Two- and Multi-particle Cumulant Measurements of \( v_n \)

and Isolation of Flow and Nonflow

in \( \sqrt{s_{NN}} = 200 \) GeV Au+Au Collisions by STAR

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Outline

• Physics motivation

• Results
  • 2- and multi-particle anisotropy
  • 2- and 4-particle $\eta-\eta$ cumulants
    $\rightarrow$ Isolation of flow and nonflow

• Summary
Azimuthal Anisotropy
Flow and Nonflow

- Hydrodynamic expansion
  \( \rightarrow \) anisotropic flow;
- Flow is sensitive to early stage of heavy ions collisions

\[
dN/d\phi \propto 1 + \sum_{n=1}^{\infty} 2\nu_n \cos(n(\phi - \Psi_{nR}))
\]
Azimuthal Anisotropy
Flow and Nonflow

- Hydrodynamic expansion
  - anisotropic flow;
- Flow is sensitive to early stage of heavy ions collisions

\[ \frac{dN}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n \cos(n(\phi - \Psi_{nR})) \]

- Event-by-event initial geometry fluctuation
  - odd harmonics
Azimuthal Anisotropy Flow and Nonflow

- Hydrodynamic expansion → anisotropic flow;
- Flow is sensitive to early stage of heavy ions collisions

\[ \frac{dN}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2 \nu_n \cos\left(n(\phi - \Psi_{nR})\right) \]

- Event-by-event initial geometry fluctuation → odd harmonics
- The reaction plane azimuthal angle is unknown → the measured anisotropies = flow(\(v\)) + flow fluctuation (\(\sigma\)) + nonflow (\(\delta\))

particle correlation unrelated to the reaction plane
Constraint on $\eta/s$

20% uncertainty in $v_2/\varepsilon \rightarrow 100\%$ uncertainty in $\eta/s$

The question is how to reduce uncertainty in $v_2/\varepsilon$:
1. $\varepsilon$ from theoretical part
2. $v_2$ from experimental part


"The extraction of $\eta/s$ from a comparison with hydrodynamics thus requires careful treatment of both fluctuation and nonflow effects"
$v_3$ vs Centrality

AuAu@200GeV

$-1<\eta<1$

$p_T<2$ GeV/c

$Q$-Cumulant Method with $\eta$-gap

- $v_3$ shows modest centrality dependence
- $v_3$ is 3× smaller than $v_2$ in peripheral to mid-central collisions

(Also see: Pandit, 1A, Tue.)

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\[ v_3 \text{ vs } \rho_T \]

- \( v_3 \): more sensitive to \( \eta/s \) than \( v_2 \)

\[ p_T^{\text{ref}} = 0 - 10 \text{ GeV}/c \]

Q-Cumulant Method

- hydro describes data trend well at \( p_T < 2 \text{ GeV}/c \)
- Data may contain nonflow

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* Schenke, Jeon, Gale, PRL 106, private commu.
Isolation of Flow and Nonflow using 2-, and 4-Particle $\eta$-$\eta$ Cumulants

Xu, LY, arXiv:1204.2815

• 2-particle cumulant:

\[
\mathcal{V}\{\eta_\alpha, \eta_\beta\} = \nu(\eta_\alpha)\nu(\eta_\beta) + \sigma(\eta_\alpha)\sigma(\eta_\beta) + \sigma'(\Delta \eta) + \delta(\Delta \eta)
\]

\[\text{flow} \quad \text{flow fluct.} \quad \Delta \eta\text{-dep fluct.} \quad \Delta \eta\text{-dep nonflow}\]

\[
\Delta \mathcal{V}\{2\} = \mathcal{V}\{-\eta_\alpha, -\eta_\beta\} - \mathcal{V}\{-\eta_\alpha, \eta_\beta\} = \Delta \sigma' + \Delta \delta
\]

\[\nu(-\eta_\beta) = \nu(\eta_\beta) \quad \sigma(-\eta_\beta) = \sigma(\eta_\beta)\]

• 4-particle cumulant:

\[
\mathcal{V}^{1/2}\{\eta_\alpha, \eta_\alpha, \eta_\beta, \eta_\beta\} = \nu(\eta_\alpha)\nu(\eta_\beta) - \sigma(\eta_\alpha)\sigma(\eta_\beta) - \sigma'(\Delta \eta)
\]

\[
\Delta \mathcal{V}\{4\}^{1/2} = -\Delta \sigma'
\]

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Isolation of Flow and Nonflow

using 2-, and 4-Particle \( \eta \)-\( \eta \) Cumulants

Xu, LY, arXiv:1204.2815

• 2-particle cumulant:

\[
\mathcal{V}\{\eta_\alpha, \eta_\beta\} = \nu(\eta_\alpha)\nu(\eta_\beta) + \sigma(\eta_\alpha)\sigma(\eta_\beta) + \sigma'(\Delta \eta) + \delta(\Delta \eta)
\]

\[
\Delta \mathcal{V}\{2\} = \mathcal{V}\{-\eta_\alpha, -\eta_\beta\} - \mathcal{V}\{-\eta_\alpha, \eta_\beta\} = \Delta \sigma' + \Delta \delta
\]

\[
\begin{align*}
\nu(-\eta_\beta) &= \nu(\eta_\beta) \\
\sigma(-\eta_\beta) &= \sigma(\eta_\beta)
\end{align*}
\]

• 4-particle cumulant:

\[
\mathcal{V}^{1/2}\{\eta_\alpha, \eta_\alpha, \eta_\beta, \eta_\beta\} = \nu(\eta_\alpha)\nu(\eta_\beta) - \sigma(\eta_\alpha)\sigma(\eta_\beta) - \sigma'(\Delta \eta)
\]

\[
\Delta \mathcal{V}\{4\}^{1/2} = -\Delta \sigma'
\]

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Isolation of Flow and Nonflow

using 2-, and 4-Particle $\eta$-$\eta$ Cumulants

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• 2-particle cumulant:

\[ \mathcal{V}\{\eta_{\alpha},\eta_{\beta}\} = \nu(\eta_{\alpha})\nu(\eta_{\beta}) + \sigma(\eta_{\alpha})\sigma(\eta_{\beta}) + \sigma'(\Delta \eta) + \delta(\Delta \eta) \]

\[ \Delta \mathcal{V}\{2\} = \mathcal{V}\{-\eta_{\alpha},-\eta_{\beta}\} - \mathcal{V}\{-\eta_{\alpha},\eta_{\beta}\} = \Delta \sigma' + \Delta \delta \]

- \[ \nu(-\eta_{\beta}) = \nu(\eta_{\beta}) \]
- \[ \sigma(-\eta_{\beta}) = \sigma(\eta_{\beta}) \]

• 4-particle cumulant:

\[ \mathcal{V}^{1/2}\{\eta_{\alpha},\eta_{\alpha},\eta_{\beta},\eta_{\beta}\} = \nu(\eta_{\alpha})\nu(\eta_{\beta}) - \sigma(\eta_{\alpha})\sigma(\eta_{\beta}) - \sigma'(\Delta \eta) \]

\[ \Delta \mathcal{V}\{4\}^{1/2} = -\Delta \sigma' \]

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Isolation of Flow and Nonflow

using 2-, and 4-Particle $\eta$-$\eta$ Cumulants

Xu, LY, arXiv:1204.2815

• 2-particle cumulant:

$$\mathcal{V}\{\eta_\alpha, \eta_\beta\} = \nu(\eta_\alpha)\nu(\eta_\beta) + \sigma(\eta_\alpha)\sigma(\eta_\beta) + \sigma'(\Delta \eta) + \delta(\Delta \eta)$$

where

- $\nu(\eta)$ represents the flow function,
- $\sigma(\eta)$ represents the flow fluctuation,
- $\sigma'(\Delta \eta)$ represents the $\Delta \eta$-dependent fluctuation,
- $\delta(\Delta \eta)$ represents the $\Delta \eta$-dependent nonflow.

$$\Delta \mathcal{V}\{2\} = \mathcal{V}\{-\eta_\alpha, -\eta_\beta\} - \mathcal{V}\{-\eta_\alpha, \eta_\beta\} = \Delta \sigma' + \Delta \delta$$

with

- $\nu(-\eta_\beta) = \nu(\eta_\beta)$,
- $\sigma(-\eta_\beta) = \sigma(\eta_\beta)$.

• 4-particle cumulant:

$$\mathcal{V}^{1/2}\{\eta_\alpha, \eta_\alpha, \eta_\beta, \eta_\beta\} = \nu(\eta_\alpha)\nu(\eta_\beta) - \sigma(\eta_\alpha)\sigma(\eta_\beta) - \sigma'(\Delta \eta)$$

$$\Delta \mathcal{V}\{4\}^{1/2} = -\Delta \sigma'$$

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$\Delta \eta$-dependence $\sigma'(\Delta \eta)$

Flow fluctuation appears independent of $\Delta \eta$.
\( \Delta \eta\)-dependent \( \delta(\Delta \eta) \)

- \( \delta(\Delta \eta_2) - \delta(\Delta \eta_1) \) linear in \( \Delta \eta_2 - \Delta \eta_1 \) at a given \( \Delta \eta_1 \) with similar slopes
- Intercept changes with \( \Delta \eta_1 \) exponentially

\[
\Delta \delta(\Delta \eta_1, \Delta \eta_2) = a(e^{-\Delta \eta_1/b} - e^{-\Delta \eta_2/b}) + A(e^{-\Delta \eta_1^2/2\sigma^2} - e^{-\Delta \eta_2^2/2\sigma^2})
\]

\[
\delta(\Delta \eta) = ae^{-\Delta \eta/b} + Ae^{-\Delta \eta^2/2\sigma^2}
\]
Cumulant $\mathcal{V}\{2\}$, Nonflow $\delta$, 'Flow' $\nu_n^2$

No Assumption about flow $\eta$ dependence in our analysis

The decomposed 'flow' appears to be independent of $\eta$.

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**Δη-dep Near-side Nonflow**

- Calculate $<\text{nonflow}>$ of all $(\eta_\alpha, \eta_\beta)$ bins with $x < \eta$-gap < 2. ($x$ = horizontal axis).

\begin{itemize}
  \item With $|\Delta \eta| > 0.7$, significant nonflow still exits.
\end{itemize}

AuAu@200GeV 20-30%

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'Flow' and Nonflow vs Centrality

Au+Au@200GeV

\[ \sqrt{\frac{\delta_2}{v_2}} \sim 20\% \text{ for } |\Delta \eta| > 0.7 \]
\[ \delta_2 / v_2^2 \sim 4\% \]
'Flow' vs $\eta$

$\mathcal{V}_2\{2\} = \delta_2 + \left\langle \nu_2^2 \right\rangle$

$\left\langle \nu_2^2 \right\rangle$

decomposed flow

$\mathcal{V}_2\{4\}^{1/2} = \left\langle \nu_2^2 \right\rangle - 2\sigma_2^2$

raw 4-particle cumulant

raw 2-particle cumulant

• Flow seems independent of $\eta$. Note no assumption of $\eta$ dependence in our approach.

• (Fluctuation / flow)$^2 \sim 13\%$

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4- and 6-Particle Cumulant

Assuming the flow fluctuations are Gaussian, we have two options:

1. $v_x, v_y$ are Gaussian: $v\{6\} = v\{4\}$
   Voloshin, Poskanzer, Tang, Wang, PLB


$$v_n\{6\} / v_n\{4\} \approx 1 - \sigma^6 / 3\langle v_n \rangle^6$$,
if $\sigma^2 / \langle v_n \rangle^2 << 1$

* No-weight applied, non-uniform acceptance corrected. Systematic errors estimated by applying weight and no acceptance correction.
4- and 6-Particle Cumulant

Assuming the flow fluctuations are Gaussian, we have two options:

1. $v_x, v_y$ are Gaussian: $v\{6\} = v\{4\}$  
   Voloshin, Poskanzer, Tang, Wang, PLB

2. $v$ is Gaussian: 

$$v_n\{6\}/v_n\{4\} \approx 1 - \sigma^6/3\langle v_n \rangle^6 \quad \text{if} \quad \sigma^2/\langle v_n \rangle^2 << 1$$

* No-weight applied, non-uniform acceptance corrected. Systematic errors estimated by applying weight and no acceptance correction
Summary

- 2-, 4- and 6-particle cumulants vs $p_T$, centrality are presented.
- Isolation of $\Delta \eta$-dependent (near-side nonflow) and $\Delta \eta$-independent (flow-dominant + small away-side nonflow) correlations, using 2- and 4-particle cumulants between $\eta$ bins
  - the decomposed 'flow'
    appears to be independent of $\eta$ within $\pm 1$ unit
  - nonflow estimation
    $\sim 4\%$ in $v_2^2$
  - flow fluctuation estimation
    $\sim 13\%$ in $v_2^2$

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Abstract

Azimuthal anisotropic flows \( v_n \), arising from the anisotropic collision geometry, reflect the hydrodynamic properties of the quark gluon plasma created in relativistic heavy-ion collisions. A long standing issue in \( v_n \) measurements is the contamination of nonflow, caused by intrinsic particle correlations unrelated to the collision geometry. Nonflow limits, in part, the precise extraction of the viscosity to entropy density ratio \( \eta/s \) from data-model comparisons. Isolation of flow and nonflow is critical to the interpretation of the Fourier decomposition of dihadron correlations.

In this talk we report measurements of \( v_n \) azimuthal anisotropies using the two- and multi-particle \( Q \)-cumulants method from STAR in Au+Au collisions at 200 GeV. The centrality and pT dependence of \( v_n \) will be presented. We compare the four- and six-particle cumulant measurements to gain insights on the nature of flow fluctuations [1,2]. We further analyze two- and four-particle cumulants between pseudo-rapidity (\( \eta \)) bins. Exploiting the collision symmetry about mid-rapidity, we isolate the \( \Delta \eta \)-dependent and \( \Delta \eta \)-independent correlations in the data with a data-driven method [3]. The \( \Delta \eta \)-dependent part arises from near-side nonflow correlations, such as HBT interferometry, resonance decays, and jet-correlations. The \( \Delta \eta \)-independent part is dominated by flow and flow fluctuations with relatively small contribution from away-side jet-correlations. The method does not make assumptions about the \( \eta \) dependence of flow. Our isolated \( \Delta \eta \)-independent part from data, dominated by flow, however, is found to be also \( \eta \)-independent within the STAR TPC of \( \pm 1 \) unit of pseudo-rapidity. The \( \Delta \eta \) drop in the measured two-particle cumulant appears to entirely come from nonflow. We assess the effect of the nonflow on \( \eta/s \) extraction. We reexamine the high-pT triggered dihadron correlations with background subtraction of our decomposed flows.


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Analysis Cuts
AuAu@200GeV

Year2004 data
19 million min-bias events
|\text{Vertex z}| < 30 cm
|\eta| < 1
Dca < 2 cm
nfit >= 20
nhits / nfit-pos > 0.51

Year2010 data
80 million min-bias events
|\text{Vertex z}| < 30 cm
|\text{vpdz-Vz}| < 3
|\text{Vr}| < 2
TiggerId: 260001, 260011, 260021, 260031
|\eta| < 1
Dca < 2 cm
nfit >= 15
1.02 > nhits / nfit-pos > 0.52
flag() < 1000

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2-, 4-, 6-Particle Q-Cumulant Method

- Particle azimuthal moments:
  In a single event $\delta$ is nonflow
  \[
  \langle 2 \rangle_n = \langle e^{in(\phi_i - \phi_j)} \rangle = v_n^2 + \delta_n
  \]
  \[
  \langle 4 \rangle_n = \langle e^{4\phi_i - 4\phi} \rangle \approx v_n^4 + 4v_n^2\delta_n + 2\delta_n^2
  \]
  \[
  \langle 6 \rangle_n = \langle e^{6\phi_i - 6\phi} \rangle \approx v_n^6 + 9v_n^4\delta_n + 18v_n^2\delta_n^2 + 6\delta_n^3
  \]
  - 2-, 4-, 6-particle azimuthal anisotropy

Average over all events

- $v_n \{2\}^2 \equiv \langle 2 \rangle_n$
- $v_n \{4\}^4 \equiv 2\langle 2 \rangle_n^2 - \langle 4 \rangle_n$
- $v_n \{6\}^6 \equiv (\langle 6 \rangle_n - 9\langle 2 \rangle_n \langle 4 \rangle_n + 12\langle 2 \rangle_n^3) / 4$

For exclusive region:

\[
\langle 2' \rangle_n = \langle e^{in(\phi_i - \phi_j)} \rangle = v_n v_n + \delta_n
\]


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4-Particle Cumulant between $\eta$ Bins

$$V^{1/2}(\eta_\alpha, \eta_\alpha; \eta_\beta, \eta_\beta) = \nu(\eta_\alpha)\nu(\eta_\beta) - \sigma(\eta_\alpha)\sigma(\eta_\beta) - \sigma'(\Delta\eta)$$

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2-Particle Cumulant between $\eta$ Bins

\[
V\{\eta_\alpha, \eta_\beta\} = v(\eta_\alpha)v(\eta_\beta) + \sigma(\eta_\alpha)\sigma(\eta_\beta) + \sigma'(\Delta\eta) + \delta(\Delta\eta)
\]

fluctuations

nonflow

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Nonflow Parameterization

Run-4 Au+Au 20-30% data

\[ \delta(\Delta \eta) = a \exp(-\Delta \eta/b) - k(\Delta \eta - \Delta \eta_{\text{max}}) + c \]

- c set to 0 (arbitrary)

- Intercepts drop quickly at large \( \Delta \eta_1 \) and then saturates

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Nonflow Fit Parameters

\[ \delta(\Delta \eta) = a^* \exp(-\Delta \eta/b) + A^* \exp(-\Delta \eta^2/2d^2) + c \]
$v_3$ vs $p_T$ and Model Comparison

- $v_3$ more sensitive to $\eta/s$ than $v_2$

B. Schenke, S. Jeon, and C. Gale
PRL 106, 042301 (2011)

- Top 5%, hydro under-predicts data
- Non-central hydro describes data well at $p_T < 2$ GeV
  hydro deviates from data at $p_T > 2$ GeV
- Data may contain nonflow

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$\Delta \eta$-dependence $\sigma'(\Delta \eta)$

**Au+Au@200 GeV 20-30% data**

Flow fluctuation appears independent of $\Delta \eta$.

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**Δη-dependent δ(Δη)**

**Au+Au@200GeV 20-30% data**

- δ(Δη₂)-δ(Δη₁) linear in Δη₂-Δη₁ at a given Δη₁ with similar slopes
- Intercept changes with Δη₁ exponentially

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Improved Nonflow Fit

Au+Au@200GeV 20-30% data

\[ \Delta \delta(\Delta \eta_1, \Delta \eta_2) = a^* [\exp(-\Delta \eta_1/b) - \exp(-\Delta \eta_2/b)] - k(\Delta \eta_1 - \Delta \eta_2) \]
\[ \delta(\Delta \eta) = a^* \exp(-\Delta \eta/b) - k \Delta \eta + c \]

\[ \Delta \delta(\Delta \eta_1, \Delta \eta_2) = a^* [\exp(-\Delta \eta_1/b) - \exp(-\Delta \eta_2/b)] + A^*[\exp(-\Delta \eta_1^2/2d^2) - \exp(-\Delta \eta_2^2/2d^2)] \]
\[ \delta(\Delta \eta) = a^* \exp(-\Delta \eta/b) + A^* \exp(-\Delta \eta^2/2d^2) \]

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$v_3$ vs $p_T$ and Model Comparison

- $v_3$ more sensitive to $\eta/s$ than $v_2$

- Top 5%, hydro under-predicts data
- Non-central hydro describes data well at $p_T < 2$ GeV hydro deviates from data at $p_T > 2$ GeV
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4- and 6-Particle Cumulant

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$$v_n\{6\}/v_n\{4\} \approx 1 - \frac{\sigma^6}{3\langle v_n \rangle^6}, \text{ if } \frac{\sigma^2}{\langle v_n \rangle^2} << 1$$

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