Dynamic event-by-event fluctuations and domain formation at the phase transition of QCD

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with:
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Approaches to the QCD phase diagram

- QCD calculations in the nonperturbative regime:

  LQCD
  (Z. Fodor, S.D. Katz, JHEP 0203 (2002))

  DSE
  (C. S. Fischer, J. Luecker, arXiv:1206.5191)

- Effective models of QCD:

- Heavy-ion collisions:
Phase diagrams and fluctuations in effective models

- Crossover (CO), critical point (CP), first order phase transition (FO) in (P)QM models.
- Location \((T_c, \mu_c)\) of the CP depends on model parameters, it is non-universal.
- At \(\mu_B = 0\) the phase transition can be tuned from CO to FO by changing the quark-meson coupling constant.

(E. Nakano, B. -J. Schaefer, B. Stokic, B. Friman, K. Redlich, PLB 682 (2010))
Phase diagrams and fluctuations in effective models

- Crossover (CO), critical point (CP), first order phase transition (FO) in (P)QM models.
- Location \((T_c, \mu_c)\) of the CP depends on model parameters, it is non-universal.
- At \(\mu_B = 0\) the phase transition can be tuned from CO to FO by changing the quark-meson coupling constant.
- Enhanced fluctuations in the quark number susceptibilities/kurtosis, increasing from CO to CP.

Coupling of the order parameter to pions $g\sigma\pi\pi$ and protons $G\sigma\bar{p}p \Rightarrow$ fluctuations in multiplicity distributions

$$\langle (\delta N)^2 \rangle \propto \langle (\Delta \sigma)^2 \rangle \propto \zeta^2$$

$\zeta$: correlation length of fluctuations of the order parameter, diverges at the CP

(M. Stephanov, K. Rajagopal, E. Shuryak, PRL 81 (1998), PRD 60 (1999))

Higher cumulants are more sensitive to the CP

$$\langle (\delta N)^3 \rangle \propto \zeta^{4.5}$$

$$\langle (\delta N)^4 \rangle - \langle (\delta N)^2 \rangle^2 \propto \zeta^7$$

(M. Stephanov, PLB 102 (2009), PRL 107 (2011))


(NA49, 7.2% Pb+Pb)

 UrQMD

STAR collaboration (QM2012)
Phase transitions in dynamic systems

- Critical point: large relaxation times limit the growth of $\xi$ (critical slowing down) (B. Berdnikov, K. Rajagopal, Phys. Rev. D 61 (2000))

- First order phase transition: instability of slow modes at the spinodal lines (spinodal decomposition) (I. Mishustin, PRL 82 (1999); C. Sasaki, B. Friman, K. Redlich, PRD 77 (2008))
Fluctuations are different, but all are interesting!

- Crossover: remnants of the $\mathcal{O}(4)$ criticality.
- Critical point: divergent event-by-event fluctuations in thermodynamic equilibrium.
- First order phase transition: large nonstatistical fluctuations in $\eta/p_T$ spectra in individual events.

Motivation: Heavy-ion collisions are dynamic, inhomogeneous and finite in space and time.

? Can nonequilibrium effects become strong enough to develop signals of the first order phase transition?

? Do enhanced equilibrium fluctuations at the critical point survive the dynamics?
Nonequilibrium chiral fluid dynamics - NχFD

- Langevin equation for the sigma field: damping and noise from the interaction with the quarks

\[ \partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g \rho_s + \eta \partial_t \sigma = \zeta \]

- For PQM: phenomenological dynamics for the Polyakov-loop (poster by C. Herold).
- Fluid dynamic expansion of the quark fluid = heat bath, including energy-momentum exchange \( \partial_\mu T_{q}^{\mu \nu} = S^\nu = -\partial_\mu T_\sigma^{\mu \nu} \)
- Nonequilibrium equation of state \( p = p(e, \sigma) \).
- Fluid dynamic initial conditions from UrQMD (www.urqmd.org).

Selfconsistent approach within the 2PI effective action!

(MN, S. Leupold, C. Herold, M. Bleicher, PRC 84 (2011); MN, S. Leupold, M. Bleicher, PLB 711 (2012); MN, M. Bleicher, S. Leupold, I. Mishustin, arXiv:1105.1962; and work in progress)

(related work: I. Mishustin, O. Scavenius, PRL 83 (1999); K. Paech, H. Stoecker, A. Dumitru, PRC 68 (2003))
Not included (yet)

- Simulations with pion fields
- Quantum dynamics (e.g. J. Berges et al., PRL 107 (2011) 061301)
- Fluid dynamic fluctuations and viscosities
- Final state interaction
Results - overview

- $\mu_B = 0$ (no fluctuations in net quark number)
  - Domain formation at the first order phase transition.
  - Event-by-event fluctuations at the critical point.
- $\mu_B \neq 0$
  - Trajectories in the phase diagram.
  - Formation of domains of high quark number density at the first order phase transition.
Dynamic domain formation

First order phase transition

Sigma field fluctuations: $\Delta \sigma = \sqrt{(\sigma - \sigma_{eq})^2}$

- Highly supercooled state at $t = 4.0 \text{ fm/c.}$
Dynamic domain formation
First order phase transition

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- Highly supercooled state at \( t = 4.0 \text{ fm/c} \).
- Dynamic formation of domains at \( t = 5.6 \text{ fm/c} \).
Dynamic domain formation
First order phase transition

Sigma field fluctuations: $\Delta \sigma = \sqrt{(\sigma - \sigma_{eq})^2}$

- Highly supercooled state at $t = 4.0$ fm/c.
- Dynamic formation of domains at $t = 5.6$ fm/c.
- Dynamic decay of domains at $t = 7.2$ fm/c.

This could lead to non-statistical fluctuations in hadron multiplicities.
Dynamic enhancement of event-by-event fluctuations

Event-by-event fluctuations

- Dynamic correlation length grows up to $\sim 2.5$ fm.
- Enhanced event-by-event fluctuations of the order parameter, $\langle \sigma^2 \rangle \propto \xi^2$.
- Initial fluctuations are washed out during the first 1 fm.
- Delay between the averaged $T_c$ and the peak in $\xi$ and $\langle \sigma^2 \rangle$. 

Temperature

- $G(r) \propto \exp(-r/\xi)$
- $\langle \sigma^2 \rangle \propto \xi^2$.
Trajectories and isentropes at finite $\mu_B$

QM, event averaged

PQM, one individual event

- Grey: equilibrium isentropes, color: $T-\mu_B$ trajectories from simulations.
- Fluid trajectories differ from the (equilibrium) isentropes due to interaction with the fields.
- Including the effective Polyakov-loop field leads to more structure of the trajectories, in particular at the first order phase transition (Poster by C. Herold).
Evolution of quark number density

Crossover - critical point

(initial density)

(Poster by C. Herold)
Evolution of quark number density
Crossover - critical point

during the evolution

initial density

(Poster by C. Herold)
Evolution of quark number density

Crossover - critical point
during the evolution

initial density

after $t = 12$ fm

(Poster by C. Herold)
Formation of high quark number density domains
First order phase transition

(Poster by C. Herold)
Formation of high quark number density domains during the evolution

First order phase transition

(Poster by C. Herold)
Formation of high quark number density domains during the evolution

Initial density, after $t = 12$ fm

(Poster by C. Herold)
Summary

• Dynamic domain formation ($\sigma$ and $n$) at the first order phase transition.

? Can nonequilibrium effects become strong enough to develop signals of the first order phase transition?

• Dynamic correlation length $\xi$ grows up to $\sim 2.5$ fm.

• Dynamic enhancement of event-by-event-fluctuations of the order parameter ($\sigma$) at the critical point.

? Do enhanced equilibrium fluctuations at the critical point survive the dynamics?
Summary

- Dynamic domain formation ($\sigma$ and $n$) at the first order phase transition.

  Yes, nonequilibrium dynamics can lead to signals at the first order phase transition!

- Dynamic correlation length $\xi$ grows up to $\approx 2.5$ fm.

- Dynamic enhancement of event-by-event-fluctuations of the order parameter ($\sigma$) at the critical point.

  Do enhanced equilibrium fluctuations at the critical point survive the dynamics?
Summary

- Dynamic domain formation ($\sigma$ and $n$) at the first order phase transition.
  - Yes, nonequilibrium dynamics can lead to signals at the first order phase transition!
- Dynamic correlation length $\xi$ grows up to $\simeq 2.5$ fm.
- Dynamic enhancement of event-by-event-fluctuations of the order parameter ($\sigma$) at the critical point.
  - Yes, the critical fluctuations develop even in the situation of a nonequilibrium, dynamic simulation of heavy-ion collisions!
BACKUP
Semiclassical equation of motion for the sigma field

\[ \partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g \rho_s + \eta \partial_t \sigma = \zeta \]

damping term \( \eta \) and noise \( \zeta \) for \( k = 0 \)

\[ \eta = g^2 \frac{d_q}{\pi} \left( 1 - 2n_F \left( \frac{m_\sigma}{2} \right) \right) \left( \frac{m_\sigma^2}{4} - m_q^2 \right)^{3/2} \]

\[ \langle \zeta(t) \zeta(t') \rangle = \frac{1}{V} \delta(t - t') m_\sigma \eta \coth \left( \frac{m_\sigma}{2T} \right) \]

below \( T_c \) damping by the interaction with the hard pion modes, apply \( \eta = 2.2 / \text{fm} \)

(T. S. Biro and C. Greiner, PRL 79 (1997))
Influence of the inhomogeneous system

first order phase transition

\[ N\chi FD \]

\( T \) [MeV]
\( \langle \sigma \rangle \)
\( \langle \sigma_{eq} \rangle \)
\( \sigma_{eq} \) hom.

\( T_c \)

\( 0 \)
\( 1 \)
\( 2 \)
\( 3 \)
\( 4 \)
\( 5 \)
\( 6 \)
\( 7 \)
\( 8 \)

time [fm]

correlation length, critical point

\[ \xi = 1/\langle m_\sigma \rangle \]

fit to corr.func.

\( \xi \) [fm]

\( 0 \)
\( 1 \)
\( 2 \)
\( 3 \)
\( 4 \)
\( 5 \)
\( 6 \)

\( t \) [fm]
Pion fluctuations

So far: pion fluctuations were not considered and \( \bar{\pi} = \langle \bar{\pi} \rangle = 0 \). Propagate pion fluctuations, too:

Critical point

First order phase transition

Larger pion fluctuations in a scenario with a first order phase transition!
Low mode fluctuations of the Polyakov-loop are enhanced in a CP scenario.