Scaling of v_2 at low and high p_T

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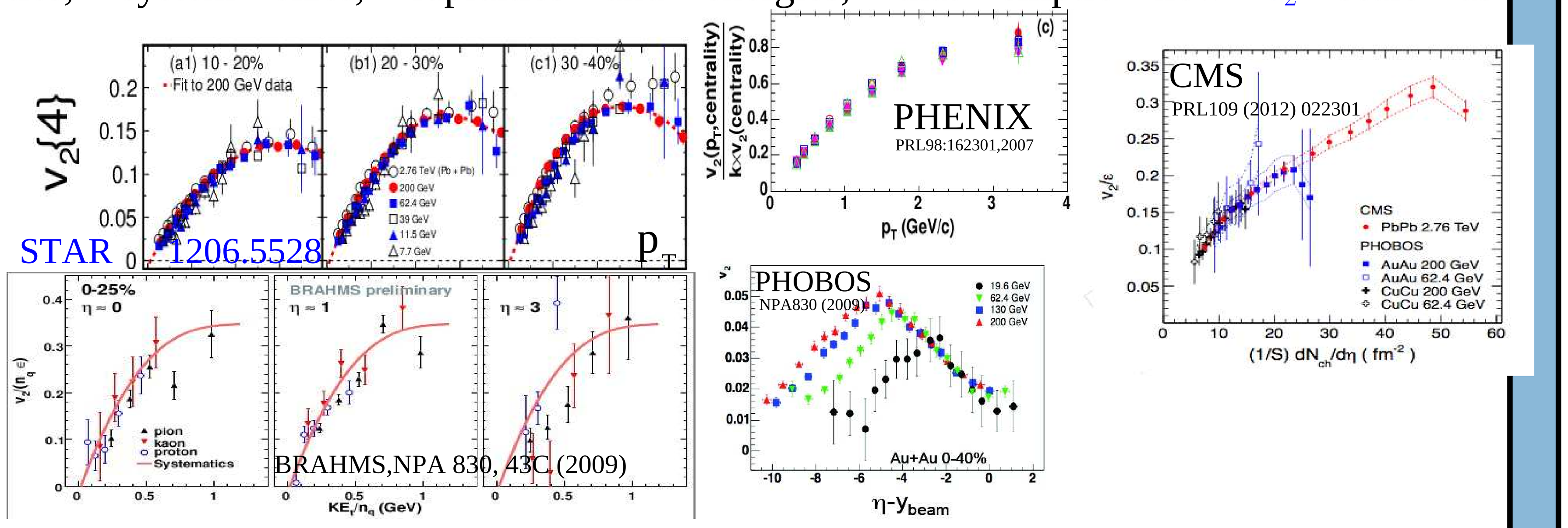
Introduction: Experimental scaling of v_2

It is an experimental statement, putting together all available centralities, system sizes, rapidities and energies, that elliptic flow v_2 scale as



$$v_2 = \epsilon(b,A) F(p_T) \quad , \quad \langle v2 \rangle = \epsilon(b,A) \int dp_T F(p_T) g\left(p_T, \langle p_T \rangle_{y,A,b,\sqrt{s}}\right)$$

$F(p_T)$ universal for all energies, g only characterized by "mean momentum" $\langle p_T \rangle \sim \frac{1}{S}\frac{dN}{dy}$ This is an experimental parametrization, "as good as the error bars" . It might break down as more precise scans in energy, rapidity etc become available. But while it stands, it makes sense to investigate to what extent do models reproduce it. Such separable and universal is a very strong constraint , generically complicated dynamics are non-separable, $(v_2 \sim v_2(b, A, p_T, \langle T_{initial} \rangle, y))$. Does separability hold in hydro+tomography [1, 2]?

How v_2 scales in hydrodynamic limit

- Approximately $\propto \epsilon$ since $v_2(\epsilon = 0) = 0$ and ϵ small+dimensionless
- Approximately $\propto c_s(T)$ since $v_2(c_s = 0) = 0$, c_s small+dimensionless+$\sim EoS$
- It is maximum for ideal hydro. Since Knudsen n. Kn small and dimensionless, $v_2 \sim v_2^{ideal}(1 - Kn)$. In turn $Kn \sim \eta/(sTR)$
- $v_2^{ideal} \sim v_2(\tau \to \infty) \times f(\tau_f/\tau_0)$. $f(\ldots)$ a monotonically increasing saturating, $\sim f(\langle p_T \rangle) \tanh(\ldots)$ in Cooper-Frye . τ_f is the freezeout time.
- $\tau_f/\tau_0 \sim (e_0/e_f)^{4\alpha} \sim (T_0/T_f)^{\alpha}$, with $\frac{1}{3}\big|_{bjorken} < \alpha < \frac{4}{3}\big|_{hubble}$.

$$\frac{v_2}{\epsilon} \sim c_s f\underbrace{\left(\frac{1}{T_f^3 \tau_0 R^2}\frac{dN}{dy}\right)}_{\sim \tanh(\ldots)}\left(1 - \mathcal{O}(1)\frac{\eta}{s}\underbrace{\frac{1}{TR}}_{no\ \ scaling}\right)$$

terms in red mix $A, \sqrt{s}$ in contrast to experimental data. Bound for Kn ? What about $c_s(T), \eta/s(T), \tau_0(\sqrt{s})$? Also in Cooper-Frye formula, $F(p_T)$ should not be universal.

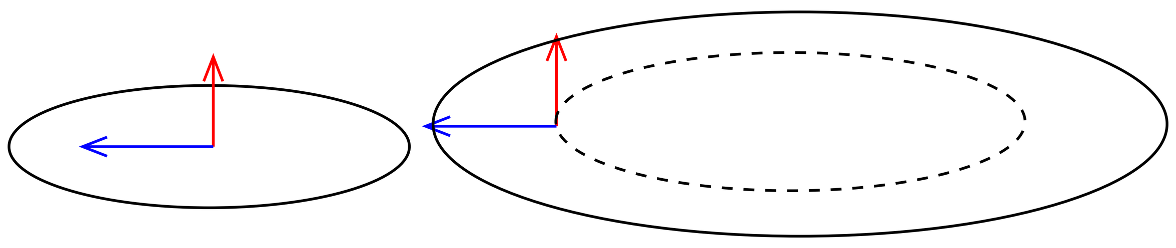
$$v_2(p_T) = \simeq \int d\phi \cos^2(2\phi)\left[\underbrace{e^{-\frac{\gamma(E - p_T u_T)}{T}}}_{=0} - \underbrace{p_T \Delta \frac{dt}{dr}}_{\sim \epsilon p_T} + \underbrace{\frac{\gamma \delta u_T(\phi) p_T}{T}}_{\sim \frac{\delta v_T}{T} p_T \sim \epsilon p_T/T} + \mathcal{O}(\epsilon^2) + \mathcal{O}(Kn)\right]$$

As long as $\frac{\delta v_T}{T} \sim \epsilon s^0$ (close to saturation of ϵ_p), $v_2(p_T)$ independent of $\sqrt{s}$.
In ideal and long-lived limit $\frac{\eta}{sT} \ll R$ in $v_2(p_T) \sim \epsilon \tanh\left(\epsilon \frac{p_T}{T_f}\right) \underset{p_T \gg T}{\longrightarrow} p_T^0$

(But $\langle p_T \rangle \sim (1/S)(dN/dy) \sim N_{part} \ln \sqrt{s}$)
Bottom line: hydro $v_2(p_T)$ and $v_2^{integrated}$ scale differently, because in hydro Fourier components of $v_T(r, \phi)$ depend on lifetime differently

p_T binned v_2 depends only on the 2nd Fourier component of $v_T(r, \phi) \sim \tanh(lifetime)$
Integrated v_2 depends on both the 0th ($\langle p_T \rangle \sim lifetime$) and 2nd component.
So if one scales, the other should not. $v_2(p_T)/\langle v_2 \rangle \sim F(p_T)$ tests this experimentally.

Low p_T vs High p_T: Tomography vs hydro



Take, as an initial condition, an elliptical distribution of matter at a given ϵ_n , run jets through it and calculate v_n . Now increase size R with constant ϵ_n .

$$\frac{v_n}{\epsilon_n}\bigg|_{tomo} \to \frac{Surface}{Volume} \to 0 \quad , \quad \frac{v_n}{\epsilon_n}\bigg|_{hydro} \to \text{constant}$$

Role of "size" $\langle R \rangle \sim A^{1/3}$ different in tomo regime , $v_2 \sim f(\langle R \rangle)$ in tomo, not in hydro.
Probe by comparing v_n in Cu-Cu vs Au-Au collisions of Same $\frac{1}{S}\frac{dN}{dy}$!
NB: "high p_T" can be defined via $\mathcal{O}(Kn)$: independently of energy $\sqrt{s}$
assuming cross section $\sigma \sim 1/Q^2$ and $\langle Q \rangle \sim \langle P - T \rangle \to_{P \gg T} P$ since momentum $P \sim p_T$

$$\frac{l_{mfp}}{R} \sim \frac{p_T^2}{sR} \quad \text{hence for} \quad s \sim \sqrt{\frac{dN}{\tau_0 R dy}} \quad , \quad \frac{p_{T1}^{tomo}(\sqrt{s_1})}{p_{T2}^{tomo}\sqrt{s_2}} \sim \left(\frac{dN_1/dy}{dN_2/dy}\right)^{1/2}$$

As $p_T \geq p_T^{tomo}$ we enter tomographic regime (NB p_T^{tomo} should go down for lower $\sqrt{s}, N_{part}, A$). Here hydro should not apply But v_2 still good observable!
Issues like Cronin effect and fragmentation, plaguing R_{AA} at low $\sqrt{s}$ don't apply to v_2

An investigation using the ABC Model [3, 4]

$$-\frac{dE}{d\tau} = f(T, p_T, \tau) \simeq \kappa p^a T^b \tau^c + \mathcal{O}\left(\frac{T}{p_T}, \frac{1}{T\tau}\right)$$

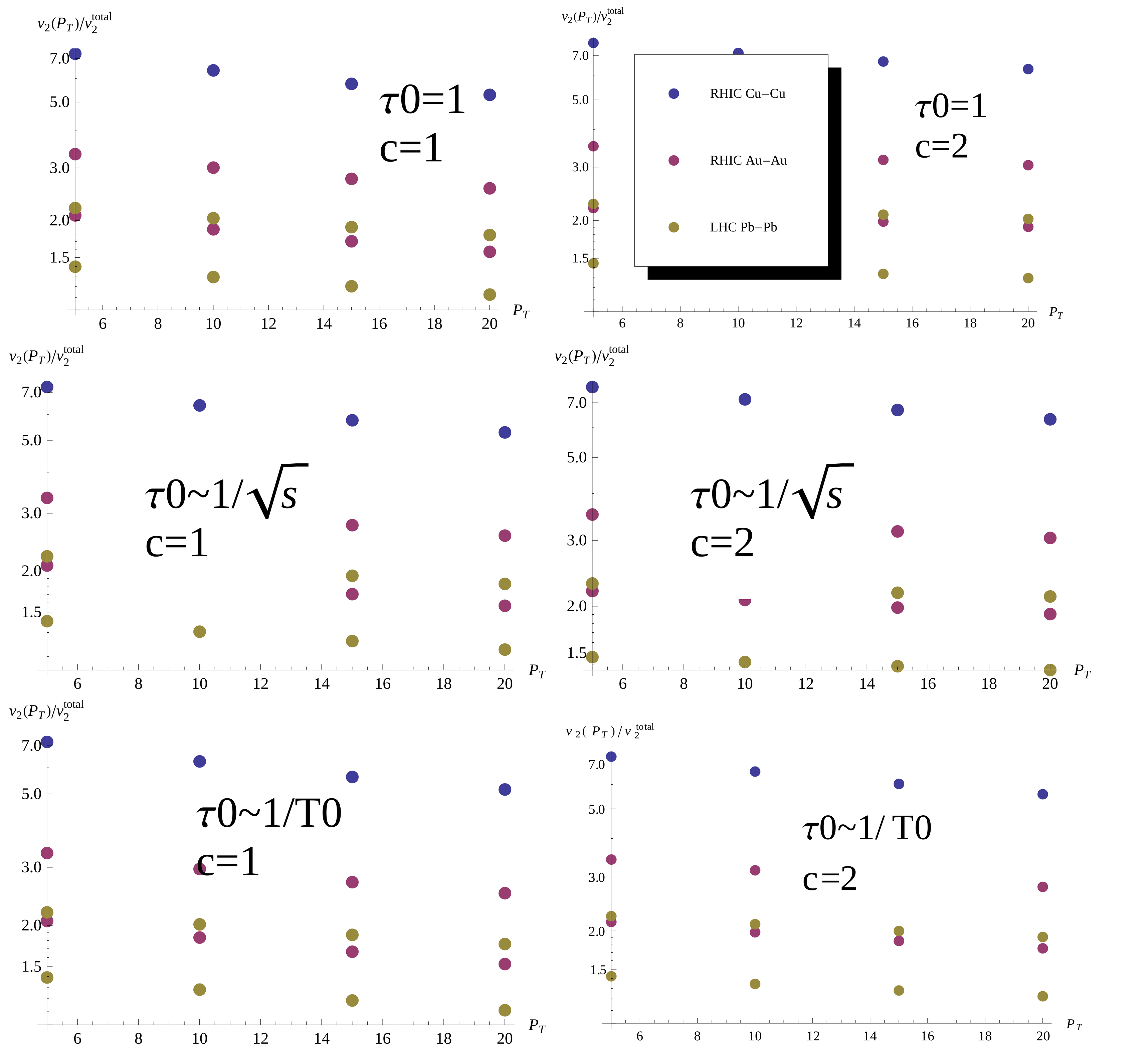
A phenomenological way of keeping track of every jet energy loss model in certain limits. $a = 1, c = 0$ parton cascade, $a = 1/3, c = 0$ Bethe Heitler, $c = 1$ LPM, $c > 2$ AdS/CFT "falling string"... Can expand in "empirical" parameters $\left(\frac{\Delta E}{E}\right)^{\pm 1}, \epsilon,$

$$v_2\left(p_T^{high}\right) \sim \frac{\langle\langle T^\alpha L^\beta \rangle_{in-plane} - \langle T^\alpha L^\beta \rangle_{out-of-plane}\rangle}{\langle\langle T^\alpha L^\beta \rangle_{in-plane} + \langle T^\alpha L^\beta \rangle_{out-of-plane}\rangle} \sim \epsilon^{\alpha_\epsilon} R^{\alpha_R(a,b,c)} p_T^{\alpha_{p_T}(a,b,c)} \left(\frac{dN}{dy}\right)^{\alpha_{dn/dy}} .$$

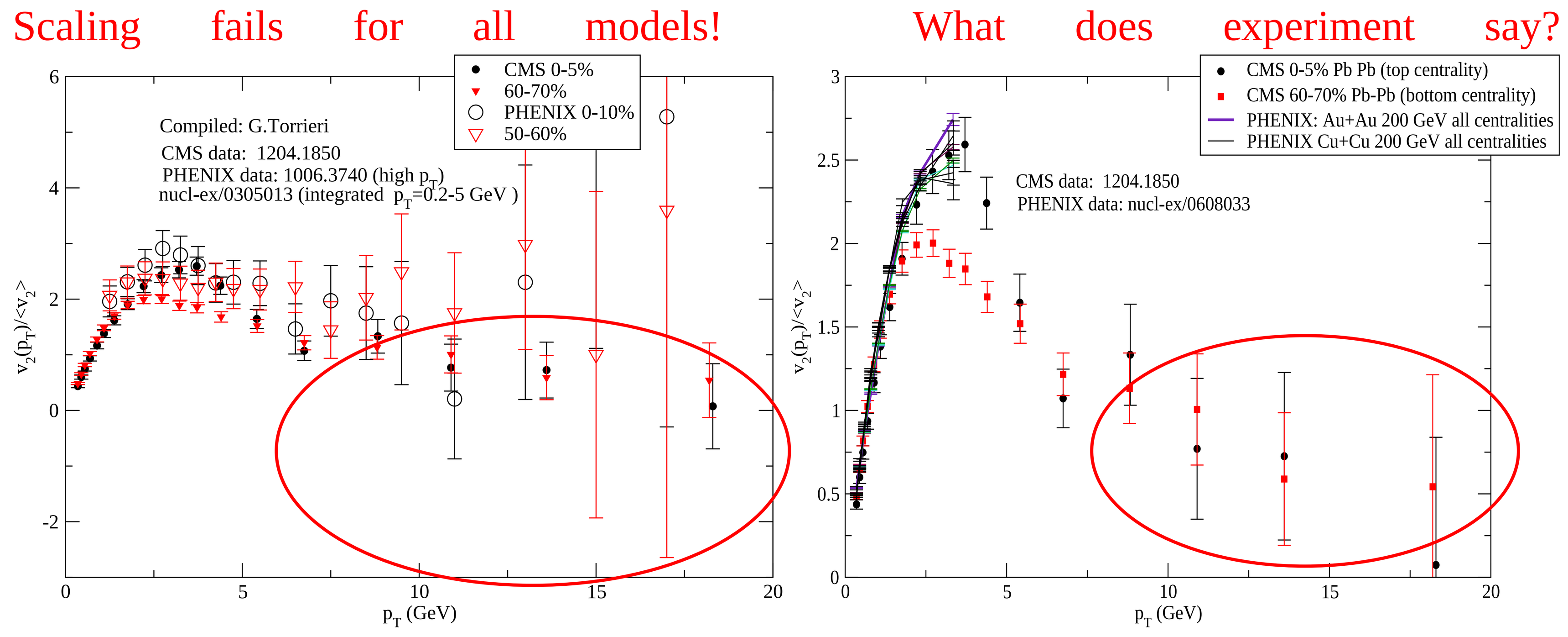
Background given by a longitudinally expanding ellipsoid, fitted to produce global multiplicity and system size

$$T(r_T, \phi, \tau, \tau_0) = T_0\left(\frac{\tau_0}{\tau}\right)^{1/3} \Theta\left(r - R\left(1 + \epsilon \cos(2\phi)\right)\right)$$

R, ϵ scanned across radii of Cu,Au,Pb , $\tau_0 = 1fm, \sqrt{s}^{-1}, T_0^{-1}$, T_0 adjusted to reproduce multiplicity. The objective is to investigate scaling of $v_2(p_T)/\langle v_2 \rangle$, the latter given by the experimental parametrization $\langle v_2 \rangle / \epsilon = 0.004(1/S)(dN/dy)$



Scaling fails for all models! What does experiment say?



Error bars too big, but only violation seen at intermediate region!

Conclusions, discussion and outlook

The "standard model" of uRICs generically predicts breaking of scaling of v_2 in p_T

Scaling of high-p_T v_2 an as yet unexploited tool, capable of detecting changes in opacity, energy loss mechanism and initial state

A useful tool to do low and high energy comparisons since Cronin effect at lower energies diminishes usefulness of R_{AA} but not v_2 . How low in energy and how high in p_T does $v_2(p_T)$ overlap ? Jump might signal change of jet-medium coupling mechanism

References

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