The Quest for the QGP
(being a short history
of the quark-gluon plasma)

Quark Matter 2012 Student Lecture

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Science Questions

✗ Uninteresting question:
  ➢ What happens when I crash two heavy nuclei together?

✓ Interesting question:
  ➢ Are there new states of matter at the highest temperatures and densities?
Fermi’s Vision

- ~1950: (Almost) included physics of 2012
- See also remarks in his “statistical model” paper

*From Fermi notes on Thermodynamics*

Matter in unusual conditions
An Updated Version of Fermi’s Diagram
Phase Diagrams

- A fundamental tool to summarize the qualitatively different phases of a complex system.
A Familiar Phase Diagram

- Our best known examples of first-order phase transitions
  - Note that here we can independently vary T and P
  - Note also presence of critical point $\Rightarrow$ vanishing of 1st order transition
Ultracold Fermi gases: Pair formation and condensation usually occur together in Fermi superfluids. The observation of a pseudogap that implies pairing above the condensation temperature in a strongly interacting Fermi gas is thus an exciting development.


Abstract: Pair formation and condensation usually occur together in Fermi superfluids. The observation of a pseudogap that implies pairing above the condensation temperature in a strongly interacting Fermi gas is thus an exciting development.
Phase Diagram of a Truly Complex System

Financial Market Phase Diagram

- Stable Markets
- Critical Risk Aversion
- Decreased Left Tail Risk
- Meta-stable Markets
- Increased Left Tail Risk
- Unstable Markets

Source: PIMCO

Leverage
Phase Diagram of a “Simple” System

![Phase Diagram](image)
Science Questions

✗ Uninteresting question:
   - What happens when I crash two gold nuclei together?

✔ Interesting question:
   - Are there new states of matter at the highest temperatures and densities?

$ Compelling question:
   - What fundamental, thermal properties of our gauge theories of nature can be investigated experimentally?
     - Hint: Gravity is a gauge theory…
The QCD Phase Diagram

The Phases of QCD

- Early Universe
- Future LHC Experiments
- Current RHIC Experiments
- RHIC Energy Scan
- Quark-Gluon Plasma
- Future FAIR Experiments
- 1st order phase transition
- Critical Point
- Hadron Gas
- Vacuum
- Nuclear Matter
- Neutron Stars
- Color Superconductor

Temperature

Baryon Chemical Potential

0 MeV 900 MeV
In principle, these diagrams are also “fundamental”, in that they result from properties of

\[ \mathcal{L} = \bar{\psi} (i \gamma^\mu D_\mu - m) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]

\[ D_\mu \equiv \partial_\mu + ieA_\mu + ieB_\mu \]

But in practice, they are (deeply) emergent, and depend on “unnatural” ratios:

- \( m_p/m_e = 1836.15 \)
- Fine-tuning (to reach unitary limit)
"Natural" Appearance of the QCD Phase Transition

\[ g_*(T) = \frac{1}{\pi^2 T^4/30} \sum_{\text{species}} \int_0^\infty \frac{E_i(p)}{e^{(E_i - \mu_i)/T_i} \pm 1} \frac{d^3 p}{(2\pi)^3} \]

(Effective number of degrees-of-freedom per relativistic particle)

Fig. 3.5: The evolution of\( g_*(T) \) as a function of temperature in the \( SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \) theory.
To Be Fair...

- **Order** of QCD transition sensitive to $m_s/m_{u,d}$
  
  □ E. Laermann and O. Philipsen, hep-ph0303042

- M. Stephanov, today: “Our world is not ideal”

- But qualitative features persist for “all” $m_s/m_{u,d}$
Robustness of QCD Phase Transformation

Q. Why is there “always” a phase change in QCD?

A. Asymptotic freedom:
Quantum Chromodynamics

- 1973 = Birth of QCD
- Gross, Politzer, Wilczek
- Explained
  - Asymptotic freedom
  - Infrared slavery

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Asymptotically Free Gauge Theories. I*

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Frank Wilczek

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(Received 23 July 1973)

Asymptotically free gauge theories of the strong interactions are constructed and analyzed. The reasons for doing this are recounted, including a review of renormalization-group techniques and their application to scaling phenomena. The renormalization-group equations are derived for Yang-Mills theories. The parameters that enter into the equations are calculated to lowest order and it is shown that these theories are asymptotically free. More specifically the effective coupling constant, which determines the ultraviolet behavior of the theory, vanishes for large spacelike momenta. Fermions are incorporated and the construction of realistic models is discussed. We propose that the strong interactions be mediated by a “color” gauge group which commutes with $SU(3) \times SU(3)$. The problem of symmetry breaking is discussed. It appears likely that this would have a dynamical origin. It is suggested that the gauge symmetry might not be broken and that the severe infrared singularities are a consequence of using an incorrect physical state. The deep-inelastic structure functions, as well as the electron-positron total annihilation cross section are analyzed. Scaling obtains up to calculable logarithmic corrections, and the naive light-cone or parton-model results follow. The problems of incorporating scalar mesons and breaking the symmetry by the Higgs mechanism are explained in detail.
Quantum Chromodynamics (QCD)

\[ L = \frac{1}{4 g^2} \mathcal{G}_{\mu
\nu} \mathcal{G}^{\mu\nu} + \sum \bar{\psi}_i \left( \gamma^\mu D_\mu + m_i \right) \psi_i \]

where

\[ \mathcal{G}^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g^{\nu\xi} \mathcal{F}_{\mu\xi} \]

and

\[ D_\mu = \partial_\mu + i t^a A_\mu \]

\[ \alpha_s(Q^2) = \frac{g^2(Q^2)}{4\pi} \]

Now That's it!
Quantum Chromodynamics (QCD)

- **Strong coupling** → non-perturbative
  "confinement"

- **Weak coupling** → perturbative
  "Rutherford scattering"
QCD’s Running Coupling “Constant”

- **Strong coupling** → non-perturbative “confinement”
- **Weak coupling** → perturbative “Rutherford scattering”
A. What Sets the Scale?

- Simple answer – Quantum Mechanics:
  - $\hbar c = 200 \text{ MeV-fm} = 0.2 \text{ GeV-fm}$
  - So $\hbar / (1 \text{ fm}) = 200 \text{ MeV} \sim \text{light hadron masses}$

- Deeper answer – vacuum condensate that establishes confinement scale $\Lambda \sim 200 \text{ MeV}$

$$
\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_F) \log(Q^2/\Lambda^2)} \sim \frac{1}{\log(Q^2/\Lambda^2)} \quad \Lambda \approx \frac{\hbar c}{r_o} \approx 0.2 \text{ GeV}
$$
High Energy Physics:
- Highest possible momentum transfer $Q$
  probing smallest possible distances $r$:
- $Q \gg 20$ GeV $\Rightarrow r << 0.01$ fm
Discovery of QCD Jets ~1983

- Rutherford scattering of asymptotically free quarks and gluons.
Another Approach to Asymptotic Freedom

- **High Energy Physics:**
  - Highest possible momentum transfer $Q$ probing smallest possible distances $r$:
  - $Q >> 20 \text{ GeV} \Rightarrow r << 0.01 \text{ fm}$

- **Nuclear Physics**
  - Study *bulk* behavior of hadronic matter at highest possible temperatures $T$ and/or densities
  - Turns out that $T \sim 0.2 \text{ GeV}$ suffices
    $\sim 2 \times 10^{12} \text{ K}$
“QCD” Prediction of Deconfining Phase Transition


“...matter at densities higher than nuclear consists of a quark soup. The quarks become free at sufficiently high density.”
Shuryak publishes first “review” of thermal QCD—and coins a phrase:

“Because of the apparent analogy with similar phenomena in atomic physics, we may call this phase of matter the QCD (or quark-gluon) plasma.”

QGP

12-Aug-12
**“Plasma” Leads To Prejudice**

Plasma is a state of matter similar to gas in which a certain portion of the particles are ionized… plasma is a state of matter similar to gas in which a certain portion of the particles are ionized.
From QCD Made Simple (F. Wilczek, August, 2000 Physics Today):

Hoping to bypass this forbidding mess, we invoke a procedure that is often useful in theoretical physics. I call it the Jesuit Stratagem, inspired by what I'm told is a credal tenet of the Order: "It is more blessed to ask forgiveness than permission." The stratagem tells you to make clear-cut simplifying assumptions, work out their consequences, and check to see that you don't run into contradictions. In this spirit we tentatively assume that we can describe high-temperature QCD starting with free quarks and gluons...

But there is a second, more rigorous level that remains a challenge for the future. Using fundamental aspects of QCD theory, similar to those I discussed in connection with jets, one can make quantitative predictions for the emission of various kinds of "hard" radiation from a quark-gluon plasma. We will not have done justice to the concept of a weakly interacting plasma of quarks and gluons until some of these predictions are confirmed by experiment.
A series of experiments using CERN's lead beam have presented compelling evidence for the existence of a new state of quark-gluon matter in which quarks, instead of being bound up into more complex particles such as protons and neutrons, are liberated to roam freely.

Present theoretical ideas provide a more precise picture for this new state of matter: it should be a quark-gluon plasma (QGP), in which quarks and gluons, the fundamental constituents of matter, are no longer confined within the dimensions of the nucleon, but free to move around over a volume in which a high enough temperature and/or density prevails.

Quarks and gluons would then freely roam within the volume of the fireball created by the collision.

A common assessment of the collected data leads us to conclude that we now have compelling evidence that a new state of matter has indeed been created, at energy densities which had never been reached over appreciable volumes in laboratory experiments before and which exceed by more than a factor 20 that of normal nuclear matter. The new state of matter found in heavy ion collisions at the SPS features many of the characteristics of the theoretically predicted quark-gluon plasma.

Even if a full characterization of the initial collision stage is presently not yet possible, the data provide strong evidence that it consists of deconfined quarks and gluons.

(All emphasis added by WAZ)
Lattice Results (Prejudice) circa 2000:

- Rapid rise in d.o.f at $T \sim 170$ MeV seen as “near” the Stefan-Boltzmann limit for free quarks and gluons.

\[
\frac{\epsilon}{T^4} \sim \frac{\text{d.o.f}}{3}
\]
RHIC’s Two Major Discoveries ➔ Paradigm Shift

- Discovery of strong “elliptic” flow:
  - 473 citations

- Discovery of strong “jet quenching”
  - 657 citations
• Understood as manifestations of

**strongly-coupled** Quark-Gluon Plasma (sQGP):

- *Not* “freely roaming” quarks and gluons

• Quite the opposite: strongly-coupled to the point of having (near) the minimal (?) value of viscosity to entropy density ratio $\eta/s$. 
The conceptual beauty of asymptotic freedom was (is) hugely prejudicial for the QCD phase transition:

- Rather than ice melting to liquid water

- Thought it would be like the sublimation of frozen CO$_2$ to gas, i.e., QCD phase transformation like “sublimation” of (p,n) $\rightarrow$ (q,g)
Could This Have Been Anticipated?

- Perhaps:

\[ \alpha_s(Q^2) \equiv \frac{g^2(Q^2)}{4\pi} \]

\[ <p> \sim 3T \]
\[ T \sim 200 \text{ MeV} \]
\[ \Rightarrow Q < \sim 1 \text{ GeV} \]
\[ \Rightarrow \alpha_s(Q^2) \sim 0.5 \]
\[ \Rightarrow g(Q^2) \sim 2.5 \]

\[ Q \sim (3-6)T \]

\[ \sim 3T \quad \sim 3T \]
Could This Have Been Anticipated?

- Perhaps:
  - “Real” plasma physicists always want to know the value of the classical coupling parameter $\Gamma$:

    $$\Gamma \equiv \frac{\langle Potential\ Energy \rangle}{\langle Kinetic\ Energy \rangle}$$

  - A (very) rough estimate:

    $$\Gamma \equiv \frac{\langle Potential\ Energy \rangle}{\langle Kinetic\ Energy \rangle} = \frac{\text{Debye\ Mass}}{\langle Kinetic\ Energy \rangle} \sim \frac{gT}{3T} \sim 1$$

- Caveat: Not a useful measure in a truly relativistic system (but $\eta/s$ is?)
Actually, It Was Anticipated (in 1982)

- This property has been known long enough to be forgotten several times:
  - 1982: Gordon Baym, proceedings of Quark Matter '82:
    "...non-perturbative effects appear to play a big role. A hint of trouble can be seen from the first order result for the entropy density (N_f = 3)

\[
s(T) = \frac{19\pi^2}{9} \left\{ 1 - \frac{54}{19\pi} \alpha_s(T) + ... \right\} T^4
\]

which turns negative for \( \alpha_s > 1.1 \)"
Actually, It Was Anticipated (in 1992)

- This property has been known long enough to be forgotten several times:

  “For plasma conditions realistically obtainable in the nuclear collisions ($T \sim 250$ MeV, $g = \sqrt{4\pi\alpha_s} = 2$) the effective gluon mass $m_g^* \sim 300$ MeV. We must conclude, therefore, that the notion of almost free gluons (and quarks) in the high temperature phase of QCD is quite far from the truth. Certainly one has $m_g^* << T$ when $g << 1$, but this condition is never really satisfied in QCD, because $g \sim 1/2$ even at the Planck scale ($10^{19}$ GeV), and $g < 1$ only at energies above 100 GeV.”
Actually, It Was Anticipated (in 2002)

- This property has been known long enough to be forgotten several times:
  - 2002: Ulrich Heinz, Proceedings of PANIC conference:
    “Perturbative mechanisms seem unable to explain the phenomenologically required very short thermalization time scale, pointing to strong non-perturbative dynamics in the QGP even at or above $2T_c$... The quark-hadron phase transition is arguably the most strongly coupled regime of QCD.”
Fermi Knew It (in 1950)

- Fermi (1950)
  - Lays groundwork for statistical approach to particle production in strong interactions:
    - “Since the interactions of the pion field are strong, we may expect that rapidly this energy will be distributed among the various degrees of freedom present in this volume according to statistical laws.”
    - (Emphasis added by WAZ)
Landau Knew It (in 1955)

- Landau (1955) significant extension of Fermi’s approach
- Considers fundamental roles of
  - hydrodynamic evolution
  - entropy

  “The defects of Fermi’s theory arise mainly because the expansion of the compound system is not correctly taken into account... (The) expansion of the system can be considered on the basis of relativistic hydrodynamics.”

  (Emphasis added by WAZ)
1) Use of hydro relies on $R/\lambda \gg 1$

2) Negligible viscosity $\eta$ equivalent to large Reynolds number $Re \gg 1$

$$Re \equiv \frac{\rho VR}{\eta} \sim \frac{VR}{\nu_{th}} \lambda$$

so for relativistic system $V \sim \nu_{th}$ and

$Re \gg 1 \Rightarrow \frac{R}{\lambda} \gg 1$; see #1
Strong interactions imply small viscosity:

- Viscosity $\eta \sim$ Transverse momentum diffusion
  
  $\sim n \langle p \rangle \lambda$, $n$ = number density
  
  $\lambda$ = mean free path
  
  $\langle p \rangle$ = mean momentum
  
  $= 1 / n \sigma$

  $\sigma = $ cross section

- So

  $\eta \sim n \langle p \rangle \lambda \sim \langle p \rangle / \sigma$

- Large interparticle cross section $\Rightarrow$ small viscosity
What Is The Viscosity at RHIC?

- “Perfect fluid” (that is, “ideal hydrodynamics”) defined as “zero viscosity”.

\[
\begin{align*}
\eta_{QGP} & \sim 2 \times 10^{11} \text{ Pa} \cdot \text{s} \\
\eta_{H_2O} & \sim 1 \times 10^{-3} \text{ Pa} \cdot \text{s} \\
\eta_{Pitch} & \sim 2.3 \times 10^8 \text{ Pa} \cdot \text{s} \\
\eta_{Glass(A.P.)} & \sim 10^{12} \text{ Pa} \cdot \text{s}
\end{align*}
\]

\[
\frac{\eta_{QGP}}{\eta_{H_2O}} \sim 2 \times 10^{14}
\]
Any engineer will tell you

- Kinematic viscosity $\eta / \rho \sim [\text{Velocity}] \times [\text{Length}]$ is what matters

Any relativist will tell you

- $\rho \rightarrow \varepsilon + p$

Any thermodynamicist will tell you

- $\varepsilon + p = sT \quad (\text{at } \mu_B = 0)$

So $\eta / \rho \rightarrow \eta / (\varepsilon + p) \rightarrow (\eta / sT) = (\eta / s) (1/T)$

$\sim (\text{damping coefficient}) \times (\text{thermal time})$
How Small Can We Make \( \eta/s \) ?

- Recall
  - Viscosity \( \eta \sim \text{Transverse momentum diffusion} \)
    \[ \sim n \langle p \rangle \lambda \]
  - Entropy density \( s \sim 4n \) (massless non-interacting quanta)

- Then
  \[
  \frac{\eta}{s} \sim \frac{n \langle p \rangle \lambda}{4n} \sim \frac{1}{4} \langle p \rangle \lambda
  \]

- But uncertainty principle
  \[
  \langle p \rangle \lambda \geq \frac{\hbar}{2}
  \]

- Which in turn implies
  \[
  \frac{\eta}{s} \geq \frac{\hbar}{8}
  \]
Miklos Gyulassy and Pawel Danielewicz:

- *Dissipative Phenomena In Quark-Gluon Plasmas*
  P. Danielewicz, M. Gyulassy

noted restrictions on smallest allowed $\eta$:

- Most restrictive:
  - $\lambda > \hbar / <p> \implies \eta > \sim n / 3$
  - But recall $s = 3.6$ $n$ for the quanta they were considering
    - $\implies \eta/s > 1 / (3.6 \times 3) \sim \hbar / (4 \pi)$
    - $\sim 0.1 \hbar$

Before estimating $\lambda_i$ via Eq. (3.2) we note several physical constraints on $\lambda_i$. First, the uncertainty principle implies that quanta transporting typical momenta $<p>$ cannot be localized to distances smaller than $<p>^{-1}$. Hence, it is meaningless to speak about mean free paths smaller than $<p>^{-1}$. Requiring $\lambda_i > <p>^{-1}$ leads to the lower bound

$$\eta \geq \frac{1}{3} n,$$

(3.3)

where $n = \sum n_i$ is the total density of quanta. What seems amazing about (3.3) is that it is independent of dynamical details. There is a finite viscosity regardless of how large is the free-space cross section between the quanta. See Refs. 21 and 22 for examples illustrating how the thermalization rate of many-body systems is limited by the uncertainty principle.
An Aside on Reynolds Number

- Show that \( Re \sim \frac{1}{4} \frac{TR}{(\eta / s)} \)

- So that if \( \frac{\eta}{s} \geq \frac{\hbar}{4\pi} \)

- Then \( Re \geq \sim \pi \frac{TR}{\hbar} \sim 6\pi \)

for Au+Au or Pb+Pb collisions
So the “perfect fluid” observed at RHIC with

\[ \left( \frac{\eta}{s} \right)_{RHIC} \sim 0.1 \hbar \]

was immediately recognized as confirming the 1985 uncertainty principle estimate of Danielewicz and Gyulassy.

Except that’s not what happened…
Instead …

- In 2003-4 a new estimate (bound?) appeared from the AdS/CFT correspondence in string theory (!):


\[ \frac{\eta}{s} \geq \frac{\hbar}{4\pi} \]

- in a rigorous calculation with no (apparent) appeal to the uncertainty principle.
Graviton with 5-momentum $k$ in bulk satisfies $k \cdot k = 0 \rightarrow$

described by 4 numbers

Those 4 numbers describe virtual gauge quanta on 4-d boundary

(CFT = Conformal Field Theory)

(AdS = Anti de Sitter space)

A hard (strongly-coupled) gauge theory calculation is dual to an easy semi-classical gravitational calculation.

(Adopted from S. Brodsky figure)
The “dual” field theory (CFT) is at best a very distant cousin of QCD:

- No confinement
- No running coupling constant
- Many more (supersymmetric) degrees of freedom

But at the same time

- The AdS/CFT calculation is an exact result
- In a strongly (infinitely) coupled theory
- Also shows infinitely-coupled entropy density is \( \frac{3}{4} \) that of free entropy density(!)
The Paradigm Shift - Before

- Recall Stefan-Boltzmann \( \varepsilon = \frac{T^4}{3} \) per d.o.f.

\[ \varepsilon_{SB} = \varepsilon_0 \frac{T^4}{\varepsilon} \]

\[ \varepsilon \approx \varepsilon_0 \frac{T^4}{\varepsilon} \]

QGP

ALMOST asymptotically free

"Pion" gas

\[ \varepsilon/T^4: N_\tau = 4 \]

\[ 3p/T^4: N_\tau = 4 \]

12-Aug-12
Recall Stefan-Boltzmann

\[ \epsilon = \frac{T^4}{3} \sim \frac{1}{\epsilon} T^4 \text{ per d.o.f.} \]
Enormously successful first heavy ion run(s)

- Clearly established that the bulk remains at or near the $\eta/s$ bound
- Dramatically extended $Q^2$ reach
- Discovered a new form of jet quenching
- Performed quantitative Upsilon spectroscopy
- (Etc., etc.)

Asymptotically free quarks and gluons when?
Asymptotically Free When?

- LHC dramatically extends our reach.
- Understood quantitatively?
- Perhaps:

(next page)
LHC dramatically extends our reach.

\[ \alpha_s^{\text{LHC}} = (0.8 - 0.9)\alpha_s^{\text{RHIC}} \]

\[ (\xi/s)^{\text{LHC}} = \sim 2 \ (\xi/s)^{\text{RHIC}} = 0.08 \]
Asymptotically Free When?

● But for the “bulk” at RHIC or at the LHC

It is hardly clear at what point we would declare “Free at last, free at last!”

● QM12: The test of all theory is experiment.

⇒ Enjoy the test!