

The Quest for the QGP  
(being a short history  
of the quark-gluon plasma)

Quark Matter 2012 Student Lecture

W.A. Zajc  
Columbia University

# Science Questions

## × Uninteresting question:

- ▶ What happens when I crash two heavy nuclei together?

## ✓ Interesting question:

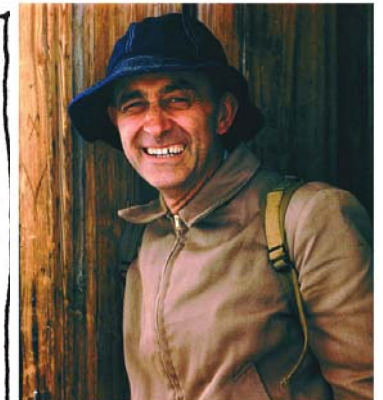
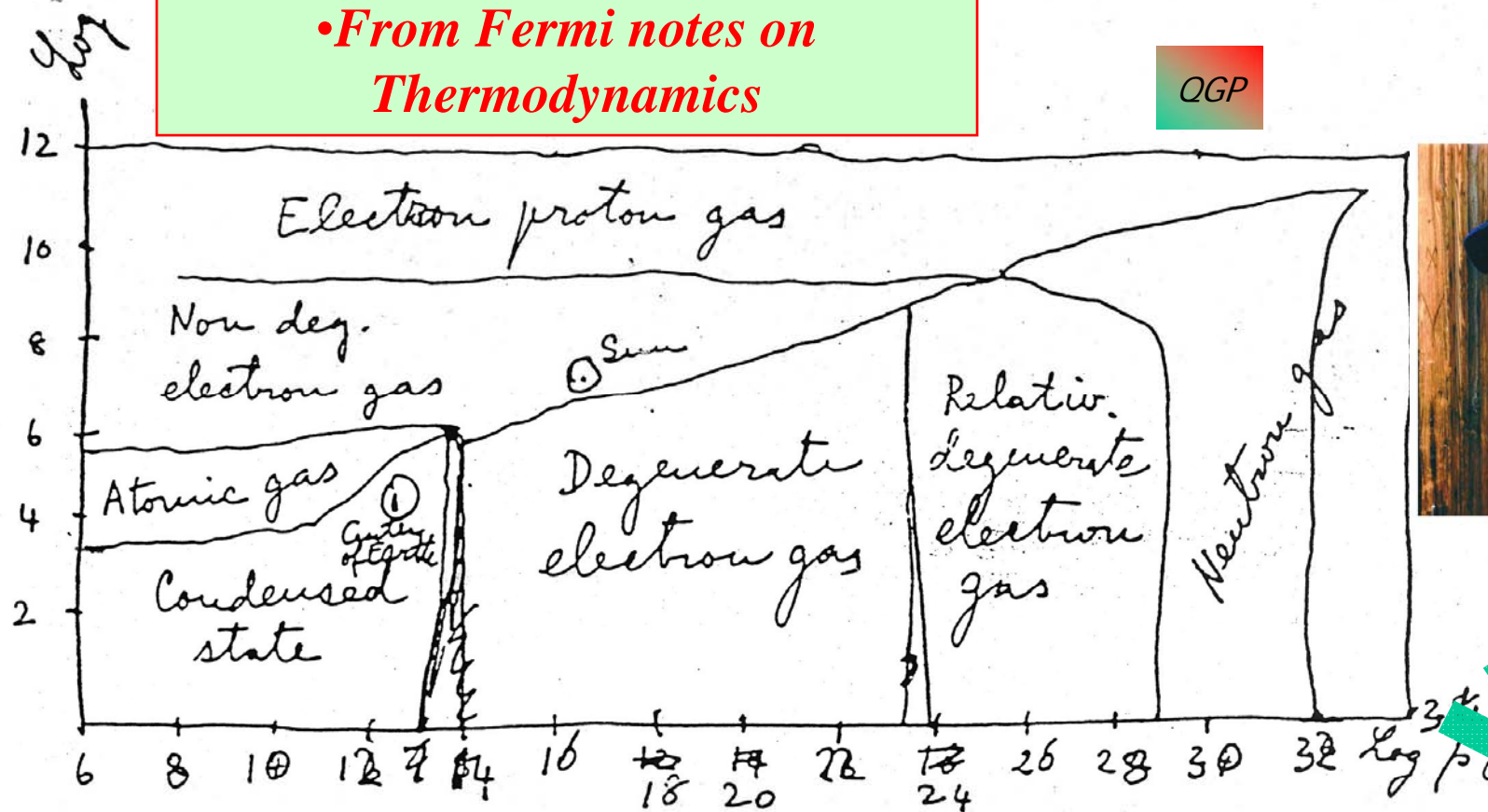
- ➡ Are there new states of matter at the highest temperatures and densities?

# Fermi's Vision

- ~1950: (Almost) included physics of 2012
- See also remarks in his "statistical model" paper

• From Fermi notes on  
*Thermodynamics*

QGP

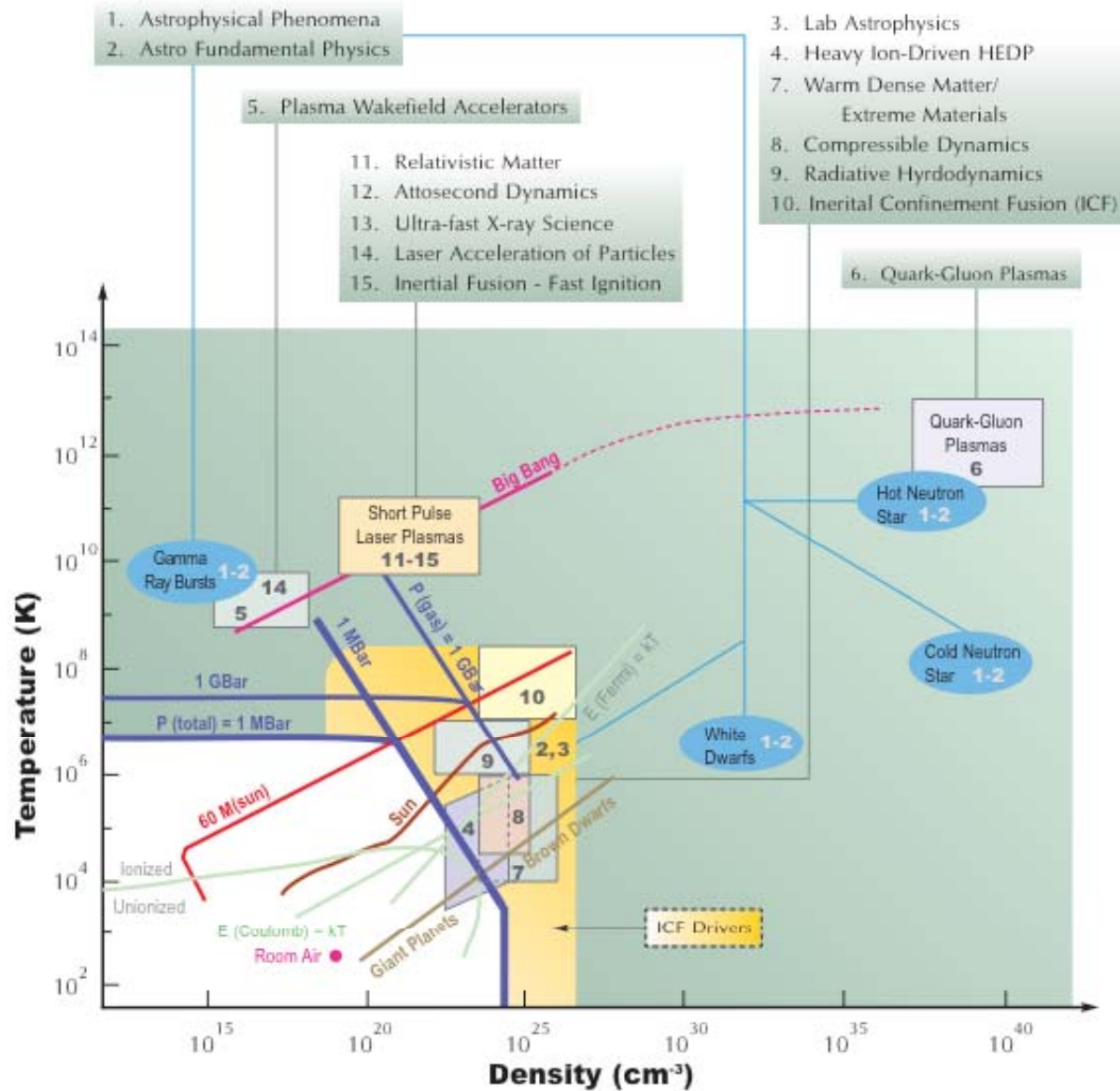


• dyne/cm<sup>2</sup> !

• (Thanks to  
• A. Melissinos)

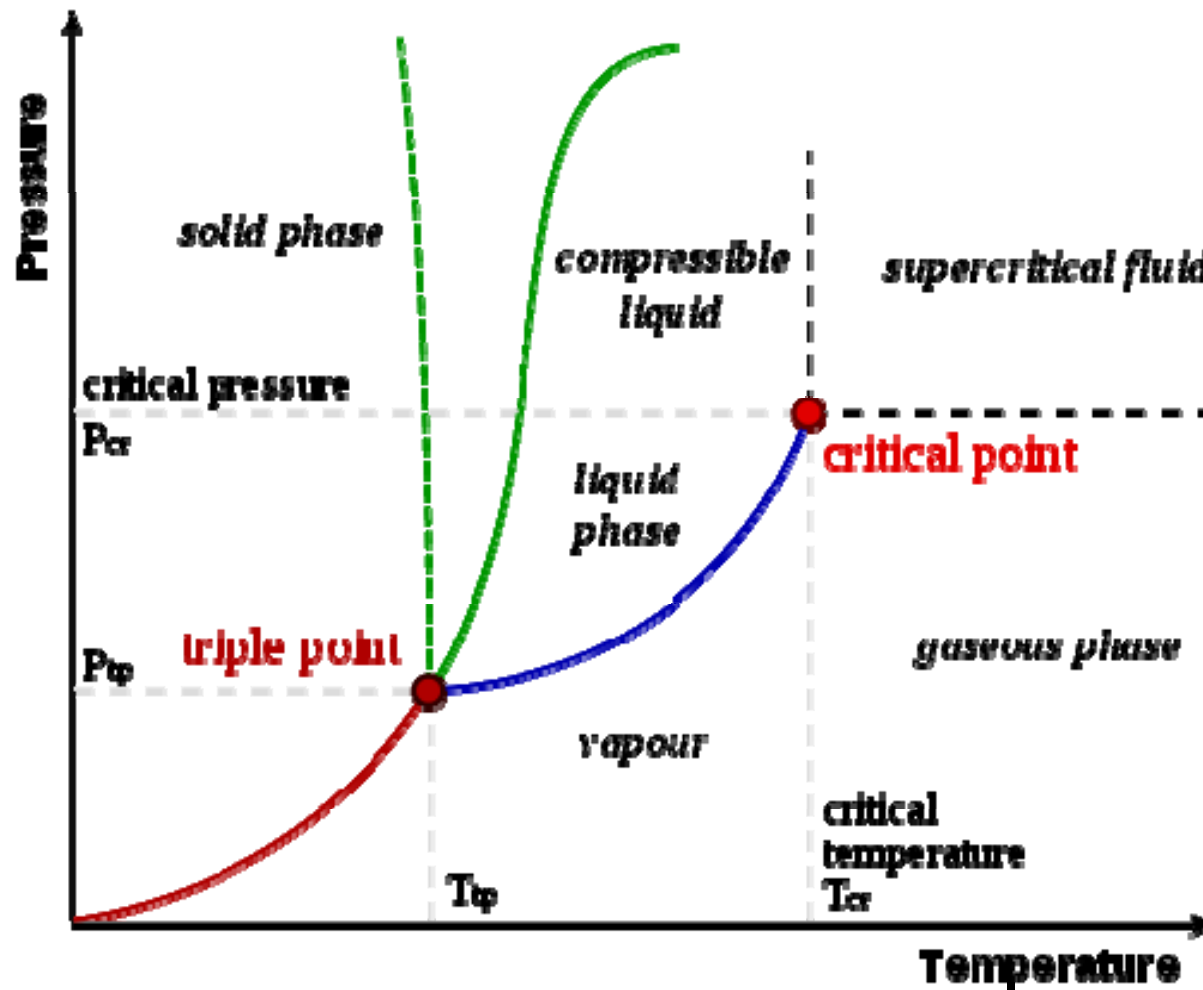
• A. Zaje

# An Updated Version of Fermi's Diagram

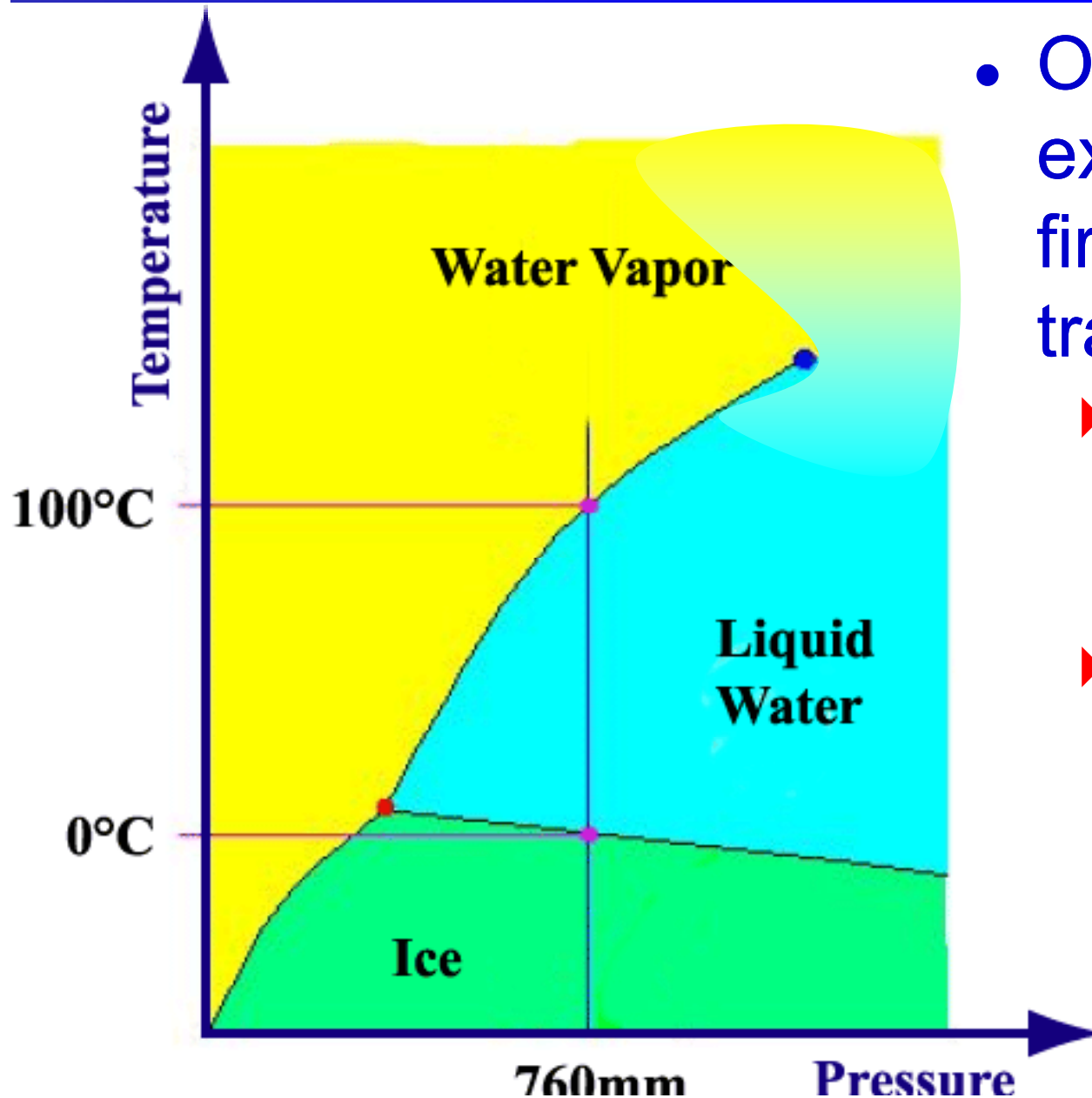


# Phase Diagrams

- A fundamental tool to summarize the qualitatively different phases of a complex system.



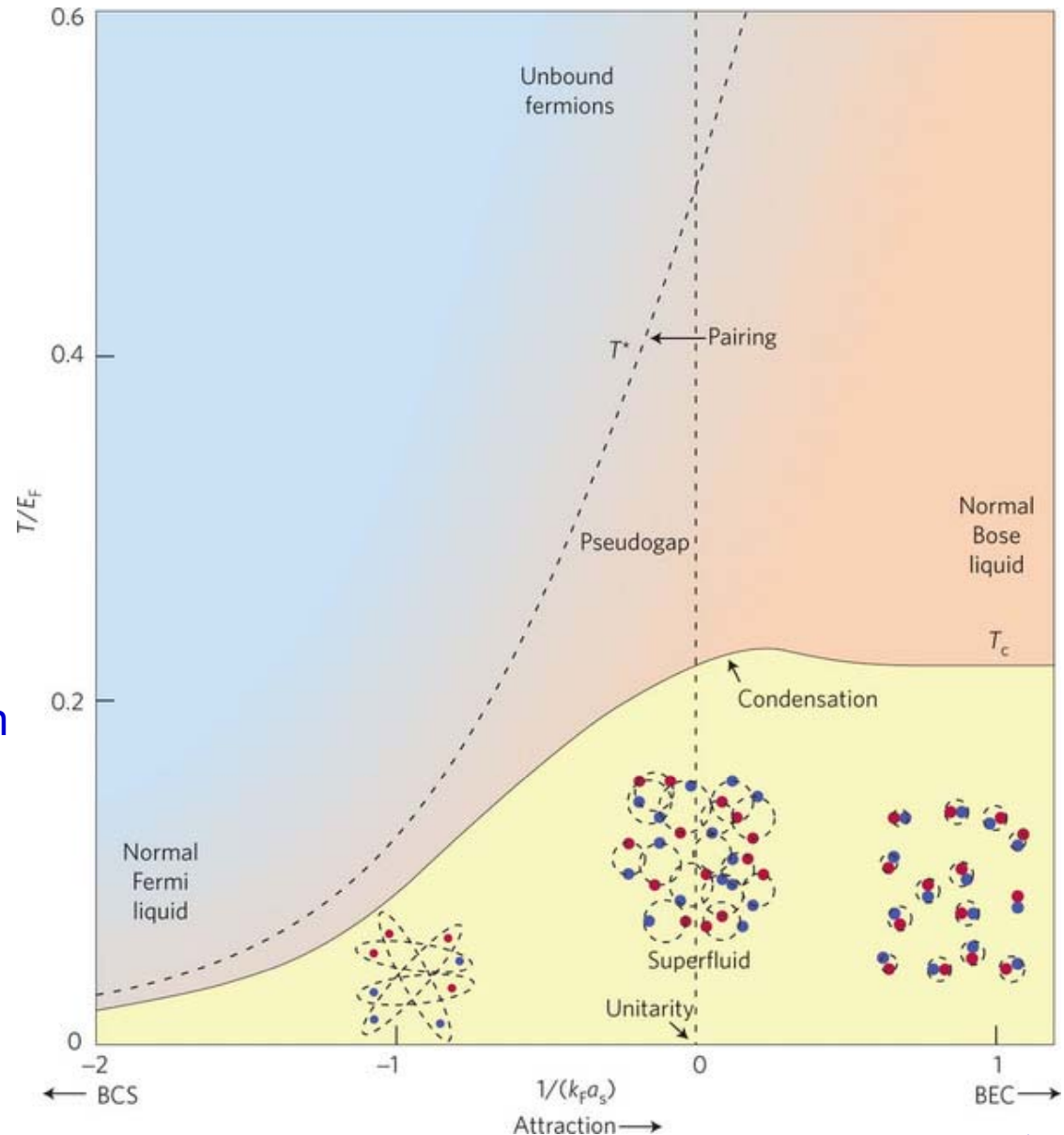
# A Familiar Phase Diagram



- Our best known examples of first-order phase transitions
  - ▶ Note that here we can independently vary  $T$  and  $P$
  - ▶ Note also presence of critical point  $\Rightarrow$  vanishing of 1<sup>st</sup> order transition

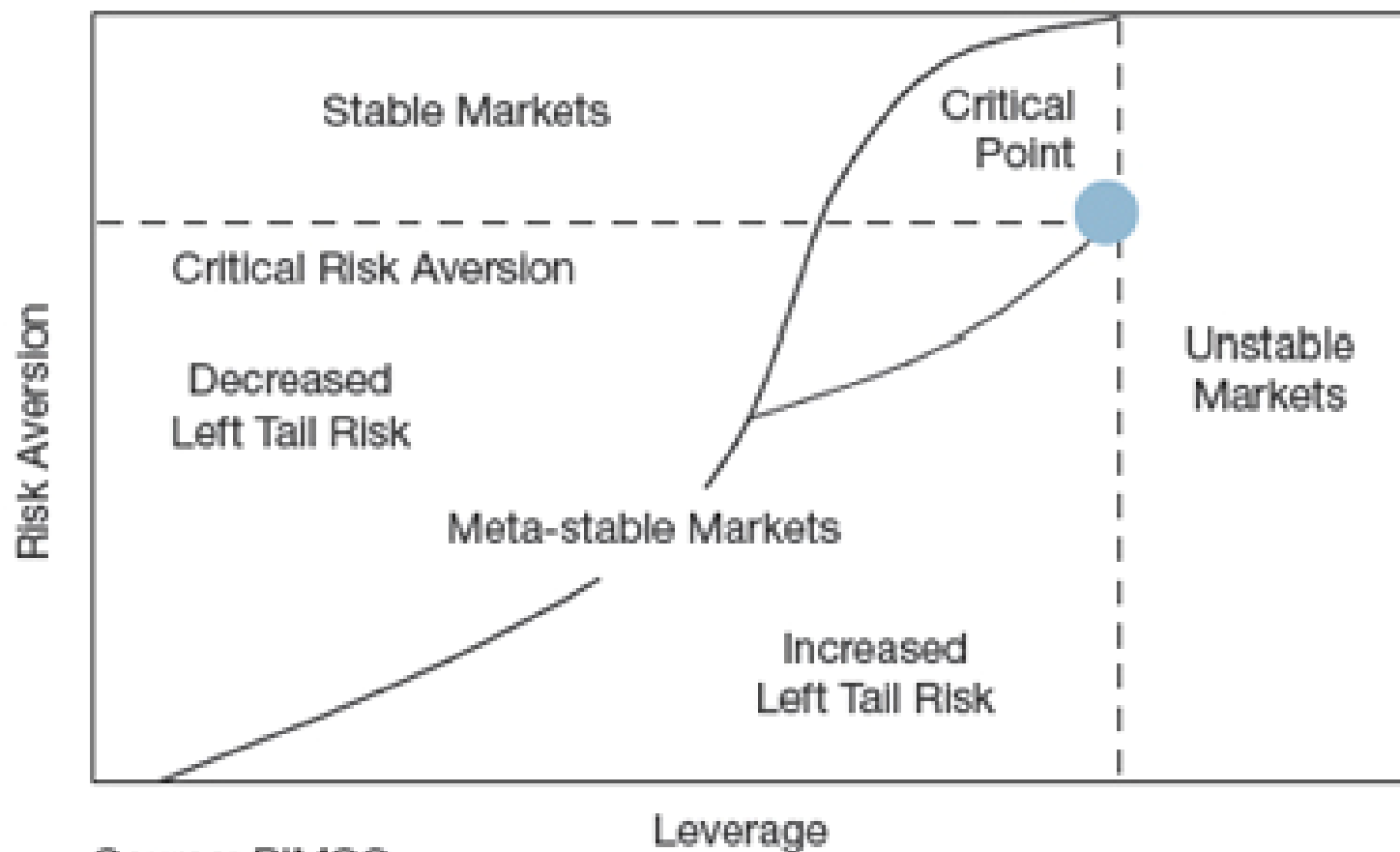
# Ultracold Fermi Gas in Unitary Limit

- Ultracold Fermi gases:  
Pre-pairing for condensation,  
Mohit Randeria,  
[Nature Physics 6, 561-562](#)  
(2010)
- **Abstract:** Pair formation and condensation usually occur together in Fermi superfluids. The observation of a pseudogap that implies pairing above the condensation temperature in a strongly interacting Fermi gas is thus an exciting development.



# Phase Diagram of a Truly Complex System

## Financial Market Phase Diagram

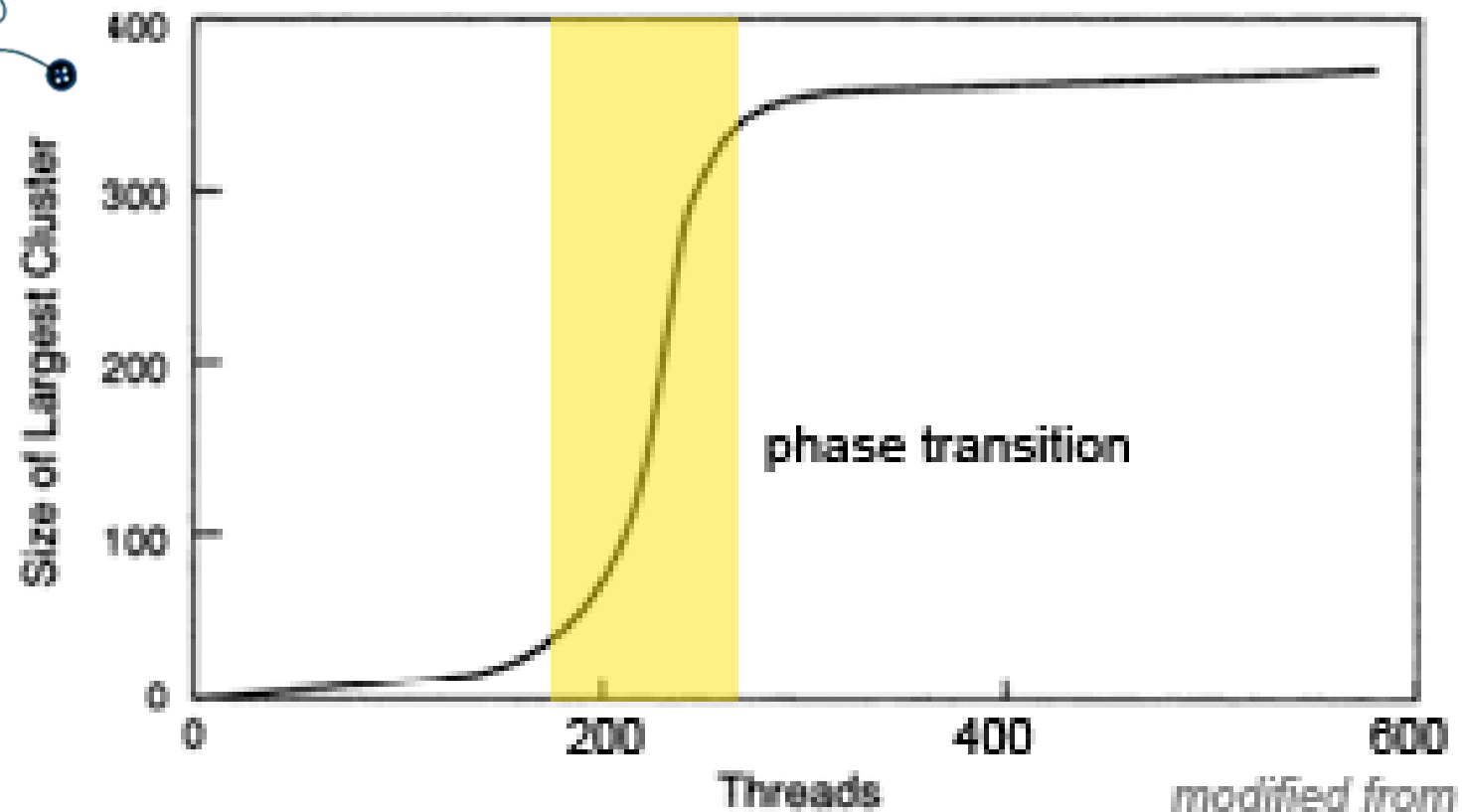
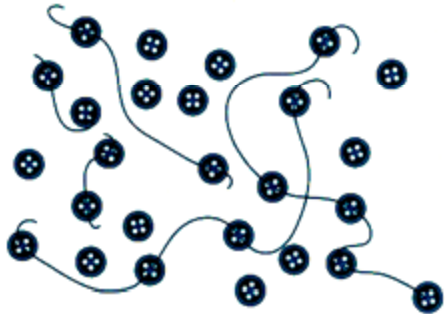


Source: PIMCO

Leverage



# Phase Diagram of a “Simple” System



*modified from  
Kauffman 1995*

# Science Questions

## × Uninteresting question:

- ▶ What happens when I crash two gold nuclei together?

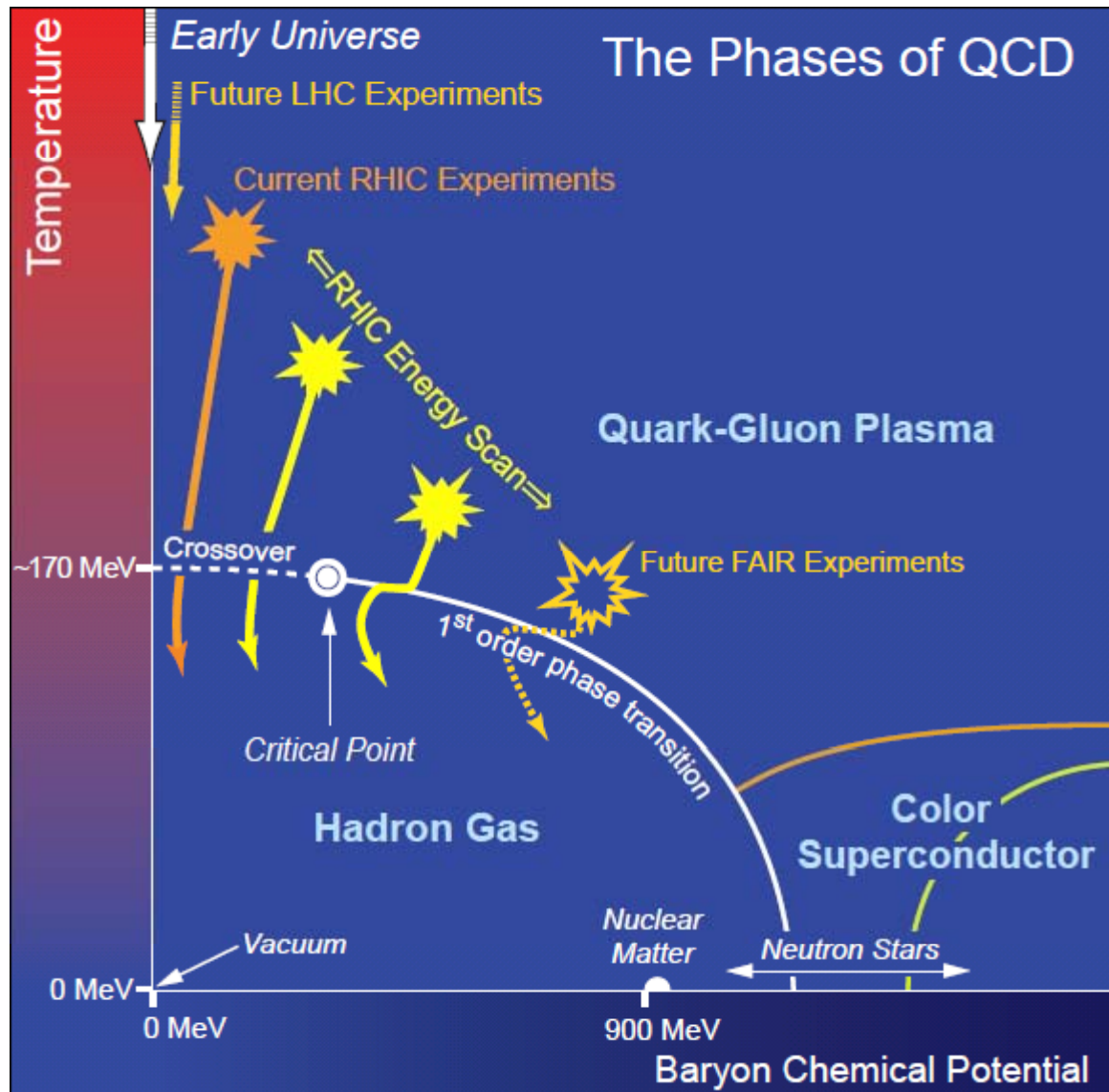
## ✓ Interesting question:

- ➡ Are there new states of matter at the highest temperatures and densities?

## \$ Compelling question:

- ☞ What fundamental, *thermal* properties of our gauge theories of nature can be investigated experimentally?
  - ☞ Hint: *Gravity* is a gauge theory...

# The QCD Phase Diagram



# On the Meaning of *Fundamental*

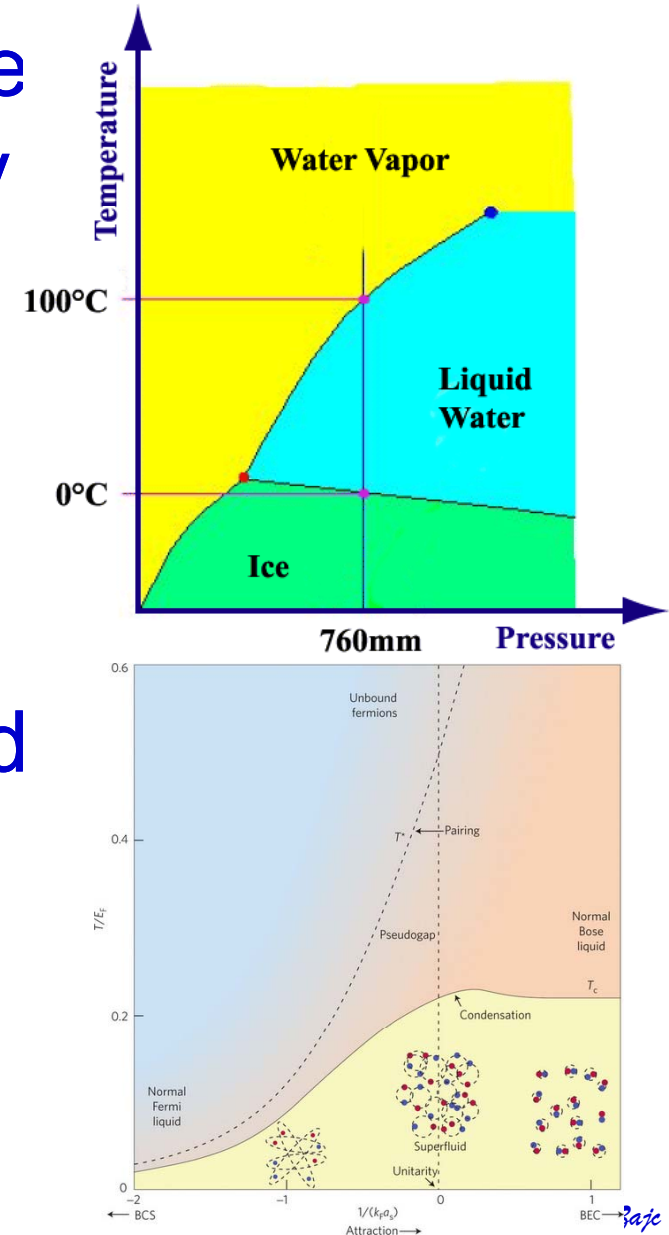
- In principle, these diagrams are also “fundamental”, in that they result from properties of

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$D_\mu \equiv \partial_\mu + ieA_\mu + ieB_\mu$$

- But in practice, they are (deeply) emergent, and depend on “unnatural” ratios:

- ▶  $m_p/m_e = 1836.15$
- ▶ Fine-tuning  
(to reach unitary limit)



# “Natural” Appearance of the QCD Phase Transition

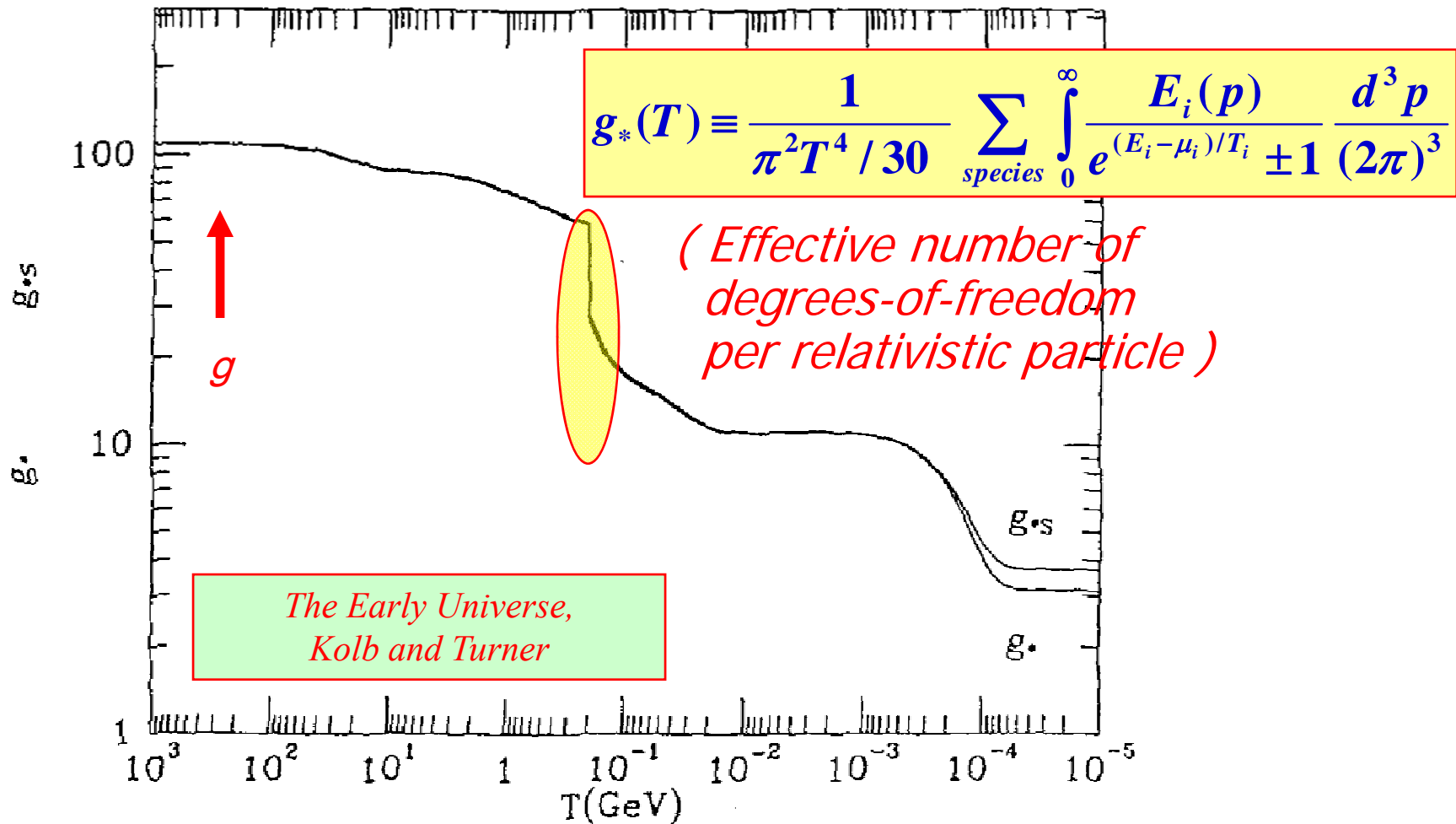
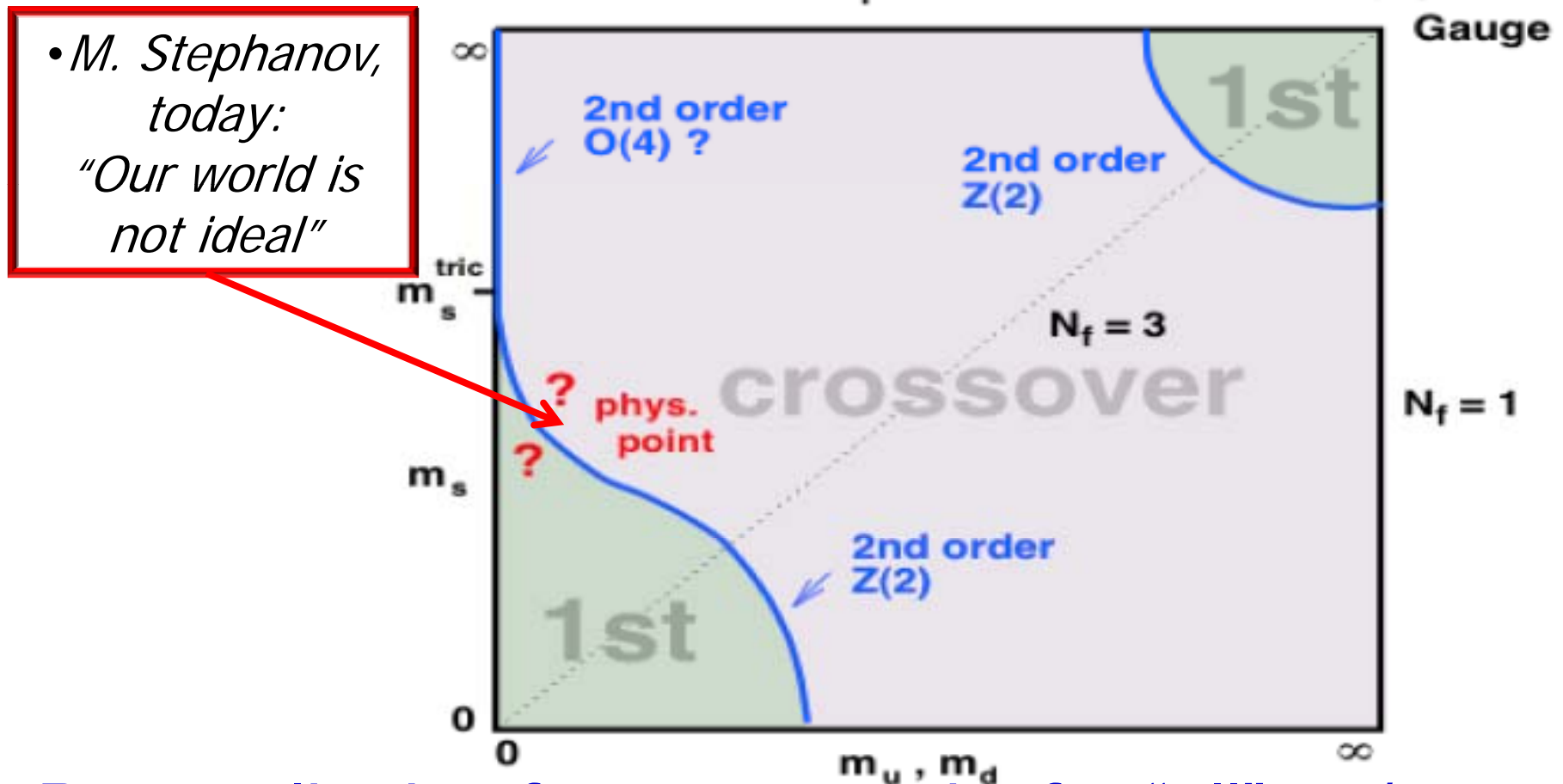


Fig. 3.5: The evolution of  $g_*(T)$  as a function of temperature in the  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  theory.

# To Be Fair...

- **Order** of QCD transition sensitive to  $m_s/m_{u,d}$

□ E. Laermann and O. Philipsen, hep-ph0303042

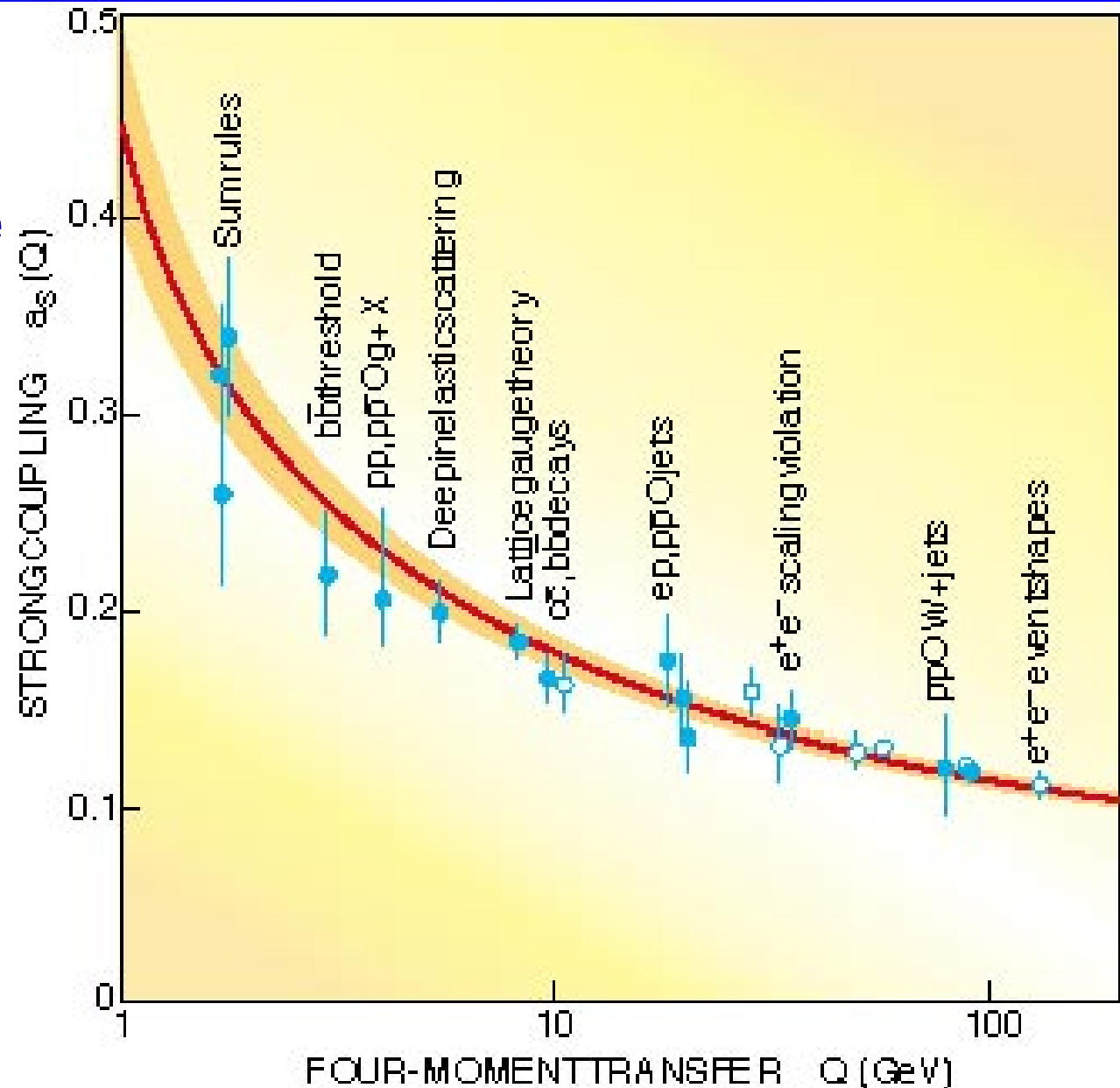


- But qualitative features persist for "all"  $m_s/m_{u,d}$

# Robustness of QCD Phase Transformation

Q. Why is there “always” a phase change in QCD?

A. Asymptotic freedom:



# Quantum Chromodynamics

- 1973 = Birth of QCD
- Gross, Politzer, Wilczek
- Explained
  - ▶ Asymptotic freedom
  - ▶ Infrared slavery

PHYSICAL REVIEW D

VOLUME 8, NUMBER 10

15 NOVEMBER 1973

## Asymptotically Free Gauge Theories. I\*

David J. Gross<sup>†</sup>

National Accelerator Laboratory, P. O. Box 500, Batavia, Illinois 60510  
and Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

Frank Wilczek

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

(Received 23 July 1973)

Asymptotically free gauge theories of the strong interactions are constructed and analyzed. The reasons for doing this are recounted, including a review of renormalization-group techniques and their application to scaling phenomena. The renormalization-group equations are derived for Yang-Mills theories. The parameters that enter into the equations are calculated to lowest order and it is shown that these theories are asymptotically free. More specifically the effective coupling constant, which determines the ultraviolet behavior of the theory, vanishes for large spacelike momenta. Fermions are incorporated and the construction of realistic models is discussed. We propose that the strong interactions be mediated by a "color" gauge group which commutes with  $SU(3) \times SU(3)$ . The problem of symmetry breaking is discussed. It appears likely that this would have a dynamical origin. It is suggested that the gauge symmetry might not be broken and that the severe infrared singularities prevent the occurrence of noncolor singlet physical states. The deep-inelastic structure functions, as well as the electron-positron total annihilation cross section are analyzed. Scaling obtains up to calculable logarithmic corrections, and the naive light-cone or parton-model results follow. The problems of incorporating scalar mesons and breaking the symmetry by the Higgs mechanism are explained in detail.

VOLUME 30, NUMBER 26

PHYSICAL REVIEW LETTERS

25 JUNE 1973

<sup>14</sup>Y. Nambu and G. Jona-Lasino, *Phys. Rev.* **122**, 345 (1961); S. Coleman and E. Weinberg, *Phys. Rev. D* **7**, 1888 (1973).

<sup>15</sup>K. Symanzik (to be published) has recently suggested that one consider a  $\lambda\phi^4$  theory with a negative  $\lambda$  to achieve UV stability at  $\lambda=0$ . However, one can show, using the renormalization-group equations, that in such theory the ground-state energy is unbounded from below (S. Coleman, private communication).

<sup>16</sup>W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, CERN Report No. CERN-TH-1538, 1972 (to be published).

<sup>17</sup>H. Georgi and S. L. Glashow, *Phys. Rev. Lett.* **28**, 1494 (1972); S. Weinberg, *Phys. Rev. D* **5**, 1962 (1972).

<sup>18</sup>For a review of this program, see S. L. Adler, in Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois, 1972 (to be published).

## Reliable Perturbative Results for Strong Interactions?\*

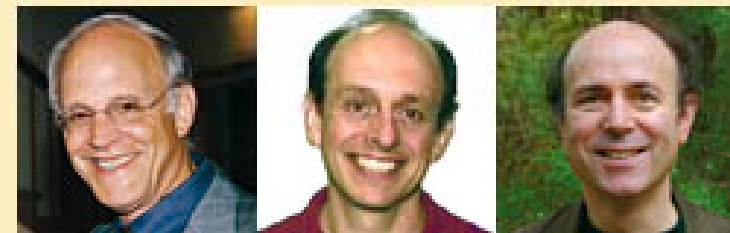
H. David Politzer

Jefferson Physical Laboratories, Harvard University, Cambridge, Massachusetts 02138  
(Received 3 May 1973)

An explicit calculation shows perturbation theory to be arbitrarily good for the deep Euclidean Green's functions of any Yang-Mills theory and of many Yang-Mills theories with fermions. Under the hypothesis that spontaneous symmetry breakdown is of dynamical origin, these symmetric Green's functions are the asymptotic forms of the physically significant spontaneously broken solution, whose coupling could be strong.

Renormalization-group techniques hold great promise for studying short-distance and strong-coupling problems in field theory.<sup>1,2</sup> Symanzik<sup>2</sup>

goes to zero, compensating for the fact that there are more and more of them. But the large- $\rho^2$  divergence represents a real breakdown of



David J. Gross  
Kavli Institute  
for Theoretical  
Physics  
University of  
California, Santa  
Barbara, USA

H. David  
Politzer  
California  
Institute of  
Technology  
(Caltech),  
Pasadena,  
USA

Frank  
Wilczek  
Massachusetts  
Institute of  
Technology  
(MIT),  
Cambridge,  
USA



# Quantum Chromodynamics (QCD)

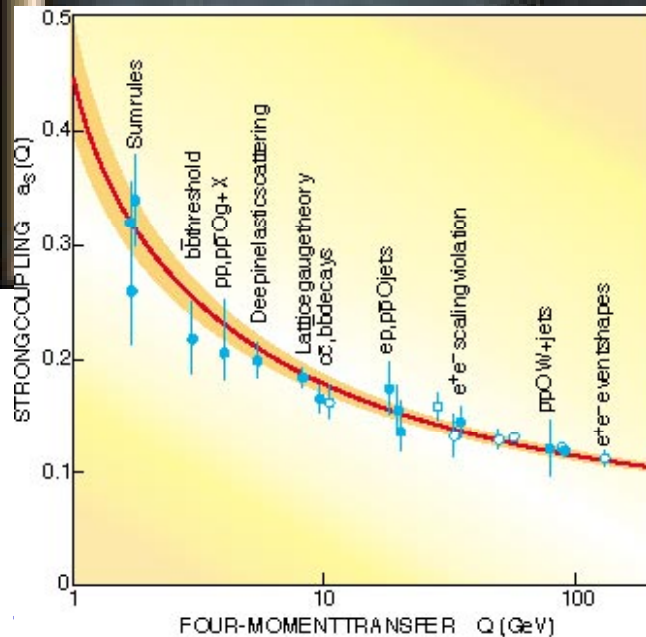
$$\mathcal{L} = \frac{1}{4g^2} G_{\mu\nu}^a G_{\mu\nu}^a + \sum_j \bar{\psi}_j (i \not{\partial} \not{D}_\mu + m_j) \psi_j$$

$$\text{where } G_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf_{abc} A_\mu^b A_\nu^c$$

$$\text{and } D_\mu = \partial_\mu + i t^a A_\mu^a$$

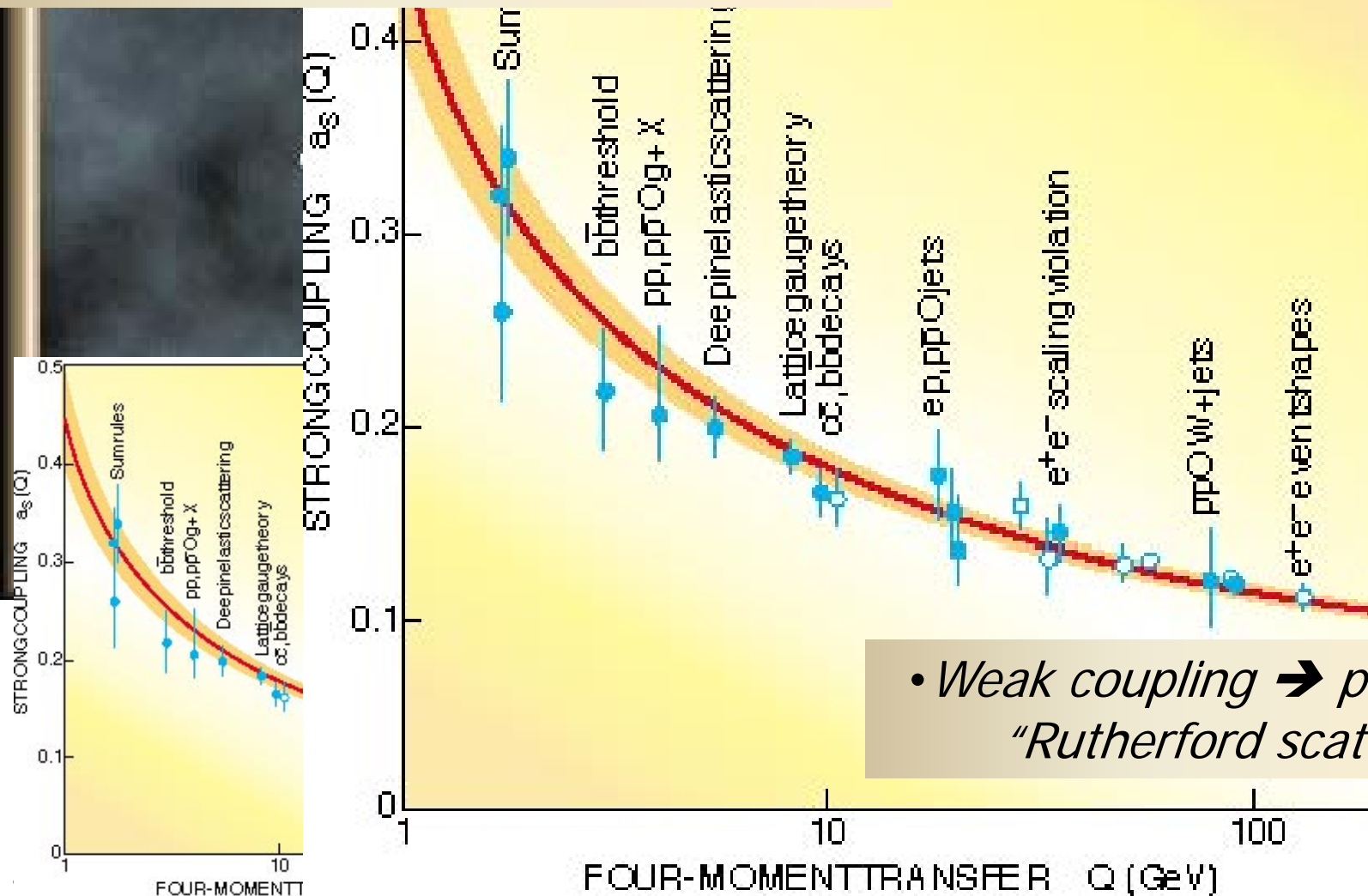
$$\alpha_s(Q^2) = \frac{g^2(Q^2)}{4\pi}$$

Now That's it!



# Quantum Chromodynamics (QCD)

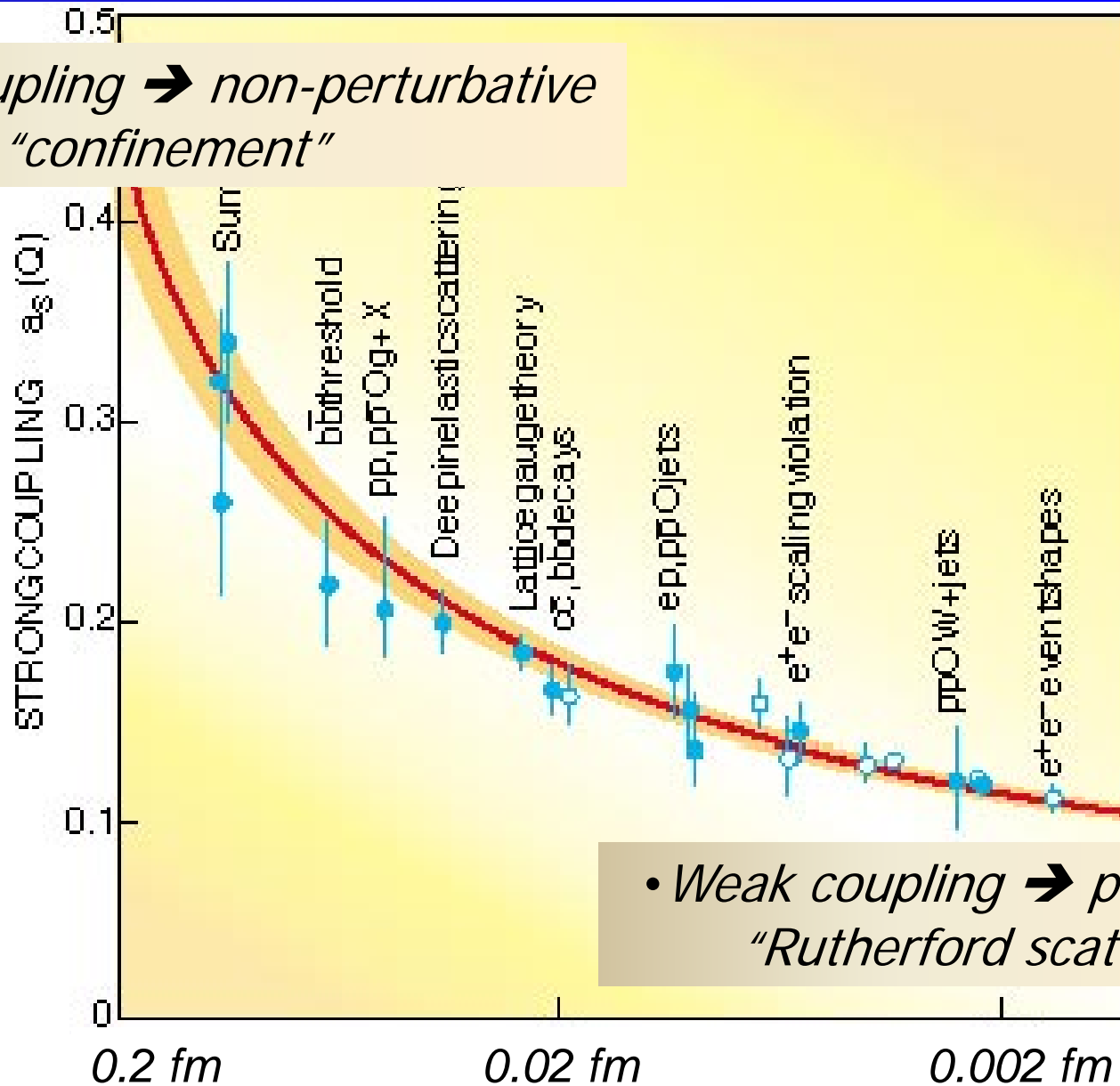
• *Strong coupling*  $\rightarrow$  *non-perturbative*  
*"confinement"*



• *Weak coupling*  $\rightarrow$  *perturbative*  
*"Rutherford scattering"*

# QCD's Running Coupling "Constant"

- *Strong coupling*  $\rightarrow$  *non-perturbative*  
*"confinement"*



- *Weak coupling*  $\rightarrow$  *perturbative*  
*"Rutherford scattering"*

# A. What Sets the Scale ?

- Simple answer – Quantum Mechanics:
  - ▶  $\hbar c = 200 \text{ MeV-fm} = 0.2 \text{ GeV-fm}$
  - ▶ So  $\hbar / (1 \text{ fm}) = 200 \text{ MeV} \sim \text{light hadron masses}$
- Deeper answer – vacuum condensate that establishes confinement scale  $\Lambda \sim 200 \text{ MeV}$

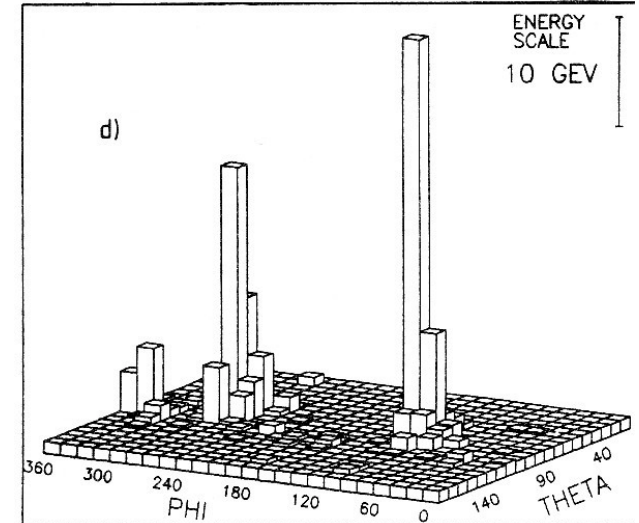
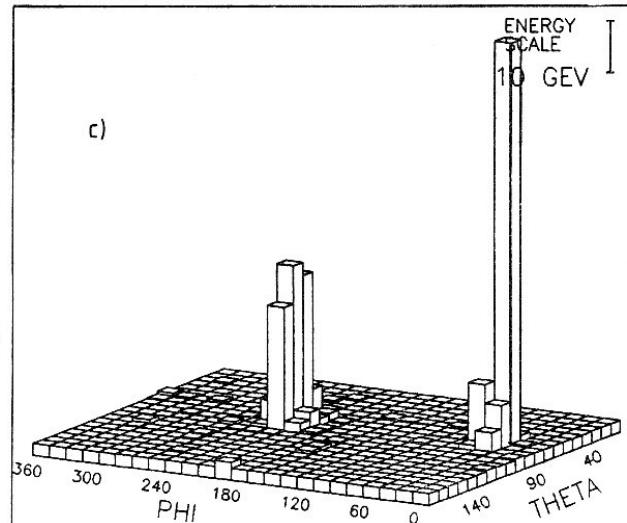
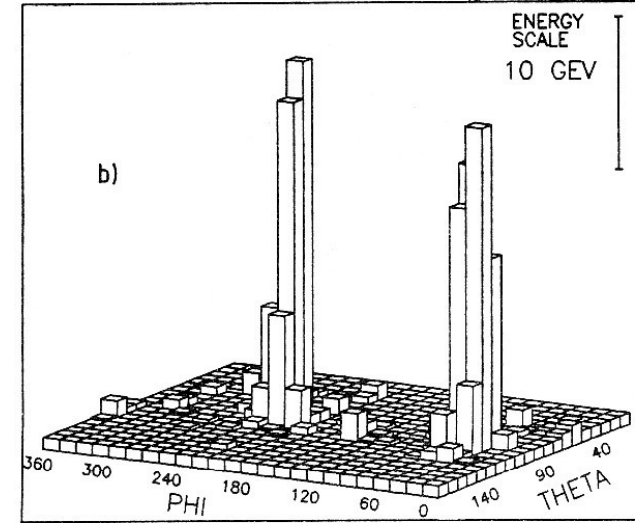
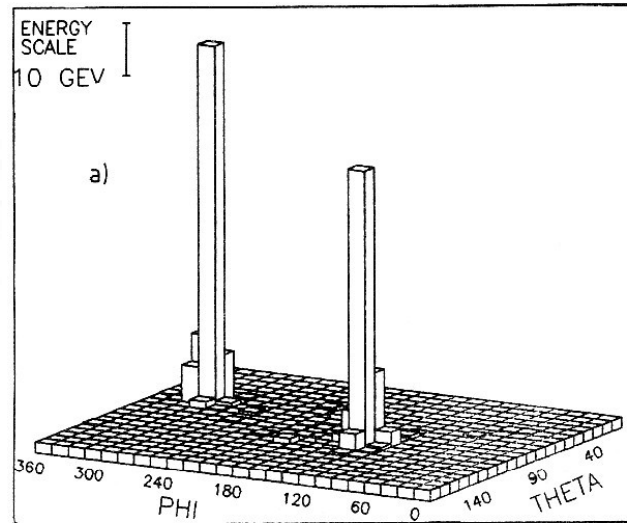
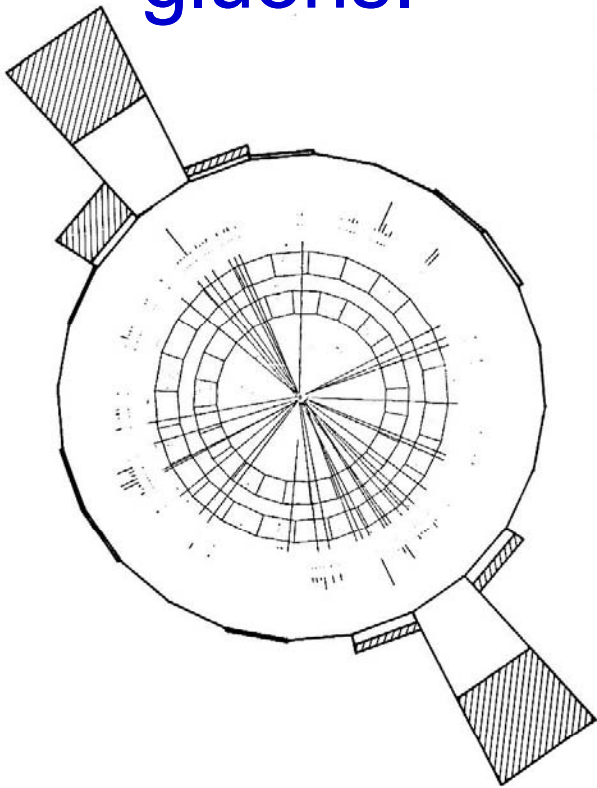
$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_F) \log\left(\frac{Q^2}{\Lambda^2}\right)} \sim \frac{1}{\log\left(\frac{Q^2}{\Lambda^2}\right)} \quad \Lambda \approx \frac{\hbar c}{r_0} \approx 0.2 \text{ GeV}$$

# One Approach to Asymptotic Freedom

- High Energy Physics:
  - ▶ Highest possible momentum transfer  $Q$   
probing smallest possible distances  $r$  :
  - ▶  $Q \gg 20 \text{ GeV} \rightarrow r \ll 0.01 \text{ fm}$

# Discovery of QCD Jets ~1983

- Rutherford scattering of *asymptotically free* quarks and gluons.



# Another Approach to Asymptotic Freedom

- High Energy Physics:
  - ▶ Highest possible momentum transfer  $Q$  probing smallest possible distances  $r$ :
  - ▶  $Q \gg 20 \text{ GeV} \rightarrow r \ll 0.01 \text{ fm}$
- Nuclear Physics
  - ▶ Study *bulk* behavior of hadronic matter at highest possible temperatures  $T$  and/or densities
  - ▶ Turns out that  $T \sim 0.2 \text{ GeV}$  suffices  
 $\sim 2 \times 10^{12} \text{ K}$

# “QCD” Prediction of Deconfining Phase Transition

 *Note false dichotomy!*

- 1975: J.C. Collins and M.J. Perry,  
*Superdense Matter or Asymptotically Free Quarks?*,  
Phys. Rev. Lett. 34, 1353.

“...matter at densities higher than nuclear consists of a quark soup.  
The quarks become free at sufficiently high density.”

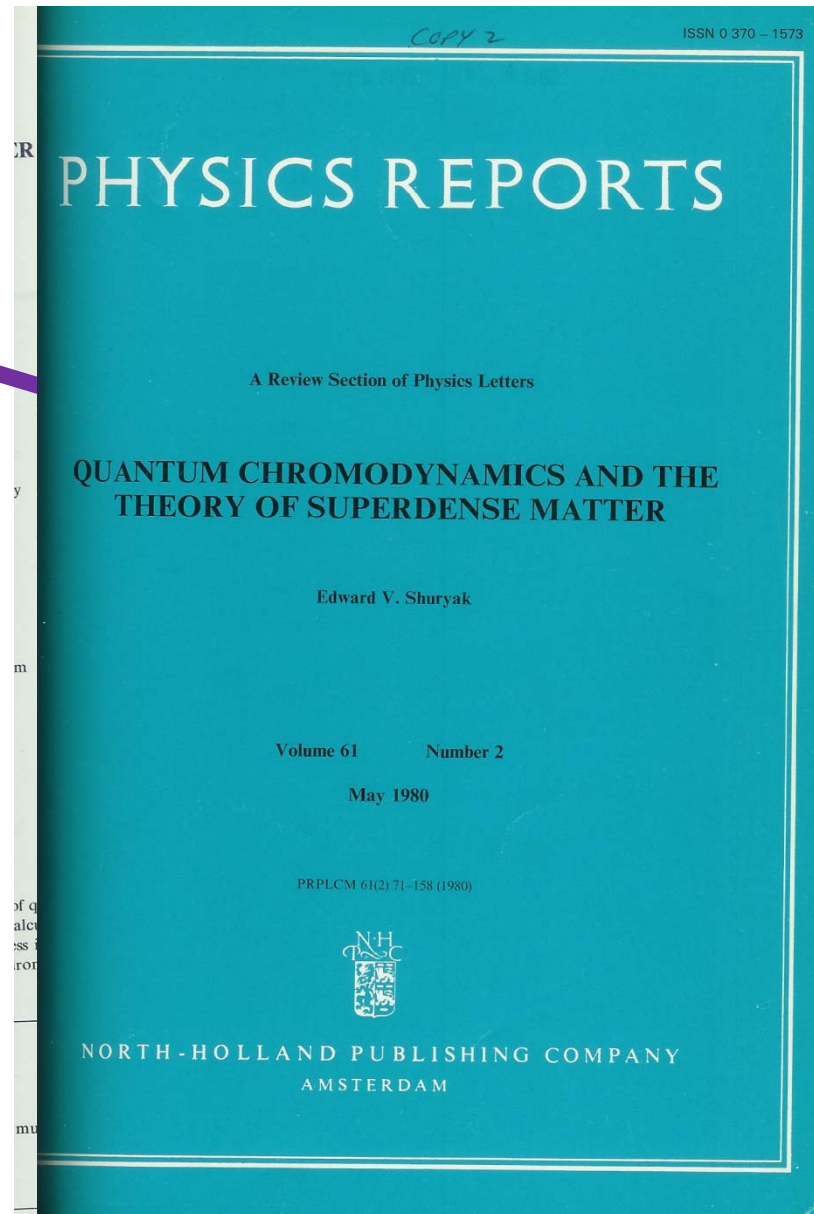


# Naming It

- Shuryak publishes first “review” of thermal QCD- and coins a phrase:

“Because of the apparent analogy with similar phenomena in atomic physics, we may call this phase of matter the QCD (or quark-gluon) plasma.”

**QGP**



to called quantum vector fields, the nomenclature and our hearts by the dynamics (QED). Now, relying upon the typical hadronic neutrons, etc.), but an analogy with (or quark-gluon) from the methods

ferences between ent in the physical ion (the so-called provide a complete ions of the gauge essed and, in the trol the vacuum

tions, in which a they are not too phase transition

tron stars. Such ory by means of he present work. ere goes beyond finite and homo- or discussing the ach is the recent rons [5.16, 5.17]. are not properly high speed. One d not to go into scuss ideas more sensive and self-

lpful discussions A.D. Linde, A.B. . Zakharov and

*W.A. Zajc*

# “Plasma” Leads To Prejudice

en.wikipedia.org/wiki/Plasma\_(physics)#Ultracold\_plasma

WIKIPEDIA  
The Free Encyclopedia

Article Talk

Plasma (physics)  
From Wikipedia, the free encyclopedia

For other uses, see *Plasma*.

plasma is a state of matter similar to gas in which a certain portion of the particles are ionized... plasma is a state of matter similar to gas in which a certain portion of the particles are ionized.

► Toolbox

► Print/export


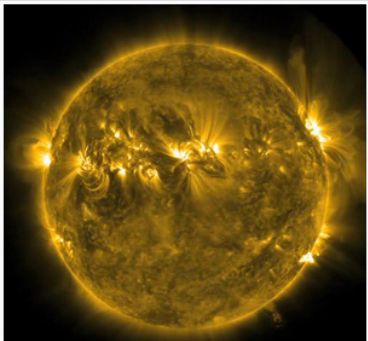
▼ Languages

- Afrikaans
- العربية
- Aragonés
- АваргаӀ
- Azərbaycanca
- Беларуская
- беларуская (тарашкевіца)
- Български
- Bosanski
- Català
- Česky
- Сұтмағ
- Dansk
- Deutsch
- Eesti
- Ελληνικό
- Español
- Esperanto
- Euskara
- فارسی

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    - 2.4.1 Thermal vs. non-thermal plasmas
  - 2.5 Potentials
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  - 2.7 Comparison of plasma and gas phases
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  - 3.3 Electric fields and circuits
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- 5 Artificial plasmas
  - 5.1 Generation of artificial plasma

Plasma globe, illustrating some of the more complex phenomena of a plasma, including *filamentation*. The colors are a result of relaxation of electrons in excited states to lower energy states after they have recombined with ions. These processes emit light in a spectrum characteristic of the gas being excited.

W. A. Zajc

# The Tyranny of Asymptotic Freedom

- From QCD Made Simple  
(F. Wilczek, August, 2000 Physics Today ):  
Hoping to bypass this forbidding mess, we invoke a procedure that is often useful in theoretical physics. I call it the **Jesuit Stratagem**, inspired by what I'm told is a credal tenet of the Order: "**It is more blessed to ask forgiveness than permission.**" The stratagem tells you to make clear-cut simplifying assumptions, work out their consequences, and check to see that you don't run into contradictions. **In this spirit we tentatively assume that we can describe high-temperature QCD starting with *free* quarks and gluons...**  
But there is a second, more rigorous level that remains a challenge for the future. Using fundamental aspects of QCD theory, similar to those I discussed in connection with jets, one can make quantitative predictions for the emission of various kinds of "hard" radiation from a quark-gluon plasma. **We will not have done justice to the concept of a *weakly interacting plasma of quarks and gluons* until some of these predictions are confirmed by experiment.**

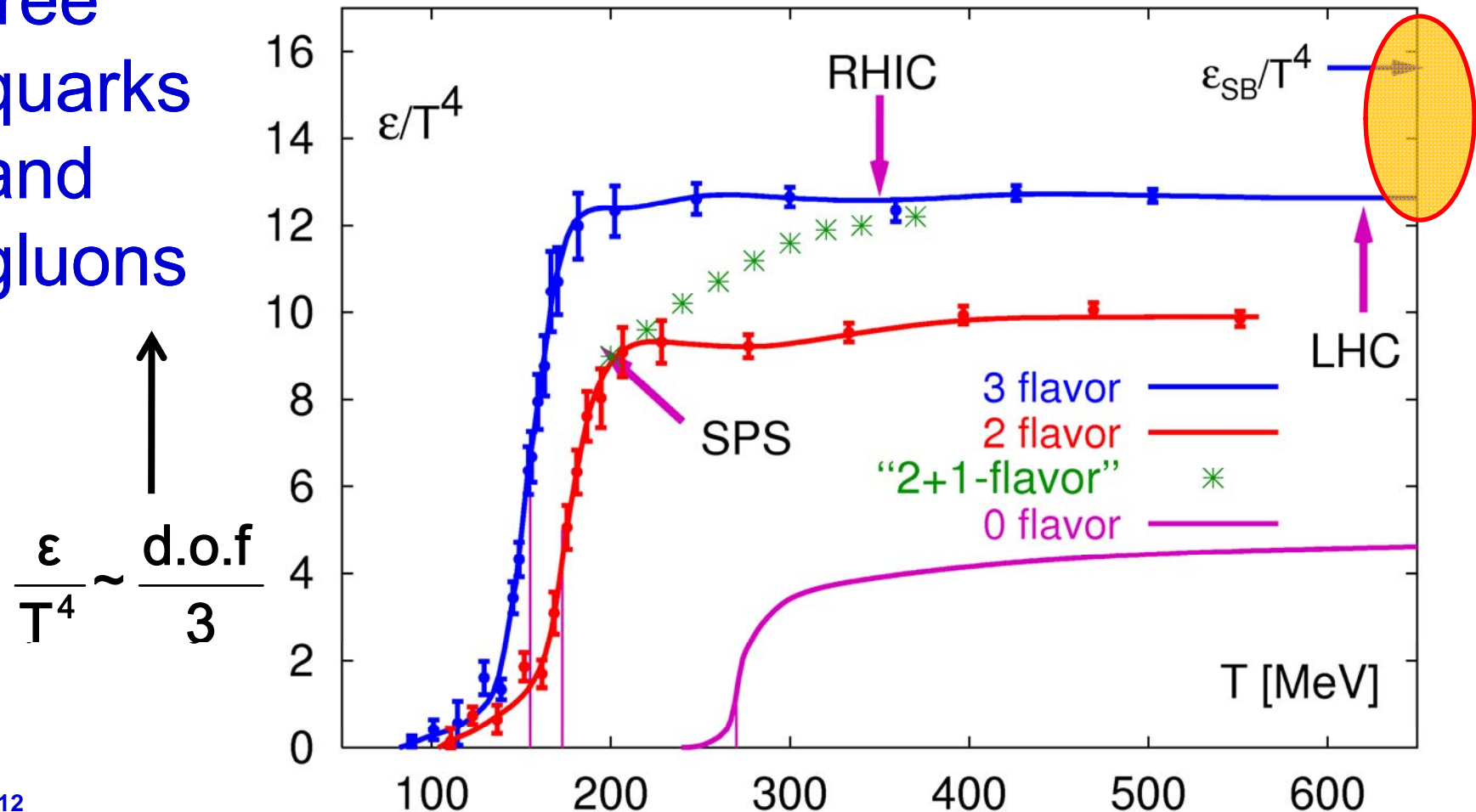
# Also From 2000: The Science Statement

- A series of experiments using CERN's lead beam have presented compelling evidence for the existence of a new state of quark-gluon matter in which *quarks*, instead of being bound up into more complex particles such as protons and neutrons, *are liberated to roam freely*.
- Present theoretical ideas provide a more precise picture for this new state of matter: it should be a quark-gluon plasma (QGP), in which quarks and gluons, the fundamental constituents of matter, are no longer confined within the dimensions of the nucleon, *but free to move around over a volume in which a high enough temperature and/or density prevails*.
- *Quarks and gluons would then freely roam* within the volume of the fireball created by the collision.
- A common assessment of the collected data leads us to conclude that we now have compelling evidence that a new state of matter has indeed been created, at energy densities which had never been reached over appreciable volumes in laboratory experiments before and which exceed by more than a factor 20 that of normal nuclear matter. The new state of matter found in heavy ion collisions at the SPS *features many of the characteristics of the theoretically predicted quark-gluon plasma*.
- Even if a full characterization of the initial collision stage is presently not yet possible, *the data provide strong evidence that it consists of deconfined quarks and gluons*.

 (All emphasis added by WAZ)

# Lattice Results (Prejudice) circa 2000:

- Rapid rise in d.o.f at  $T \sim 170$  MeV seen as “near” the Stefan-Boltzmann limit for free quarks and gluons



# RHIC's Two Major Discoveries → *Paradigm Shift*

## • Discovery of strong “elliptic” flow:

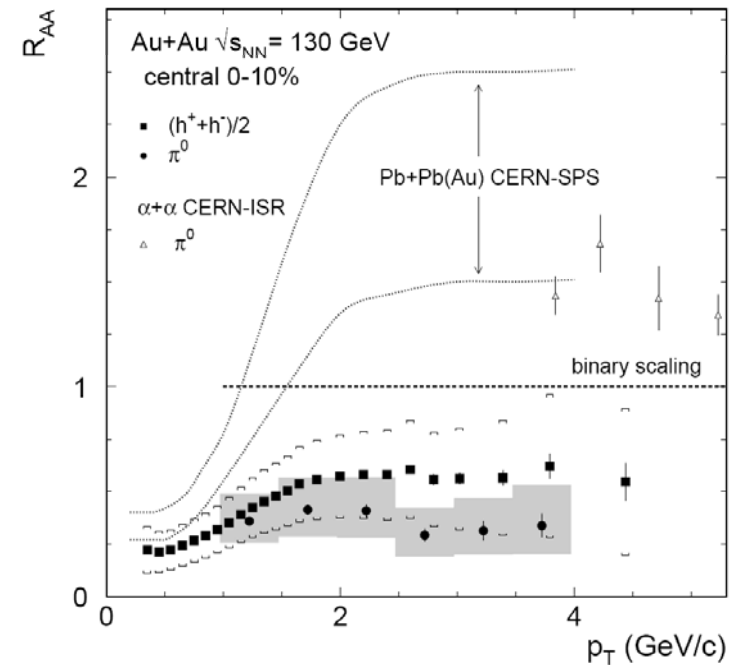
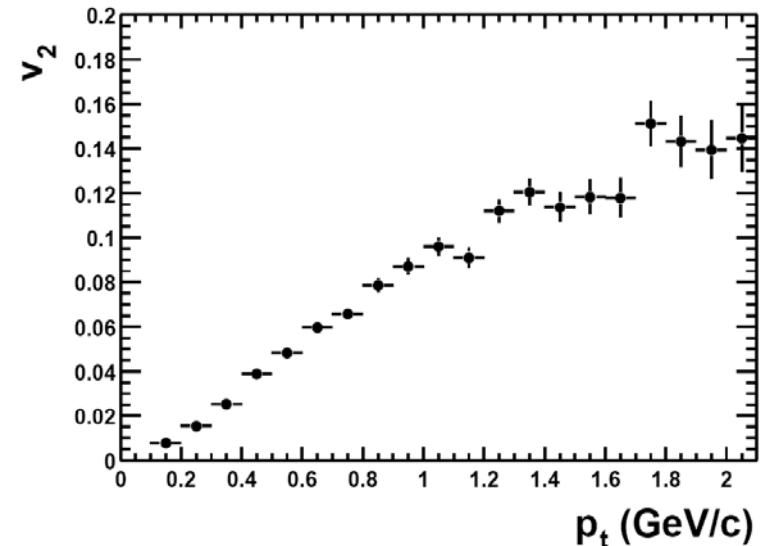
- ▶ Elliptic flow in Au + Au collisions at  $\sqrt{s_{NN}} = 130$  GeV, STAR Collaboration, (K.H. Ackermann *et al.*), [Phys.Rev.Lett.86:402-407,2001](#)

▶ 473 citations

## • Discovery of strong “jet quenching”

- ▶ Suppression of hadrons with large transverse momentum in central Au+Au collisions at  $\sqrt{s_{NN}} = 130$  GeV, PHENIX Collaboration (K. Adcox *et al.*), [Phys.Rev.Lett.88:022301,2002](#)

▶ 657 citations



# The New Paradigm

- Understood as manifestations of *strongly-coupled* Quark-Gluon Plasma (sQGP):
- *Not* “freely roaming” quarks and gluons
- Quite the opposite: strongly-coupled to the point of having (near) the minimal (?) value of viscosity to entropy density ratio  $\eta/s$  .

# The Tyranny of Asymptotic Freedom

- The conceptual beauty of asymptotic freedom was (is) hugely prejudicial for the QCD phase transition:
  - ▶ Rather than ice melting to liquid water
  - ▶ Thought it would be like the sublimation of frozen  $\text{CO}_2$  to gas, i.e, QCD phase transformation like “sublimation” of  $(p,n) \rightarrow (q,g)$





# Could This Have Been Anticipated?

- Perhaps:

$$\alpha_s(Q^2) \equiv \frac{g^2(Q^2)}{4\pi}$$

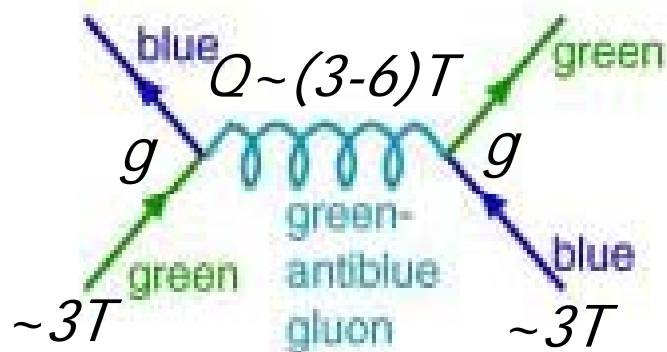
▶  $\langle p \rangle \sim 3T$

▶  $T \sim 200 \text{ MeV}$

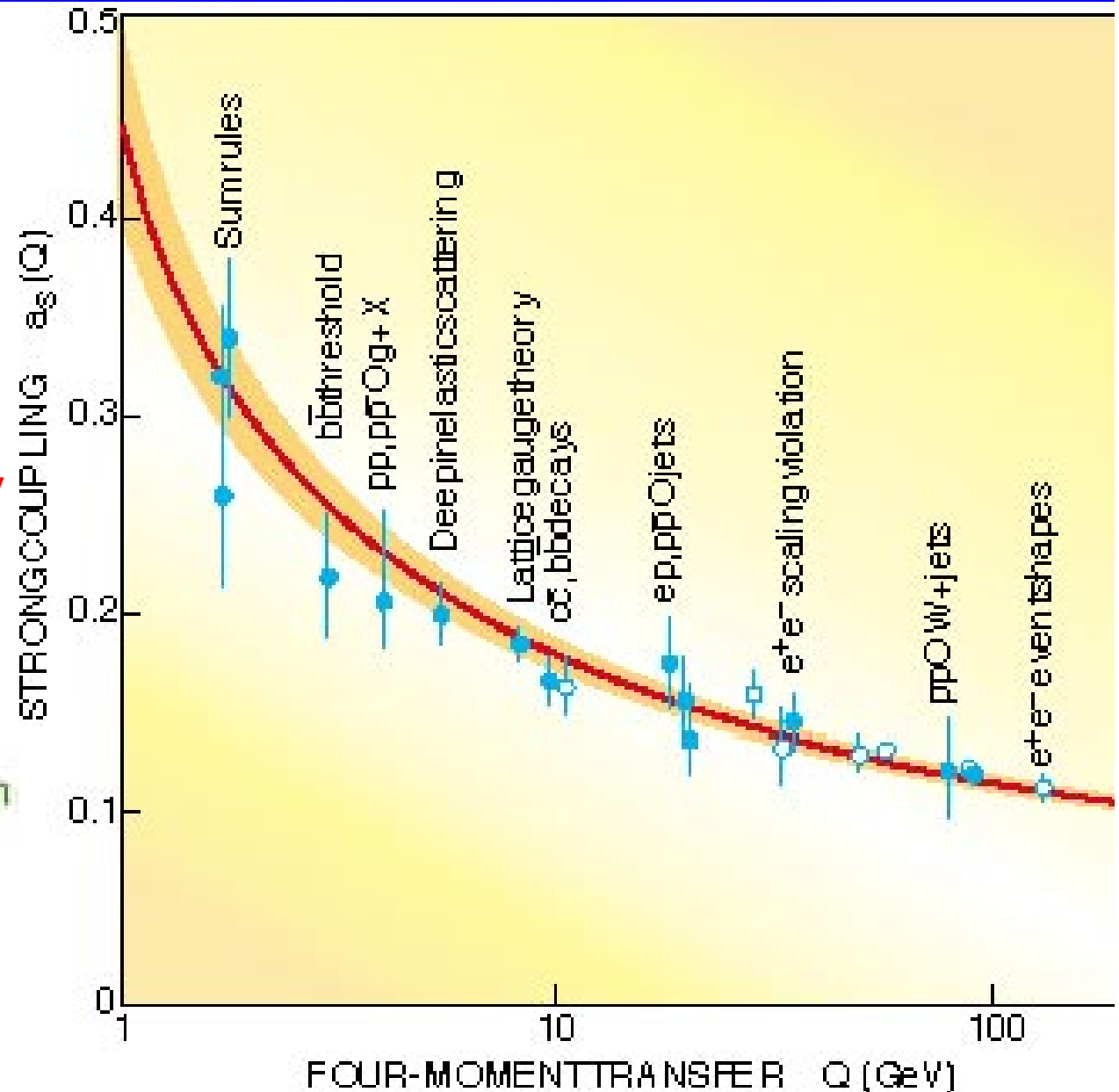
⇒  $Q < \sim 1 \text{ GeV}$

⇒  $\alpha_s(Q^2) \sim 0.5$

⇒  $g(Q^2) \sim 2.5$



12-Aug-12



# Could This Have Been Anticipated?

- Perhaps:

- ▶ “Real” plasma physicists always want to know the value of the classical coupling parameter  $\Gamma$ :

$$\Gamma \equiv \frac{\langle \text{Potential Energy} \rangle}{\langle \text{Kinetic Energy} \rangle}$$

- ▶ A (very) rough estimate:

$$\Gamma \equiv \frac{\langle \text{Potential Energy} \rangle}{\langle \text{Kinetic Energy} \rangle} = \frac{\text{Debye Mass}}{\langle \text{Kinetic Energy} \rangle} \sim \frac{gT}{3T} \sim 1$$

- Caveat: Not a useful measure in a truly relativistic system (but  $\eta/s$  is?)

# Actually, It Was Anticipated (in 1982)

- This property has been known long enough to be forgotten several times:

▶ 1982: Gordon Baym,  
proceedings of Quark Matter '82:

“...non-perturbative effects appear to play a big role. A hint of trouble can be seen from the first order result for the entropy density ( $N_f = 3$ )

$$s(T) = \frac{19\pi^2}{9} \left\{ 1 - \frac{54}{19\pi} \alpha_s(T) + \dots \right\} T^4$$

which turns negative for  $\alpha_s > 1.1$ ”

# Actually, It Was Anticipated (in 1992)

- This property has been known long enough to be forgotten several times:

- ▶ 1992: Berndt Mueller,  
Proc. of NATO Advanced Study Institute

“For plasma conditions realistically obtainable in the nuclear collisions ( $T \sim 250$  MeV,  $g = \sqrt{4\pi\alpha_s} = 2$ ) the effective gluon mass  $m_g^* \sim 300$  MeV. **We must conclude, therefore, that the notion of almost free gluons (and quarks) in the high temperature phase of QCD is quite far from the truth.** Certainly one has  $m_g^* \ll T$  when  $g \ll 1$ , but this condition is never really satisfied in QCD, because  $g \sim 1/2$  even at the Planck scale ( $10^{19}$  GeV), and  $g < 1$  only at energies above 100 GeV.”

# Actually, It Was Anticipated (in 2002)

- This property has been known long enough to be forgotten several times:
  - ▶ 2002: Ulrich Heinz,  
Proceedings of PANIC conference:  
“Perturbative mechanisms seem unable to explain the phenomenologically required very short thermalization time scale, pointing to strong non-perturbative dynamics in the QGP even at or above  $2T_c$ .... The quark-hadron phase transition is arguably the most strongly coupled regime of QCD.”

# Fermi Knew It (in 1950)

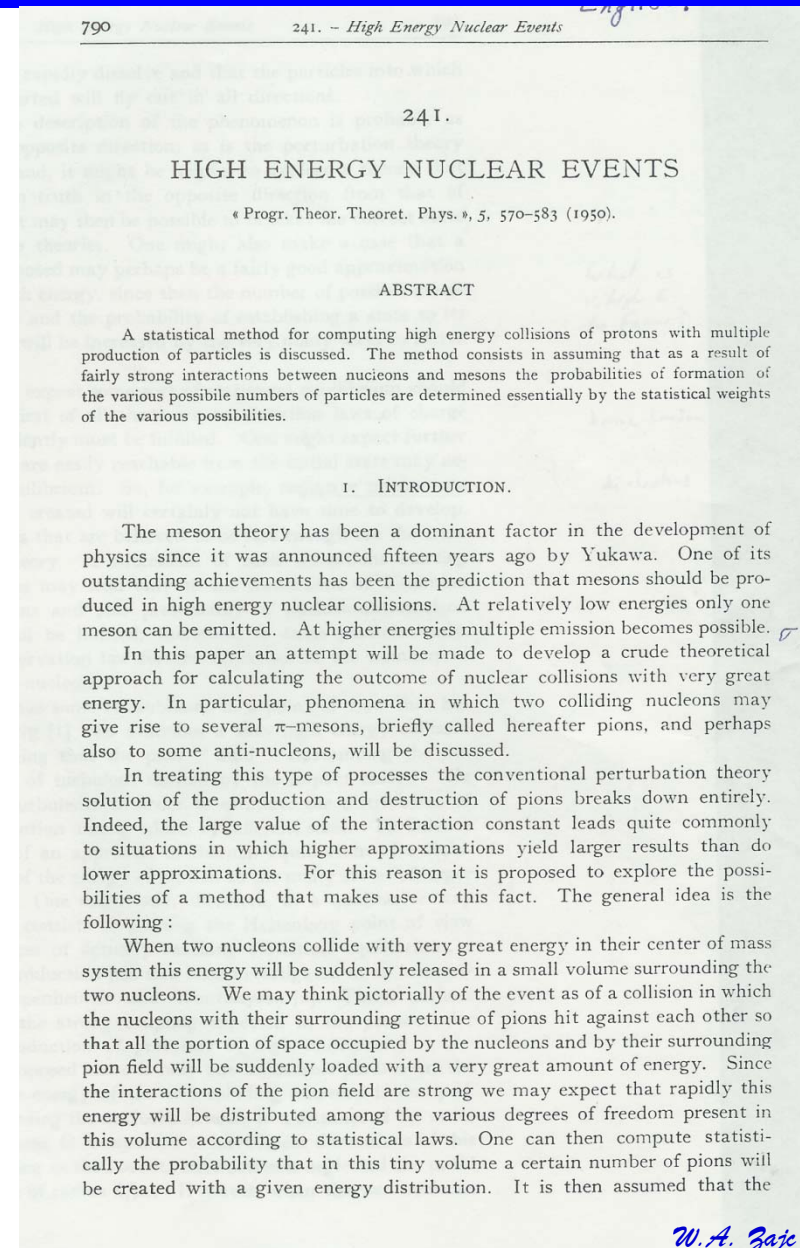
- Fermi (1950)

- ▶ “High Energy Nuclear Events”, Prog. Theor. Phys. 5, 570 (1950)

- ▶ Lays groundwork for statistical approach to particle production in strong interactions:

- “Since the interactions of the pion field are *strong*, we may expect that rapidly this energy will be distributed among the various degrees of freedom present in this volume according to statistical laws.”

- (Emphasis added by WAZ)



# Landau Knew It (in 1955)

- Landau (1955) significant extension of Fermi's approach
- Considers fundamental roles of
  - ▶ hydrodynamic evolution
  - ▶ entropy
- “The defects of Fermi's theory arise mainly because the expansion of the compound system is not correctly taken into account... (The) expansion of the system can be considered on the basis of relativistic hydrodynamics.”
- (Emphasis added by WAZ)

## 88. A HYDRODYNAMIC THEORY OF MULTIPLE FORMATION OF PARTICLES

### 1. INTRODUCTION

Experiment shows that in collisions of very fast particles a large number of new particles are formed in multi-prong stars. The energy of the particles which produce such stars is of the order of  $10^{12}$  eV or more. A characteristic feature is that such collisions occur not only between a nucleon and a nucleus but also between two nucleons. For example, the formation of two mesons in neutron-proton collisions has been observed at comparatively low energies, of the order of  $10^9$  eV, in cosmotron experiments<sup>1</sup>.

Fermi<sup>2,3</sup> originated the ingenious idea of considering the collision process at very high energies by the use of thermodynamic methods. The main points of his theory are as follows.

(1) It is assumed that, when two nucleons of very high energy collide, energy is released in a very small volume  $V$  in their centre of mass system. Since the nuclear interaction is very strong and the volume is small, the distribution of energy will be determined by statistical laws. The collision of high-energy particles may therefore be treated without recourse to any specific theories of nuclear interaction.

(2) The volume  $V$  in which energy is released is determined by the dimensions of the meson cloud around the nucleons, whose radius is  $\hbar/\mu c$ ,  $\mu$  being the mass of the pion. But since the nucleons are moving at very high speeds, the meson cloud surrounding them will undergo a Lorentz contraction in the direction of motion. Thus the volume  $V$  will be, in order of magnitude,

$$V = \frac{4\pi}{3} \left( \frac{\hbar}{\mu c} \right)^3 \frac{2M c^2}{E'}, \quad (1.1)$$

where  $M$  is the mass of a nucleon and  $E'$  the nucleon energy in the centre of mass system.

(3) Fermi assumes that particles are formed, in accordance with the laws of statistical equilibrium, in the volume  $V$  at the instant of collision. The particles formed do not interact further with one another, but leave the volume in a “frozen” state.

С. З. Беленький и Л. Д. Ландау, Гидродинамическая теория множественного образования частиц, *Успехи Физических Наук*, 56, 309 (1955).

S. Z. Belenkij and L. D. Landau, Hydrodynamic theory of multiple production of particles, *Nuovo Cimento*, Supplement, 3, 15 (1956).

# Landau on Viscosity

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(4) Fermi calculations lead to the same results as those obtained from the study of collisions. In momentum collisions an obtained.

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(2) The second stage of the collision consists in the expansion of the system. Here the hydrodynamic approach must be used, and the expansion may be regarded as the motion of an ideal fluid (zero viscosity and zero thermal con-

† The conditions of applicability of thermodynamics and hydrodynamics are comprised in the requirement  $l/L < 1$ , where  $l$  is the "mean free path" and  $L$  the least dimension of the system.

1) Use of hydro relies on  $R/\lambda \gg 1$

2) Negligible viscosity  $\eta$  equivalent to large Reynolds number  $\mathcal{R}_e \gg 1$

$$\mathcal{R}_e \equiv \rho V R / \eta \sim V R / v_{th} \lambda$$

so for relativistic system  $V \sim v_{th}$  and

$$\mathcal{R}_e \gg 1 \Rightarrow R / \lambda \gg 1 ; \text{ see \#1}$$

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(2.3)

† This may be made clear by the following qualitative arguments. If viscosity and thermal conductivity are to be negligible, the Reynolds number  $L V / l v$  must be much greater than unity. Here  $L$  is the least dimension of the system,  $V$  the "macroscopic" velocity,  $v$  the "molecular" velocity and  $l$  the mean free path. Since  $V$  and  $v$  are of the order of  $c$ , the condition  $R > 1$  corresponds to  $l/L < 1$ .



# A Funny Thing About Viscosity

- **Strong** interactions imply **small** viscosity:
  - ▶ Viscosity  $\eta \sim$  Transverse momentum diffusion  
 $\sim n \langle p \rangle \lambda$  ,  $n$  = number density  
 $\langle p \rangle$  = mean momentum  
 $\lambda$  = mean free path  
 $= 1 / n\sigma$  ,  
 $\sigma$  = cross section
  - ▶ So  

$$\eta \sim n \langle p \rangle \lambda \sim \langle p \rangle / \sigma$$
  - ▶ **Large** interparticle cross section  $\rightarrow$  **small** viscosity

# What Is The Viscosity at RHIC?

- “Perfect fluid” (that is, “ideal hydrodynamics”) defined as “zero viscosity”.

$$\left. \begin{array}{l} \eta_{QGP} \sim 2 \times 10^{11} \text{ Pa} \cdot \text{s} \\ \eta_{H_2O} \sim 1 \times 10^{-3} \text{ Pa} \cdot \text{s} \end{array} \right\} \Rightarrow \frac{\eta_{QGP}}{\eta_{H_2O}} \sim 2 \times 10^{14}$$

$$\eta_{Pitch} \sim 2.3 \times 10^8 \text{ Pa} \cdot \text{s}$$

$$\eta_{Glass(A.P.)} \sim 10^{12} \text{ Pa} \cdot \text{s}$$



# Small Viscosity Compared to What ?

- Any engineer will tell you
  - ▶ *Kinematic viscosity*  $\eta / \rho \sim [\text{Velocity}] \times [\text{Length}]$   
is what matters
- Any relativist will tell you
  - ▶  $\rho \rightarrow \varepsilon + p$
- Any thermodynamicist will tell you
  - ▶  $\varepsilon + p = s T$  ( at  $\mu_B = 0$  )
- So  $\eta/\rho \rightarrow \eta/(\varepsilon + p) \rightarrow (\eta/sT) = (\eta/s) (1/T)$   
 $\sim$  (damping coefficient)  $\times$  (thermal time)

# How Small Can We Make $\eta/s$ ?

- Recall

- ▶ Viscosity  $\eta \sim$  Transverse momentum diffusion  
 $\sim n \langle p \rangle \lambda$

- ▶ Entropy density  $s \sim 4 n$  (massless non-interacting quanta)

- Then 
$$\frac{\eta}{s} \sim \frac{n \langle p \rangle \lambda}{4n} \sim \frac{1}{4} \langle p \rangle \lambda$$

- But uncertainty principle  $\rightarrow \langle p \rangle \lambda \geq \frac{\hbar}{2}$

- Which in turn implies 
$$\frac{\eta}{s} \geq \frac{\hbar}{8}$$

# This Is Yet Another Thing We Forgot (1985)

- Miklos Gyulassy and Pawel Danielewicz:

- ▶ *Dissipative Phenomena In Quark-Gluon Plasmas*  
P. Danielewicz, M. Gyulassy  
Phys.Rev. D31, 53, 1985.



noted restrictions on smallest allowed  $\eta$ :

- Most restrictive:
  - $\lambda > \hbar / \langle p \rangle \Rightarrow \eta > \sim n / 3$
  - But recall  $s = 3.6 n$  for the quanta they were considering
  - $\Rightarrow \eta/s > 1 / (3.6 \times 3) \sim \hbar / (4 \pi)$   
 $\sim 0.1 \hbar$

Before estimating  $\lambda_i$  via Eq. (3.2) we note several physical constraints on  $\lambda_i$ . First, the uncertainty principle implies that quanta transporting typical momenta  $\langle p \rangle$  cannot be localized to distances smaller than  $\langle p \rangle^{-1}$ . Hence, it is meaningless to speak about mean free paths smaller than  $\langle p \rangle^{-1}$ . Requiring  $\lambda_i \gtrsim \langle p \rangle_i^{-1}$  leads to the lower bound

$$\eta \gtrsim \frac{1}{3} n, \quad (3.3)$$

where  $n = \sum n_i$  is the total density of quanta. What seems amazing about (3.3) is that it is independent of dynamical details. There is a finite viscosity regardless of how large is the free-space cross section between the quanta. See Refs. 21 and 22 for examples illustrating how the thermalization rate of many-body systems is limited by the uncertainty principle.

# An Aside on Reynolds Number

- Show that  $\Re e \sim \frac{1}{4} \frac{T R}{(\eta / s)}$
- So that if  $\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$
- Then  $\Re e \geq \sim \pi \frac{T R}{\hbar} \sim 6\pi$

for Au+Au or Pb+Pb collisions

# Alternative History

- So the “perfect fluid” observed at RHIC with

$$\left( \frac{\eta}{s} \right)_{RHIC} \sim 0.1 \hbar$$

was immediately recognized as confirming the 1985 uncertainty principle estimate of Danielewicz and Gyulassy

- Except that's not what happened...

# Instead ...

- In 2003-4 a new estimate (bound?) appeared from the AdS/CFT correspondence in string theory (!):

▶ *Viscosity in Strongly Interacting Quantum Field Theories from Black Hole Physics,*

P. Kovtun, D.T. Son, A.O. Starinets,

[hep-th/0405231](http://arxiv.org/abs/hep-th/0405231)

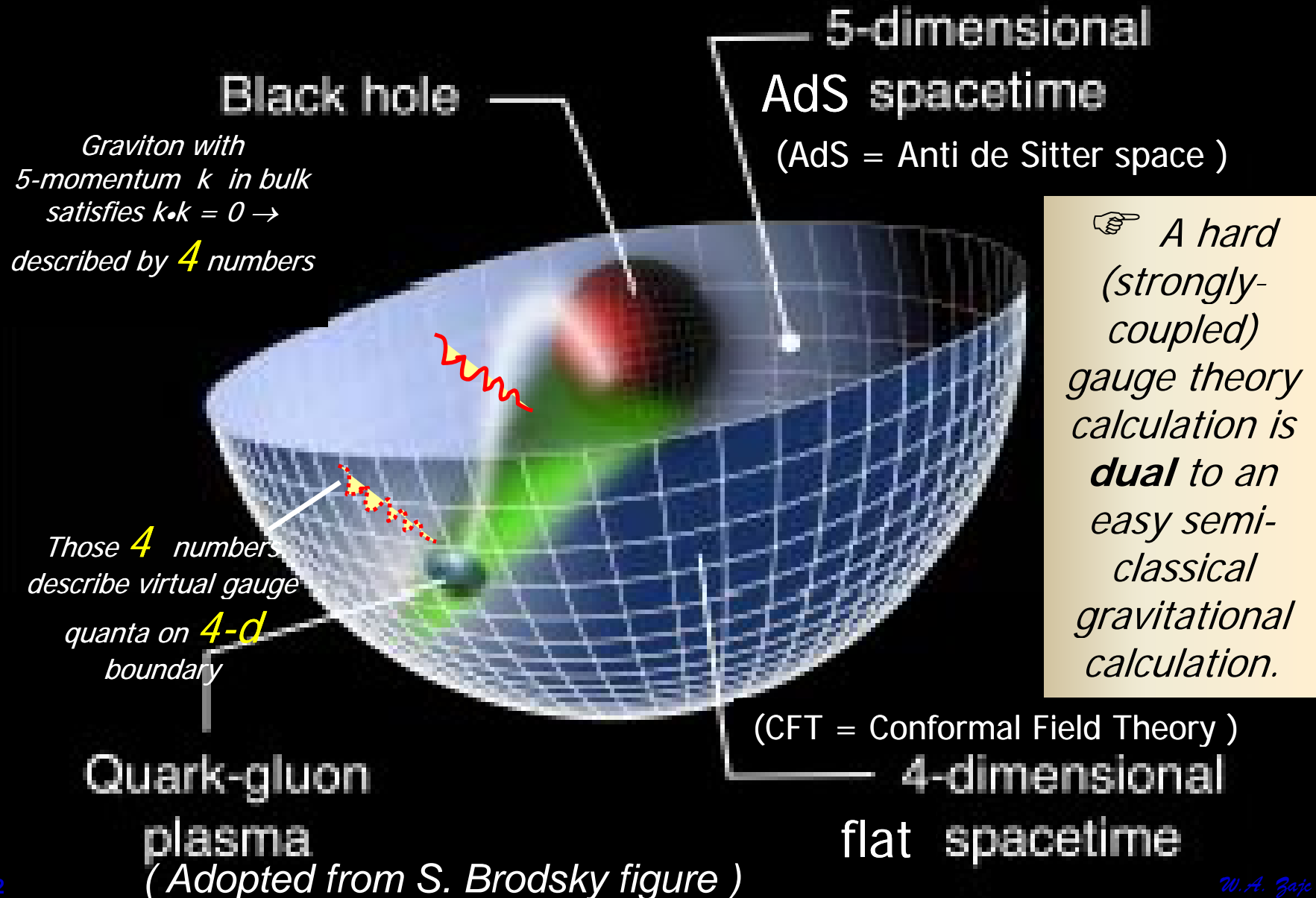
(968 citations)

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$$

- ▶ in a rigorous calculation with no (apparent) appeal to the uncertainty principle.



# AdS / CFT in a Picture

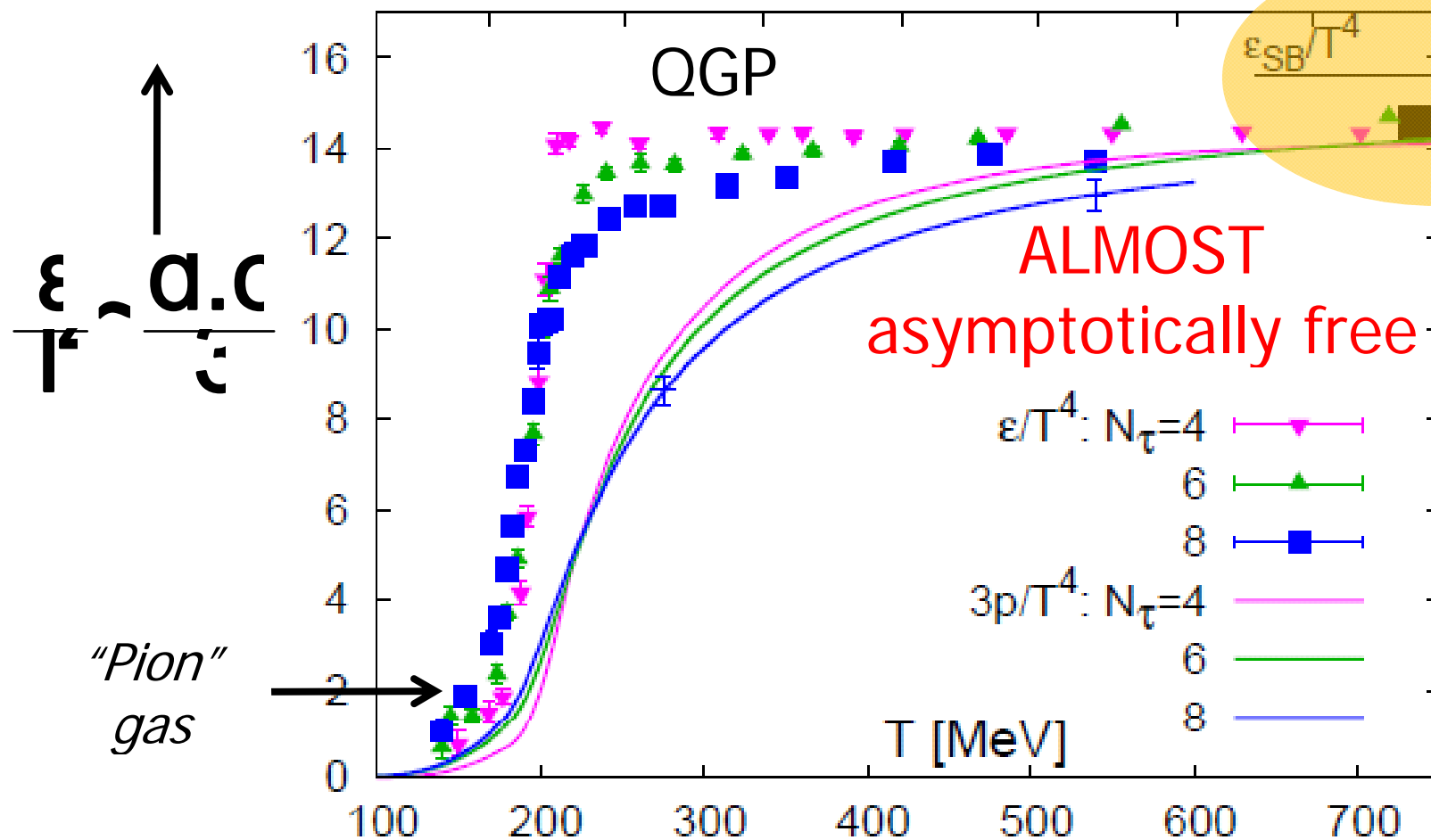


# The Required Disclaimer

- The “dual” field theory (CFT) is at best a very distant cousin of QCD:
  - ▶ No confinement
  - ▶ No running coupling constant
  - ▶ Many more (supersymmetric) degrees of freedom
- But at the same time
  - ▶ The AdS/CFT calculation is an exact result
  - ▶ In a strongly (infinitely) coupled theory
  - ▶ Also shows infinitely-coupled entropy density is  $\frac{3}{4}$  that of free entropy density(!)

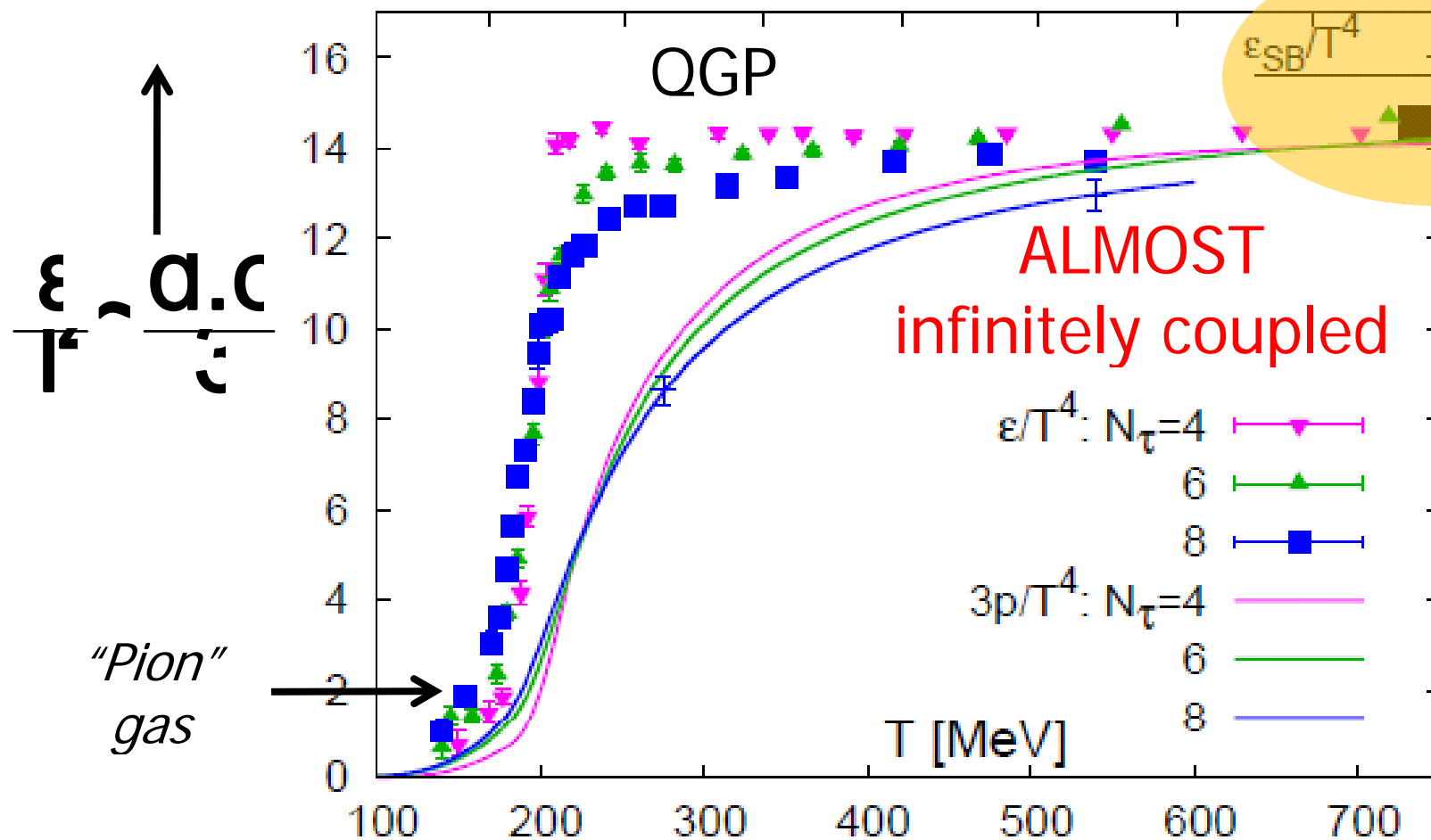
# The Paradigm Shift - Before

- Recall Stefan-Boltzmann  $\rightarrow \epsilon = \frac{T^4}{3} \sim \frac{1}{3} T^4$  *per* d.o.f.



# The Paradigm Shift – After

- Recall Stefan-Boltzmann  $\rightarrow \epsilon = \frac{T^4}{3} \sim \frac{1}{3} T^4$  *per* d.o.f.

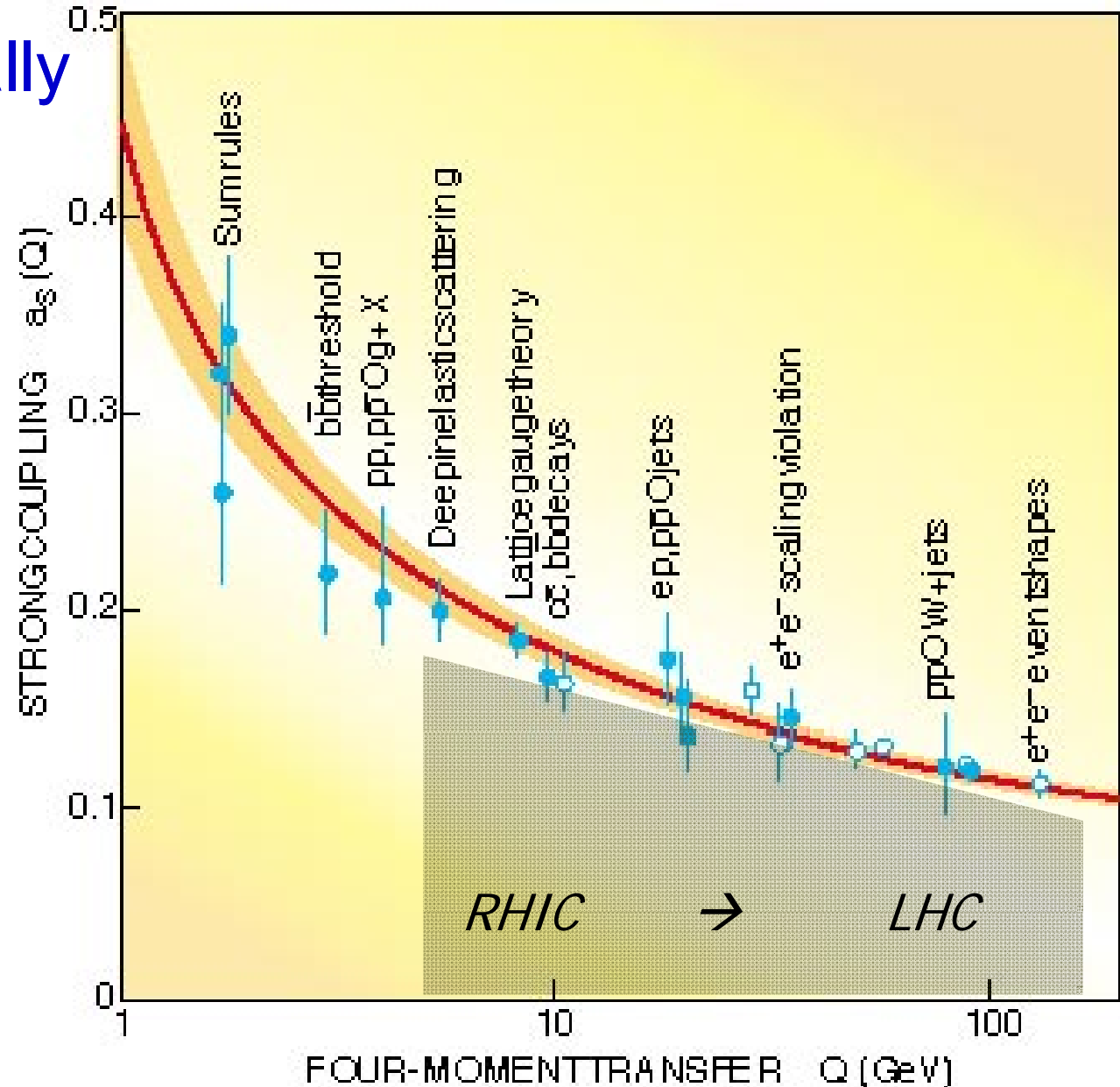


# The LHC Regime

- Enormously successful first heavy ion run(s)
  - ▶ Clearly established that the bulk remains at or near the  $\eta/s$  bound
  - ▶ Dramatically extended  $Q^2$  reach
  - ▶ Discovered a new form of jet quenching
  - ▶ Performed quantitative Upsilon spectroscopy
  - ▶ (Etc., etc.)
- Asymptotically free quarks and gluons when?

# Asymptotically Free When?

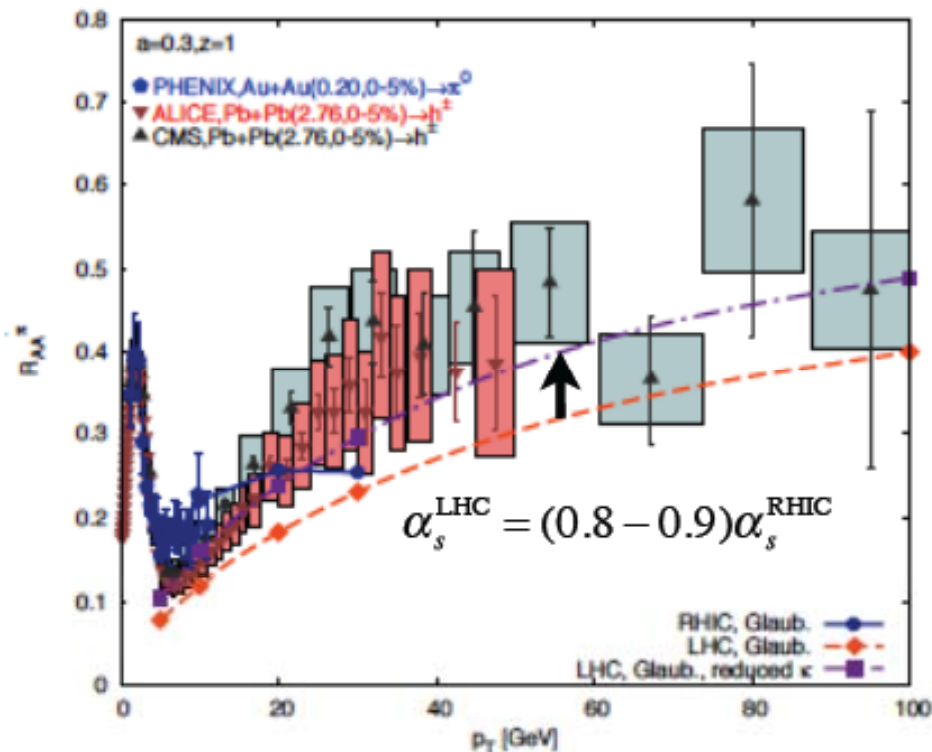
- LHC dramatically extends our reach.
- Understood quantitatively?
- Perhaps:  
(next page)



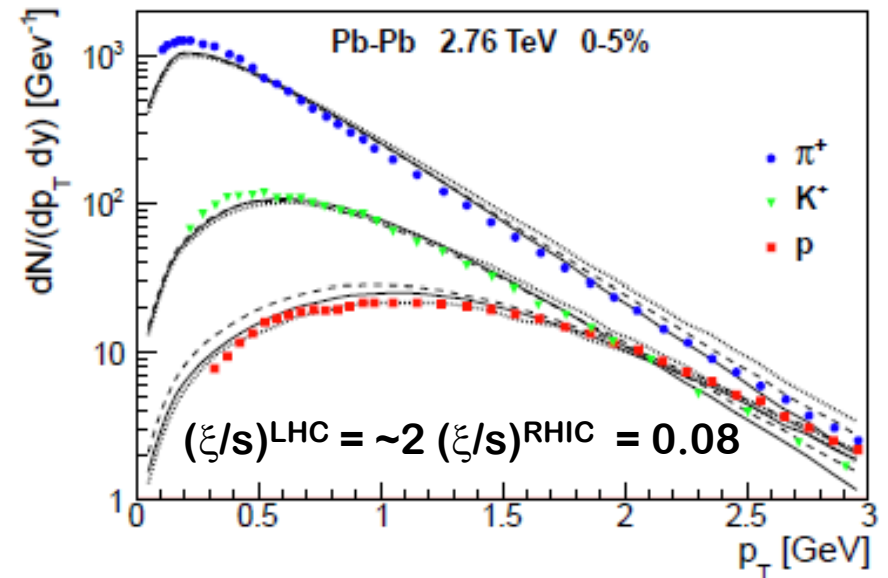
# Asymptotically Free When?

- LHC dramatically extends our reach.

Betz & Gyulassy, arXiv:1201.0281

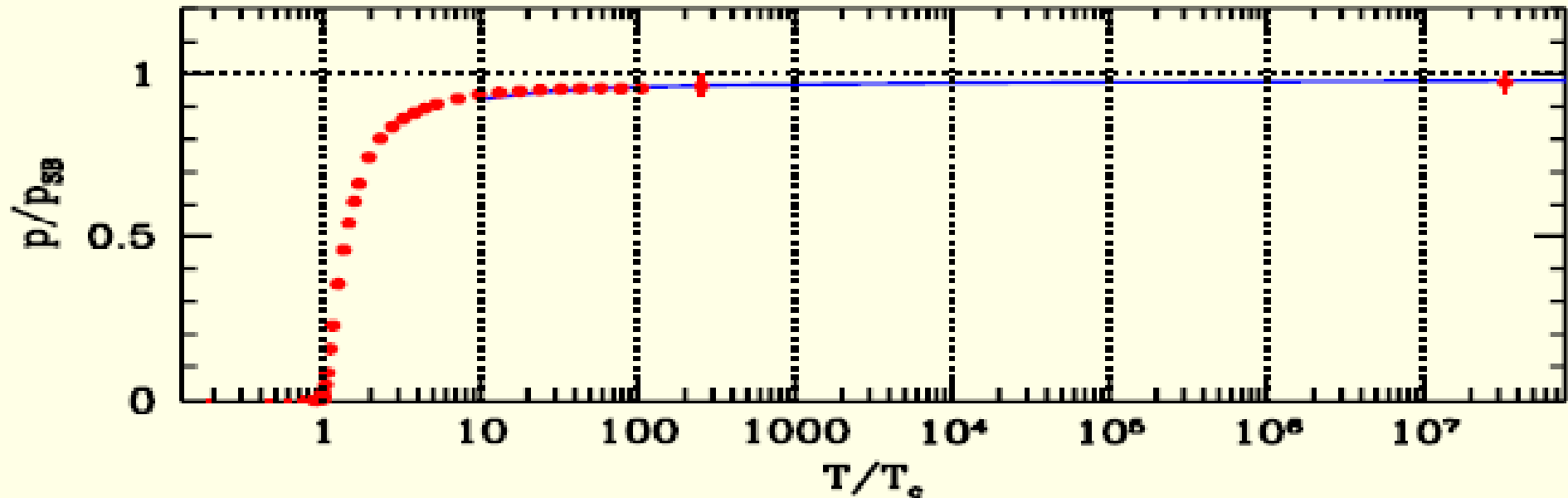


Bozek, Broniowski & Wyskiel-Piekarska, arXiv:1207:3176



# Asymptotically Free When?

- But for the “bulk” at RHIC or at the LHC



- It is hardly clear at what point we would declare “Free at last, free at last!”
- QM12: *The test of all theory is experiment.*



*Enjoy the test !*