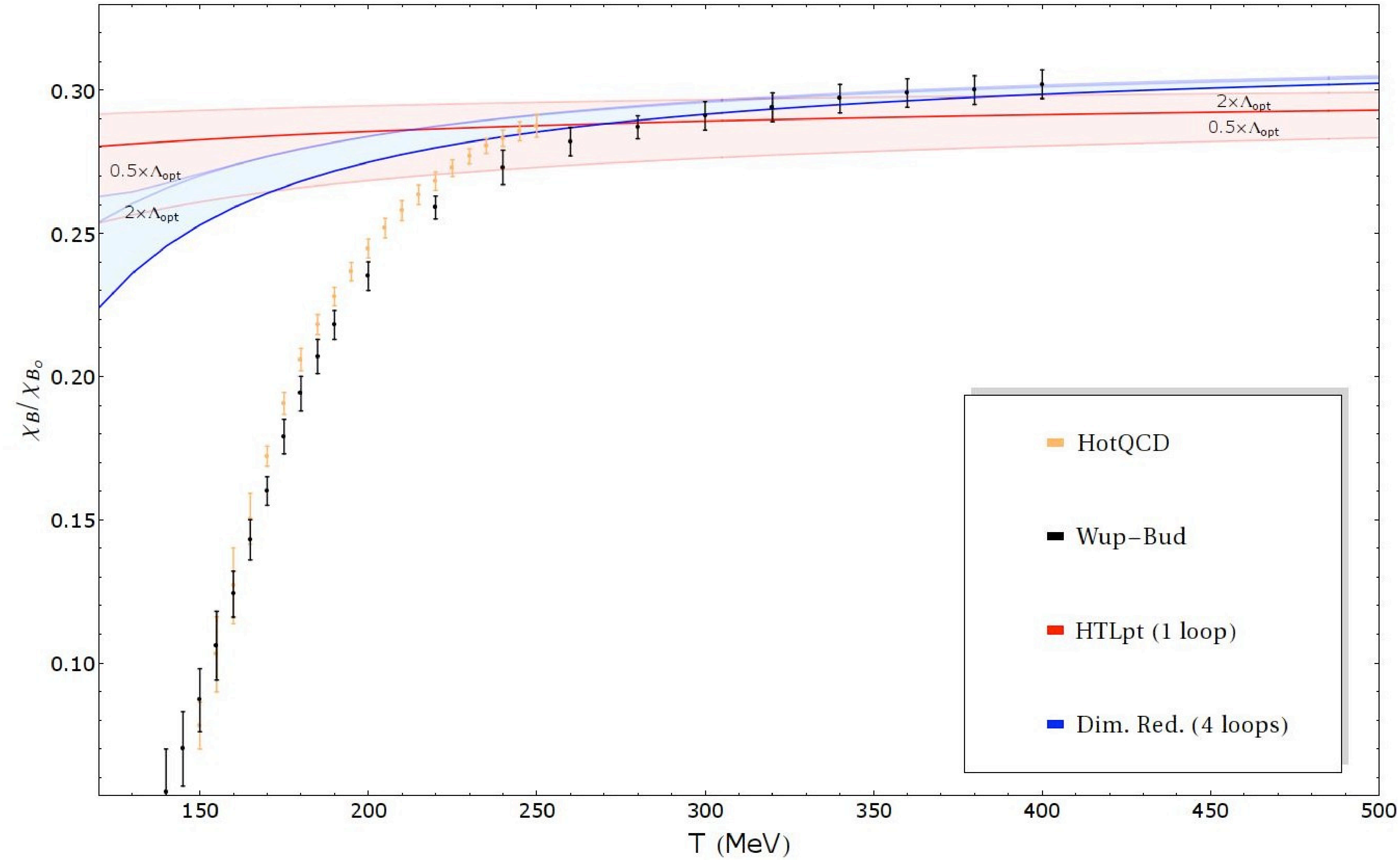
Do we agree on the fluctuations?

Comparison of the published continuum results:



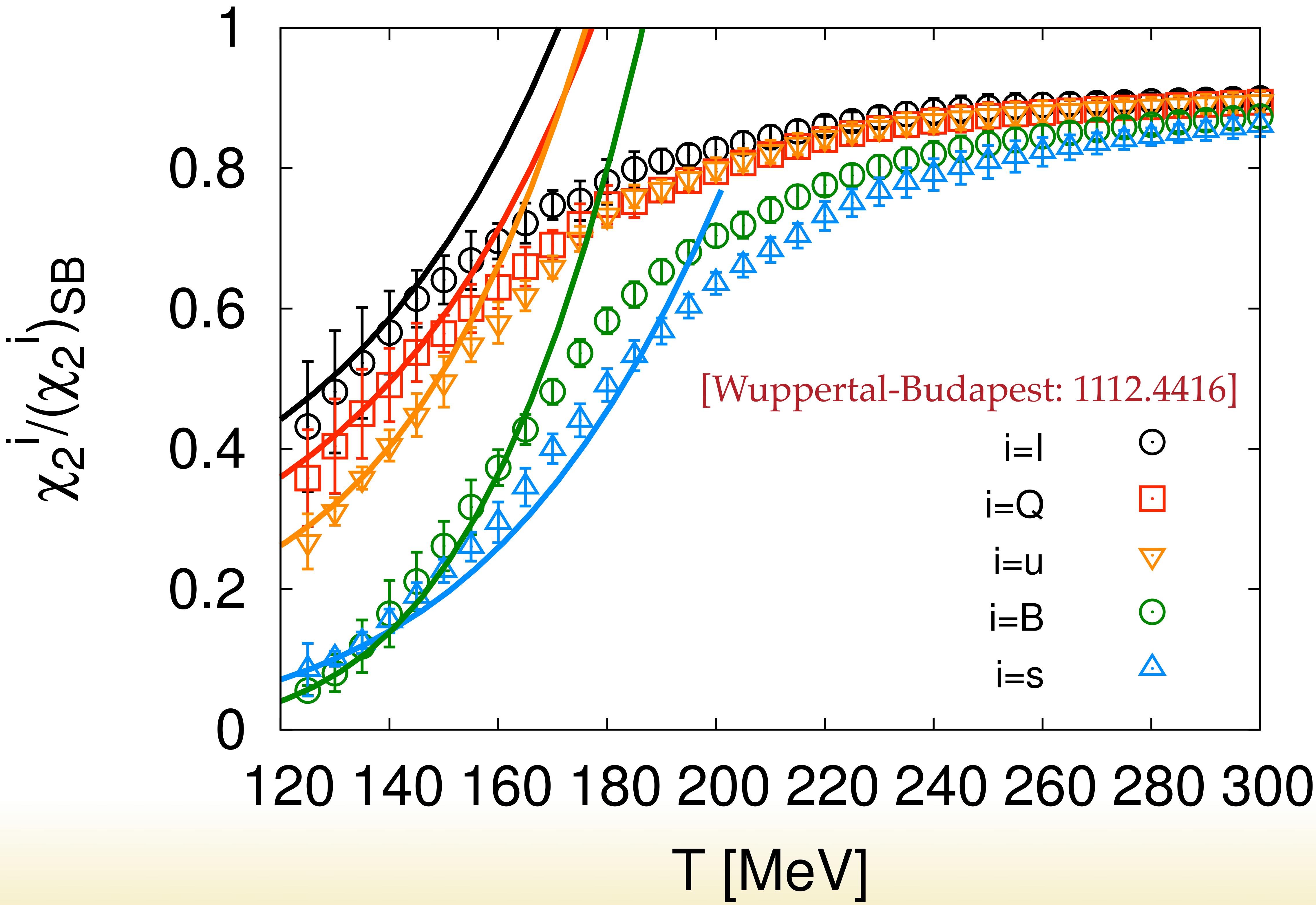
Lattice vs analytical techniques

Baryon number susceptibility:
comparison of lattice result with hard thermal loop and dimensional reduction.

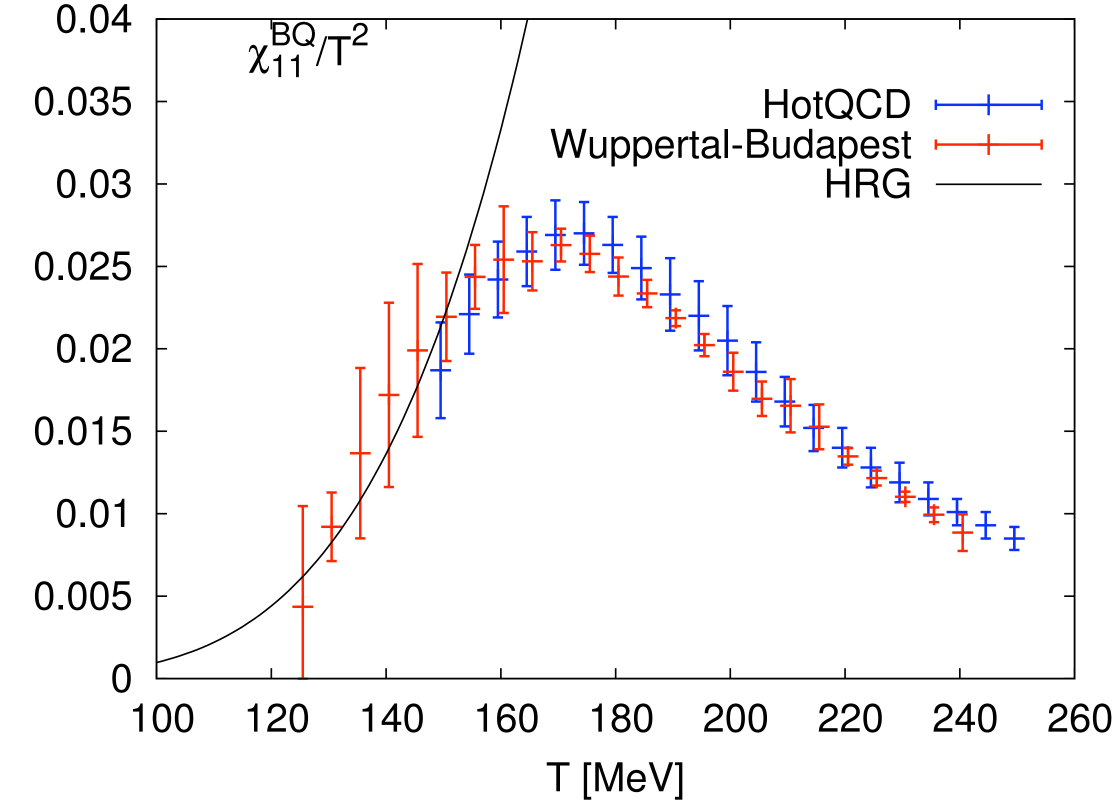


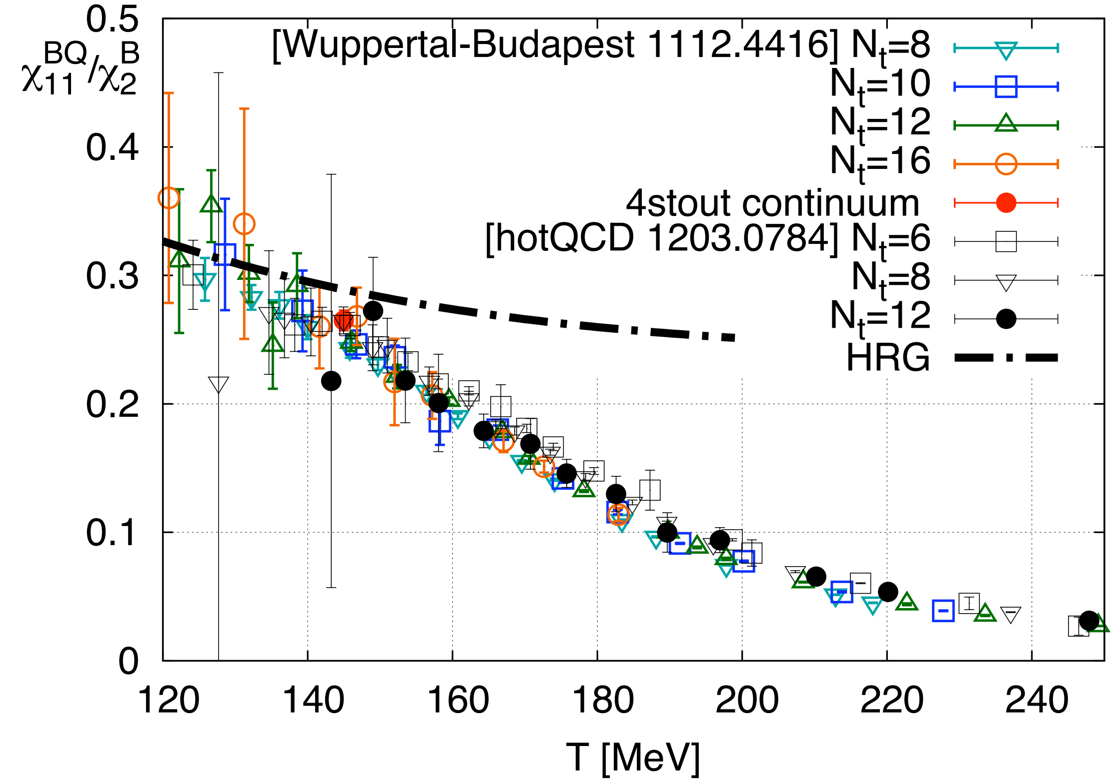
[Andersen,Mogliacci,Su,Vuorinen 2012]

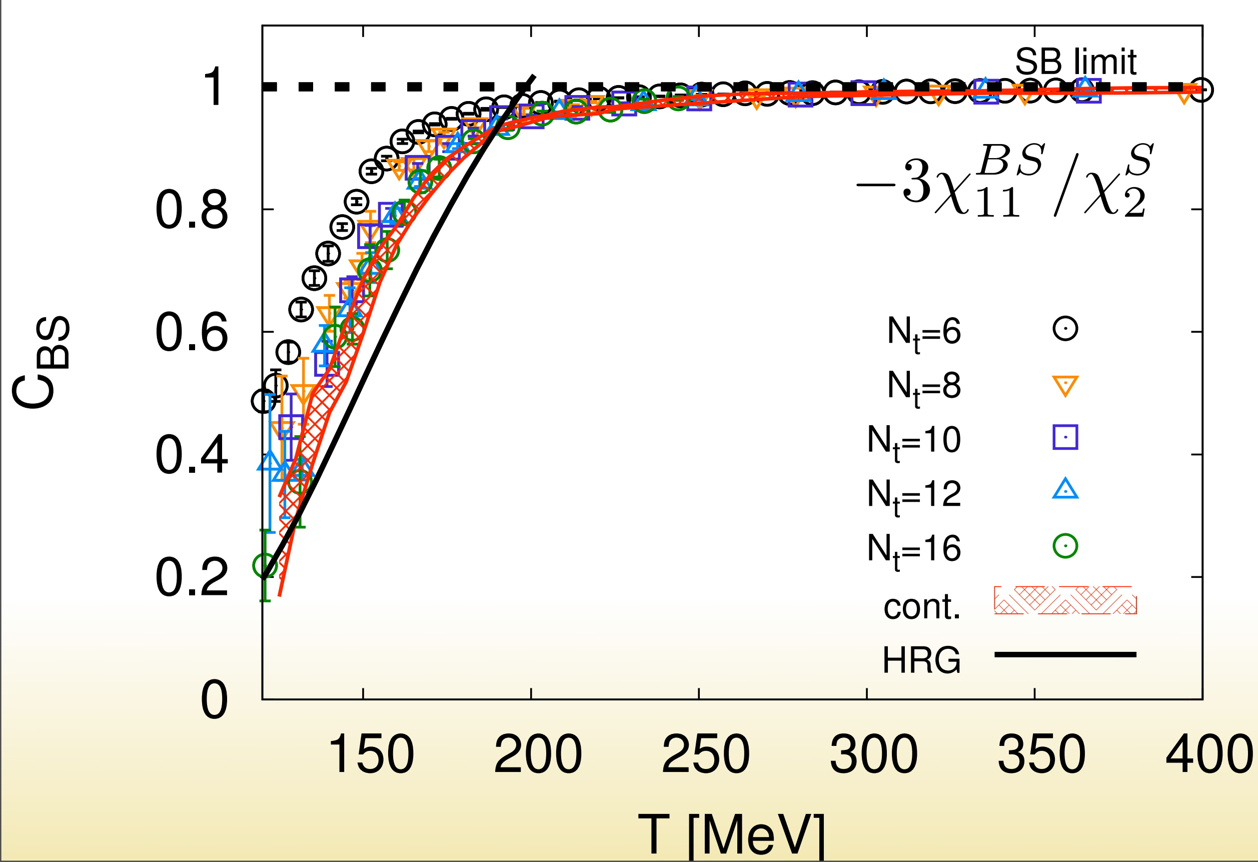
All diagonal fluctuations



Off-diagonal correlators







WB data goes down to temperatures where an agreement with HRG can be demonstrated (T~130 MeV) (deficit at T=145 Mev: 10% [4stout result])

χ_{11}^{BQ}/χ_2^B and C^{BS} are almost linear in the freeze-out range.

Freeze-out conditions

Even/odd ratios are useful for μ-determination

$$\frac{\sigma_B^2}{M_B} \equiv \frac{\chi^B_{2,\mu}}{\chi^B_{1,\mu}} = \frac{T}{\mu_B}\left[\frac{1+\frac{1}{2}\frac{\chi_4^B}{\chi_2^B}(\mu_B/T)^2+\ldots}{1+\frac{1}{6}\frac{\chi_4^B}{\chi_2^B}(\mu_B/T)^2+\ldots}\right]$$

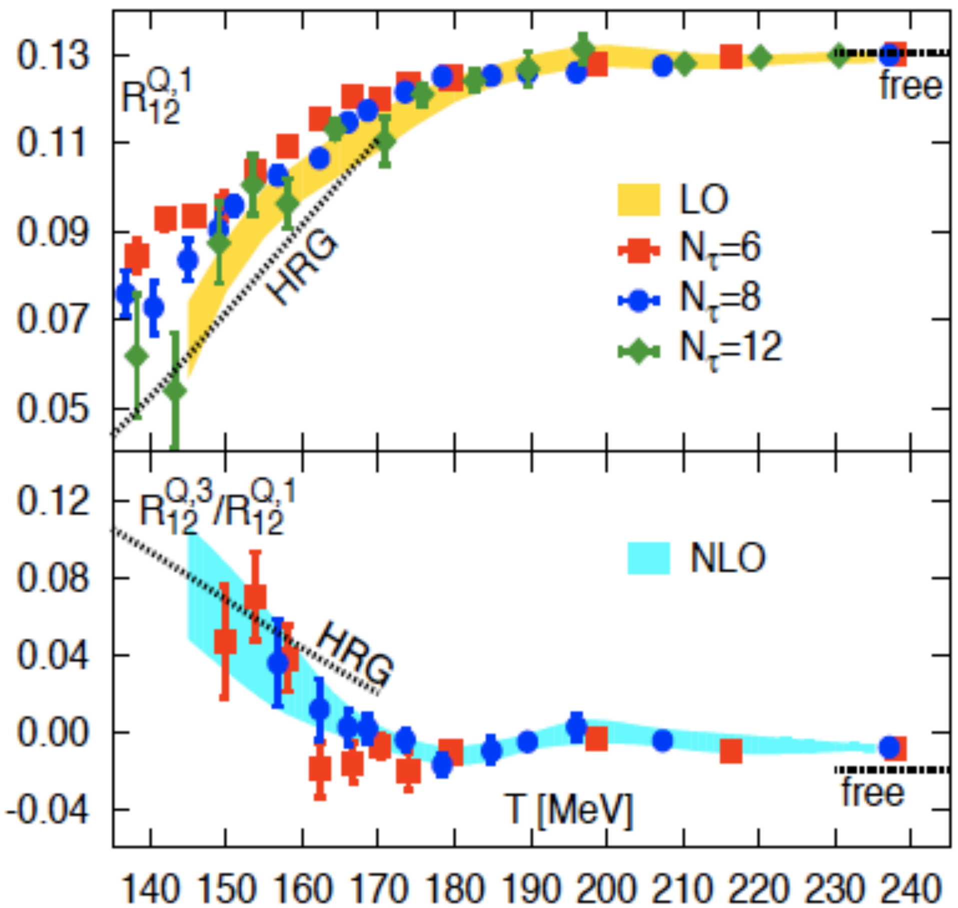
Even/even or odd/odd are useful for T fitting e.g. χ_4^Q/χ_2^Q or χ_3^Q/χ_1^Q

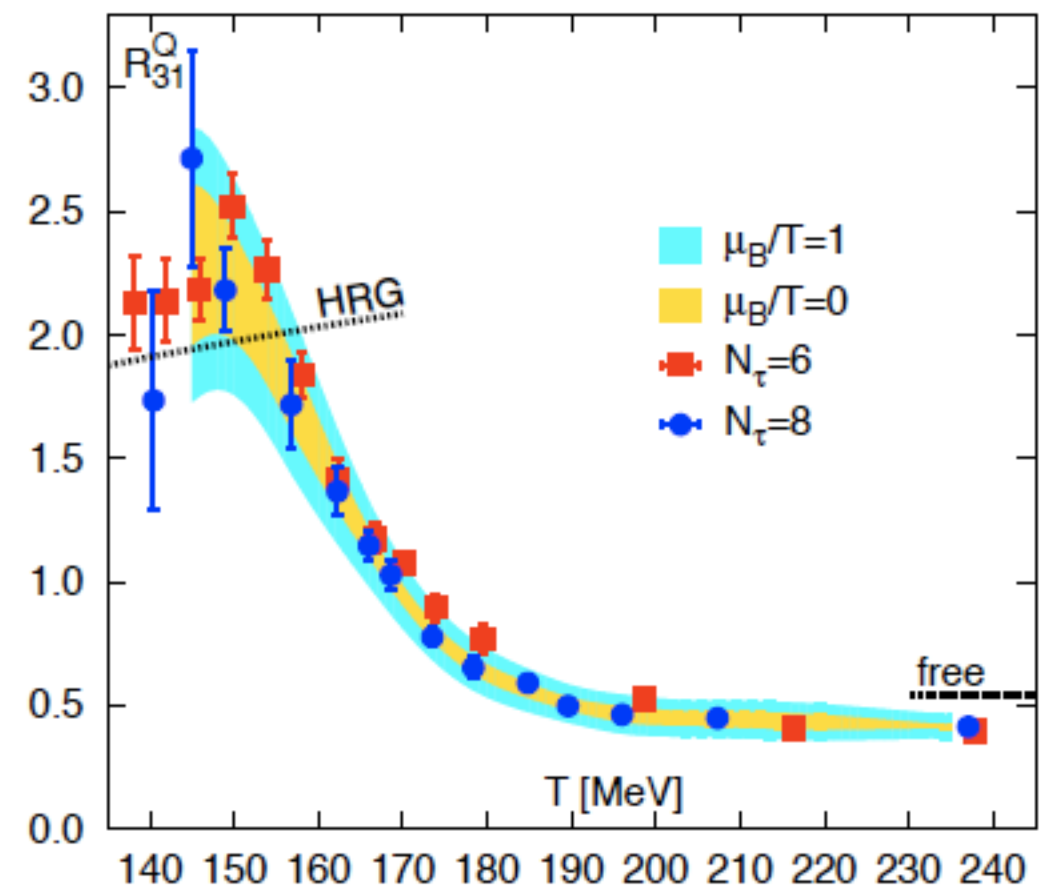
[Karsch 1202.4173]

A set of these ratios have been determined very recently:

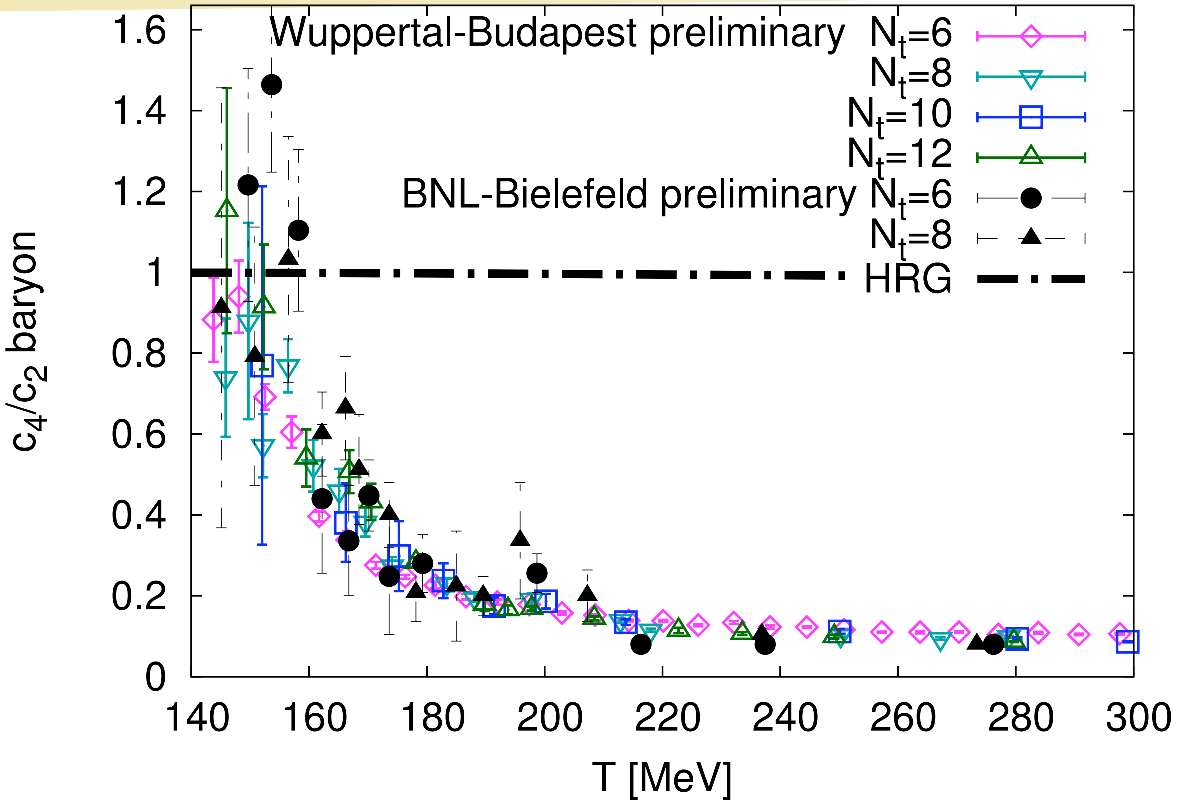
[Bazavov et al 1208.1220] [Mukherjee QM12]

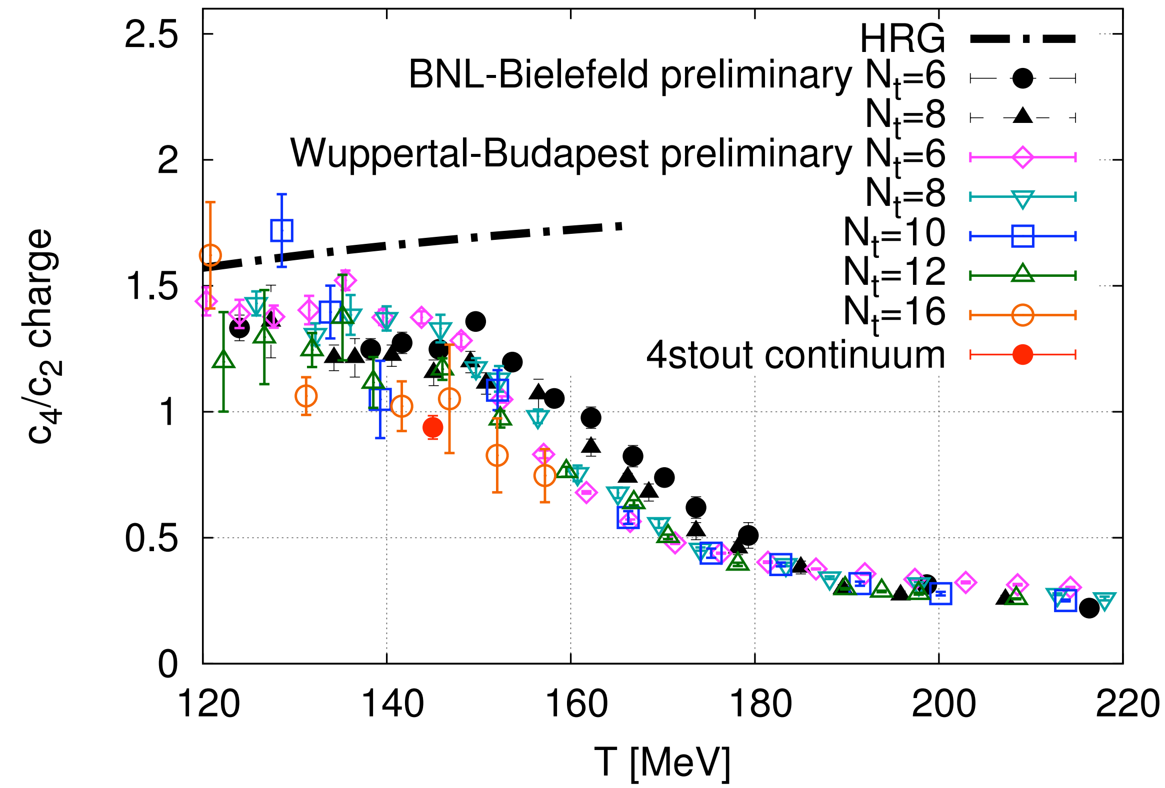
These have been determined for small μ: $R^X_{nm} = \chi^X_{n,\mu}/\chi^X_{m,\mu} = R^{X,0}_{nm} + \frac{\mu_B}{T} R^{X,1}_{nm} + \ldots$

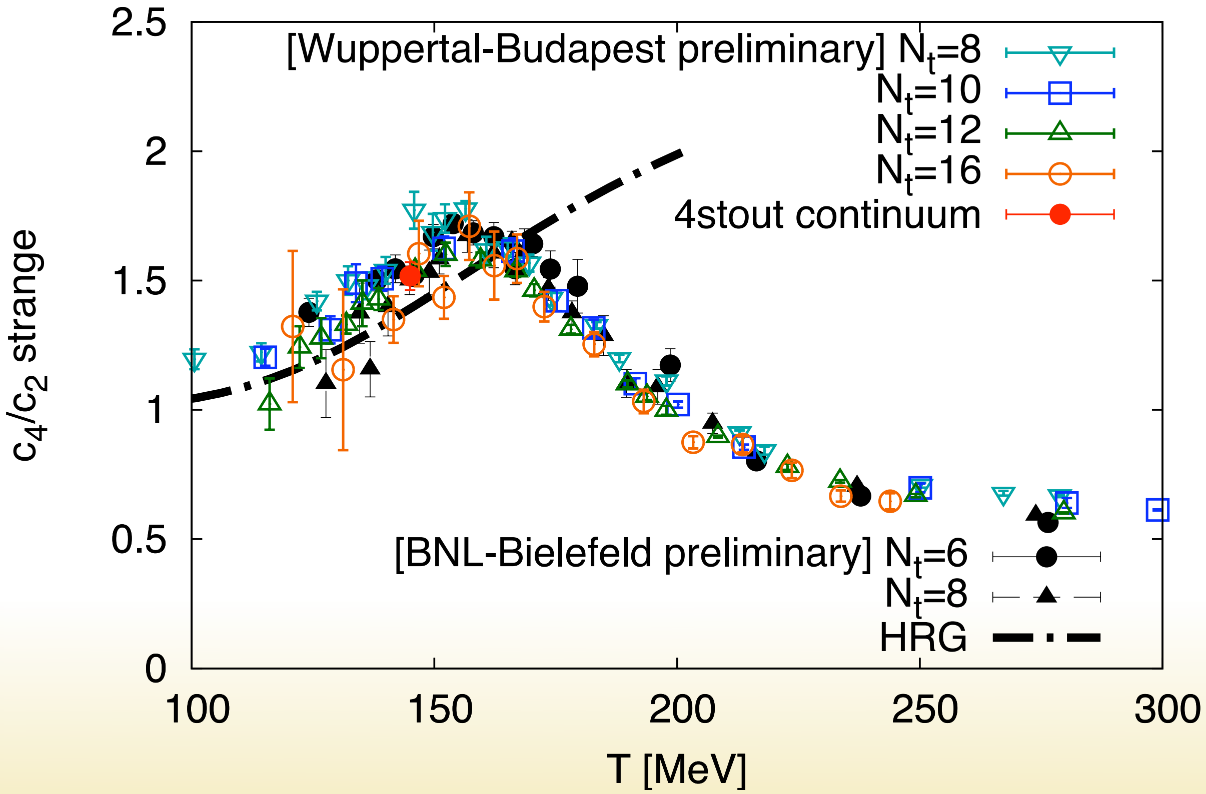




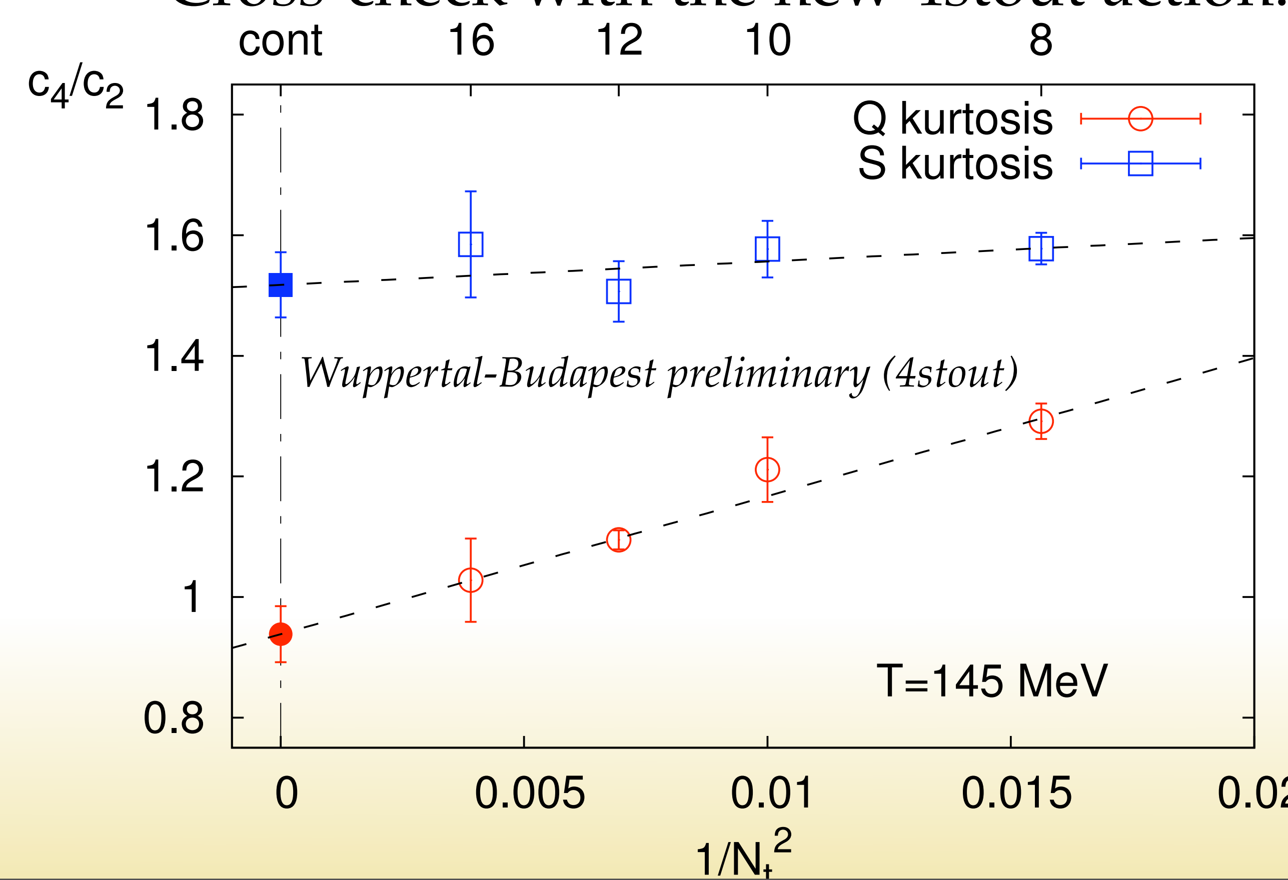
Kurtosis: c_4/c_2



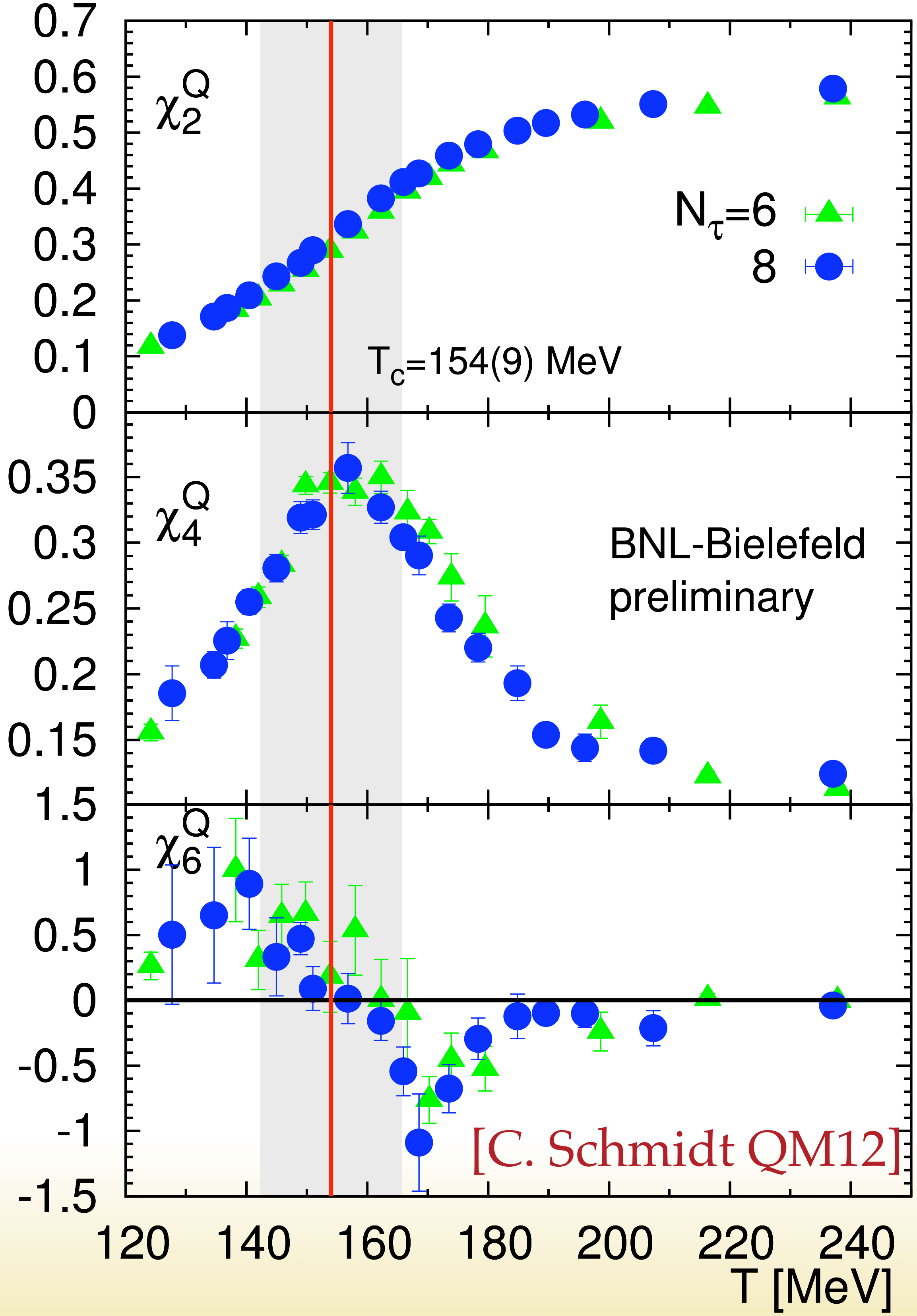




Cross-check with the new 4stout action:



Higher order cumulants



From the 6th order onwards, the cumulants change sign and diverge in the chiral limit. This behaviour was manifest already with the physical quark masses, even on a coarse lattice.

The baryon 6th cumulant and other mixed 6th order derivatives have also been determined (not shown). As of today these are rather noisy and inconclusive.

Nevertheless, they can be used to estimate the order of magnitude of finite-mu corrections of the kurtosis.

[S. Mukherjee QM12]

Summary

- ***T_c*** dispute has settled: the new HISQ data reproduces Wuppertal-Budapest prediction.
- The chiral **O(4) scaling** is visible already slightly above the physical quark mass.
- The transition line has a known **curvature**, no sign for a nearby critical end point.
- T_c decreases in an **external magnetic field** and remains a crossover.
- **Equation of state**: 50 % discrepancy is now down at 20 %. N_t=16 entropy is needed.
- The Budapest-Wuppertal **EoS** is extended to **finite chemical potential** at leading order.
- The effect of the **charm quark** is visible above ~300 MeV *(it is not yet a continuum result)*
- The **width of net charge/strangeness/baryon** distribution is predicted in the continuum limit by both the HotQCD and Budapest-Wuppertal collaborations
- HotQCD's fluctuation data has been used to define **freeze-out conditions** at small chemical potentials.
- Higher cumulants (**curtosis,skewness**) has been presented both by HotQCD and W-B.

In the near future:
equation of state and higher cumulants will be available at increasing chemical potentials
Lattice QCD is mapping the phase diagram with continuum/physical results.