Non-Equilibrium Phase Transitions in Field Theories Applications on the Quark Meson Model

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H-QM Helmholtz Research School Quark Matter Studies

We want to investigate non-equilibrium dynamics in a linear sigma model to study the chiral phase transition in QCD.
In high-energy heavy-ion collisions, the quark-gluon-plasma is supposed to go from a hot, chiral-restored and deconfined phase to a cold, chiral-broken and confined phase. While there are many calculations in thermal and chemical equilibrium, we use a dynamical 3+1D linear sigma model with constituent quarks to examine the evolution of equilibrium and non-equilibrium scenarios.
In a first attempt we employ a mean-field ansatz to reproduce the thermodynamical properties of the model, in a later ansatz we use a 2-PI effective action to handle non-equilibrium field effects.

The model: Quark meson model with constituent quarks

We use a linear sigma model with constituent quarks and a non-vanishing mean-field $<\sigma>\neq 0$:

$$\mathcal{L} = \bar{\psi}\left[i\partial\!\!\!/ - g\left(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau}\right)\right]\psi \qquad (1)$$

$$+\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - U\left(\sigma,\vec{\pi}\right) \qquad (2)$$

with two quark flavours $q=(u,d)$. The meson potential with explicit symmetry breaking:

$$U\left(\sigma,\vec{\pi}\right) = \frac{\lambda^2}{4!}\left(\sigma^2+\pi^2-\nu^2\right)^2 - f_\pi m_\pi^2\sigma - U_0 \qquad (3)$$

Model Parameter

λ^2	$= 20$	coupling parameter
g	$\approx 3\ldots6$	quark-sigma coupling
U_0	$= m_\pi^4/\left(4\lambda^2\right) - f_\pi^2 m_\pi^2$	ground state term
f_π	$= 93$ MeV	pion decay constant
m_π	$= 138$ MeV	pion mass
ν^2	$= f_\pi^2 - m_\pi^2/\lambda^2$	field shift term

Mean-field equation of motions for the meson fields in classical approximation:

$$\partial_\mu\partial^\mu\,\sigma + \lambda^2\left(\sigma^2+\vec{\pi}^2-\nu^2\right)\sigma + g\langle\bar{\psi}\psi\rangle - f_\pi m_\pi^2 = 0 \qquad (4)$$

$$\partial_\mu\partial^\mu\,\vec{\pi} + \lambda^2\left(\sigma^2+\vec{\pi}^2-\nu^2\right)\vec{\pi} + g\langle\bar{\psi}\imath\gamma_5\psi\rangle = 0 \qquad (5)$$

with the one-loop scalar and pseudo-scalar quark density, acting as a source for the fields:

$$\langle\bar{\psi}\psi(\mathbf{r})\rangle = g\sigma(\mathbf{r})\int d^3\mathbf{p}\,\frac{f(\mathbf{r},\mathbf{p})+\bar{f}(\mathbf{r},\mathbf{p})}{E(\mathbf{r},\mathbf{p})} \qquad (6)$$

Quarks and anti-quarks are propagated via Vlasov-Equation in test-particle approximation:

$$\left[\partial_t + \frac{p}{E(t,\mathbf{r},\mathbf{p})}\cdot\nabla_\mathbf{r} - \nabla_\mathbf{r}E(t,\mathbf{r},\mathbf{p})\nabla_\mathbf{p}\right] f(t,\mathbf{r},\mathbf{p}) = 0, \qquad (7)$$

with a time and space dependent mass term:

$$E(t,\mathbf{r},\mathbf{p}) = \sqrt{\mathbf{p}(t)^2 + M(\mathbf{r})^2} \qquad (8)$$

$$M(t,\mathbf{r})^2 = g^2\left[\sigma(t,\mathbf{r})^2 + \vec{\pi}(t,\mathbf{r})^2\right] \qquad (9)$$

Thermodynamics

The thermodynamical properties of the linear sigma model can be deduced from the dynamical simulation by initializing the model isotropically with the steady state solution:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla_{\vec{r}}^2\right)\sigma_0(t,\vec{r}) \equiv 0$$

This is done by solving equations 4 and 5 self-consistently:

$$\left[g^2\int \mathrm{d}^3\mathbf{p}\frac{f(t,\mathbf{r},\mathbf{p})+\bar{f}(t,\mathbf{r},\mathbf{p})}{E(t,\mathbf{r},\mathbf{p})} + \lambda^2\left(\sigma_0^2-\nu^2\right) + \right]\sigma_0 - f_\pi m_\pi^2 = 0 \qquad (10)$$

with the thermal Fermi distribution $f(t,\mathbf{r},\mathbf{p})$ for quarks and $\bar{f}(t,\mathbf{r},\mathbf{p})$ for anti-quarks.

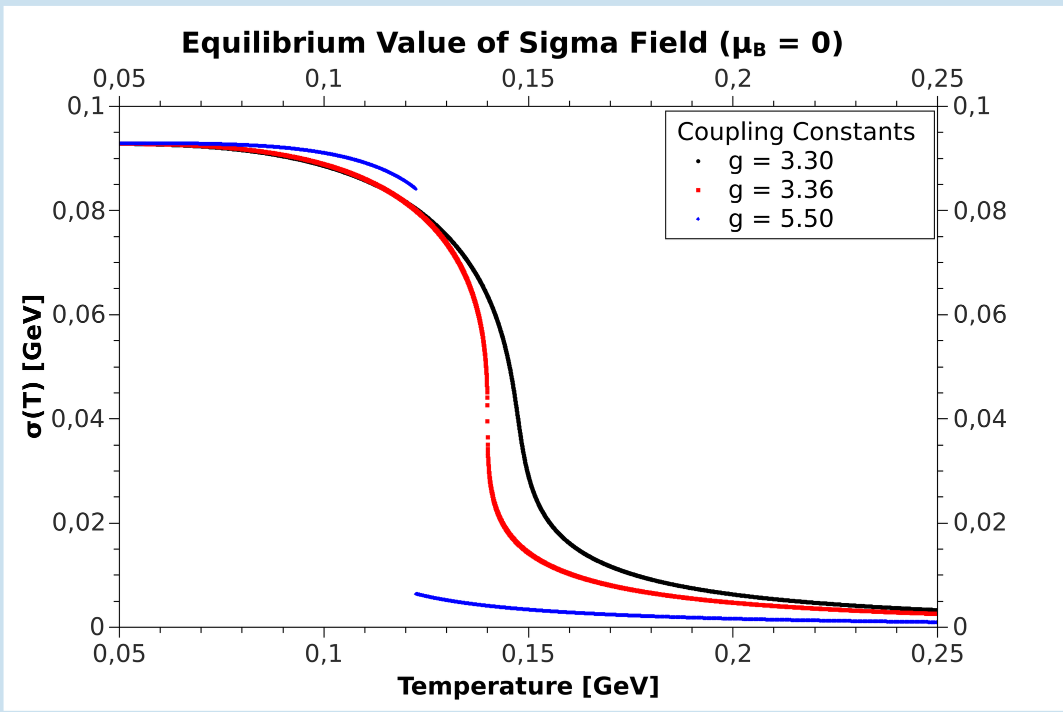

Figure 1: Phase transition of the order parameter $\sigma(T)$. Depending on the coupling strength g, different order of the phase transition can be observed: $g=3.3$ for a cross-over, $g=3.63$ for a second order and $g=5.5$ for a first order transition.

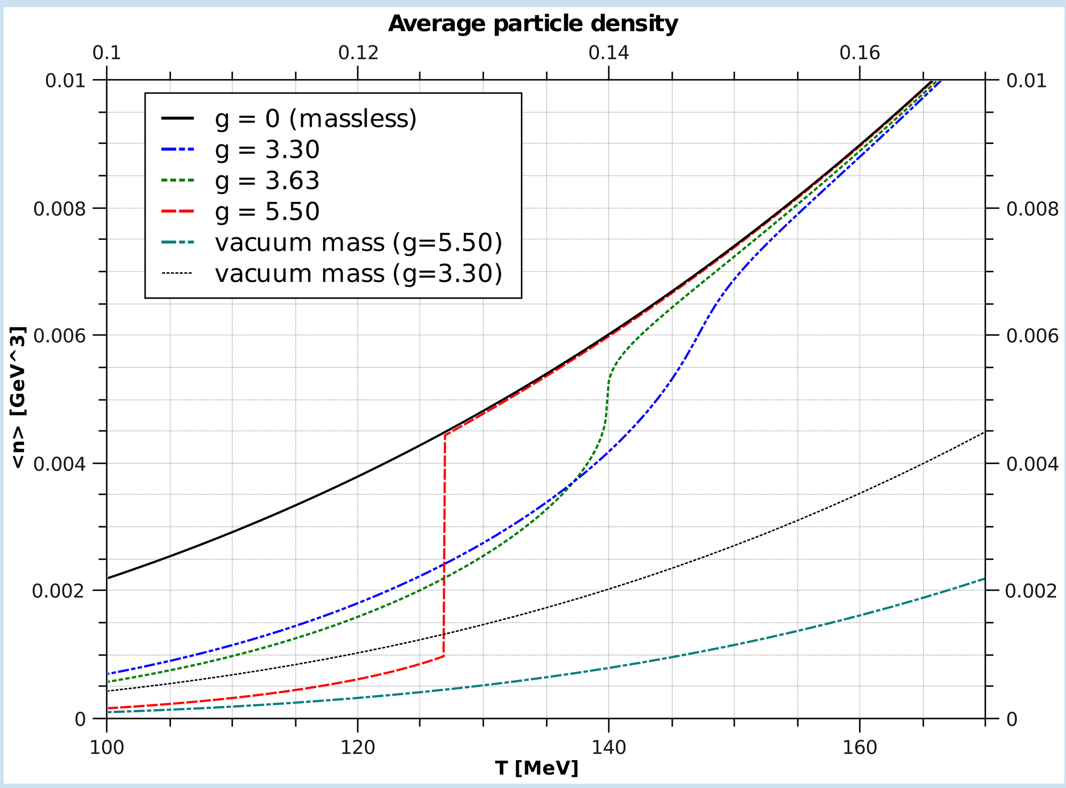

Figure 2: The average particle density is directly coupled to the meson fields and is therefore affected by the phase transition. For a first order phase transition, an instant jump in the temperature dependence is seen.

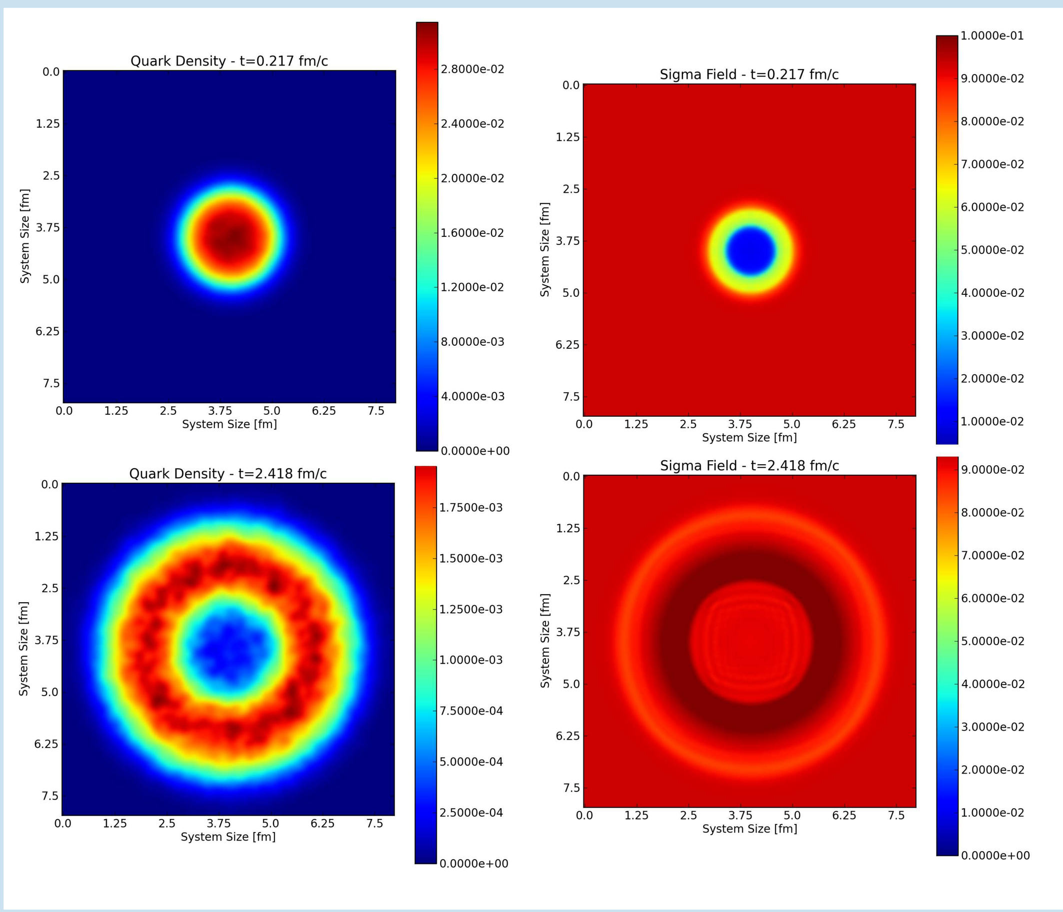

Figure 3: Transport simulation of the expansion of an hot and dense droplet. The droplet is initialized with a Wood-Saxon like thermal distribution. The expansion of the droplet leads to shell-like structures.

Non-Equilibrium Effects

By employing the Vlasov-equation (7), the total particle number in the system is conserved. Particle number conservation is found in many comparable studies, which is a reasonable approximation if chemical processes are slow in comparison to spatial expansion processes. However, a deviation from chemical equilibrium can lead to dramatic effects in the behavior of the phase transition.
Note that calculations for figure 4 are done for a medium with a slow temperature change and figure 5 for a medium with a slow expansion rate. In relativistic expansion scenarios the results can be different qualitatively and quantitatively. If the expansion rate is faster than the chemical equilibration rate, the expansion can drive an initial equilibrated system far from chemical equilibrium. This would have an huge impact on the phase diagram and the phase transition. Additionally, real-time effects will have an effect on the shape and strength of fluctuations. Both are subject of further investigation.

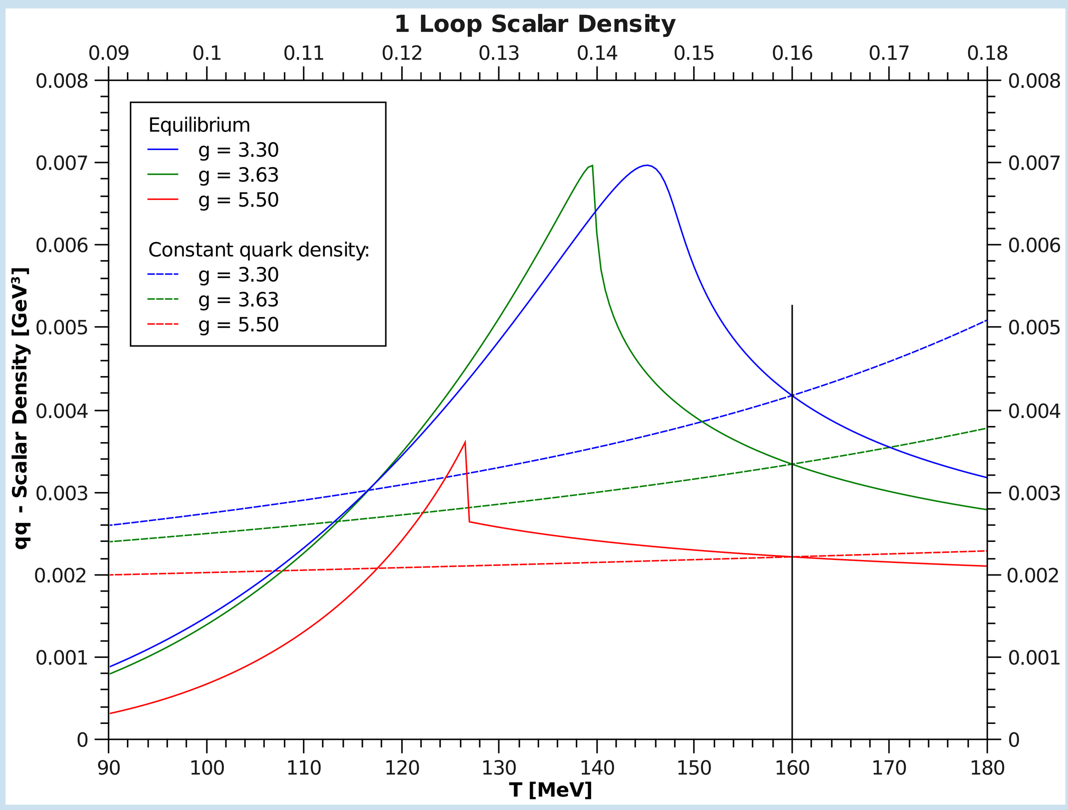

Figure 4: Linear sigma model in the thermal limit. If chemical processes suppressed (conservation of the particle density), the chiral phase transition vanishes.

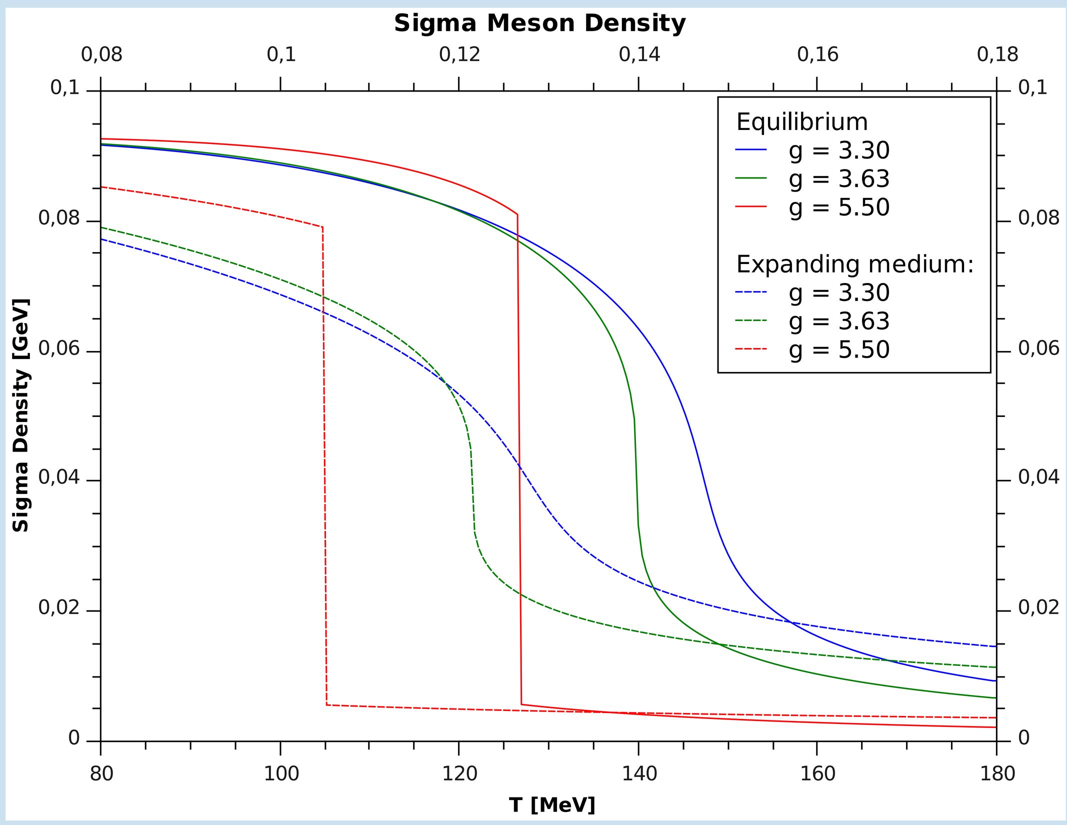

Figure 5: Adiabatic expansion of a thermal medium with suppressed chemical processes. If the temperature and particle density can decrease by medium-expansion, a pseudo-phase transition can be observed. The temperature of this transition is significantly different from the transition temperature of the full-equilibrium medium.

Conclusions and Outlook

We have seen in the last section, that non-equilibrium effects can have a dramatic effect on the phase transition. Chemical processes have a huge impact on the equilibration of the system and can only be neglected in few scenarios. To extend the model towards a more realistic frame, the 2-PI effective action formalism is used to derive self-consistent cross-sections and equations of motions:

$$\Gamma = S + \frac{\imath}{2}\left(\mathrm{Tr}\ln G^{-1} + \mathrm{Tr}G_0^{-1}G\right) - \imath\left(\mathrm{Tr}\ln\Delta^{-1} + \mathrm{Tr}\Delta_0^{-1}\Delta\right) + \Gamma_2$$

Beside the already included one-loop contributions in (4-6), the following contributions will be included:

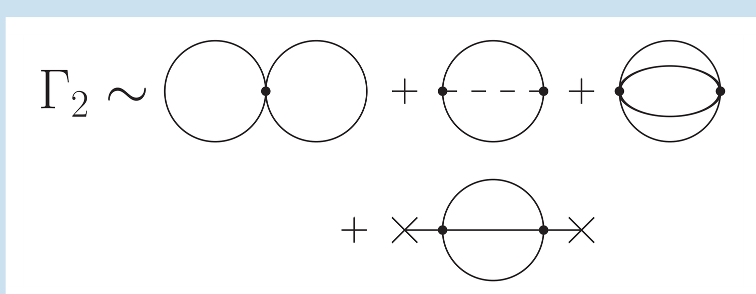

Figure 6: Diagrammatic contributions for Γ_2 in the 2-PI formalism. The diagram in the second line includes Ψ dependent contributions.

Beside numerical calculations, an analytical study of the critical dynamics near the QCD critical point by means of the dynamic renormalization group is in progress. This technique is an excellent tool to investigate critical slowing down near and at the phase transition.

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