Initial state fluctuations and higher harmonic flow in heavy-ion collisions

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Introduction

- Large elliptic flow has indicated fluid behavior of matter created at RHIC in early 2000’s. BNL announces “perfect liquid” in 2005 press release.
- The importance of fluctuations was realized later and analysis of odd flow harmonics began in 2010 since B. Alver, G. Roland, Phys. Rev. C81, 054905.
- Analysis of all flow harmonics can help determine:
  - Initial state properties
  - Transport properties of the QGP (and hadron gas)
- I will discuss systematics within event-by-event hydrodynamics
  - Present a QCD based model for the initial state including geometric and color charge fluctuations
  - Make first comparisons to experimental data

Initial state fluctuations: MC-Glauber model

To study systematics we use a simple geometric model
Later, we improve significantly on this

- Sample Woods-Saxon distributions to determine all nucleon positions (green and red circles)
- Sample impact parameter $b$ and overlap nuclei

- Nucleon-nucleon collision occurs if distance is $< \sqrt{\sigma_{NN}/\pi}$
- At position of collision add 2D-Gaussian energy density distribution with width $\sigma_0$ (blue blobs)
  $\sigma_0$ (e.g. 0.4 fm) is a model parameter

$\Psi_{PP2}$ and $\Psi_{PP3}$ are participant planes for ellipticity and triangularity
Hydrodynamic evolution

Given the initial energy density distribution we solve

\[ \partial_{\mu} T^{\mu\nu} = 0 \]

\[ T^{\mu\nu} = (\epsilon + P)u^{\mu}u^{\nu} - Pg^{\mu\nu} + \pi^{\mu\nu} \]

using only shear viscosity: \( \pi^{\mu\nu} = 0 \)

initial  \hspace{1cm} evolve to  \hspace{1cm} ideal  \hspace{1cm} \eta/s = 0.16

\[ \tau = 6 \text{ fm/c} \]


3+1D event-by-event relativistic viscous hydrodynamic simulation
Flow analysis  

After Cooper-Frye freeze-out and resonance decays in each event we compute

\[ v_n = \langle \cos[n(\phi - \psi_n)] \rangle \]

with the event-plane angle \( \psi_n = \frac{1}{n} \arctan \left( \frac{\langle \sin(n\phi) \rangle}{\langle \cos(n\phi) \rangle} \right) \)

**Sensitivity of event averaged** \( v_n \) **on**

**viscosity**

**initial state granularity**

Sensitivity to viscosity and initial state structure increases with \( n \)
New model for the initial state

To make use of $v_n$ measurements we need a more rigorous understanding of the initial state and its fluctuations

Gluon saturation $\rightarrow$ strong gluon fields, large occupation numbers at $k_T \leq Q_s$ $\rightarrow$ classical field approximation

Solve classical Yang-Mills equations event-by-event, including geometric and color charge fluctuations
Color charge densities of incoming nuclei

- Sample nucleon positions from **Woods-Saxon** distributions.
- Use **IP-Sat model** fit to HERA data to get $Q_s^2(x, b_\perp)$ for each nucleon. The color charge density squared $g^2\mu^2$ is proportional to $Q_s^2$.
- Add all $g^2\mu^2(x_\perp)$ in each nucleus to obtain $g^2\mu_1^2(x_\perp)$ and $g^2\mu_2^2(x_\perp)$.

Sample $\rho^a$ from local Gaussian distribution for each nucleus

$$\langle \rho^a(x_\perp)\rho^b(y_\perp) \rangle = \delta^{ab}\delta^2(x_\perp - y_\perp)g^2\mu^2(x_\perp)$$
Gauge fields before the collision

Color currents:

\[ J_1^{\nu} = \delta^{\mu+} \rho_1(x^-, x_{\perp}) \]
\[ [D_{\mu}, F^{\mu\nu}] = J_1^{\nu} \]

\[ J_2^{\nu} = \delta^{\mu-} \rho_2(x^+, x_{\perp}) \]
\[ [D_{\mu}, F^{\mu\nu}] = J_2^{\nu} \]

Correlations and fluctuations in the gluon fields:

Shown is the correlator of the Wilson lines

\[ C_{(1,2)}(x_{\perp}) = \frac{1}{N_c} \text{Re}[\text{tr}(V(1, 2)^\dagger(0, 0)V(1, 2)(x, y))] \]

The length scale of fluctuations is \(1/Q_s\) - not the nucleon size
Energy density

Solve for gauge fields after the collision in the forward lightcone
Compute energy density in the fields at \( \tau = 0 \) and later times with CYM evolution

Very different initial energy density distributions in the models
MC-KLN: Drescher, Nara, nucl-th/0611017
mckln-3.52 from http://physics.baruch.cuny.edu/files/CGC/CGC_IC.html with defaults, energy density scaling
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\( \frac{dN_g}{dy} \) at finite time \( \tau = 0.4 \text{ fm} \) in transverse Coulomb gauge \( \partial_i A^i = 0 \)

\( N_{\text{part}} \) from MC-Glauber with \( \sigma_{NN} = 42 \text{ mb} \) and 64 mb respectively.


Scaled by \( 2/3 \) to compare to charged particles.

Some freedom in normalization - will need to account for entropy production.
$dN_g/dy$ at finite time $\tau = 0.4 \text{ fm}$ in transverse Coulomb gauge $\partial_i A^i = 0$.

$N_{\text{part}}$ from MC-Glauber with $\sigma_{NN} = 42 \text{ mb}$ and $64 \text{ mb}$ respectively.

Scaled by $2/3$ to compare to charged particles.

Some freedom in normalization - will need to account for entropy production.
**Multiplicity**  B. Schenke, P. Tribedy, R. Venugopalan, arXiv:1206.6805

\[ P(dN_g/dy) \] at time \( \tau = 0.4 \text{ fm} \) with \( P(b) \) from a Glauber model


Glasma model gives a convolution of negative binomial distributions
No need to put them in by hand
Eccentricities

\[ \varepsilon_n = \frac{\sqrt{\langle r^n \cos(n\phi) \rangle^2 + \langle r^n \sin(n\phi) \rangle^2}}{\langle r^n \rangle} \]

Averages are weighted by the energy density

- \( \varepsilon_n \) larger in Glasma model for odd \( n \)
- \( \varepsilon_n \) smaller in Glasma model for \( n = 2 \) (for \( b > 3 \text{ fm} \))
- About equal for \( n = 4 \), larger for \( n = 6 \)
Compute all components of $T^{\mu\nu}$
Determine energy density and $(u^x, u^y)$ at $\tau > 0$ fm from $u_\mu T^{\mu\nu} = \varepsilon u^\nu$ as input for hydrodynamic simulations.

Energy density and $(u_x, u_y)$ at $\tau = 0.4$ fm/c

No instabilities (need full 3+1D Yang-Mills for that): system is far from equilibrium - cannot yet match $\Pi^{\mu\nu}$
Centrality selection and flow

Glasma centrality selection

$P(dN_g/dy)$

$Glasma$ centrality selection

0-5%
5-10%
10-20%
20-30%
30-40%
40-50%
50-60%

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Centrality selection and flow

Glasma centrality selection

P(dN_g/dy)

dN_g/dy

Glasma centrality selection

0-5%
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20-30%
30-40%
40-50%
50-60%

Hydro evolution

MUSIC

ATLAS 20-30%, EP

\tau_{\text{switch}} = 0.2 \text{ fm/c}

\eta/s =0.2

\langle v_n^2 \rangle^{1/2}

p_T [GeV]

Distribution of b in 20-30% central bin

P(b)

b [fm]

\langle v_n^2 \rangle^{1/2}

p_T [GeV]
Centrality selection and flow

Glasma centrality selection

Hydro evolution

MUSIC

Experimental data:
Event-by-event distributions of $v_n$

comparing to all new ATLAS data:

see talk by Jiangyong Jia in Session 4A, today, 11:20 am

Preliminary results: Statistics to be improved.

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Preliminary results: Statistics to be improved.
Effect of initial flow

Weak effect of initial flow on hadron $v_n(p_T)$

Expect stronger effect for photon $v_n$: Photons are mostly produced early at high temperatures

Effect of different switching time $0.4\, \text{fm}/c$ is very weak

Experimental data:
Temperature dependent $\eta/s$


$v_n(p_T)$ for given $\eta/s(T)$ indistinguishable from constant $\eta/s = 0.2$

More detailed study needed

Experimental data:
Directed flow $v_1$

Experimental data:
Summary and conclusions

- Higher flow harmonics are sensitive to viscosity and fluctuating initial states

- **IP-Glasma model**
  - includes geometric and color charge fluctuations
  - produces negative binomial fluctuations
  - has different eccentricities than previous CGC based models
  - provides initial flow profile from the non-equilibrium stage
  - describes flow coefficients up to at least $v_5$ with $\eta/s = 0.2$

- Initial flow has weak effect on hadronic $v_n$
  Photon study underway

- Temperature dependent $\eta/s$ not distinguishable from average $\eta/s$

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Gauge fields before the collision

Color currents:

\[ J_1^\nu = \delta^{\mu+} \rho_1 (x^-, x_\perp) \]
\[ [D_\mu, F^{\mu\nu}] = J_1^\nu \]
\[ J_2^\nu = \delta^{\mu-} \rho_2 (x^+, x_\perp) \]
\[ [D_\mu, F^{\mu\nu}] = J_2^\nu \]

Solution in covariant gauge:

\[ A^+_{\text{cov}(1,2)} (x^-, x_\perp) = - \frac{g \rho_{(1,2)} (x^-, x_\perp)}{\nabla_\perp^2 + m^2} \]

with infrared cutoff \( m \) of order \( \Lambda_{\text{QCD}} \).

Solution in light cone gauge:

\[ A^+_{(1,2)} (x_\perp) = A^-_{(1,2)} (x_\perp) = 0 \]
\[ A^i_{(1,2)} (x_\perp) = \frac{i}{g} V_{(1,2)} (x_\perp) \partial_i V_{(1,2)}^\dagger (x_\perp) \]

\( V \) is the path-ordered exponential of \( A^+_{\text{cov}(1,2)} \)
Gauge fields before the collision

The correlator of the Wilson lines

\[ C_{(1,2)}(x_\perp) = \frac{1}{N_c} \text{Re}[\text{tr}(V(1,2)^\dagger(0,0)V(1,2)(x,y))] \]

with

\[ V_{(1,2)}(x_\perp) = P \exp \left( -ig \int dx_- \frac{\rho_{(1,2)}(x^-, x_\perp)}{\nabla_\perp^2 + m^2} \right) \]

shows the degree of correlations and fluctuations in the gluon fields.

The length scale of fluctuations is \(1/Q_s\). Not the nucleon size.
Gauge fields after the collision (Glasma)

Initial condition on the lightcone: require that fields match smoothly on the lightcone.

Solution:

\[ A^i_\mu |_{\tau=0} = A^i_\mu \big|_{(1)} + A^i_\mu \big|_{(2)} \]

\[ A^\eta_\mu |_{\tau=0} = \frac{ig}{2} [A^i_\mu \big|_{(1)}, A^i_\mu \big|_{(2)}] \]

On the lattice the Wilson lines in the future lightcone are obtained from the condition:

\[ \text{tr} \left\{ t^a \left[ \left( U^i_\mu (1) + U^i_\mu (2) \right) \left( 1 + U^i_\mu (3) \right) - \left( 1 + U^i_\mu (3) \right) \left( U^{i\dagger}_\mu (1) + U^{i\dagger}_\mu (2) \right) \right] \right\} = 0 \]

where \( t^a \) are the generators of \( SU(N_c) \) in the fundamental representation. Solve iteratively.


\[ U^i_{(1,2),j} = V^i_{(1,2),j} V^{i\dagger}_{(1,2),j} + \epsilon_i \]  (gauge transform of 1: pure gauge)
Negative binomial fluctuations

Fluctuations in the total energy per unit rapidity produce negative binomial distribution (NBD).


\[ P(n) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\bar{n}^nk^k}{(\bar{n}+k)^{n+k}} \]

Good, since multiplicity in pp collisions can be described well with NBD.

In AA, convolution of NBDs at all impact parameters describes data well too.

P. Tribedy and R. Venugopalan

MC-KLN does not do that - these fluctuations need to be put in by hand.

see Dumitru and Nara arXiv:1201.6382
**Negative binomial fluctuations**

Extract \( k \) and \( \bar{n} \) using a fit with

\[
P(n) = \frac{\Gamma(k + n)}{\Gamma(k)\Gamma(n + 1)} \frac{\bar{n}^n k^k}{(\bar{n} + k)^{n+k}}
\]

at fixed impact parameters.

Ratio of \( k/\bar{n} \) is \( > 1 \) for small \( b \) and becomes small \( \sim 0.14 \) for large \( b \).

That is close to the value extracted for \( p + p \) collisions: Dumitru and Nara arXiv:1201.6382
NBDs and Glasma flux tubes

Glasma flux tube picture:

\[ k = \zeta \frac{N_c^2 - 1}{2\pi} Q_s^2 S_\perp \]


Width of NBD is inversely proportional to the number of flux tubes \( Q_s^2 S_\perp \).

\( S_\perp = \) interaction area.

B.Schenke, P.Tribedy, R.Venugopalan, arXiv:1206.6805

\( \zeta \) should be close to constant in the flux tube picture.
NBDs and Glasma flux tubes

$\zeta$ is not constant because geometric fluctuations are very important. Were not considered in the derivation of

$$k = \zeta \frac{N^2_c - 1}{2\pi} Q_s^2 S_\perp$$

Eliminate by using smooth nucleon distributions:

![Graph showing the relationship between $\zeta$ and $Q_s^2 S_\perp$.]

B.Schenke, P.Tribedy, R.Venugopalan, arXiv:1206.6805
More centrality classes: IP-Glasma + MUSIC

\[ \langle v_n \rangle^{1/2} \]

\[ p_T \text{ [GeV]} \]

\[ \eta/s = 0.2 \]

\[ \tau_{\text{switch}} = 0.2 \text{ fm/c} \]

ATLAS 0-5%, EP

ATLAS 10-20%, EP

ATLAS 30-40%, EP

ATLAS 40-50%, EP
Using $\eta/s = 0.16$ overestimates all $v_n$

Experimental data: