Evolution of singularities in thermalization of strongly coupled gauge theory

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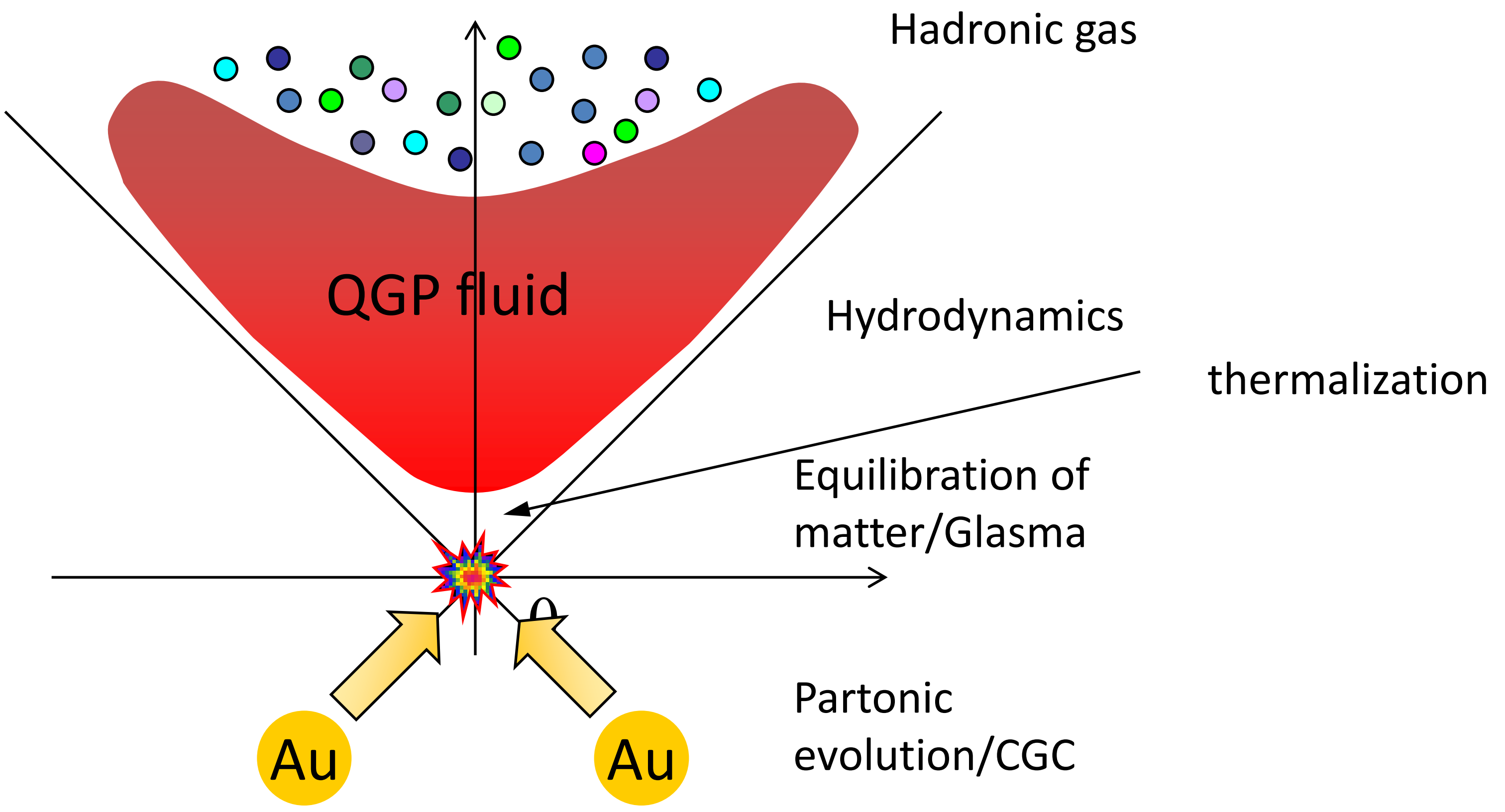
J. Erdmenger, SL: **1205.6873**
J. Erdmenger, C. Hoyos, SL: **1112.1963**
J. Erdmenger, SL, H. Ngo: **1101.5505**
SL, E. Shuryak: **0808.0910**

Stages of heavy ion collisions



AdS/CFT description

Large N_c , strong coupling λ limit of *N*=4 SYM

⟺

string theory in AdS background

Pure AdS $$ds^2 = \frac{L^2}{z^2}(-dt^2 + d\vec{x}^2 + dz^2)$$

N=4 SYM at zero temperature(vacuum)

↓

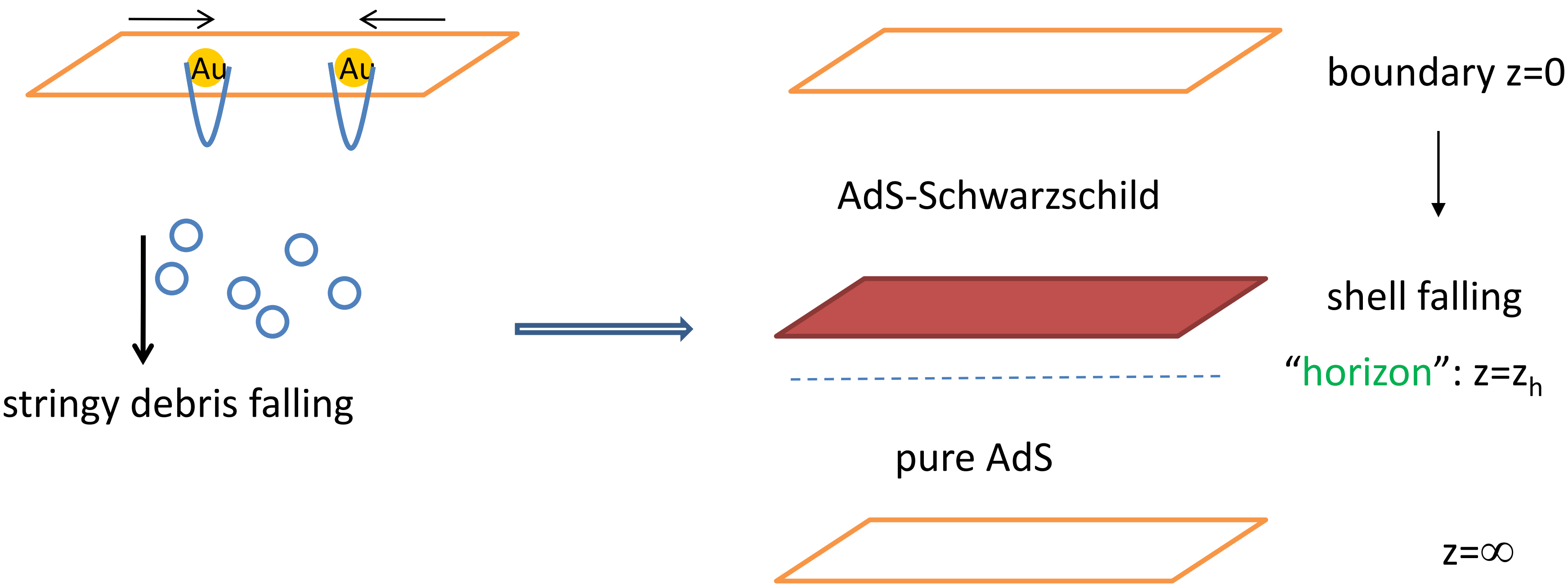
AdS-Schwarzschild $$ds^2 = \frac{L^2}{z^2}(-f(z)dt^2 + d\vec{x}^2 + dz^2 / f(z))$$

$$f = 1 - \frac{z^4}{{z_h}^4}$$

N=4 SYM at temperature (plasma) $$T = \frac{1}{\pi z_h}$$

bulk field ϕ ⟺ boundary glue ball operator TrF^2

Gravitational collapse model dual to thermalization



$$T_{\mu\nu} = diag(\varepsilon, p, p, p)$$

Homogeneous and isotropic but not thermalized

Sin, Shuryak & Zahed hep-th/0511199
SL, E. Shuryak **0808.0910** [hep-th]

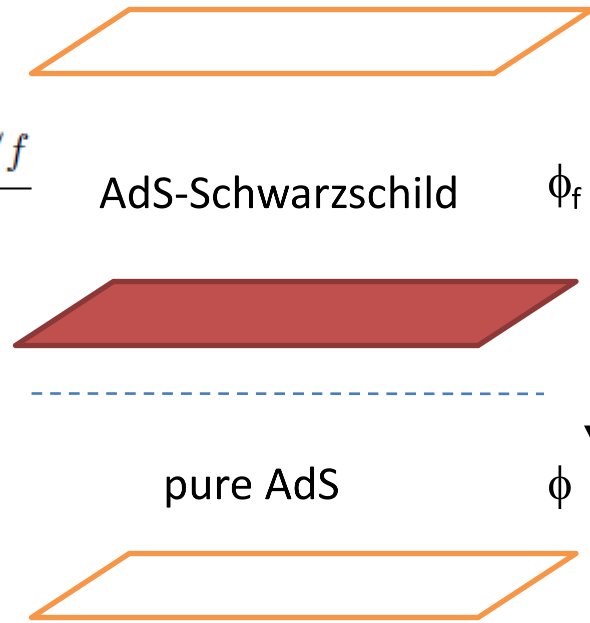
Correlator for glue ball operator from bulk scalar in collapse background

$\phi_f\ (z \to 0,t)=\delta(t-t')$

$\Box_f\ \phi_f = 0$

$$ds^2 = \frac{-f\,dt_f^2 + d\vec{x}^2 + dz^2/f}{z^2}$$

$$f = 1 - \frac{z^4}{z_h^4}$$



matching
$$\begin{cases} \phi_f = \phi \\ n_f \cdot \nabla \phi_f = n \cdot \nabla \phi \end{cases}$$

$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

$\Box\ \phi = 0$

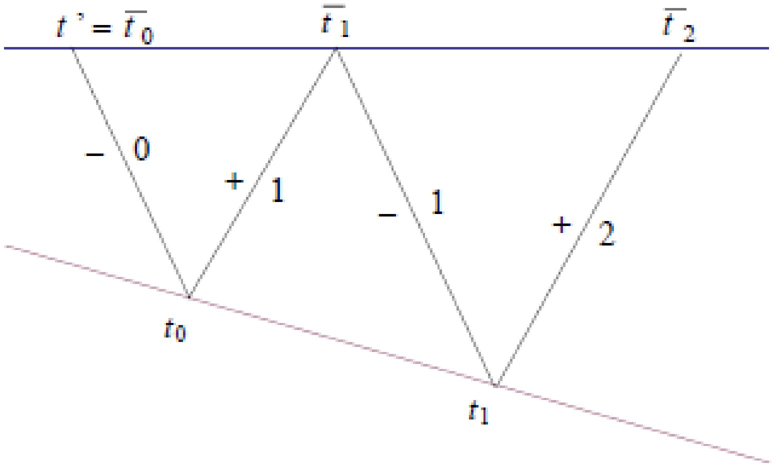
$$\phi_f(t,t',z) = \delta(t-t') + \ldots + z^4 G^R(t,t')$$

$$G^R(t,t') = \int d^3x\,\theta(t-t')\langle[O(t,x),O(t',0)]\rangle$$

two point correlator for glue ball operator at q=0

Instead of solving the PDE numerically, we use a divergence matching method

Divergence matching in pure AdS



Geometric optics in the bulk:

$\phi_f(t,t',z)$ singular near the segments (-,0), (+,1), (-,1) etc

Initial condition: $\phi^{0,-} = \phi^{0,-}_{>} + \phi^{0,-}_{<}$

$$\phi^{0,-}_{>} = \frac{B_0}{(-t+t'+z+i\varepsilon)^{5/2}}, \phi^{0,-}_{<} = \frac{-B_0}{(-t+t'+z-i\varepsilon)^{5/2}}$$

splitting between positive/negative frequency contributions

$$\vdots$$

$$\phi^{n,+}_{>} = \frac{B_n(-i)^{n-1}}{(-t+\bar{t}_n-z+i\varepsilon)^{5/2}}, \phi^{n,-}_{<} = \frac{-B_n i^{n-1}}{(-t+\bar{t}_n-z-i\varepsilon)^{5/2}}$$

$$B_n = -B_{n-1}\left(\frac{1+f'(t_{n-1})}{1-f'(t_{n-1})}\right)^c$$

$$\phi^{n,-}_{>} = \frac{B_n(-i)^{n}}{(-t+\bar{t}_n+z+i\varepsilon)^{5/2}}, \phi^{n,-}_{<} = \frac{-B_n i^{n}}{(-t+\bar{t}_n+z-i\varepsilon)^{5/2}}$$

J. Erdmenger, C. Hoyos, SL **1112.1963**

Divergence matching in pure AdS

Singular part of $G^R(t,t')$: $G^R(t,t') = \int d^3x\, \theta(t-t') \langle [O(t,x), O(t',0)] \rangle$

$$G^R(t \to \bar{t}_n, t') \sim \frac{B_n (-i)^{n-1}}{(-t+\bar{t}_n+i\varepsilon)^5} - \frac{B_n i^{n-1}}{(-t+\bar{t}_n-i\varepsilon)^5}$$

n=0 simply gives the light-cone singularity

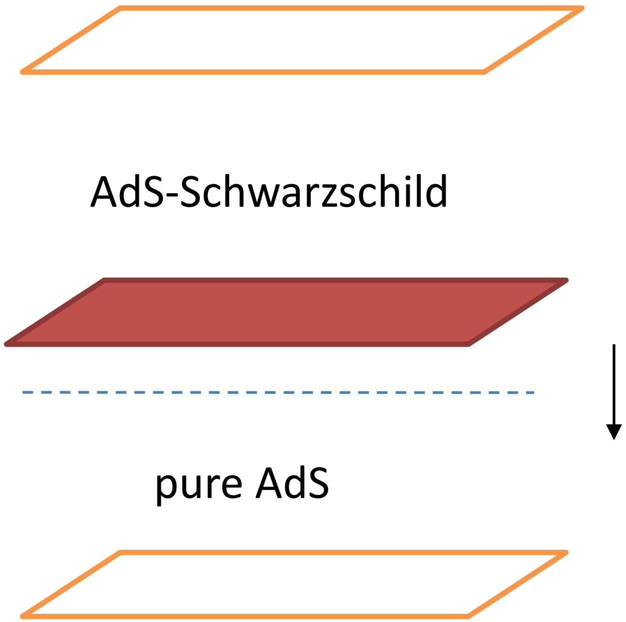
n>0 singularities contain information on the spectrum of the glue ball operator: existence of equally spaced Normal Mode in the spectrum

Elementary example: nonrelativistic harmonic oscillator

$$K(x,t,x',0) = \langle x,t | x',0 \rangle$$
$$= \sum_n \langle x | e^{-iHt} | n \rangle \langle n | x' \rangle = \sum_n e^{-i(n+1/2)\Omega t} \langle x | n \rangle \langle n | x' \rangle$$
$$= \left[\frac{m\Omega}{2i\pi \sin \Omega t} \right]^{1/2} \exp\{ \frac{im\Omega}{2\sin\Omega t} [(x^2 + x'^2)\cos\Omega t - 2xx'] \}$$

Singularities at $\Omega t = 2\pi \cdot k$

Gravitational collapse model



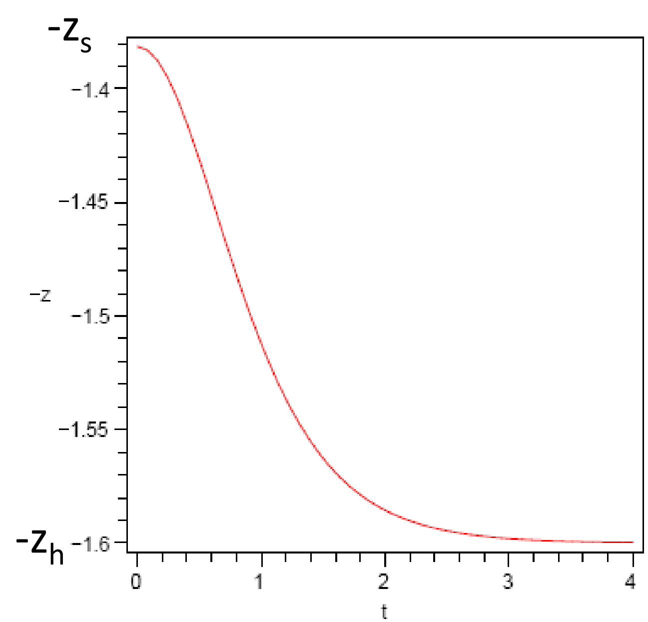
$$\text{above} : ds^2 = \frac{-f\,dt_f^2 + d\vec{x}^2 + dz^2/f}{z^2}$$

$$\text{below} : ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2},$$

$$\text{with} \quad f = 1 - \frac{z^4}{z_h^4}.$$

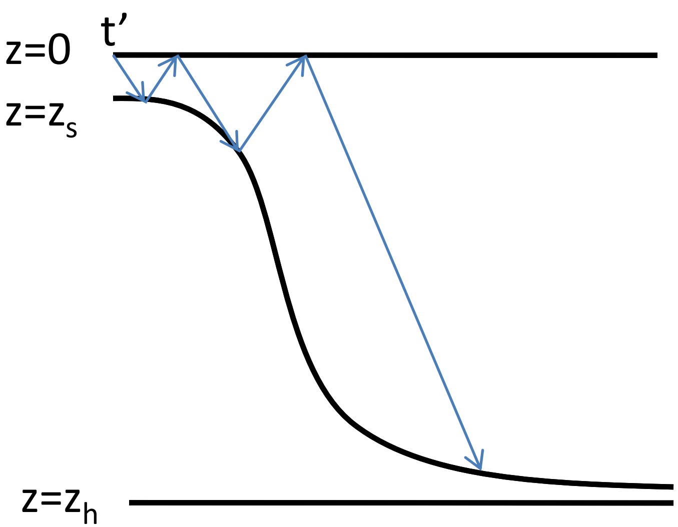
Falling trajectory of the shell by Israel junction condition:

$$[K_{ij} - \gamma_{ij} K] = \kappa S_{ij}\,, \qquad \{K_{ij}\} S^{ij} = 0.$$

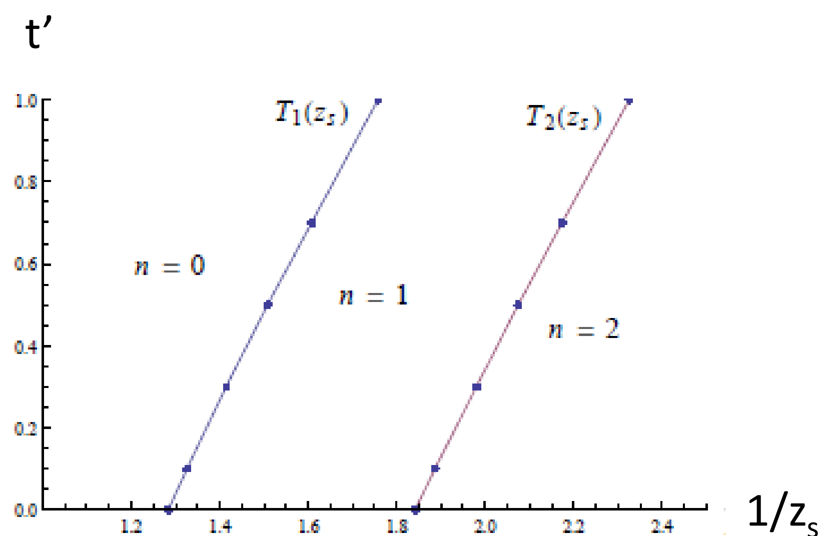


Light ray bouncing in collapse background

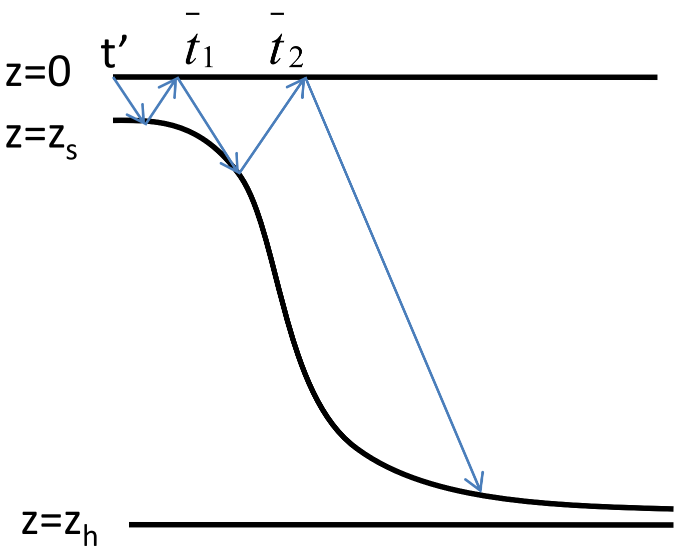
Expectation from geometric optics picture suggests singularities of $G^R(t,t')$ when the light ray starting off at t' returns to the boundary



Only finite bouncing is possible:
The warping factor freeze both the shell and the light ray near horizon



Divergence matching in collapse background

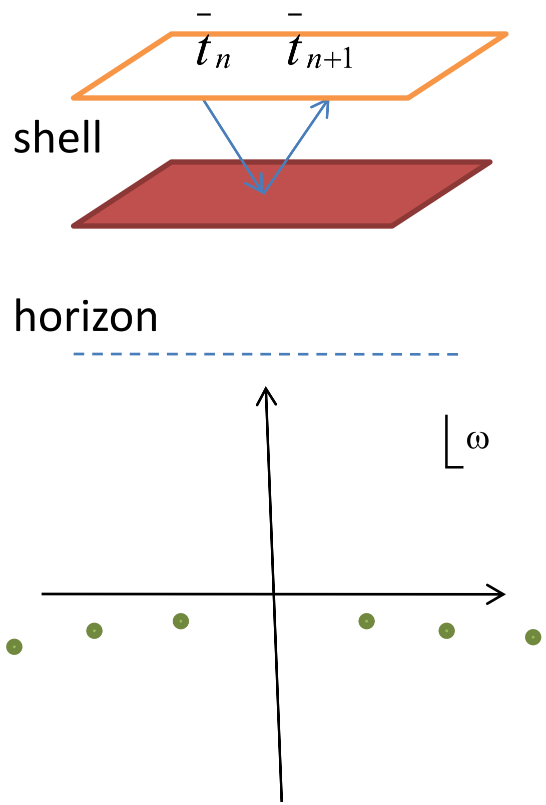


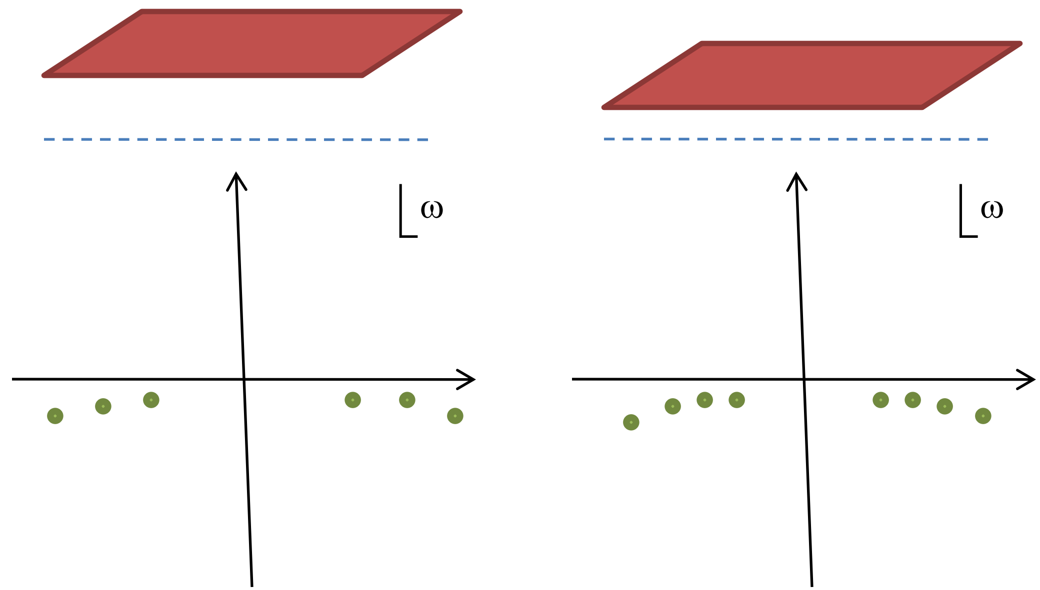
$$G^R_{>}(t \to \bar{t}_n) = \frac{A_n(-i)^{n-1}}{(-t+\bar{t}_n+i\varepsilon)^{5-n}}, G^R_{<}(t \to \bar{t}_n) = \frac{A_n i^{n-1}}{(-t+\bar{t}_n-i\varepsilon)^{5-n}}$$

$\bar{t}_n \to +\infty$ as $t' \to T_n(z_s)$

"thermalization time" $t_{th} = \frac{T_1(\pi T z_s)}{\pi T} \sim \frac{O(1)}{\pi T}$

Evolution of spectrum for quasi-static state



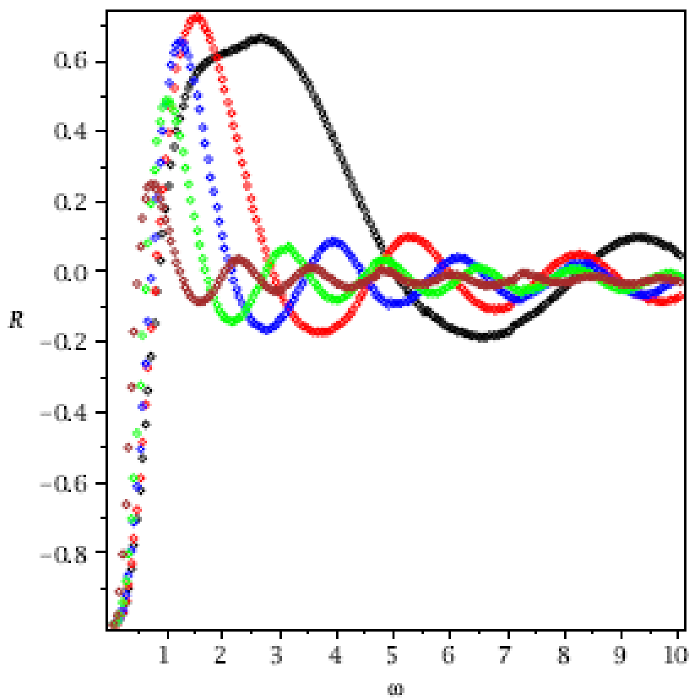


Approximate Normal Mode: Re(ω)>>Im(ω)

$$\Delta\omega \cdot (\bar{t}_{n+1} - \bar{t}_n) \approx 2\pi$$

See also Baier et al 1205.2998

Deviation from thermal spectral function for quasi-static state



$$\chi = -2\,\mathrm{Im}\,G^R(\omega)$$

$$R = \frac{\chi - \chi_{thermal}}{\chi_{thermal}}$$

Spectral function for quasi-static state oscillate around thermal spectral function

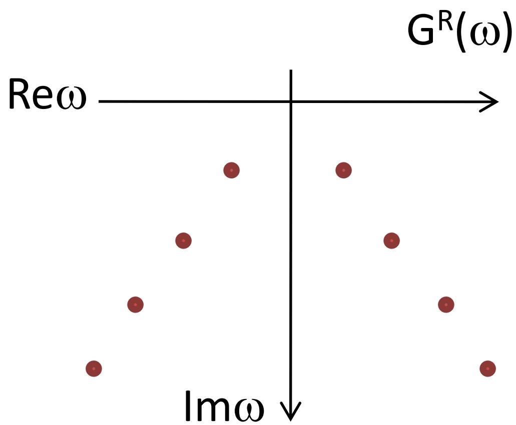
Approximate Normal Mode responsible for the oscillation

$\Delta\omega$ shrinks as the shell os lowered toward the horizon

Shell falling →

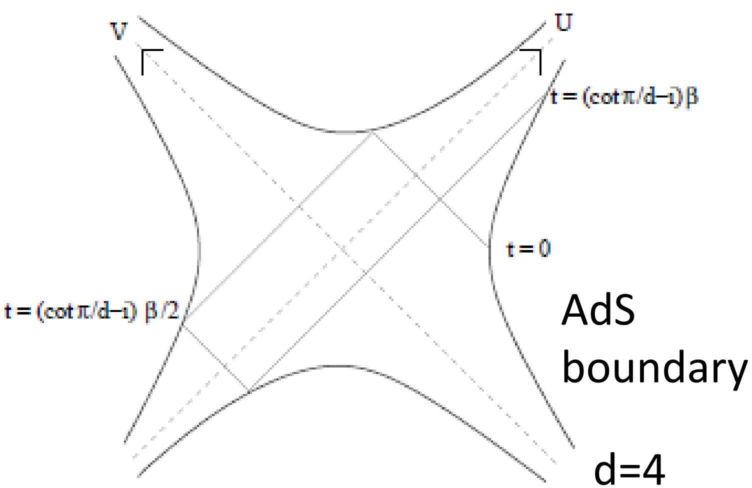
	black	red	blue	green	brown
f	0.99	0.91	0.75	0.51	0.19

Quasi-Normal Modes at end point of thermalization



QNM for thermalized QGP obtained from scalar probe in AdS-Schwarzschild black hole

Quasi Normal Mode: Re(ω) ~ Im(ω)



Bouncing light ray in Penrose diagram gives rise to complex time and therefore Quasi-Normal Modes

$$\Delta t = \frac{1}{2T}(1+i) \Rightarrow \Delta\omega = \frac{2\pi}{\Delta t} = 2\pi T(1-i)$$

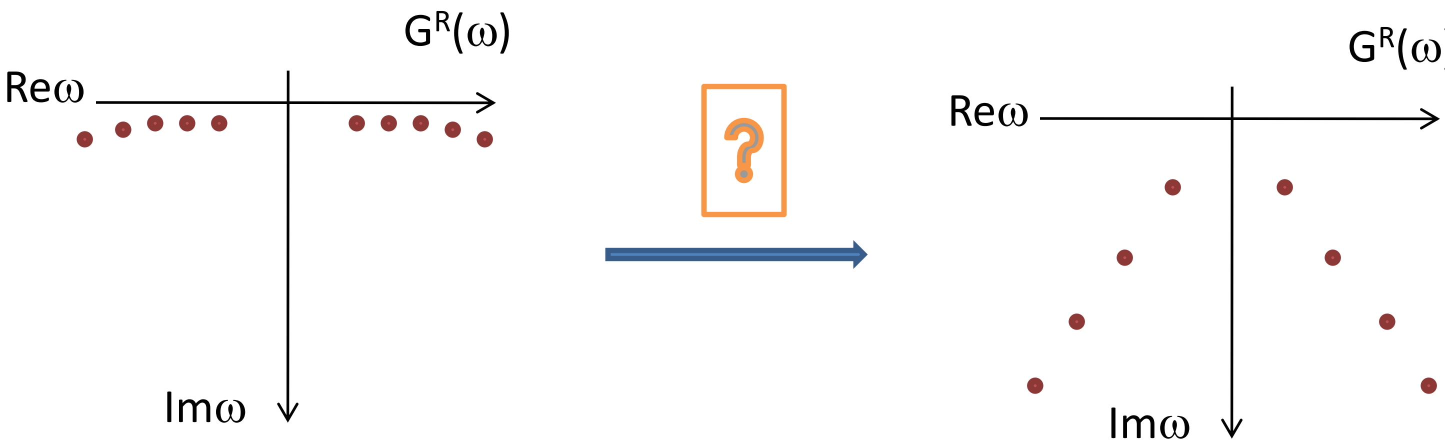
$$\omega_n = 2\pi T(1-i)n$$

Amado & Hoyos JHEP 2008

Summary

- We studied a probe scalar in a gravitational collapse model, and obtain the singular part of the unequal time correlator for glue ball operator.
- The singularities are consistent with bouncing light ray in collapse background: Finite singularities; singularities eventually disappear at late stage of thermalization.
- For quasi-static state, we have established the the singularities as from the contribution of approximate Normal Modes, which are also responsible for the oscillation in spectral function.

Evolution of QNM in thermalization



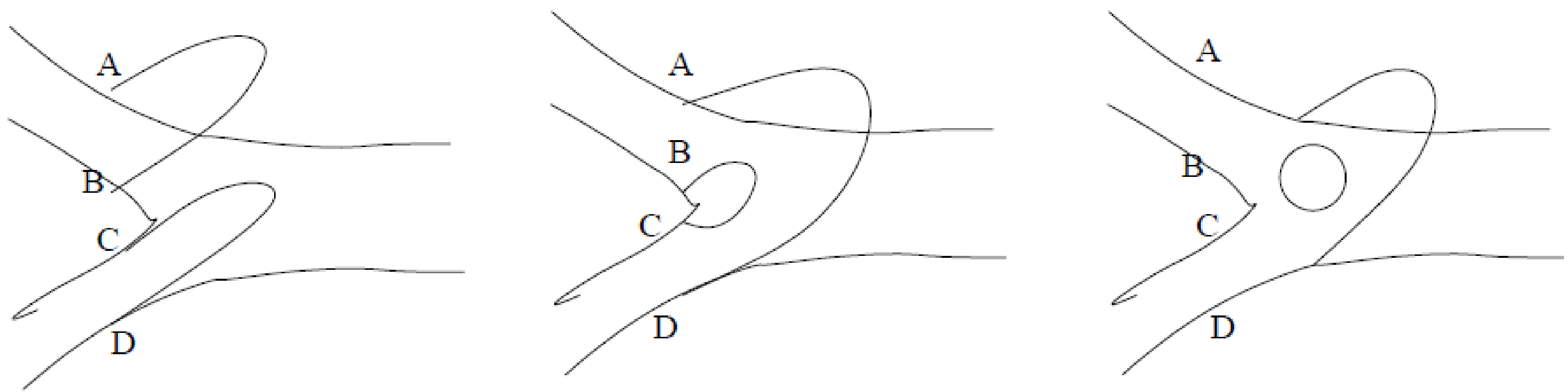
Approximate Normal Modes disappear in thermalization

Emergence of Quasi Normal Modes?

Perhaps missed by the WKB approximation

$\bar{t}_n \to +\infty$ as $t' \to T_n(z_s)$

"thermalization time" $$t_{th} = \frac{T_1(\pi T z_s)}{\pi T} \sim \frac{O(1)}{\pi T}$$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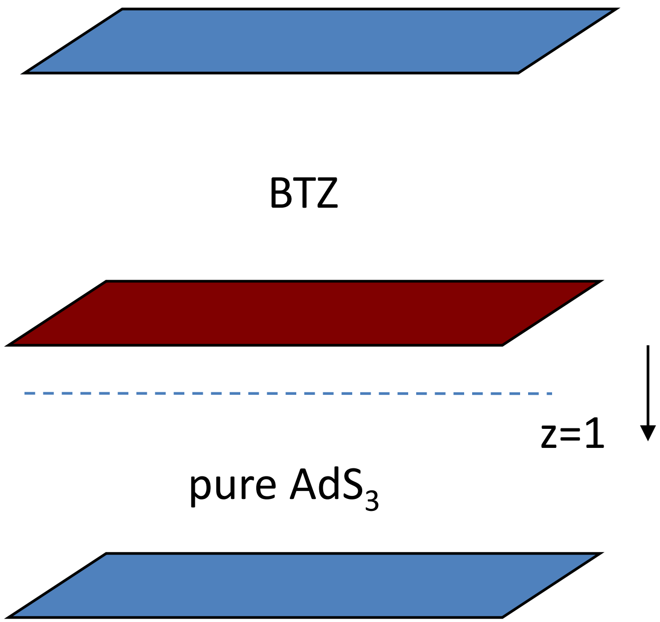
Evolution of QNM in gravitational collapse of BTZ black hole



Ingoing wave

$$G_R(\omega) = i\omega - \omega^2\psi\left(1 - \frac{i\omega}{2}\right),$$

Outgoing wave

$$G_A(\omega) = -i\omega - \omega^2\psi\left(1 + \frac{i\omega}{2}\right)$$

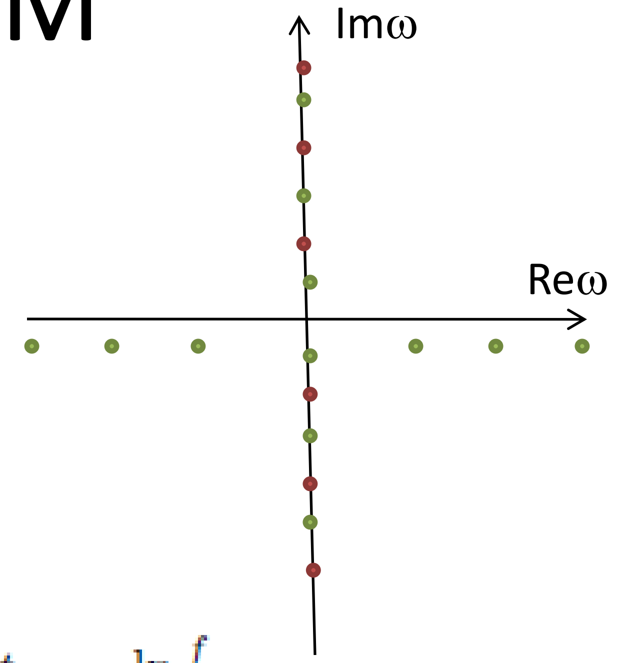
Quasi static state:

$z=z_s$

$$G_R^q(\omega) = \frac{\frac{\Gamma(1-i\omega)}{\Gamma(1-\frac{i\omega}{2})^2}G_R(\omega) - f^{-i\omega}\frac{\sqrt{f}}{4i\omega}\frac{\Gamma(1+i\omega)}{\Gamma(1+\frac{i\omega}{2})^2}G_A(\omega)}{\frac{\Gamma(1-i\omega)}{\Gamma(1-\frac{i\omega}{2})^2} - f^{-i\omega}\frac{\sqrt{f}}{4i\omega}\frac{\Gamma(1+i\omega)}{\Gamma(1+\frac{i\omega}{2})^2}}.$$

QNM given only by the vanishing of the denominator

Two sets of QNM



$$G_R^q(\omega) = \frac{\frac{\Gamma(1-i\omega)}{\Gamma(1-\frac{i\omega}{2})^2} G_R(\omega) - f^{-i\omega}\frac{\sqrt{f}}{4i\omega}\frac{\Gamma(1+i\omega)}{\Gamma(1+\frac{i\omega}{2})^2} G_A(\omega)}{\frac{\Gamma(1-i\omega)}{\Gamma(1-\frac{i\omega}{2})^2} - f^{-i\omega}\frac{\sqrt{f}}{4i\omega}\frac{\Gamma(1+i\omega)}{\Gamma(1+\frac{i\omega}{2})^2}}.$$

Set 1: $\omega = a + ib$

$$a \approx -\frac{(2n-1)\pi}{\ln\frac{f}{4}}, \quad b \approx \frac{\ln\frac{4a}{f}}{\ln\frac{f}{4}},$$

$$a \approx \frac{2n\pi}{\ln\frac{f}{4}}, \quad b \approx \frac{\ln\frac{-4a}{f}}{\ln\frac{f}{4}},$$

Asymptotically Normal Modes

$$\Delta\omega = -\frac{2\pi}{\ln\frac{f}{4}} \Rightarrow \Delta t = -\ln\frac{f}{4}$$

Agrees with results from divergence matching

Set 2: $i\omega \approx -(2n-1)$ and $i\omega \approx 2n-1$
as opposed to
$i\omega = -2n$ for retarded correlator and
$i\omega = 2n$ for advanced correlator

The QNM evolution does not seem to reduce to the pattern of the thermal state