Ivan Vitev

Electroweak Boson-tagged Jet Event Asymmetries at the Large Hadron Collider

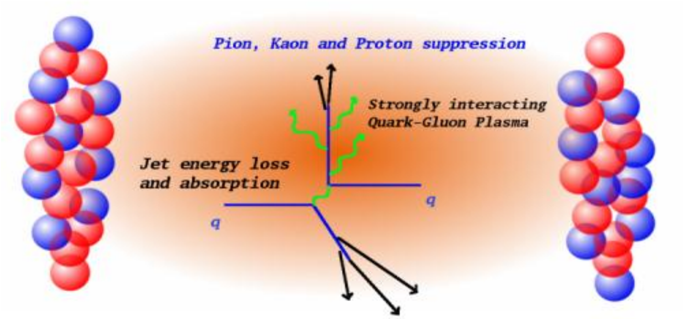
Quark Matter 2012 - Washington , DC

Thanks to my collaborators: Y. He, Z. B. Kang, R.B. Neufeld, G. Ovanesyan, R. Sharma, E. Wang, S. Wicks, H. Xing, B.W. Zhang

Outline of the talk

- Motivation: differential probes of the QGP, recent experimental LHC (and anticipated RHIC) results
- Theoretical underpinnings: medium induced splitting kernels, parton shower dissipation, generalization of jet quenching
- Z^0/γ^*-tagged jets: 2D tagged jet modification patterns, A_J asymmetry results, insensitivity to background fluctuations
- γ-tagged jets: comparisons between RHIC and LHC. 2D 2D tagged jet modification patterns Z_J momentum imbalance distributions.
- Summary and conclusions

Motivation I



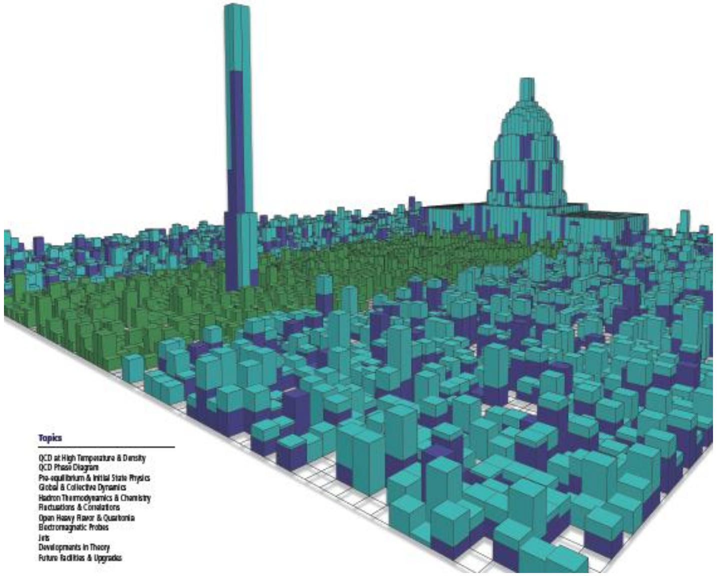
- “Jet quenching”: has been a major thrust in heavy ion physics since the early 1990s

M. Gyulassy et al. (1992)

$$R_{AA}(I_{AA}\ldots) = \frac{\mathrm{Yield}_{AA} / \langle N_{binary} \rangle_{AA}}{\mathrm{Yield}_{pp}} = \frac{1}{\langle N_{binary} \rangle_{AuAu}} \frac{d s_{AuAu} / dp_T dy}{d s_{pp} / dp_T dy}$$

In the past six years: the theory and experimental measurements of reconstructed jets in heavy ion collisions have emerged

- More differential probes of the many-body QCD dynamics at RHIC and LHC
- Complement leading particle measurements with larger discriminating power against theoretical models
- A rich variety of new observables, extended physics reach

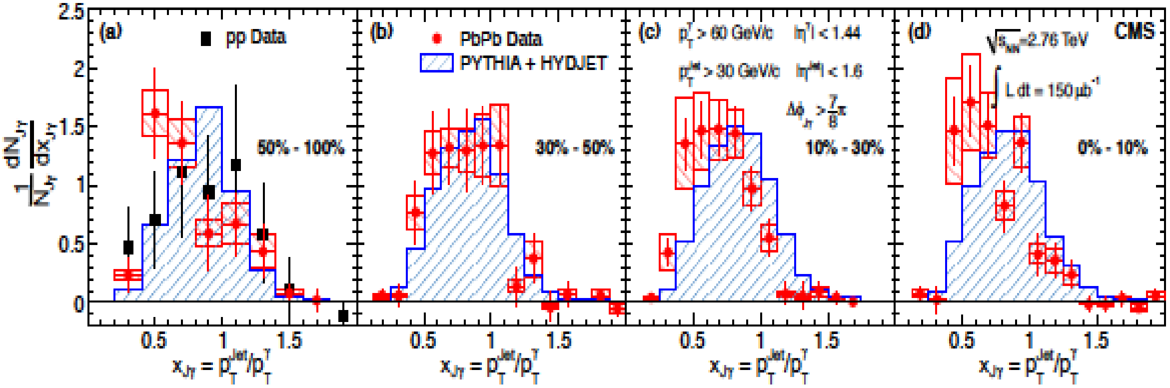


PHENIX, STAR (2008 -), ALICE, ATLAS, CMS (2010 -)

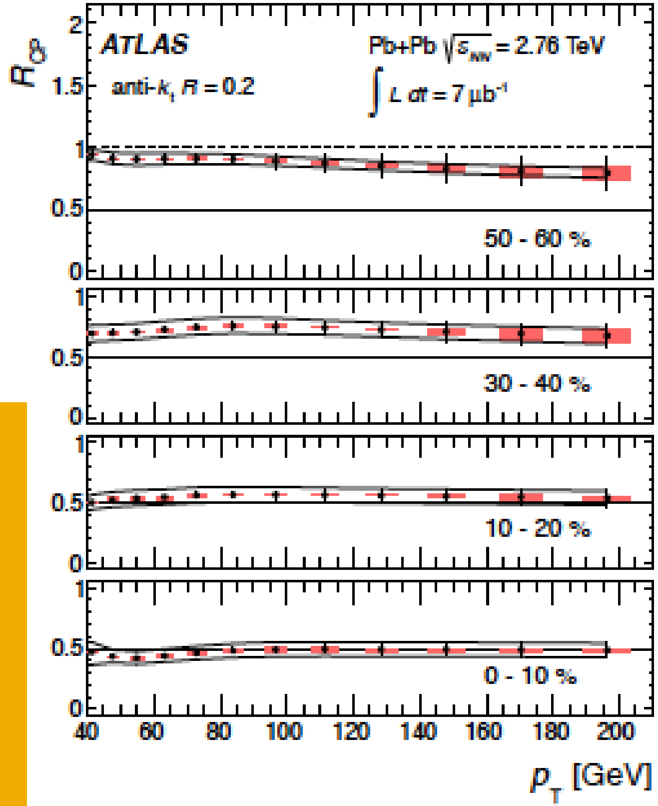
QM organizers (2012)

Motivation II

- Experimental results on the modification of reconstructed jets are now becoming readily available (especially at the LHC)



CMS Collab. (2012)



ATLAS Collab. (2012)

Jets tagged by electroweak bosons (Z^o, $W^\pm$, γ) play a special role

- Tagging provides constraints, on average, on the parton shower energy
- Help elucidate the mean parton shower energy loss
- Photon-tagged jets allow direct comparisons between RHIC and the LHC

Theoretical foundations I

- Combine the NLO pQCD production in the vacuum with the medium-induced parton splitting processes

Exact matrix elements:	FO ✓	PS ✗
Precision:	FO ✓	PS ✗
Hard region description:	FO ✓	PS ✗
Soft region description:	FO ✗	PS ✓

I.V. (2010)

→ medium

↓ vacuum

	LO	NLO	NNLO	…
LO	α_s^2	$\alpha_s^2\alpha_s^{med}$	$\alpha_s^2\alpha_s^{2\ med}$	…
NLO	α_s^3	$\alpha_s^3\alpha_s^{med}$	…	
NNLO	α_s^4	…		
…	…			

NLO tools available: MCFM, JETPHOX, ….

- Use modern effective field theory approaches to to address the in-medium parton shower formation and propagation

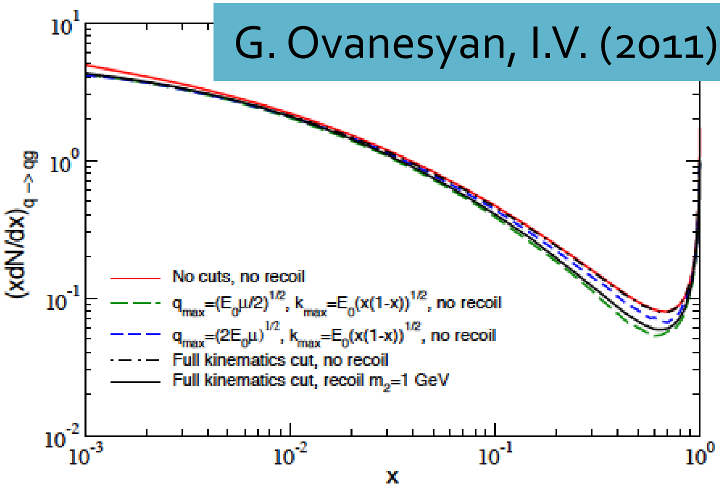
Soft Collinear Effective Theory (SCET) | Q | $p_\perp/Q$ | ψ, A → ξ_n, A_n, A_s

C. Bauer et al. (2001)

Theoretical foundations II

In-medium splitting kernels

G. Ovanesyan, I.V. (2011)



- Evaluated the medium induced splitting kernels beyond the soft gluon approximation (in that approximation ➔ reaction operator approach)

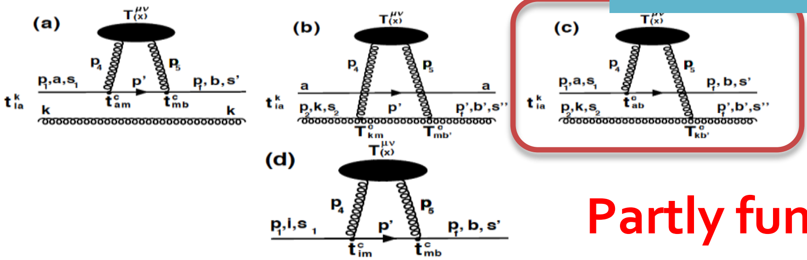
M.Gyulassy, P.Levai, I.V. (2000)

- Proved the gauge invariance of the jet broadening and radiative energy loss results
- Demonstrated the factorization of the final-state radiative corrections form the hard scattering

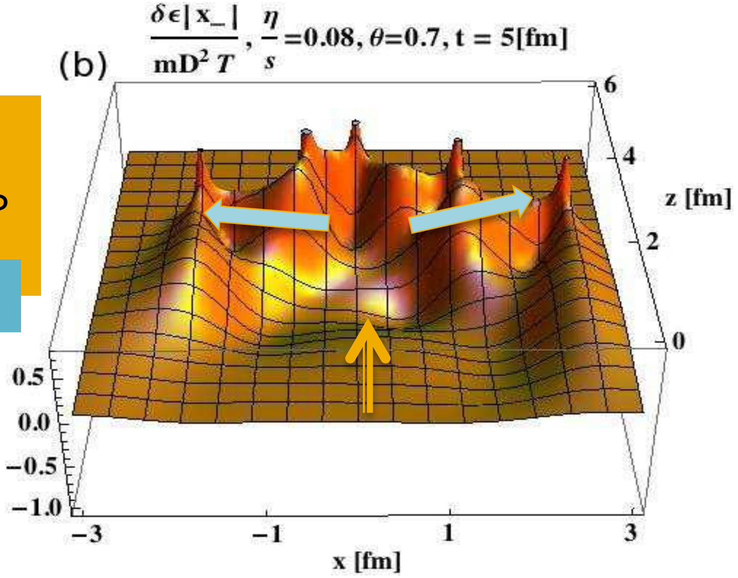
Parton shower energy dissipation in the QGP

- Included important thus far neglected interference diagrams
- Parton showers may dissipate a lot of their energy in the QGP medium

R.B. Neufeld, I.V. (2011)



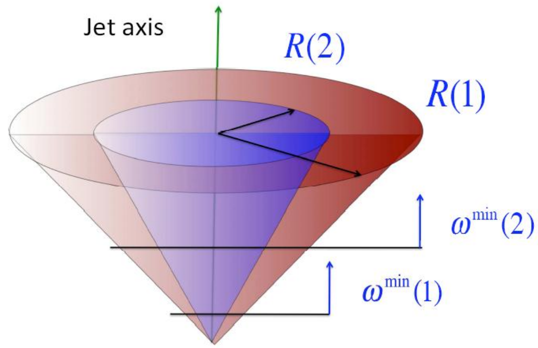
Partly funded by FES



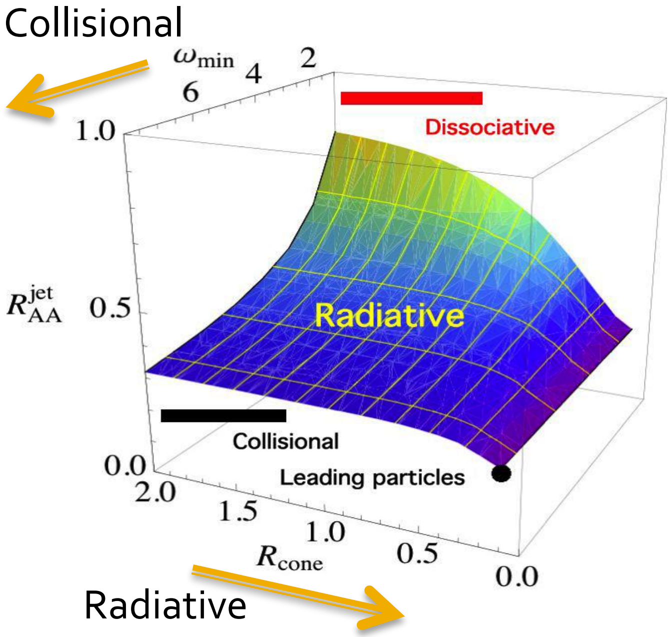
Theoretical foundations III

- Exploiting the jet variables in heavy ion collisions (R)
- Making use of intrajet observables (e.g. ψ)

I.V., S.Wicks, B.W.Zhang (2008)



- Qualitative expectations (how to interpret the experimental results)



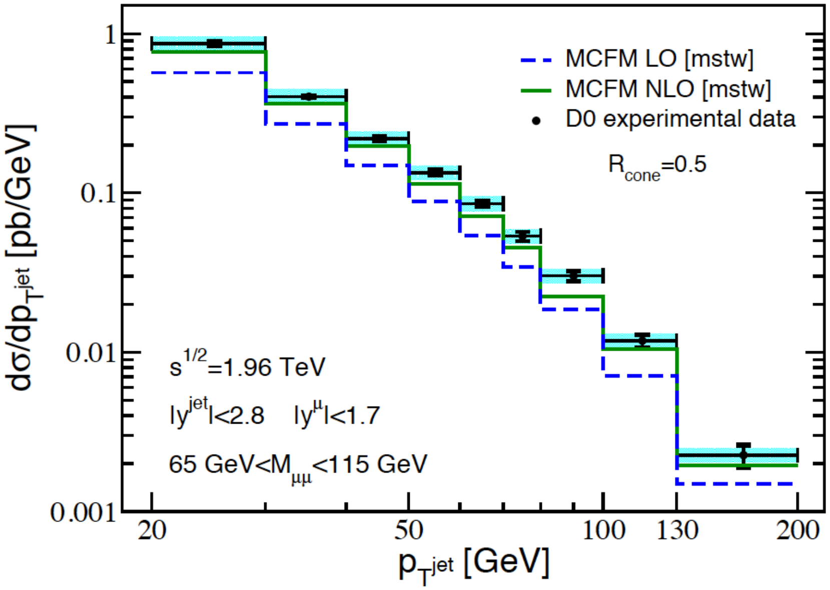
$$R^{\rm jet}_{AA}(E_T;R^{\rm max},\omega^{\rm min}) = \frac{\frac{d\sigma^{AA}(E_T;R^{\rm max},\omega^{\rm min})}{dyd^2E_T}}{\langle N_{\rm bin}\rangle \frac{d\sigma^{pp}(E_T;R^{\rm max},\omega^{\rm min})}{dyd^2E_T}}$$

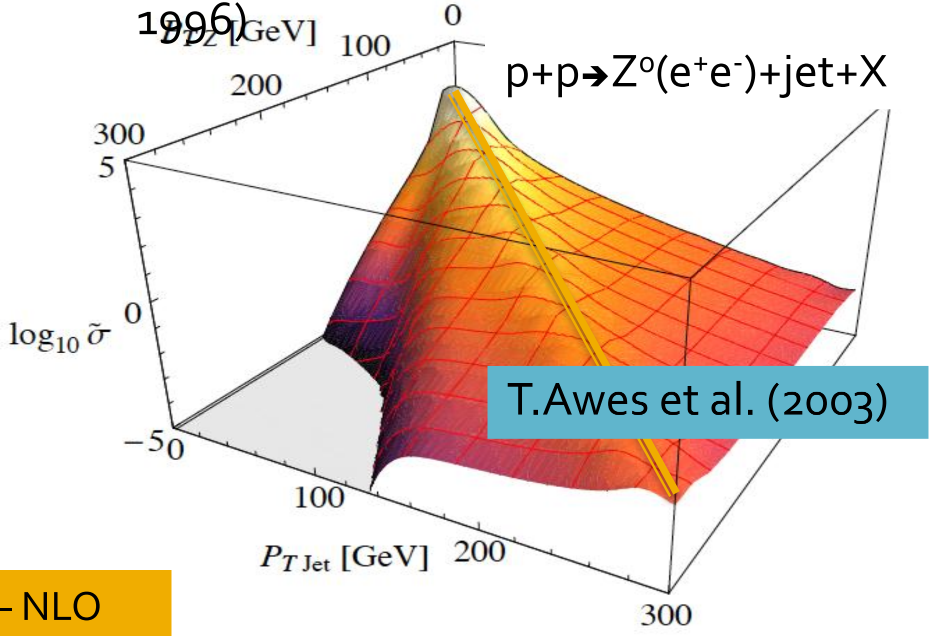
Mechanism	Signature	Status
Dissociative	~ Constant R_{AA}^{jet}=1 ? (Not well understood)	▯ No calculation
Radiative	Continuous variation of R_{AA}^{jet} with R	▯ Incl. jets at RHIC, LHC ▯ Di-jets at the LHC ▯ Z^0- , γ-tagged jets
Collisional	~ Constant R_{AA}^{jet}= $R_{AA}^{particle}$ (Large suppression)	▯ Incl. jets at LHC ▯ Di-jets at the LHC ▯ Z^0- , γ-tagged jets

Z^o-tagged jets I

- MCFM (Monte Carlo for Femtobarn processes)

J. Campbell, R.K. Ellis et al (1992, 1996)





p+p→Z^o(e^+e^-)+jet+X

T.Awes et al. (2003)

		LO	R = 0.2	R = 0.4
p_T^Z = 7-13 GeV	$\langle p_T^{jet}\rangle$	9.29	8.72	9.38
	$\Delta\langle p_T^{jet}\rangle$	2.18	4.45	4.67
p_T^Z = 92.5-112.5	$\langle p_T^{jet}\rangle$	100.79	93.91	96.63
	$\Delta\langle p_T^{jet}\rangle$	6.95	25.19	24.88

- Z^o production, integrating over large p_T range – NLO accuracy
- For the double differential cross section - lowest non-trivial order O($\alpha_s^2 G_F$), O($\alpha_s^2\alpha_{em}$)

R.B.Neufeld, I.V., B.W.Zhang (2011)

Zº-tagged jets II

- Suppression of Zº-tagged jets

$$R_{AA}^{\mathrm{Z-jet}}(p_{T\,Z}, p_{T\,\mathrm{Jet}}; R, \omega_{\min}) = \frac{\frac{d\sigma_{AA}}{dp_{T\,Z}\,dp_{T\,\mathrm{Jet}}}}{\langle N_{\mathrm{bin}}\rangle \frac{d\sigma_{pp}}{dp_{T\,Z}\,dp_{T\,\mathrm{Jet}}}}$$

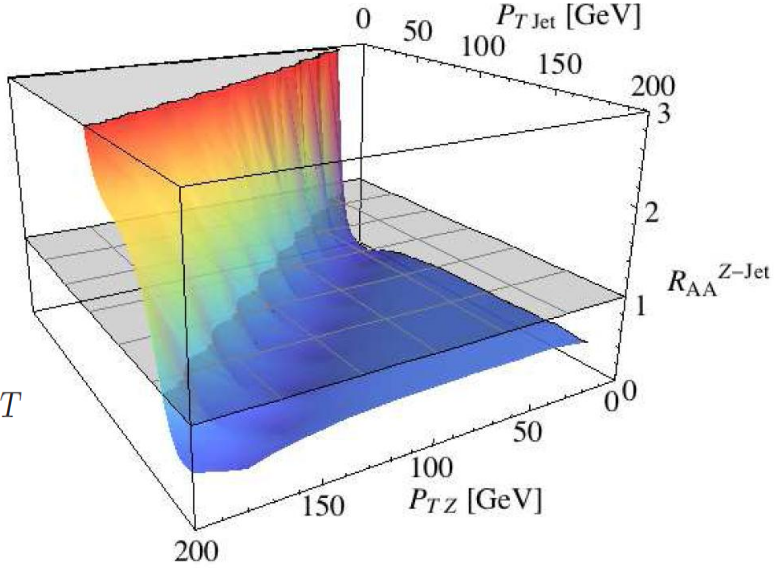
$$\frac{1}{\langle N_{\mathrm{bin}}\rangle}\frac{d\sigma_{AA}}{dp_{T\,Z}\,dp_{T\,\mathrm{Jet}}} = \sum_{q,g}\int_0^1 d\epsilon \frac{P_{q,g}(\epsilon)}{1-(1-f(\omega_{\min},R))\epsilon} \times \frac{d\sigma^{q,g}\left(\frac{p_{T\,\mathrm{Jet}}}{1-(1-f(\omega_{\min},R))\epsilon}\right)}{dp_{T\,Z}\,dp_T} . \quad (3)$$

Superposition of proto-jets of initially higher transverse momentum

$$p_{T\,\mathrm{Jet}} = [1-(1-f(\omega_{\min},R))\epsilon\,]p_T$$

Energy loss fraction (both radiative and collisional) from the point of view of the jet

$$(1-f(\omega_{\min},R))$$



R.B.Neufeld, I.V. (2012)

- Derivative signatures of jet quenching

Broadening and enhancement on the di-jet or tagged-jet asymmetry distributions. Shift of the momentum fraction distributions. All related by Jacobian transformations (carry a subset of the information in $R^{2(\mathrm{tagged})\text{-jet}}_{AA}$)

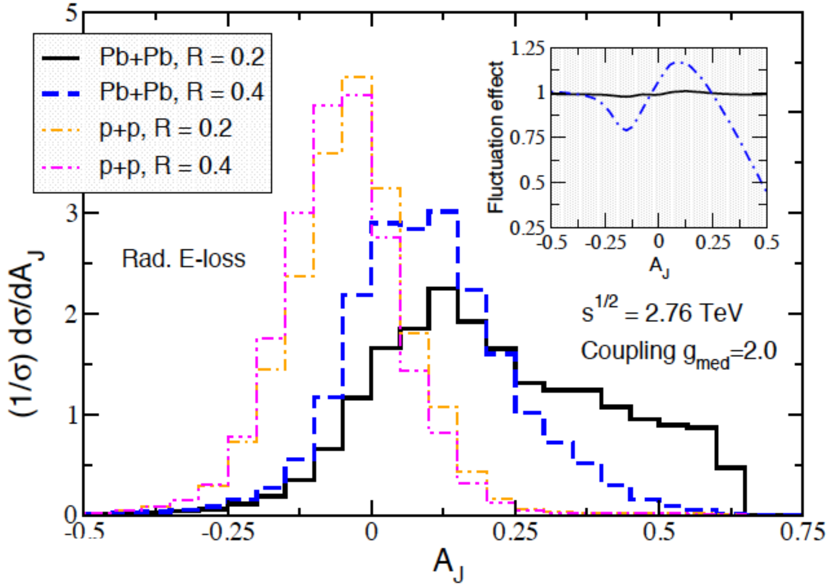
Z^{o}-tagged jets III

- Evaluating the Z^{o}-tagged jet asymmetry distribution $A_J = \frac{p_{T\ Z} - p_{T\ Jet}}{p_{T\ Z} + p_{T\ Jet}}$

$$\frac{d\sigma}{dA_J} = \int_{p_{T\ \mathrm{Jet\ min}}}^{p_{T\ \mathrm{Jet\ max}}} dp_{T\ \mathrm{Jet}} \frac{2p_{T\ \mathrm{Jet}}}{(1-A_J)^2} \frac{d\sigma}{dp_{T\ Z}\,dp_{T\ \mathrm{Jet}}}$$

$p_{T\ Z} \in (80, 100)$ GeV $\quad p_{T\ \mathrm{Jet}} > 20$ GeV

Clear dependence of the asymmetry distribution on the jet cone radius R when we only include radiative energy loss



- Effect of background fluctuations is small

M. Cacciari et al. (2011)

$$\frac{d\sigma^{\mathrm{fluc.}}_{AA}}{dp_{T\ Z}\,dp_{T\ \mathrm{Jet}}} = \int d\delta p_T \frac{d\sigma_{AA}(p_{T\ \mathrm{Jet}} - \delta p_T)}{dp_{T\ Z}\,dp_{T\ \mathrm{Jet}}} \mathcal{N}(\delta p_T; \Delta p_T^2)$$

~ normal distribution

Not significant effect for the Z/gamma tagged jet events

$\Delta p_T(R=0.4)=11\ GeV$

ALICE Collab. (2012)

Z°-tagged jets IV

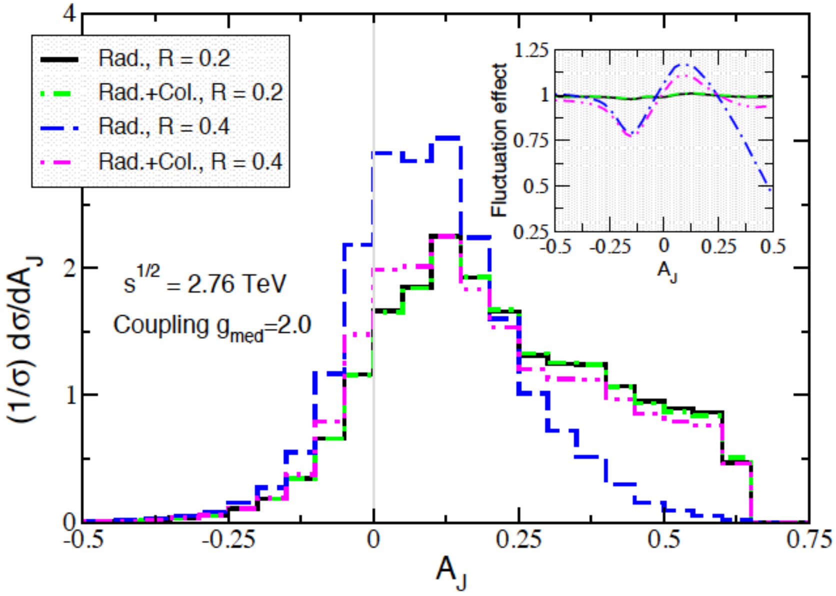
- Collisional dissipation of the parton shower energy in the QGP

When both collisional and radiative energy losses are included the jet radius dependence is very small

Collisional interactions are very effective in dissipating the energy of the in-medium parton shower

System	$\langle A_J \rangle_{\text{no fluct.}}$	$\langle A_J \rangle_{\text{fluct.}}$
p+p with R=0.2	-0.025	-0.025
p+p with R=0.4	-0.040	-0.040
Pb+Pb, rad, R=0.2, g_{med}=1.8	0.190	0.189
Pb+Pb, rad, R=0.2, g_{med}=2.0	0.229	0.228
Pb+Pb, rad, R=0.2, g_{med}=2.2	0.274	0.272
Pb+Pb, rad, R=0.4, g_{med}=2.0	0.115	0.132
Pb+Pb, rad+col, R=0.2, g_{med}=2.0	0.229	0.229
Pb+Pb, rad+col, R=0.4, g_{med}=2.0	0.211	0.214

On this example, on average ~ 35% jet momentum shift



- The jet momentum imbalance

$$Z_J(=p_{T\ jet}/p_{T\ Z}) = \frac{1-A_J}{1+A_J}$$

(Asymmetry enhancement ↔ momentum imbalance down shift)

Photon-tagged jets I

- Photon-tagged jets combine the advantages of Z°/γ* -tagged jets with the ability to probe possibly different plasmas at very different C.M. energies at RHIC and LHC

Prompt Direct Fragmentation

$$d\sigma^{\gamma} = d\sigma^{(D)}(\mu_R,\mu_f) + \sum_{k=q,\bar{q},g} d\sigma_k^{(F)}(\mu_R,\mu_f,\mu_{fr}) \otimes D_{\gamma/k}(\mu_{fr})$$

To reach to early photon production: Isolation radius (and transverse energy /momentum cut)

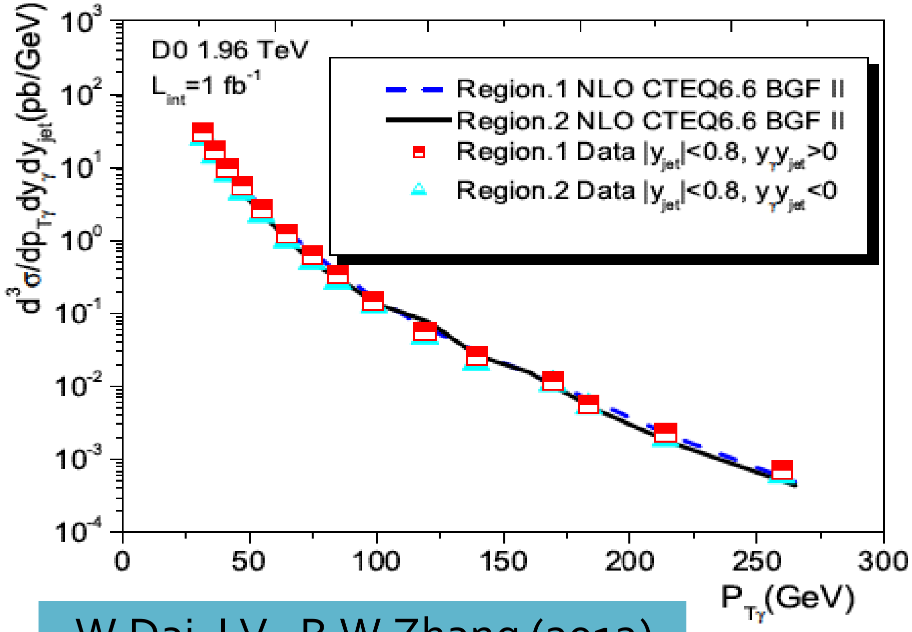
$$R_{\mathrm{iso.}} = \sqrt{(y-y_\gamma)^2+(\phi-\phi_\gamma)^2}$$

The calculation will deal with isolated photons to $O(\alpha_s^2\alpha_{em})$, $O(\alpha_s\alpha_s\alpha_{em})$

NLO calculations (e.g. JETPHOX) gives a satisfactory production picture in p+p

S. Catani et al. (2002)

Z. Belghobsi et al. (2002)



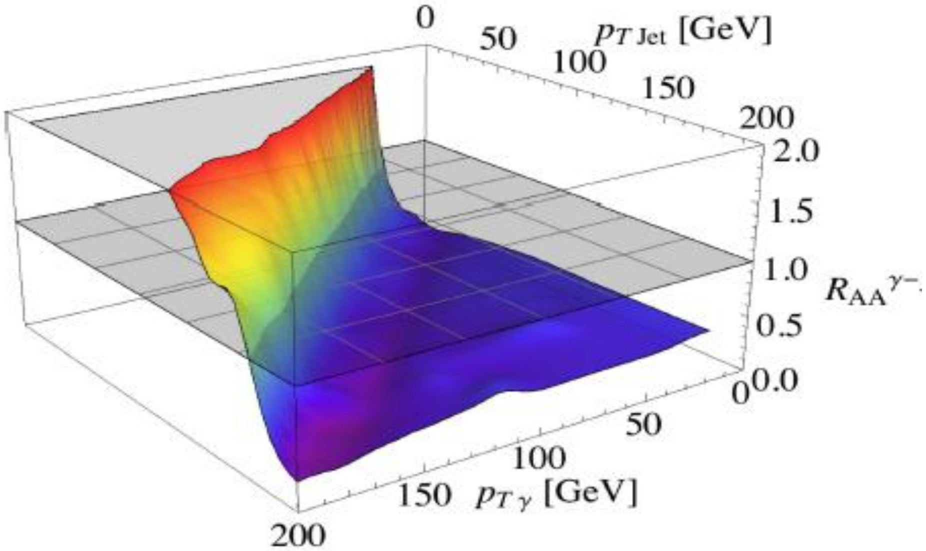
W.Dai, I.V., B.W.Zhang (2012)

Photon-tagged jets II

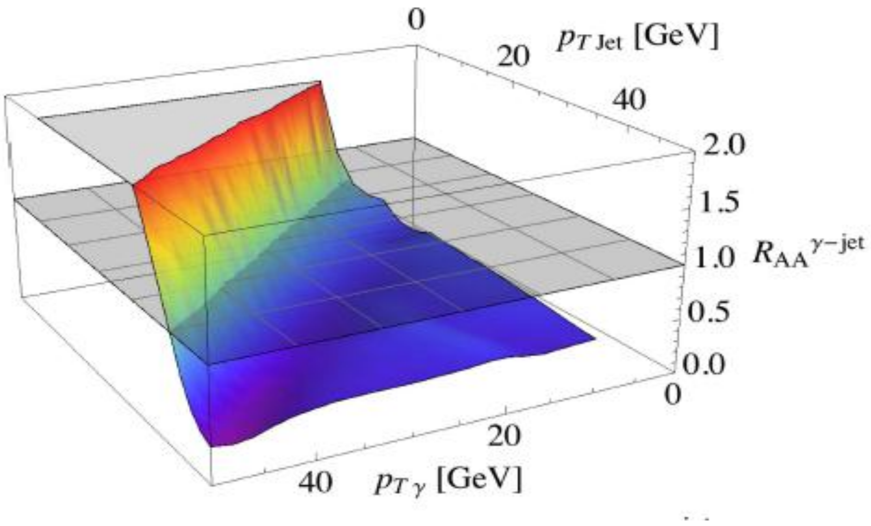
- 2D suppression of gamma-tagged jet R_{AA}

$$R_{AA}^{\gamma-\text{jet}}(p_{T_{\text{jet}}}, p_{T_\gamma}; R) = \frac{\frac{d\sigma^{AA}}{dp_{T_\gamma}\, dp_{T_{\text{jet}}}}}{\langle N_{bin}\rangle \frac{d\sigma^{pp}}{dp_{T_\gamma}\, dp_{T_{\text{jet}}}}} .$$

LHC $s^{1/2}$ = 2.76 TeV



RHIC $s^{1/2}$ = 200 GeV



- Similar pattern of suppression of gamma-tagged jets with subtle but important differences
- Somewhat larger suppression at the LHC along the main diagonal reflective of the larger QGP density/T/… At RHIC, at high p_T (edge of kinematic phase space) amplified CNM and QGP effects

Photon-tagged jets III

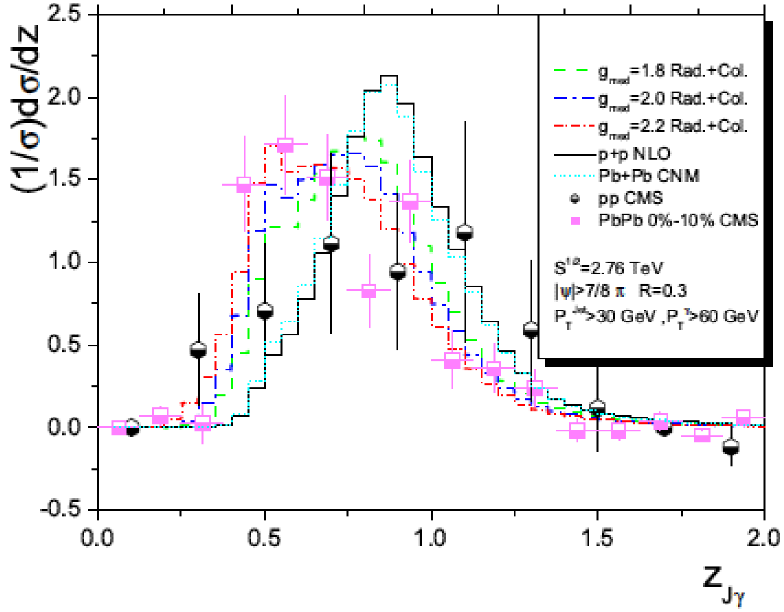
- Evaluating the γ-Jet momentum imbalance distribution.

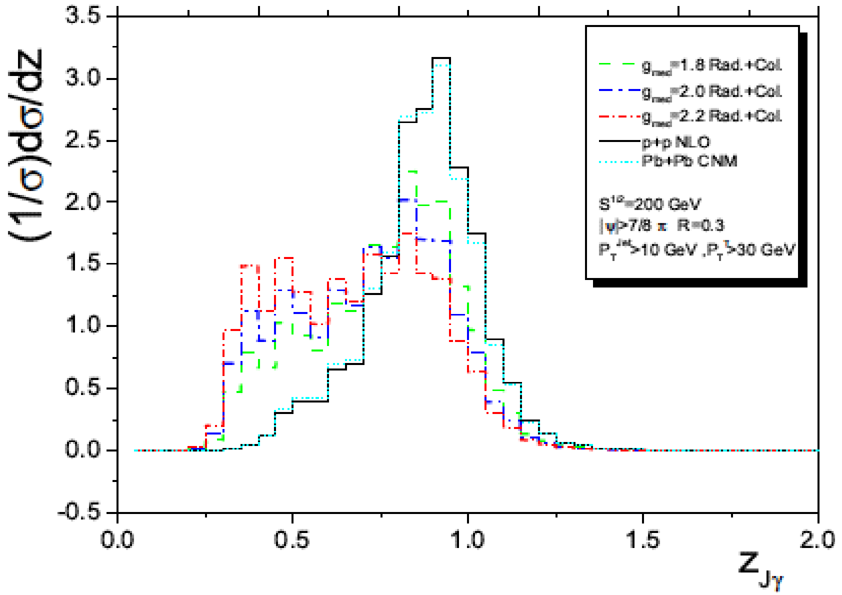
Imbalance variable

W.Dai, I.V., B.W.Zhang (2012)

$$Z_J = \frac{p_{T\ jet}}{p_{T\ g}}$$

$$\frac{d\sigma}{dz_{J\gamma}} = \int_{p_{T_{\rm jet}}^{min}}^{p_{T_{\rm jet}}^{max}} dp_{T_{\rm jet}} \frac{p_{T_{\rm jet}}}{z_{J\gamma}^2} \frac{d\sigma[z_{J\gamma}, p_{T_\gamma}(z_{J\gamma}, p_{T_{\rm jet}})]}{dp_{T_\gamma} dp_{T_{\rm jet}}}$$





- Theoretical simulations with jet-medium couplings that predict the inclusive jet suppression g≈2 can describe quantitatively the modification of event asymmetry distributions

Photon-tagged jets IV

- Kinematic cuts play a role

- **If we use the ATLAS cuts**

γ energy: 60-90 GeV, Jet $p_T > 25$ GeV

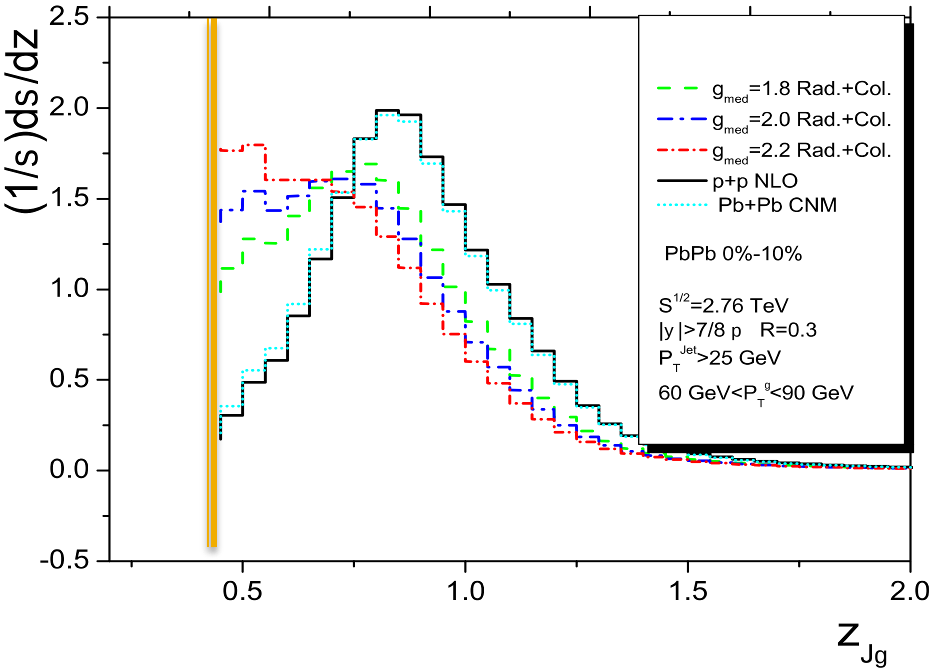
$Z > 25/60 = 0.42$ 10% central

W.Dai, I.V., B.W.Zhang private communication with ATLAS (2012)

- Quantifying the mean momentum imbalance shifts

$$\langle z_{J\gamma}\rangle = \int dz_{J\gamma} z_{J\gamma} \frac{1}{\sigma}\frac{d\sigma}{dz_{J\gamma}}$$

- RHIC can shows somewhat larger broadening and larger shifts reflective of the LHC even with the more generous cuts.



System	$\langle z_{J\gamma}\rangle$LHC	$\langle z_{J\gamma}\rangle$RHIC
p+p	0.94	0.90
A+A, CNM	0.94	0.89
A+A, $g_{med} = 1.8$,Rad.+Col	0.84	0.78
A+A, $g_{med} = 2.0$,Rad.+Col	0.80	0.74
A+A, $g_{med} = 2.2$,Rad.+Col	0.71	0.70

Summary of references for the presented work

Subject	ArXiv	Journal
The original paper on theory of jets in A+A, cross sections and shapes (LO)	arXiv:0810.2807 [hep-ph]	JHEP 0811 (2008) 093
NLO calculation of inclusive jets at RHIC, separating IS, FS effects	arXiv:0910.1090 [hep-ph]	Phys.Rev.Lett. 104 (2010) 132001
NLO calculation of Z^0 tagged jets, inclusive Z^0 at the LHC	arXiv:1006.2389 [hep-ph]	Phys.Rev. C83 (2011) 034902
NLO calculation of inclusive jets and $O(\alpha_s^3)$ di-jets at the LHC, di-jet asymmetry	arXiv:1105.2566 [hep-ph]	Phys.Lett. B713 (2012) 224
SCET theory of jet propagation in matter, gauge invariance, factorization, large x	arXiv:1103.1074 [hep-ph]	JHEP 1106 (2011) 080
Parton showers a sources of energy momentum deposition in the QGP	arXiv:1105.2067 [hep-ph]	Phys.Rev. C86 (2012) 024905
Medium-induced splitting kernels form SCET	arXiv:1109.5619 [hep-ph]	Phys.Lett. B706 (2012) 371
Z^0 - tagged jet modification and event asymmetries to $O(\alpha_s^2 G_F)$ LHC	arXiv:1202.5556 [hep-ph]	Phys.Rev.Lett. 108 (2012) 242001
γ - tagged jet modification, momentum imbalance distributions and shifts to $O(\alpha_s^2\alpha_{em})$ RHIC, LHC	arXiv:1207.5177 [hep-ph]	-

Conclusions

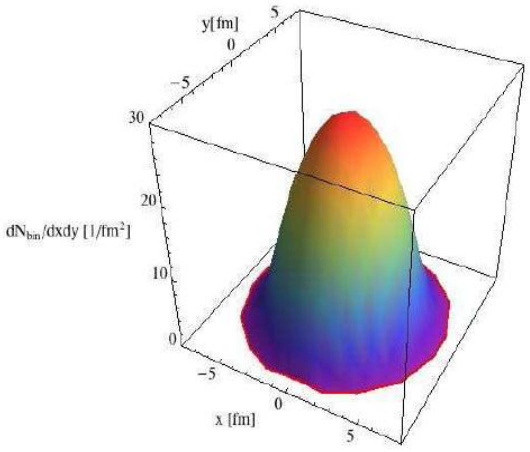
- Electroweak tagged jets open a new channel for QGP tomography at RHIC and the LHC.
- Theoretically attractive, NLO codes in p+p collisions available. We can comparable to inclusive jets, varying the tagging particles and p_T ranges access "q", "g" jets. "Tune" jets to probe "different" QGPs at different C.M. energies
- Readily experimentally accessible, can provide average constraints on the parton shower energy. Allow different characterizations of the event: asymmetry distributions, momentum imbalance distributions
- We have calculated the 2D Z^0/γ-tagged jet distributions to $O(\alpha_s^2 G_F)$ $O(\alpha_s^2 \alpha_{em})$ and combined with radiative and collisional processes in the medium
- Investigated the sensitivity of the tagged-jet asymmetry and momentum imbalance distributions. Data is becoming available.
- A range of jet-medium couplings g = 2 (10%) used to predict the inclusive jet suppression quantitatively describes the imbalance modification measured by CMS. Results "produced" for ATLAS
- By investigation these new channels in detail (R, p_T cuts) we can gain insight in the relative contribution of radiative and collisional energy loss mechanism, quark vs gluon initiated jets

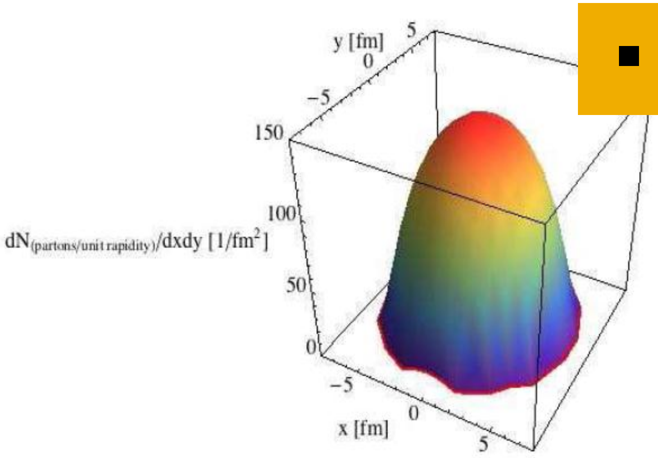
Inclusive jet cross sections in A+A reactions

- Jet cross sections with cold nuclear matter and final-state parton energy loss effect are calculated for different R

$$\frac{\sigma^{AA}(R,\omega^{\min})}{d^2E_T dy} = \int_{\epsilon=0}^{1} d\epsilon \sum_{q,g} P_{q,g}(\epsilon) \frac{1}{(1-(1-f_{q,g})\cdot\epsilon)^2} \frac{\sigma^{NN}_{q,g}(R,\omega^{\min})}{d^2E'_T dy}$$

$$|J_i(\epsilon_i)| = 1/\left(1-[1-f(R_i,p^{\min}_{T\,i})_{q,g}]\epsilon_i\right)$$





I. Vitev et al (2008)

- Calculate in real time

Fraction of the energy redistributed inside the jet

$$f(R_i,p^{\min}_{T\,i})_{q,g} = \frac{\int_0^{R_i} dr \int_{p^{\min}_{T\,i}}^{E_{T\,i}} d\omega \frac{dI^{\text{rad}}_{q,g}(i)}{d\omega dr}}{\int_0^{R\,i^\infty} dr \int_0^{E_{T\,i}} d\omega \frac{dI^{\text{rad}}_{q,g}(i)}{d\omega dr}}$$

The probability to lose energy due to multiple gluon emission

$$\int_0^1 P_{q,g}(\epsilon_i)d\epsilon_i = 1\,, \quad \int_0^1 \epsilon_i P_{q,g}(\epsilon_i)d\epsilon_i = \frac{\Delta E_{q,g\,i}}{E_i}$$

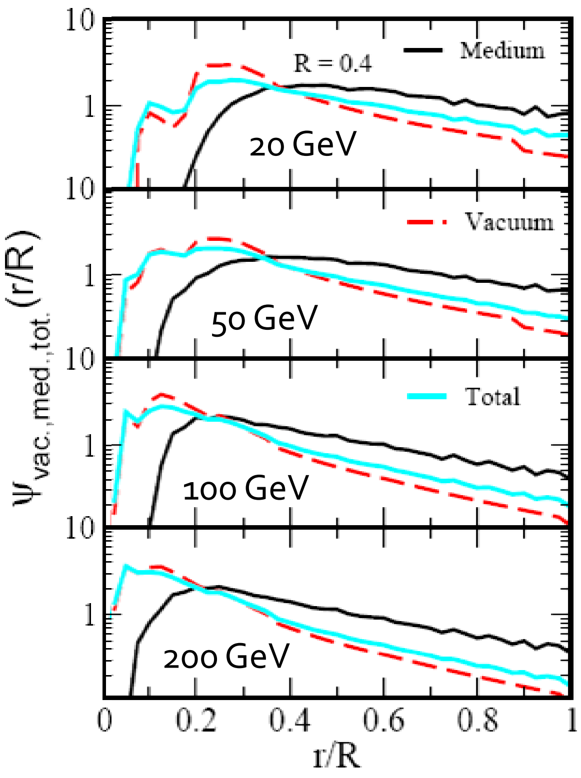
- Calculate

$$R^{\text{jet}}_{AA}(E_T;R,p_T^{\min}) = \frac{\frac{d\sigma^{AA}(E_T;R,p_T^{\min})}{dyd^2E_T}}{\langle N_{\text{bin}}\rangle \frac{d\sigma^{pp}(E_T;R,p_T^{\min})}{dyd^2E_T}}$$

QGP – modified jet shapes

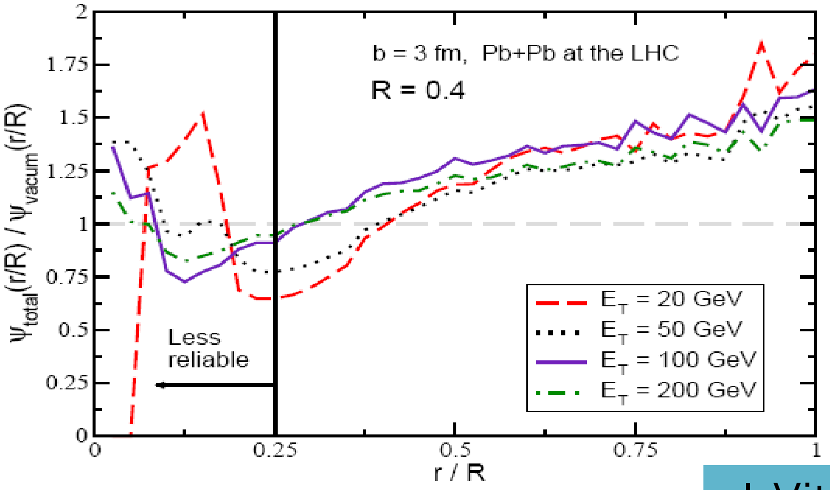
$$\Psi_{\rm int}(r;R) = \frac{\sum_i (E_T)_i \Theta(r-(R_{\rm jet})_i)}{\sum_i (E_T)_i \Theta(R-(R_{\rm jet})_i)}$$

$$\psi(r;R) = \frac{d\Psi_{\rm int}(r;R)}{dr}$$



- Surprisingly, there is no big difference between the jet shape in vacuum and the total jet shape in the medium
- Take a ratio of the differential jet shapes

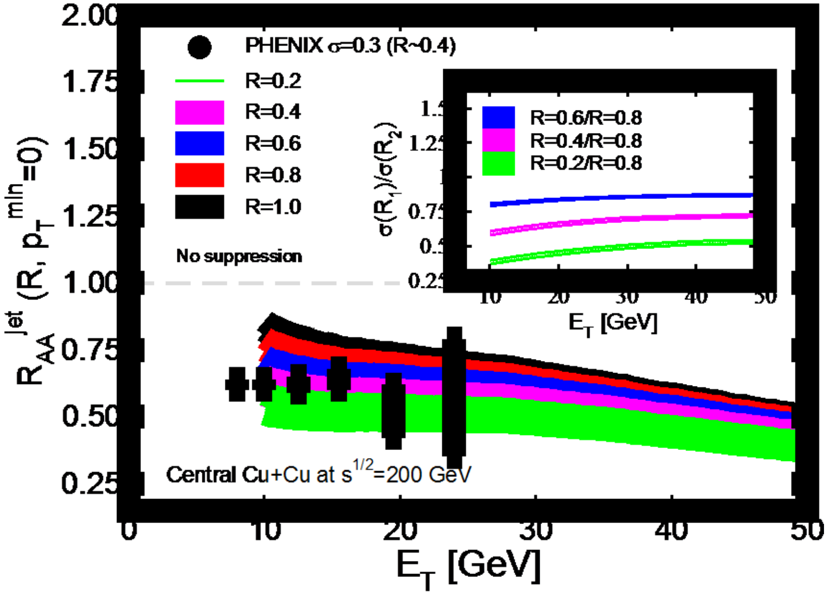
R=0.4	Vacuum	Complete E-loss	Realistic Case
$\langle r/R\rangle$, E_T=20GeV	0.41	0.57	0.45
$\langle r/R\rangle$, E_T=50GeV	0.35	0.53	0.38
$\langle r/R\rangle$, E_T=100GeV	0.28	0.42	0.32
$\langle r/R\rangle$, E_T=200GeV	0.25	0.42	0.28



I. Vitev et al. (2008)

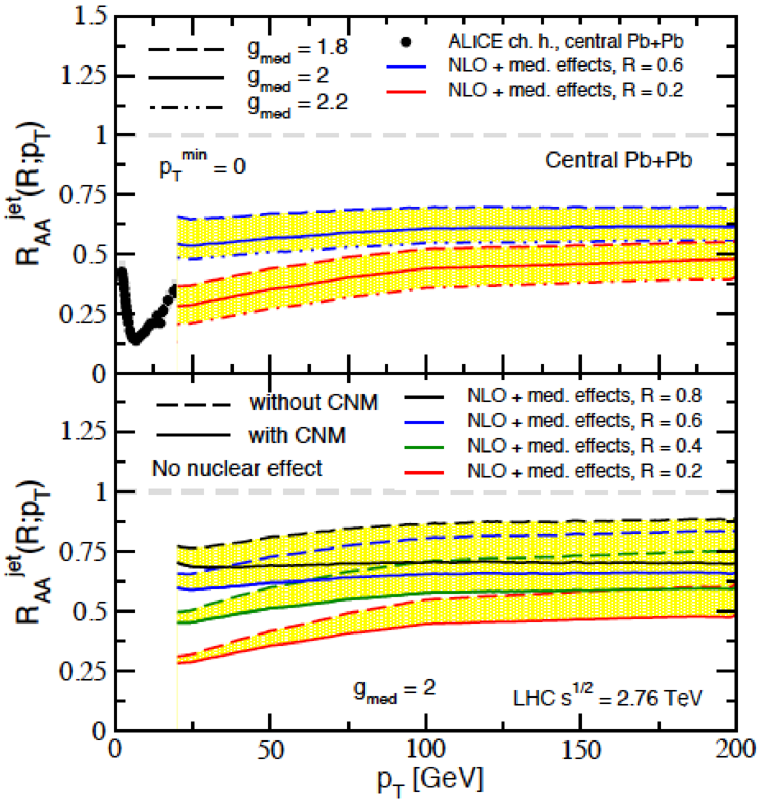
Jet cross sections in A+A reactions at RHIC and LHC

- Jet R_{AA} with cold nuclear matter and final-state parton energy loss effect are calculated for different R



I. Vitev et al (2009)

Y. Lai (2009)



R_{AA} – *CNM effects, QGP quenching and R dependence in p+p*

σ(R_1)/σ(R_2) in A+A – *QGP quenching and R dependence in p+p*

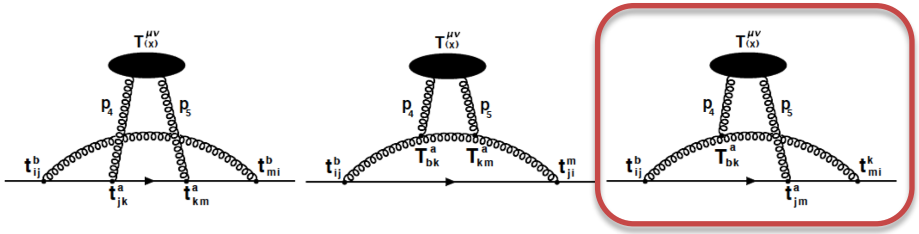
Y. He et al. (2011)

K. Amadot et al. (2011)

III. Parton showers as sources of energy deposition in the QGP

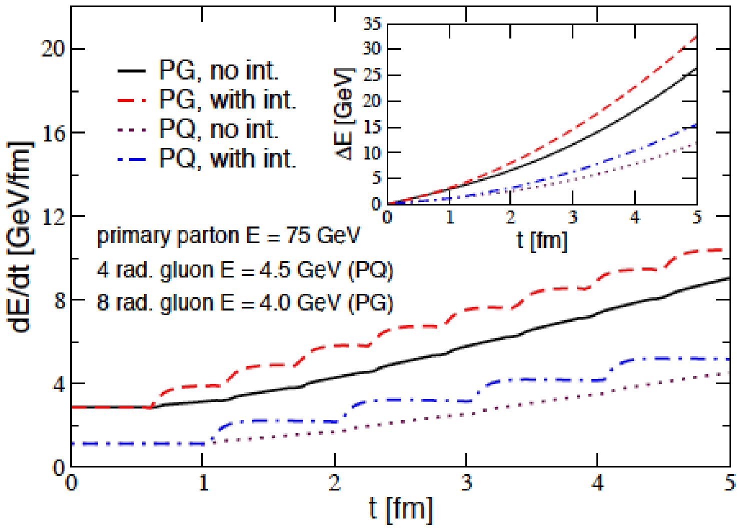
- The first theory calculation to describe a splitting parton system as a source term, including quantum color interference effects
- Think of it schematically as the energy transferred to the QGP through collisional interactions at scales ~ T, gT, …

R.B. Neufeld et al. (2011)



- Calculated diagrammatically from the divergence of the energy-momentum tensor (EMT)
- Simple intuitive interpretation of the result

$$\begin{aligned}\partial_\mu T^{\mu\nu} &= C_p J^\nu_a(x,u_1,u_1) + C_A J^\nu(x,u_2,u_2) \\ &\quad -\frac{C_A}{2}\left[J^\nu(x,u_1,u_2) + J^\nu(x,u_2,u_1)\right]\end{aligned}$$



- 10-20 GeV from the shower energy can be transmitted to the QGP
- See poster by Bryon Neufeld

III. The ambiguity of jet/background separation

- There is no first-principles understanding of heavy ion dynamics at all scales and consequently jet/medium separation

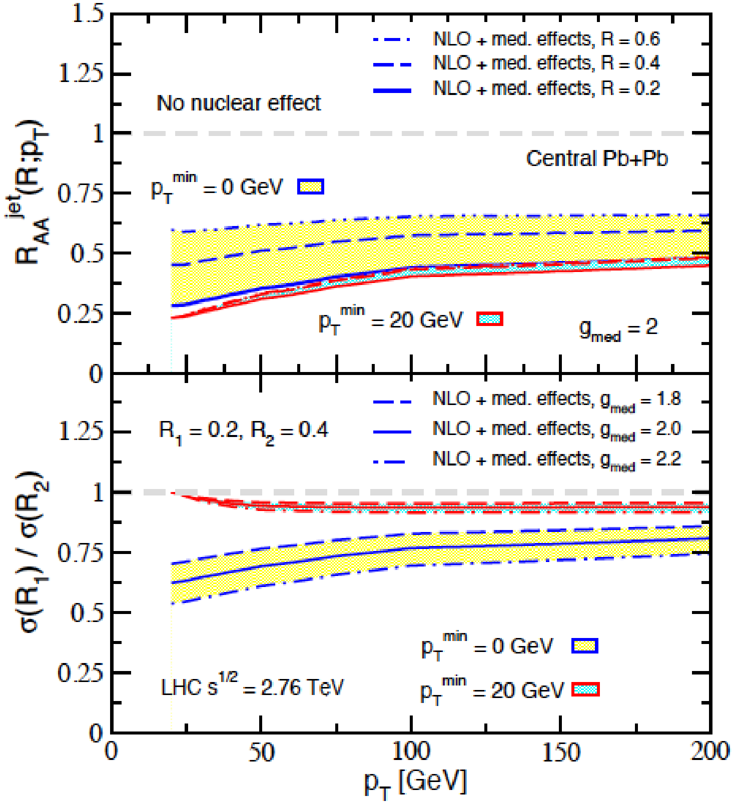
- Background fluctuations may affect jet observabled

M. Cacciari et al. (2011)

- Part of the jet energy may be misinterpreted as background
- It may also diffuse outside R through collisional processes

In our approach we can simulate these scenarios with the cut p_T^{min}

Can easily wipe out the R dependence of jet observables (also for di-jets)



Constrain NP corrections in p+p

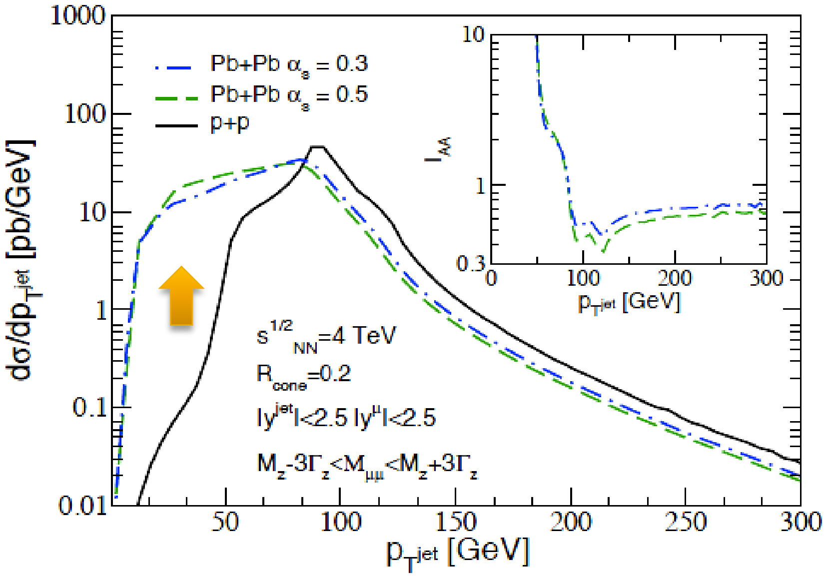
$$\frac{d\sigma^{\text{hadron}}}{dE_{T1}\cdots dE_{Tn}} = \frac{d\sigma^{\text{parton}}}{dE_{T1}\cdots dE_{Tn}} \prod_{i=1}^{n} f_{\text{NP}}(E_{T\,i}, R_i)$$

Y. He et al. (2011)

Quenching of Z^o/γ^*- tagged jets at the LHC, inclusive Z^o

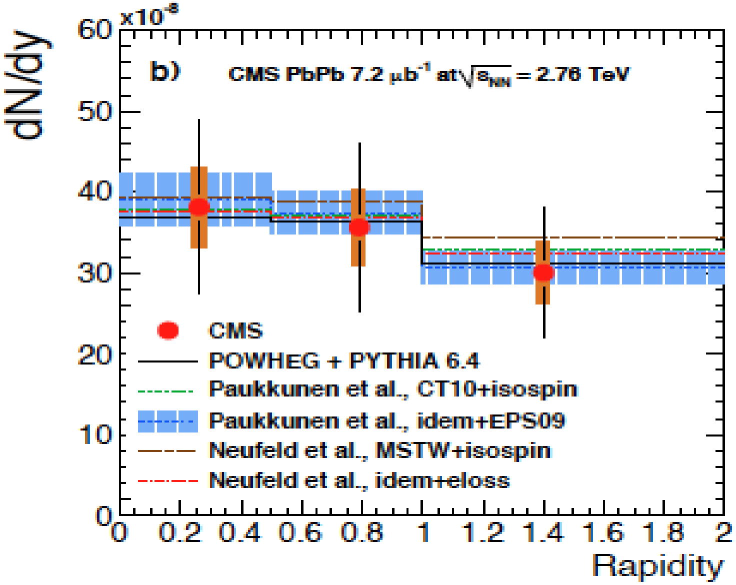
- Quenched Z^o-tagged jet cross section

Strong redistribution of the energy and enhanced I_{AA} below the trigger p_T



- Associated with the part of phase space of quickly increasing with p_T cross section

- Inclusive Z^o production has also been evaluated



R.B. Neufeld et al. (2010)

S.Chatrchyan et al. (2011)

Isospin +3%, CNM energy loss -6%

Soft Collinear Effective Theory

- Galuber gluons (transverse to the jet direction)

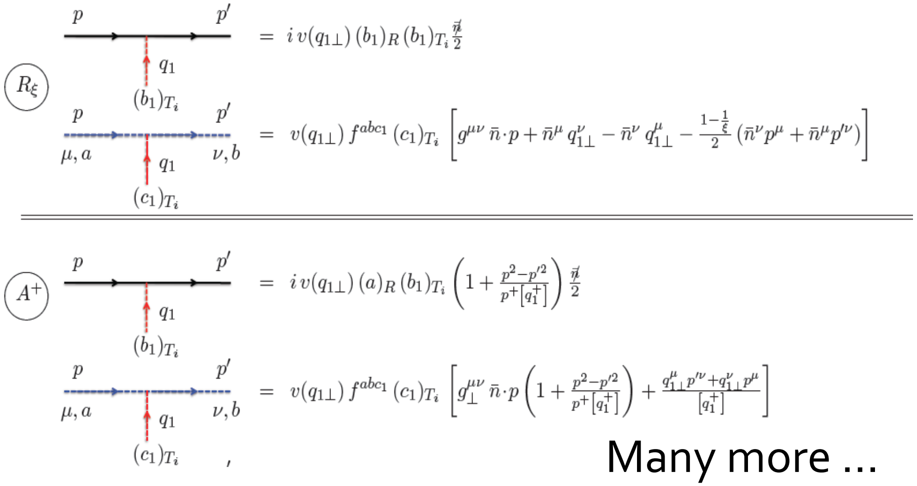
A. Majumder et al. (2009)

G. Ovanesyan et al. (2011)

$$\mathcal{L}_{\mathrm{SCET_G}}(\xi_n, A_n, A_G) = \mathcal{L}_{\mathrm{SCET}}(\xi_n, A_n) + \mathcal{L}_{\mathrm{G}}(\xi_n, A_n, A_G)$$

$$\mathcal{L}_{\mathrm{G}}(\xi_n, A_n, \eta) = \sum_{p,p',q} e^{-i(p-p'+q)x} \left(\bar{\xi}_{n,p'} \Gamma^{\mu,a}_{\mathrm{qqA_G}} \frac{\bar{\not{n}}}{2} \xi_{n,p} - i\Gamma^{\mu\nu\lambda,abc}_{\mathrm{ggA_G}} \left(A^c_{n,p'}\right)_\lambda \left(A^b_{n,p}\right)_\nu \right) \bar{\eta}\, \Gamma^{\delta,a}_{\mathrm{s}}\, \eta\, \Delta_{\mu\delta}(q)$$

- Complete Feynman rules in the soft, collinear and hybrid gauges



$$= i\, v(q_{1\perp})\, (b_1)_R\, (b_1)_{T_i} \frac{\bar{\not{n}}}{2}$$

$$= v(q_{1\perp})\, f^{abc_1}\, (c_1)_{T_i} \left[g^{\mu\nu}\, \bar{n}\cdot p + \bar{n}^\mu q^\nu_{1\perp} - \bar{n}^\nu q^\mu_{1\perp} - \frac{1-\frac{1}{\xi}}{2} \left(\bar{n}^\nu p^\mu + \bar{n}^\mu p'^\nu\right) \right]$$

$$= i\, v(q_{1\perp})\, (a)_R\, (b_1)_{T_i} \left(1 + \frac{p^2 - p'^2}{p^+[q_1^+]}\right) \frac{\bar{\not{n}}}{2}$$

$$= v(q_{1\perp})\, f^{abc_1}\, (c_1)_{T_i} \left[g^{\mu\nu}_\perp\, \bar{n}\cdot p \left(1 + \frac{p^2 - p'^2}{p^+[q_1^+]}\right) + \frac{q^\mu_{1\perp} p'^\nu + q^\nu_{1\perp} p^\mu}{[q_1^+]} \right] ,$$

Many more …

- First proof of gauge invariance of the broadening/radiative energy loss results

- Showed factorization of the final-state process-dependent radiative corrections and the hard scattering cross section, calculated large-x

$$\int \frac{d^2 q_\perp}{(2\pi)^2} |\tilde{v}(q_\perp)|^2\, \mathrm{Tr}\left(\frac{\not{n}}{2} \bar{n}\cdot p\, J \bar{J}\, \frac{g^2}{d_R d_T} \left[\rho^{\mathrm{SB}} + \rho^{\mathrm{DB}}\right] \right) \qquad \rho = \sum_{i=1}^{2} c_i \left(F_i\, \mathbb{I} + G_i\, \Sigma^3\right)$$

- See talk by G. Ovanesyan