Mapping the hydrodynamic response to the initial geometry in heavy-ion collisions

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with

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Outline

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     Smooth & Realistic Initial Conditions
2. Mapping the hydrodynamic response
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   - The elliptic flow case;
   - Generalization to higher harmonics
   - Improving the predictor
3. Conclusion

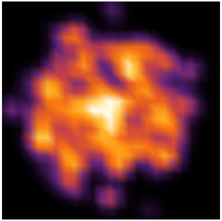
Motivation

- The azimuthal distribution of outgoing particles in a hydro event can be written as

$$\frac{2\pi}{N}\frac{dN}{d\phi_p} = 1 + 2\sum_n v_n \cos[n(\phi_p - \Psi_n)]$$

or, equivalently: $\left\{e^{in\phi_p}\right\} = v_n e^{-in\Psi_n}$

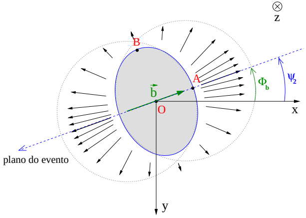
$\{\ldots\}$ =average in one event



- The largest source of uncertainty in hydro models is the initial conditions.
- Anisotropic flow v_n and the event plane Ψ_n are determined by initial conditions.
- We need to understand which properties of the initial state determine v_n and Ψ_n, so as to constrain models of initial conditions from data.

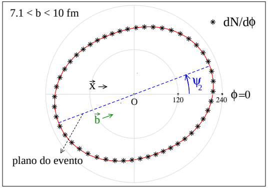
Average Initial Conditions - the smooth case

- With smooth initial conditions, the participant eccentricity ε_2 is proportional to the elliptic flow v_2,
- And the participant plane Φ_2 is aligned with the event plane Ψ_2.



$$\varepsilon_2 e^{i2\Phi_2} = -\frac{\{r^2 e^{i2\phi}\}}{\{r^2\}}.$$

$\{\cdots\}$ = average over initial density profile



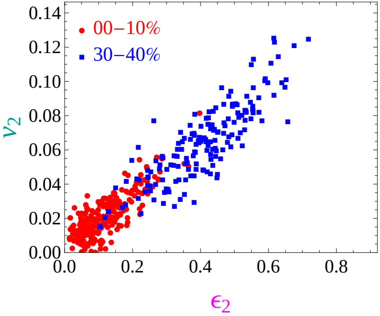
figures by R. Andrade

- But, in real collisions there are fluctuations, event-by-event.
- In event-by-event hydrodynamics are these relations, $v_2 \approx k\varepsilon_2$ and $\Psi_2 \approx \Phi_2$, valid?

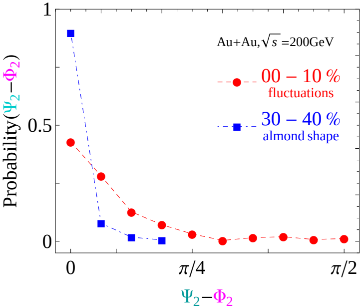
Fluctuations: Event-by-event hydro

NeXSPheRIO→
- NeXus: initial condition generator;
- SPheRIO: solves the equations of relativistic ideal hydrodynamics.

Scatter plot of v_2 versus ε_2



Distribution of $\Psi_2 - \Phi_2$



$v_2 \approx k\varepsilon_2$ and $\Psi_2 \approx \Phi_2$?

Reasonable, but not perfect.

See also, *F.G.G. et al 1110.5658, Petersen et al 1008.0625, Qiu & Heinz 1104.0650*

Our goal

- Propose a simple quantitative measure of the correlation between (v_2, Ψ_2) and (ε_2, Φ_2);
- Generalization to higher harmonics;
- Find better scaling laws.

How To Characterize The Hydrodynamic Response

- Previously, the correlation of the flow with the initial geometry was studied through

Distribution of Ψ_2-Φ_2

Scatter plot v_2 versus ε_2

Our Proposal: A GLOBAL ANALYSIS

$$v_2 e^{i2\Psi_2} = k\varepsilon_2 e^{i2\Phi_2} + \mathcal{E}$$

- k: It is the same for all events (in each centrality class).
- $\mathcal{E}$: event-by-event error.

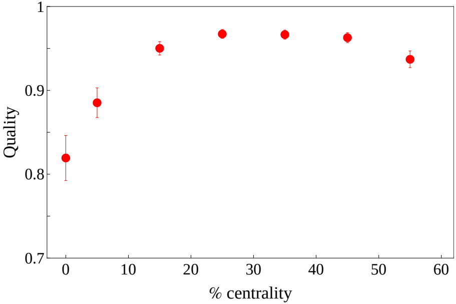
- The best linear fit is achieved minimizing the mean-square error $\langle|\mathcal{E}^2|\rangle$ ($\langle\cdots\rangle$ $\equiv$average over events).
- $k = \langle\varepsilon_2 v_2 \cos[2(\Psi_2 - \Phi_2)]\rangle / \langle\varepsilon_2^2\rangle$
- $\langle|\mathcal{E}^2|\rangle = \langle v_2^2\rangle - k^2\langle\varepsilon_2^2\rangle$

Elliptic flow as a response to the almond-shaped overlap area

- The *quality* response is given by:

$$Quality = \frac{k\sqrt{\langle \varepsilon_2^2 \rangle}}{\sqrt{\langle v_2^2 \rangle}}$$

The closer *Quality* to 1, the better the response.



- *Central collisions:*
  All anisotropies due to fluctuations
  Quality 81%

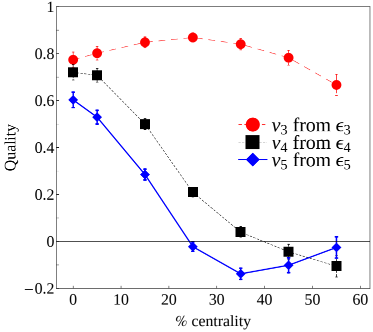
- *Mid-central collisions:*
  Elliptic flow is driven by the almond shape: Quality 95%

ε_2 is a very good predictor of v_2!

Generalization to higher harmonics

- Natural estimators are:
  $v_n e^{in\Psi_n} = k\varepsilon_n e^{in\Phi_n} + \mathcal{E}$
- Generalizing ε_n (*Petersen et al 1008.0625*).
  $$\varepsilon_n e^{in\Phi_n} = -\frac{\{r^n e^{in\phi}\}}{\{r^n\}}$$

Teaney&Yan (1010.1876) showed ε_n come from a cumulant expansion of the initial density energy



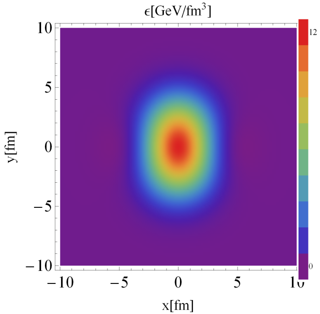
- **n=3**:
  ε_3 is a very good predictor of v_3.
- **n=4,5**:
  Good quality for central collisions

  Then decrease and even become negative: Ψ_n and Φ_n are anticorrelated.

  Qiu&Heinz, 1104.0650

The Almond Shape and v_4

With smooth IC, inspired by NeXus IC in the 30 – 40% centrality bin



n	ε_n	v_n	Φ_n	Ψ_n
2	.4	.069	0	0
4	0	.011	undf	0
odd	0	0	undf	undf

There is no ε_4, so where does v_4 come from?

- v_4 is generated by ε_2!
- Ψ_4 is in the reaction plane, as Ψ_2.

Comparing with NeXSPheRIO (30-40%), $\langle v_2 \rangle \approx .066$ and $\langle v_4 \rangle \approx .01$

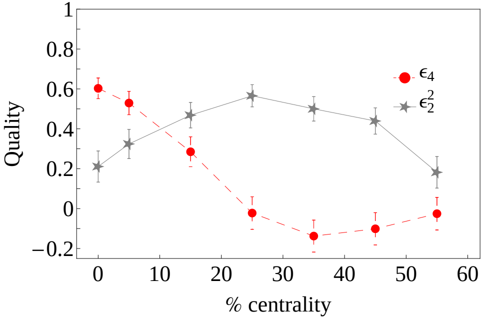
F.G.G. et al 1110.5658.

v_4 induced by ε_2^2 in event-by-event

- A natural estimator is:

$$v_4 e^{i4\Psi_4} = k\left(\varepsilon_2 e^{i2\Phi_2}\right)^2 + \mathcal{E}$$

(preserves rotational symmetry)



- For mid-central collisions, where ε_2 is large, the non-linear term is important!
- This estimator is not as good as previous estimators of v_2 and v_3.

How to improve the estimator?

Finding a better estimator

Combining both effects?

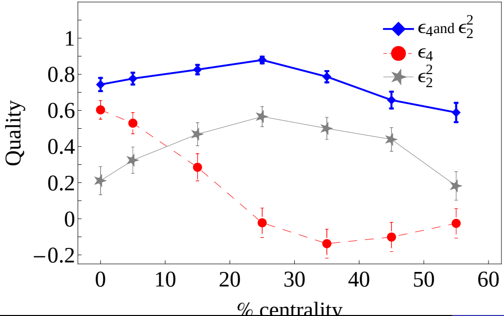
- Defining

$$v_4 e^{i4\Psi_4} = k\varepsilon_4 e^{i4\Phi_4} + k'\left(\varepsilon_2 e^{i2\Phi_2}\right)^2 + \mathcal{E}$$

- And minimizing $\langle|\mathcal{E}|^2\rangle$, with respect to k and k'.
- Then, the mean-square error is

$$\langle|\mathcal{E}^2|\rangle = \langle v_2^2\rangle - \langle|k\varepsilon_4 e^{i4\Phi_4} + k'\left(\varepsilon_2 e^{i2\Phi_2}\right)^2|^2\rangle$$

This error is always smaller than with one parameter



- The combined estimator results in an excellent predictor for all centralities!
- For v_5, it is also possible to use both, linear and non-linear, terms to obtain the best estimator: ε_5 and $\varepsilon_2\varepsilon_3$ preserves rotational symmetry

Conclusions

- We have defined a quantitative measure of the quality of estimators of v_n from initial conditions event-by-event hydro;
- v_2 can be understood as a response to the almond-shaped overlap area ε_2, even for central collisions;
- The triangularity ε_3 is a very good predictor to v_3;
- Non-linear terms are necessary to predict v_4 (and v_5) from initial energy density, for all centralities. (*See TeaneyYan 1206.1905*)
- These results provide an improved understanding of the hydro response to the initial state in realistic heavy-ion collisions.