Quantum chaos in the quantum fluid:

the spectrum of initial fluctuations in the little bang

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QM2012, Washington DC, August 13-18, 2012

Talk Outline

- **Motivation:**
  i) the unreasonable effectiveness* of hydrodynamics in heavy ion collisions
  ii) quantitative phenomenology of flow

  * to paraphrase E. P. Wigner

An ab initio weak coupling approach:

- **Classical coherence of wee partons in nuclear wavefunctions**

  (See Tuomas Lappi's talk)

- **Quantum fluctuations: Factorization, Evolution, Decoherence**

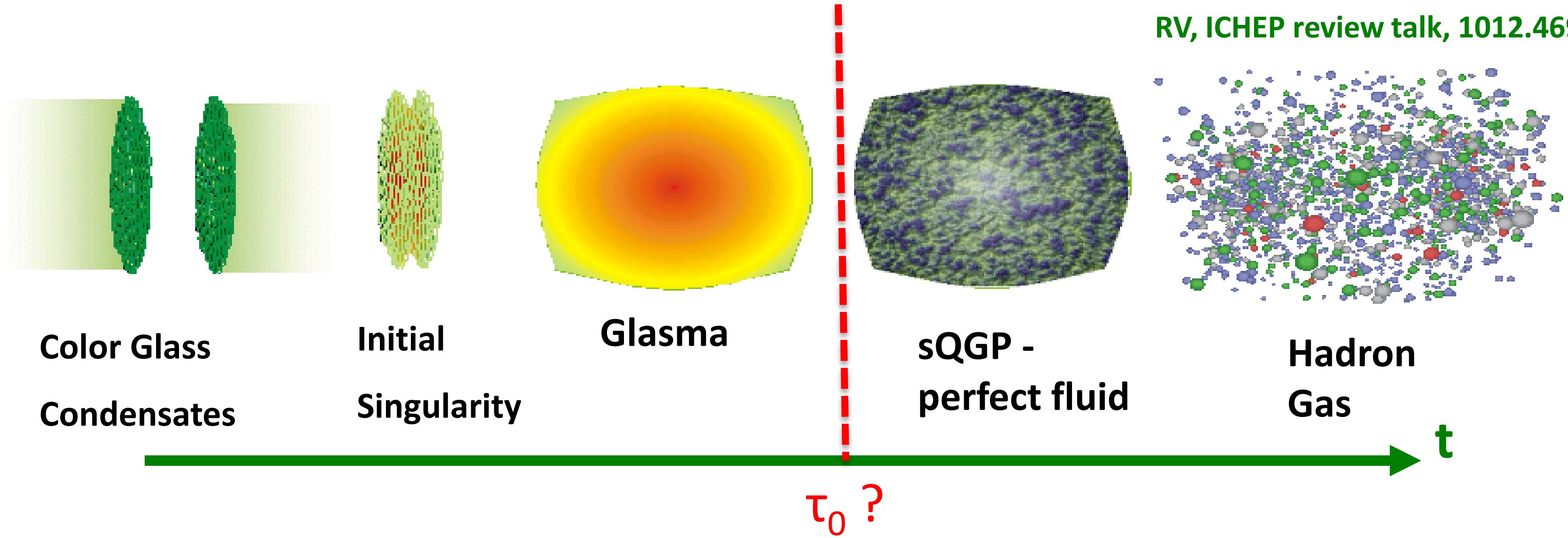
- **Isotropization, Bose-Einstein Condensation, Thermalization ?**

  (See Jinfeng Liao's talk)

This approach draws concretely on (is closely related to) concepts in small x physics, reaction-diffusion systems, QCD factorization, topological effects, plasma physics, thermodynamics and stat. mech, quantum chaos, Bose-Einstein condensates, pre-heating in inflationary cosmology

Ab initio approach to heavy ion collisions

RV, ICHEP review talk, 1012.4699

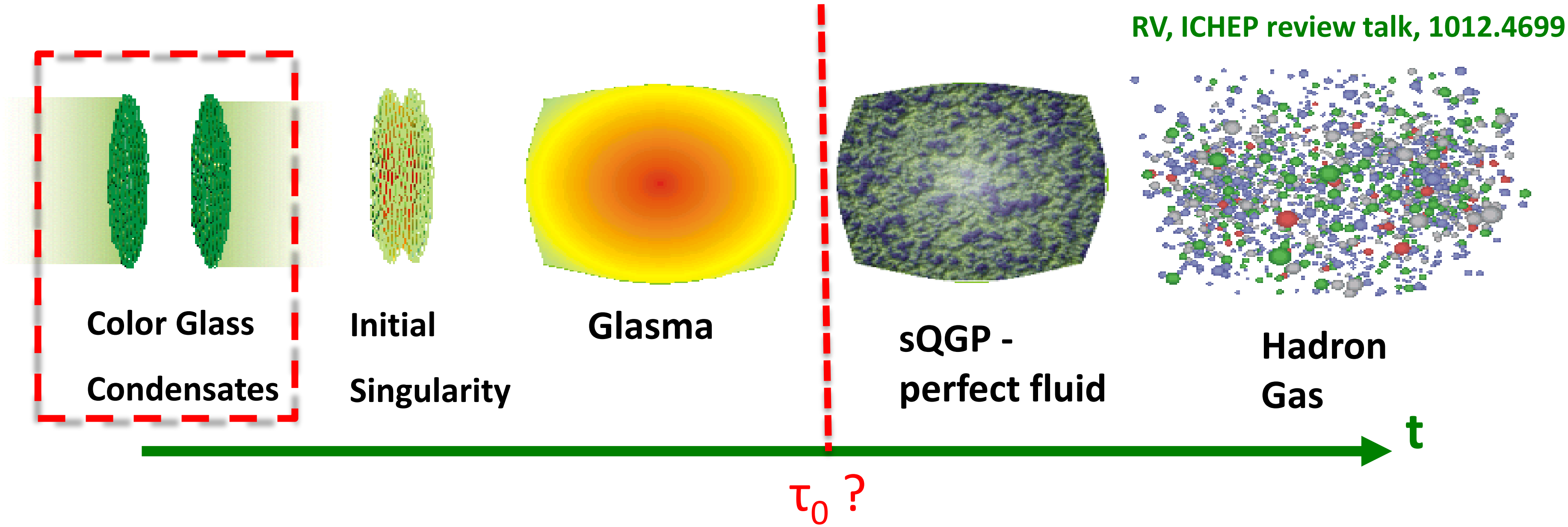


- Compute properties of relevant degrees of freedom of wave fns. in a systematic framework (as opposed to a “model”)?

- How is matter formed ? What are its non-equilibrium properties & lifetime? Can one “prove” thermalization or is the system “partially” thermal ?

- When is hydrodynamics applicable? How much jet quenching occurs in the Glasma? Are there novel topological effects (sphaleron transitions?)

Ab initio approach to heavy ion collisions



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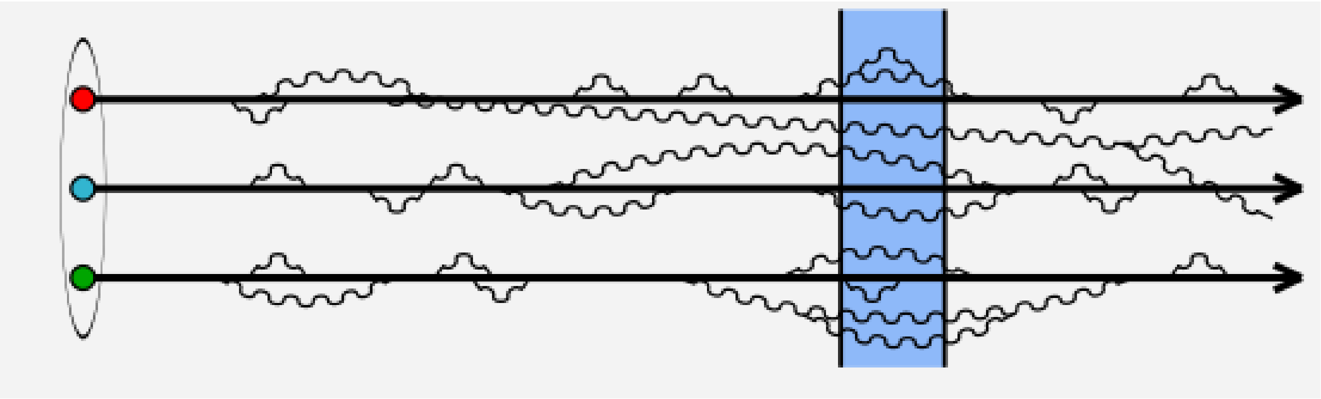
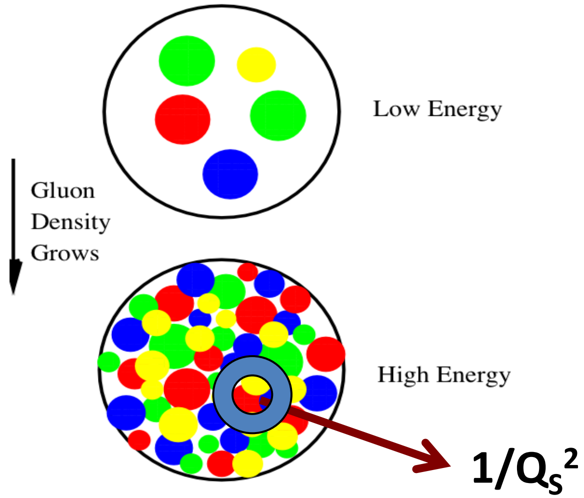
Gluon Saturation in large nuclei: classical coherence from quantum fluctuations



Wee parton fluctuations time dilated on strong interaction time scales

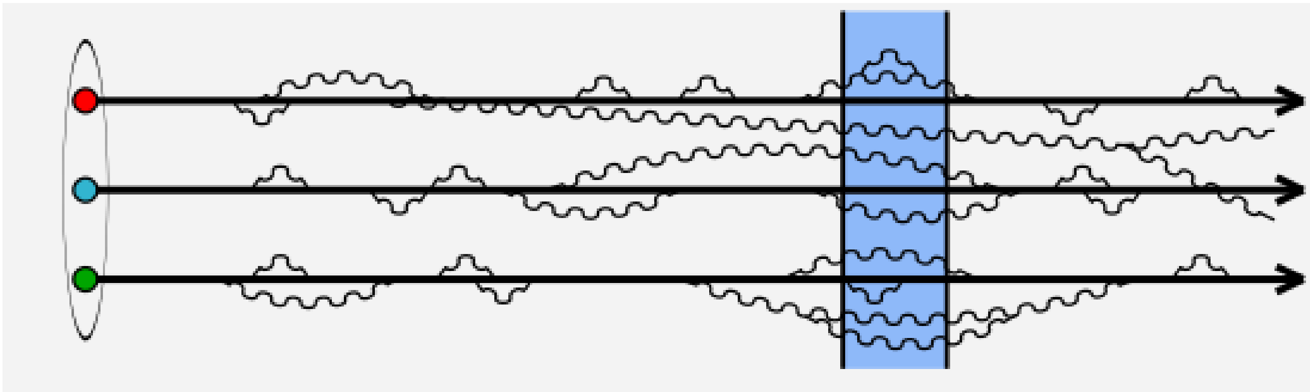


The gluon density saturates at a maximal value of ~ $1/\alpha_S$ ➔ gluon saturation

Large occupation # => classical color fields

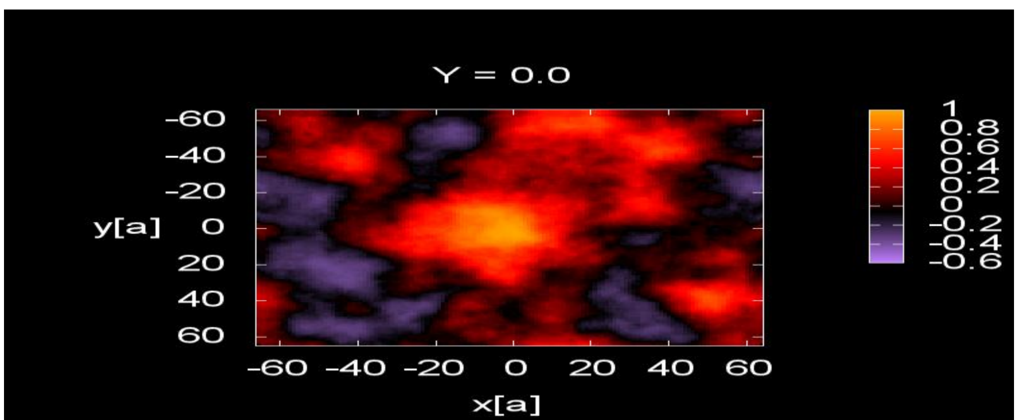
|P>_pert ➔ |P>_classical

Gluon Saturation in large nuclei:
classical coherence from quantum fluctuations



Wee parton fluctuations time dilated on strong interaction time scales

Correlator of Light-like Wilson lines Tr(V(0,0)V^dagger (x,y))

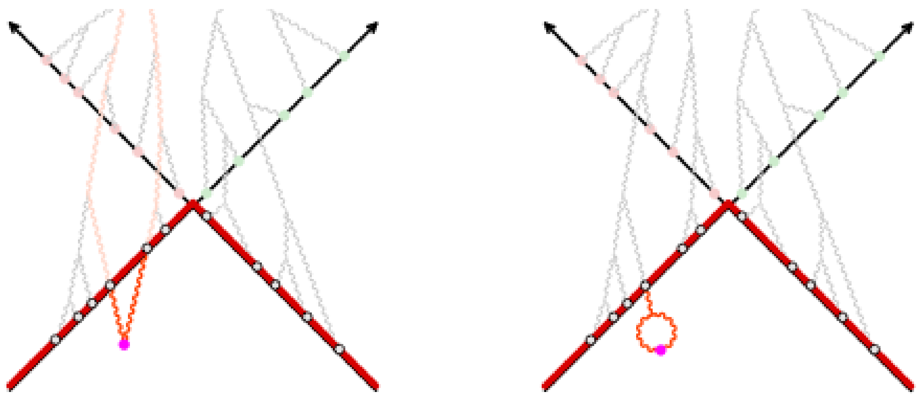


Dumitru,Jalilian-Marian,Lappi,Schenke,RV, PLB706 (2011)219

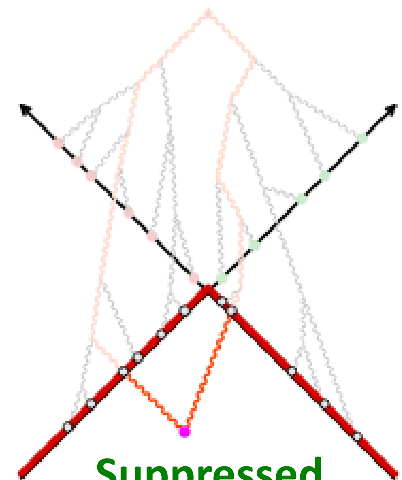
Rummukainen,Weigert (2003)

Quantum fluctuations in classical backgrounds: I

Gelis,Lappi,RV: arXiv: 0804.2630, 0807.1306, 0810.4829



Factorized into energy evolution of wavefunctions



Suppressed

JIMWLK factorization: $p^\eta=0$ (small x !) modes that are coherent with the nuclei can be factorized for inclusive observables

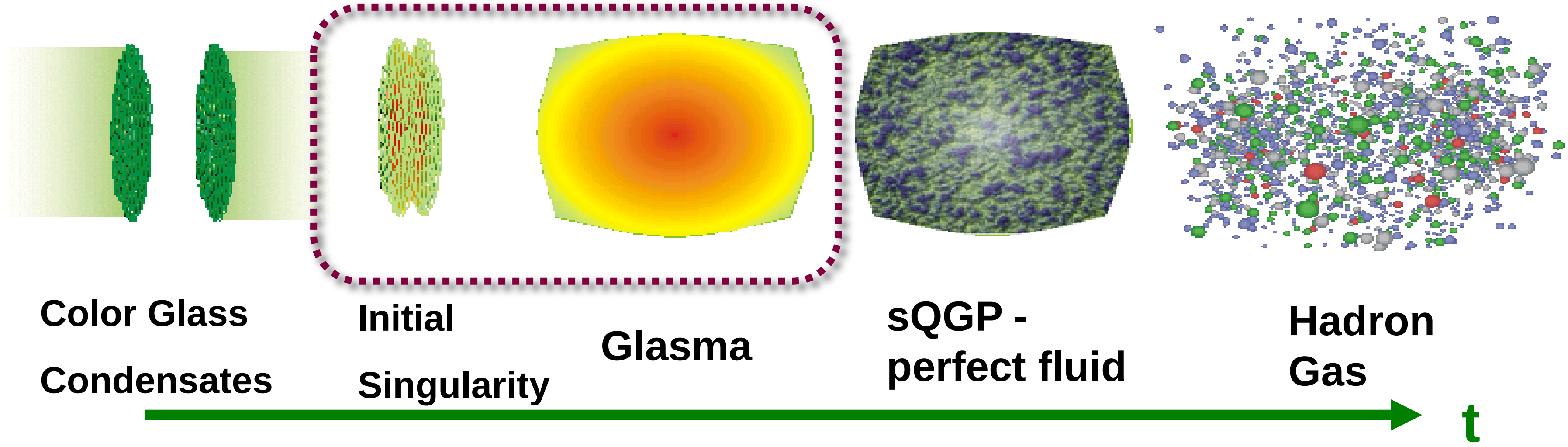
See Lappi's talk

$$\langle T^{\mu\nu}(\tau,\underline{\eta},x_\perp)\rangle_{\rm LLog} = \int [D\rho_1 d\rho_2]\, W_{Y_1}[\rho_1]\, W_{Y_2}[\rho_2]\, T^{\mu\nu}_{\rm LO}(\tau,x_\perp)$$

$$Y_1 = Y_{\rm beam} - \eta\,;\; Y_2 = Y_{\rm beam} + \eta$$

W's are universal "functional density matrices" describing distribution of large x color sources ρ_1 and ρ_2 of incoming nuclei; can be extracted from DIS or hadronic collisions

Quantum decoherence from classical coherence



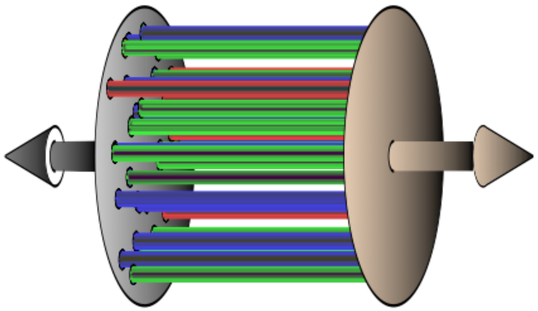
Glasma (\Glahs-maa\): ***Noun:*** **non-equilibrium matter between CGC and QGP**

Computational framework Gelis,RV NPA (2006)

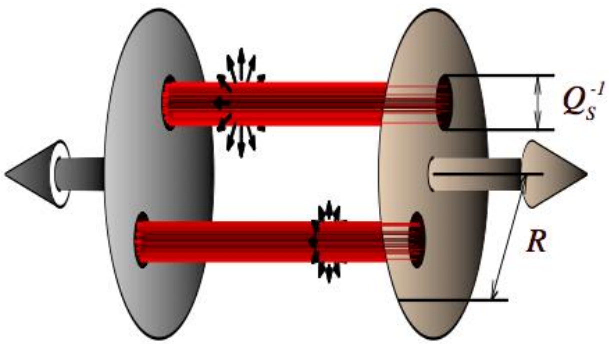
Schwinger-Keldysh: for strong time dependent sources (ρ ~ 1/g) ,
initial value problem for inclusive quantities

For eg., Schwinger mechanism for pair production, Hawking radiation, …

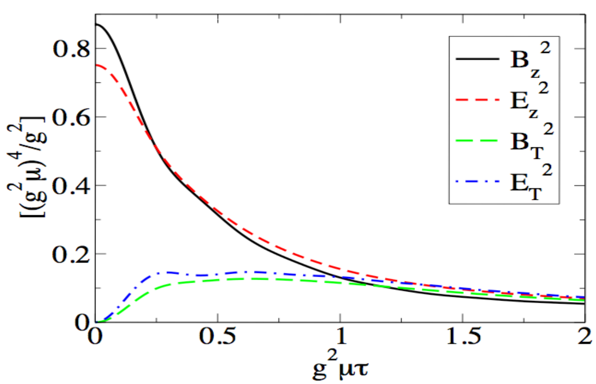
Lumpy classical configurations



Solutions of Yang-Mills equations produce (nearly) boost invariant gluon field configurations: "Glasma flux tubes"



Lumpy gluon fields are color screened in transverse plane over distances ~ $1/Q_S$
- Negative Binomial multiplicity distribution.



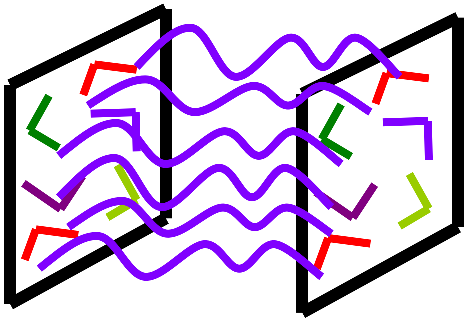
"Glasma flux tubes" have non-trivial longitudinal color E & B fields at early times
--generate Chern-Simons topological charge

See talk by Bjoern Schenke: basis of IP-Glasma model

Quantum fluctuations in classical backgrounds: II

$p^\eta \neq 0$ (generated after collision) modes grow exponentially

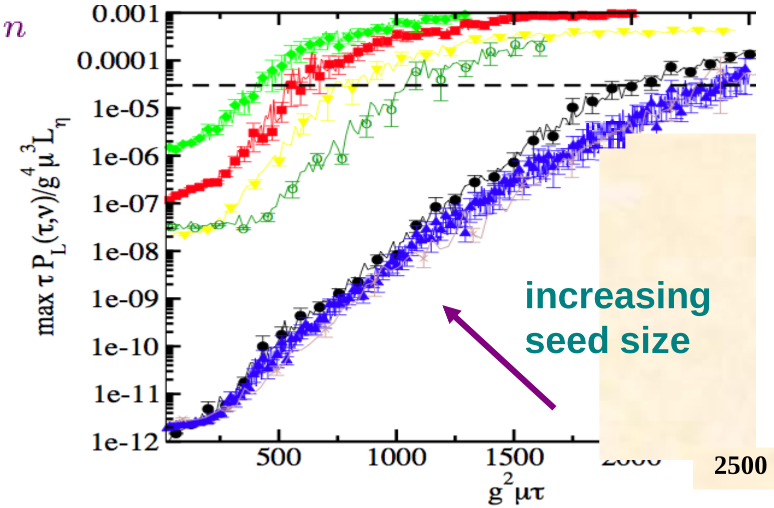
Romatschke,Venugopalan
Fukushima,Gelis,McLerran

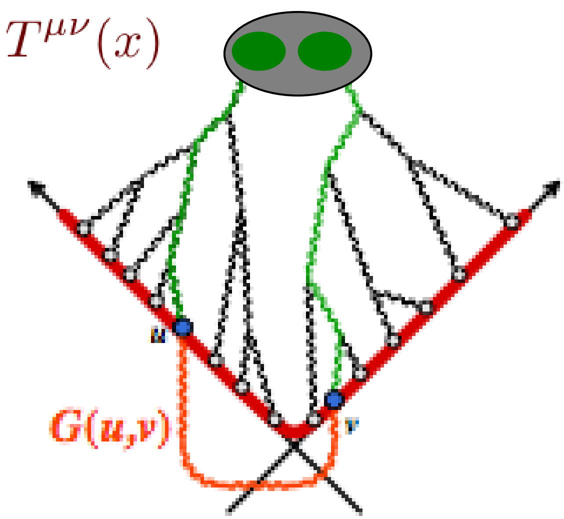


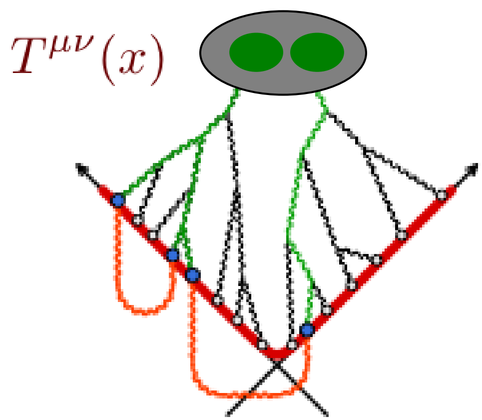
$$\left[g \exp\left(\sqrt{Q_S \tau}\right)\right]^n$$

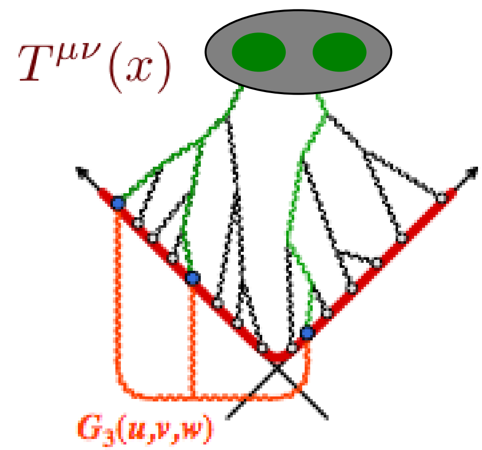
Quant. fluct.
grow exponentially
after collision

As large as classical
field at 1/Qs !









Parametrically suppressed

Exponentiate and resum these

The first fermi: a master formula

Also correlators of $T^{\mu\nu}$

✓ From solutions of B-JIMWLK

$$\langle\langle T^{\mu\nu}\rangle\rangle_{\rm LLx+Linst.} = \int [D\rho_1][D\rho_2]\, W_{\rm Y_{beam}-Y}[\rho_1]\, W_{\rm Y_{beam}+Y}[\rho_2] \times \int [da(u)]\; F_{\rm init}[a]\; T^{\mu\nu}_{\rm LO}[A_{\rm cl}(\rho_1,\rho_2)+a]$$

✧ Gauge invariant Gaussian spectrum of quantum fluctuations

✓ 3+1-D solutions of Yang-Mills equations

✧ Expression computed recently-numerical evaluation in progress

Dusling,Epelbaum,Gelis,RV

- This is what needs to be matched to viscous hydrodynamics, event-by-event
- All modeling of initial conditions for heavy ion collisions includes various degrees of over simplification relative to this “master” formula

Glasma spectrum of initial quantum fluctuations

Path integral over small fluctuations equivalent to

$$A(\tau,\eta,x_\perp) = A_{\mathrm{cl.}}(\tau,x_\perp) + \frac{1}{2}\int \frac{d\nu}{2\pi} d\mu_K\, c_{\nu K}\, e^{i\nu\eta}\, \chi_K(x_\perp)\, H^{(2)}_{i\nu}(\lambda_K \tau) + c.c$$

$$[D^2 + V''(A_{\mathrm{cl.}})]\chi_K(x_\perp) = \lambda_K^2\, \chi_K(x_\perp)$$

Gaussian random variables

$$\langle c_{\nu k} c_{\mu l} \rangle = 0$$
$$\langle c_{\nu k} c^*_{\mu l} \rangle = 2\pi\delta(\nu-\mu)\delta_{kl}$$

Berry conjecture: High lying quantum eigenstates of classically chaotic systems, linear superpositions of Gaussian random variables

Yang-Mills is a classically chaotic theory

B. Muller et al.

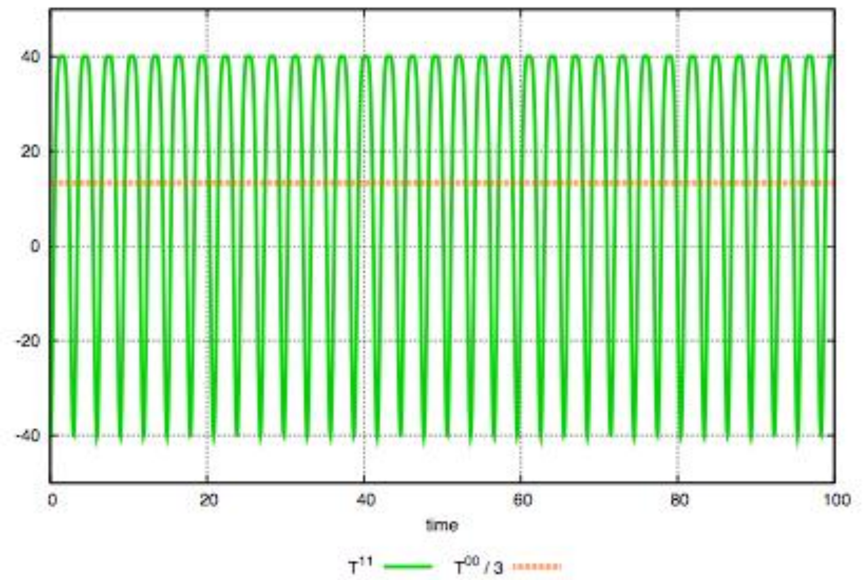
Srednicki: Systems that satisfy Berry's conjecture exhibit "eigenstate thermalization"

Also, Jarzynski, Rigol, …

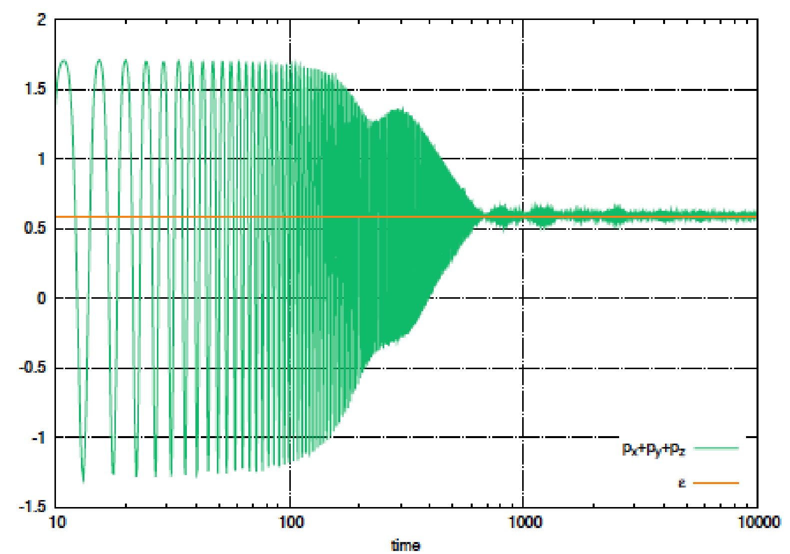
Hydrodynamics from quantum fluctuations

Dusling,Epelbaum,Gelis,RV (2011)

massless scalar Φ^4 theory:



Energy density and pressure without averaging over fluctuations



Energy density and pressure after averaging over fluctuations

➡ **Converges to single valued relation "EOS"**

Hydrodynamics from quantum fluctuations

Dusling,Epelbaum,Gelis,RV (2011)

Anatomy of phase decoherence:

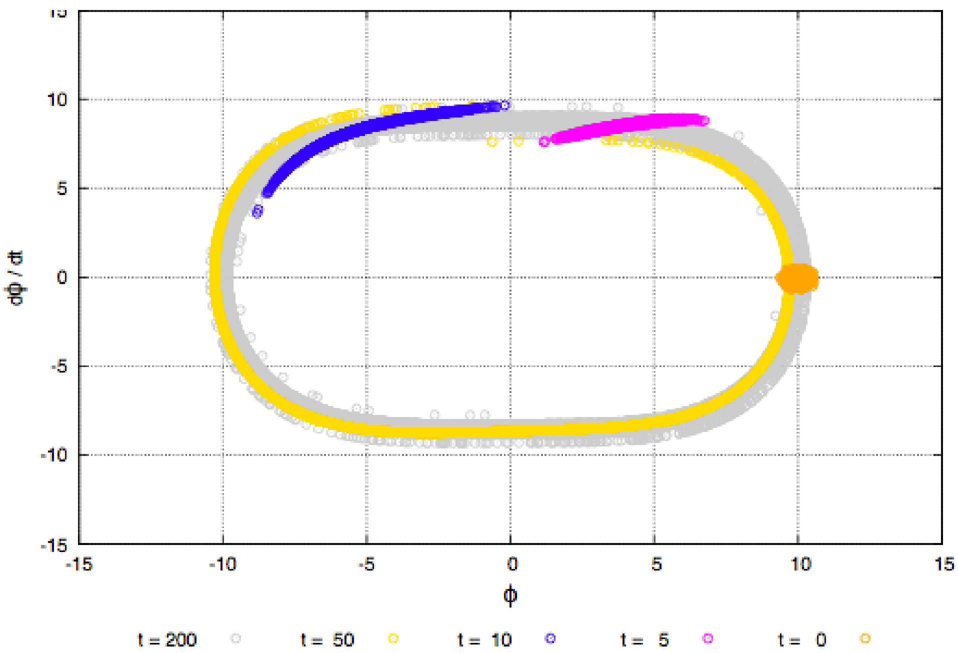
$\Delta\Theta = \Delta\omega\ t$

$T_{period} = 2\pi\ /\ \Delta\omega$

➔ $T_{period} \cong 18.2\ /\ g\ \Delta\Phi_{max}$

↓

Different field amplitudes from different initializations of the classical field



$$\langle T^\mu_\mu \rangle = \int d\phi\, d\dot\phi\ \rho_t(\phi,\dot\phi)\ T^\mu_\mu(\phi,\dot\phi) \equiv \int dE\, d\theta\ \tilde\rho_t(E,\theta)\ T^\mu_\mu(E,\theta)$$

$$\underset{t\to\infty}{\approx} \int dE\ \tilde\rho_t(E) \int d\theta\ T^\mu_\mu(E,\theta) = 0$$

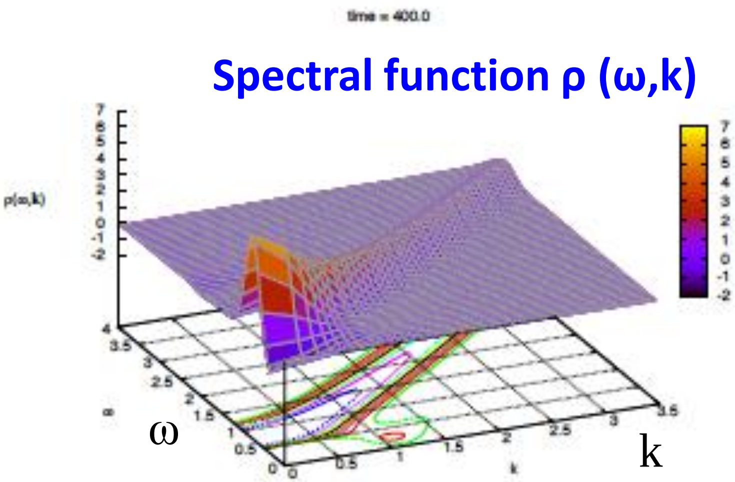
$$\int d\theta\ T^\mu_\mu(E,\theta) = \frac{2\pi}{T}\int_t^{t+T} d\tau\ T^\mu_\mu(\phi(\tau),\dot\phi(\tau)) = 0$$

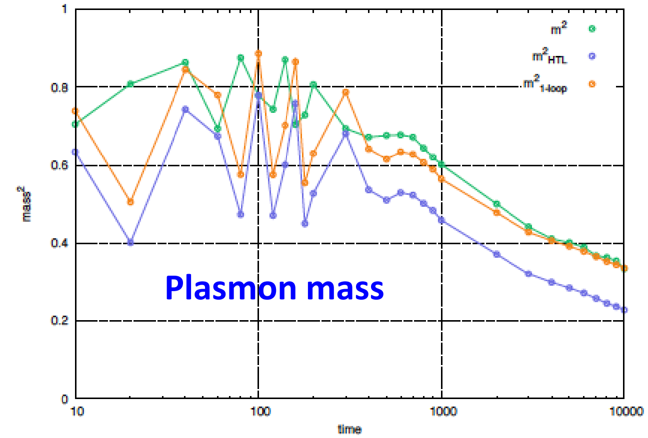
Because $T_\mu^{\ \mu}$ for scalar theory is a total derivative and ϕ is periodic

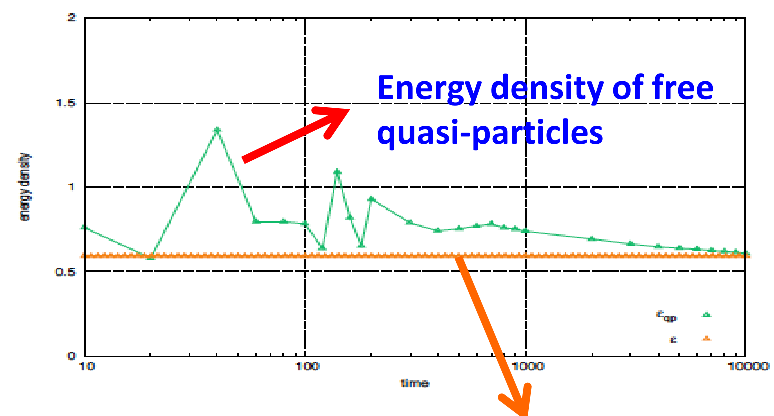
Phase decoherence leads to EOS for conformal theories

Quasi-particle description?

Epelbaum,Gelis (2011)





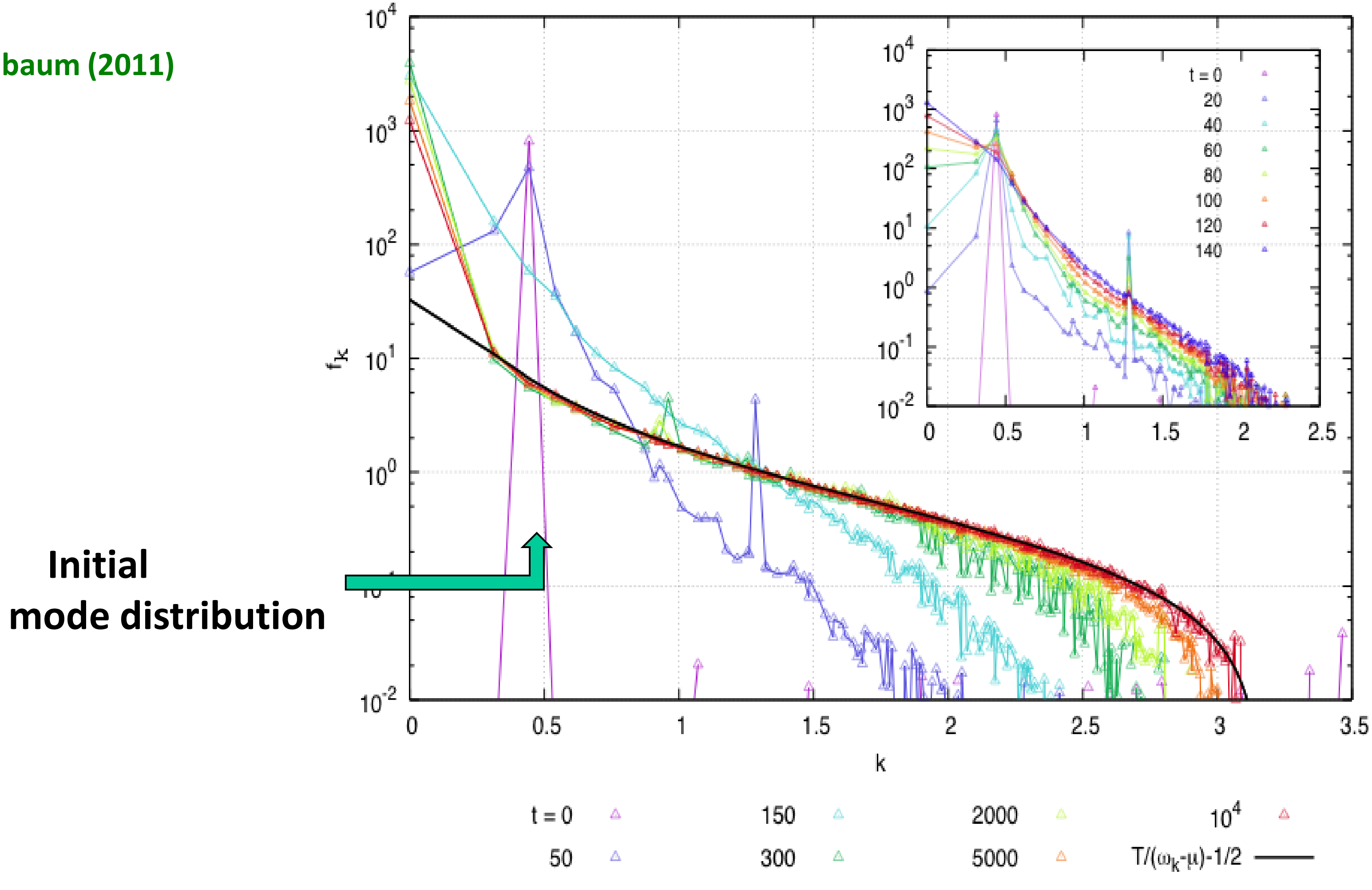


Energy density on the lattice

- At early times, no quasi-particle description
- May have quasi-particle description at late times. Effective kinetic “Boltzmann” description in terms of interacting quasi-particles at late times ?

Quasi-particle occupation number

Gelis,Epelbaum (2011)

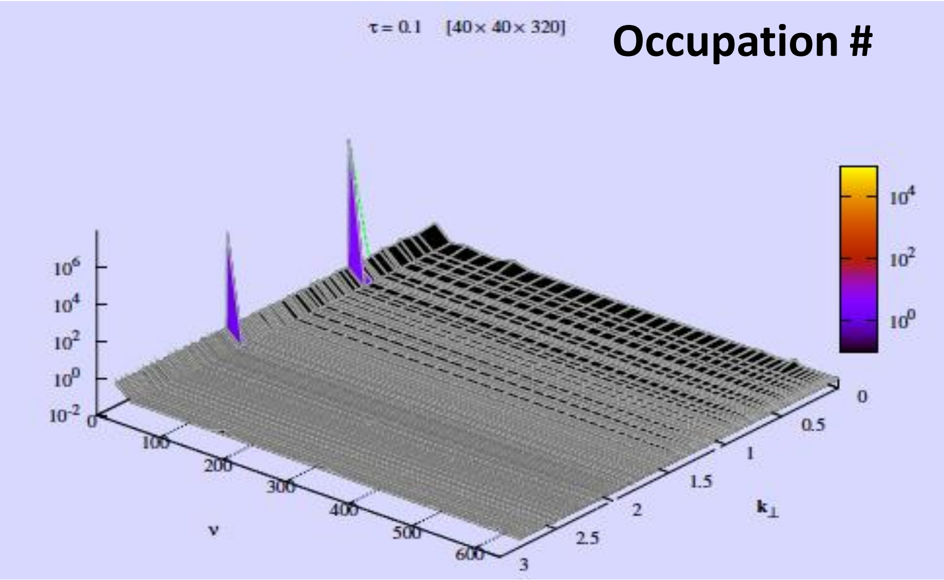


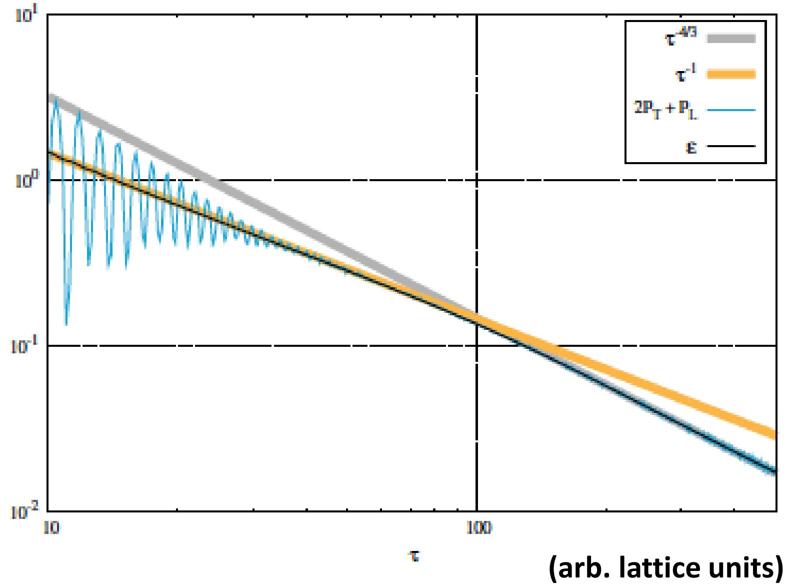
System becomes over occupied relative to a thermal distribution

Best thermal fit for $\mu \approx m_{plasmon}$

Proof of concept: isotropization of longitudinally expanding fields in scalar Φ^4

Dusling,Epelbaum,Gelis,RV, arXiv:1206.3336





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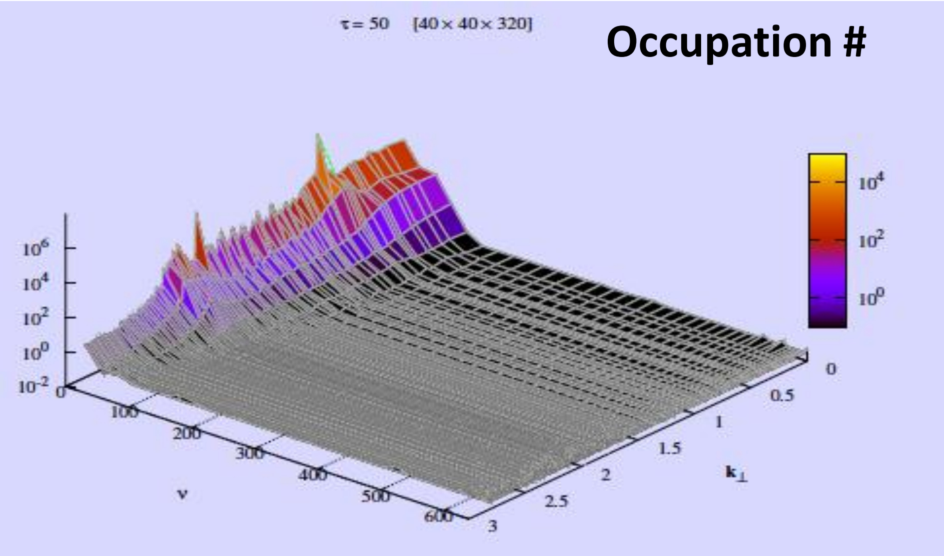
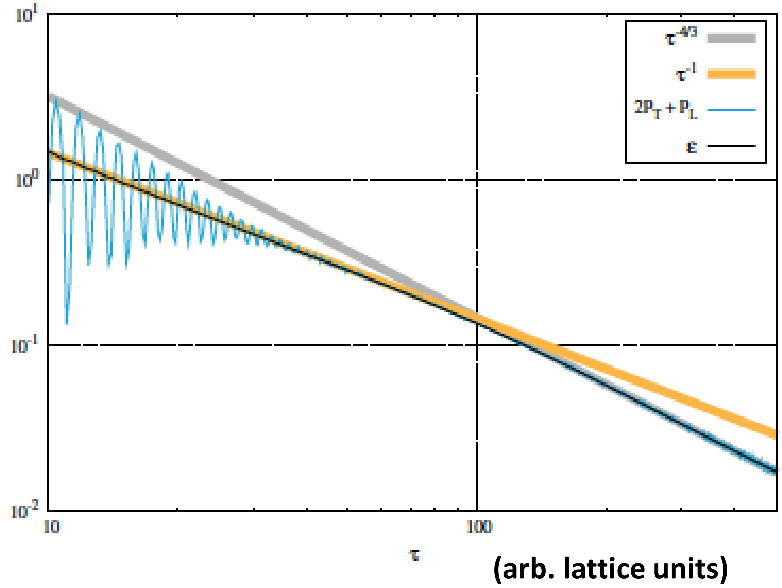
Dusling,Epelbaum,Gelis,RV, arXiv:1206.3336





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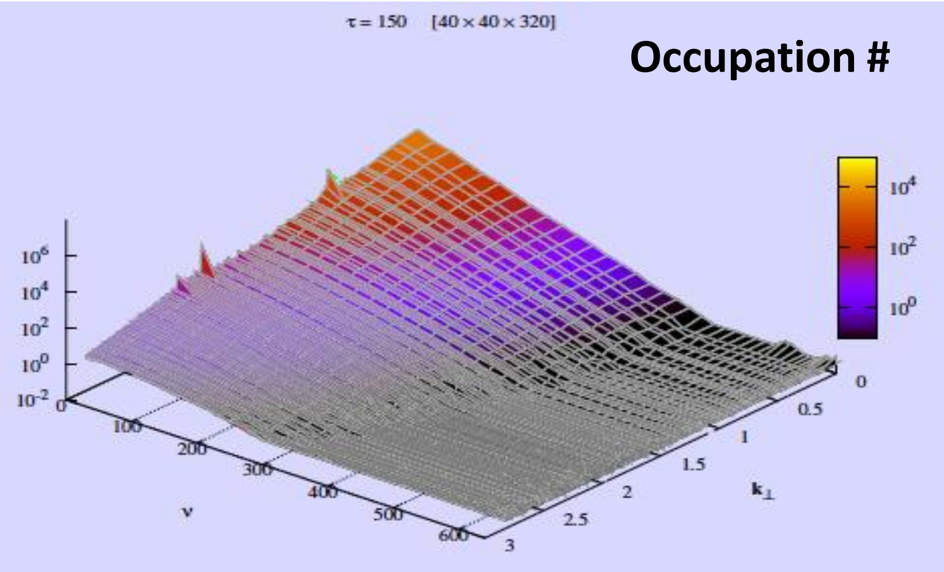

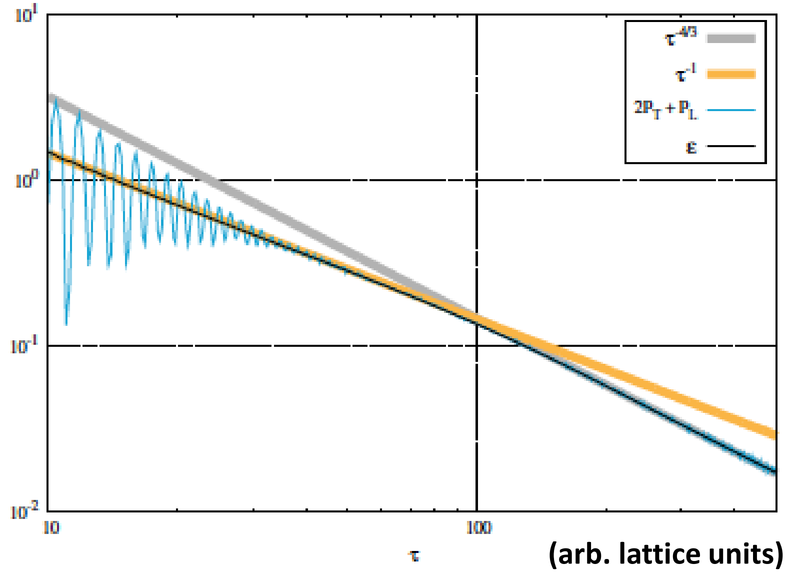

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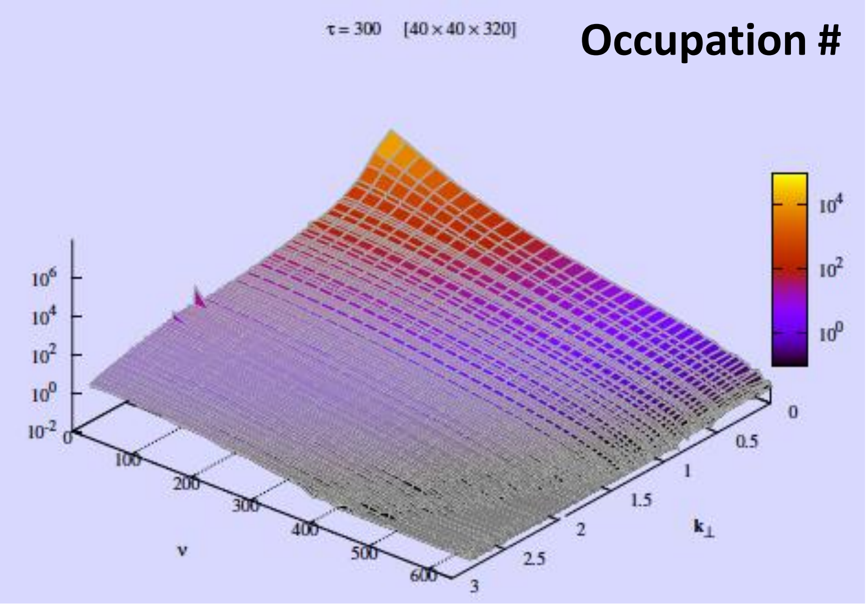
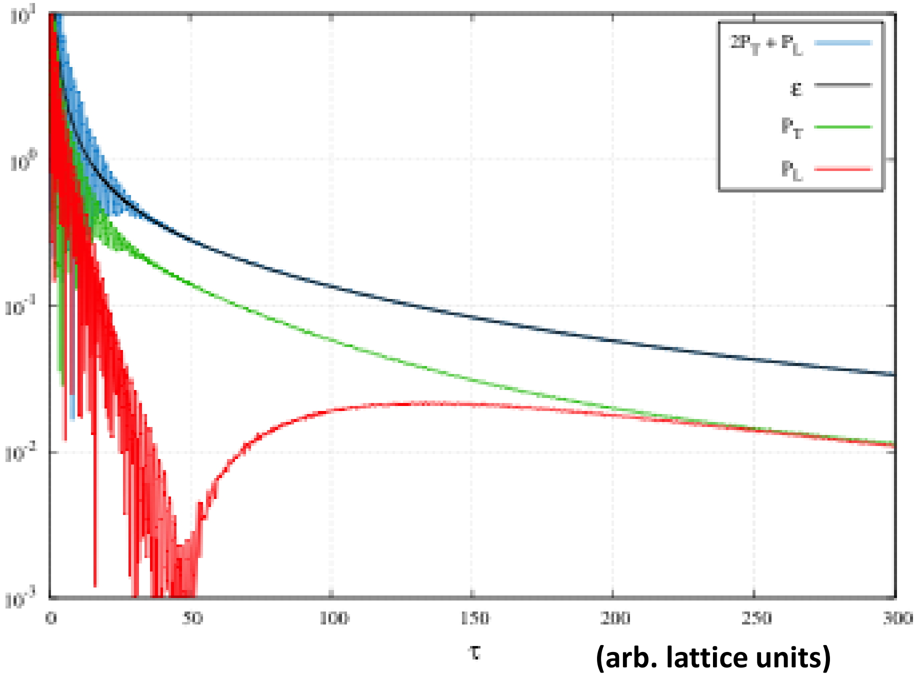




Decoherence → EOS → Isotropization

QCD: Real time evolution of quantum fluctuations

Analogous procedure to Scalar case:

Dusling,Gelis,RV:arXiv:1106.3927
Dusling,Epelbaum,Gelis,RV: in progress

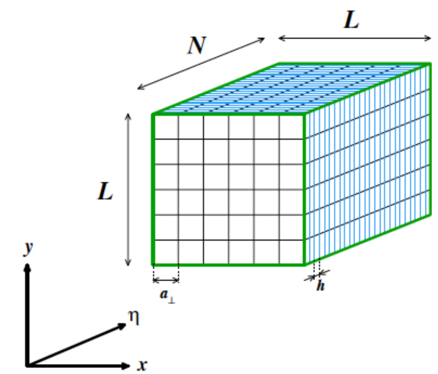
i) Determine small fluctuation eigenfunctions and eigenvalues at $\tau=0^+$

$$a^{\mu}_{\nu l 2}(\tau,\eta,x_\perp) = \frac{\sqrt{\pi}e^{\pi\nu/2}}{2Q_{\nu l 2}} \begin{pmatrix} b^x_{\nu l 2}(x_\perp) R^{(2)}_{-1,i\nu}(Q_{\nu l 2}\tau) \\ b^y_{\nu l 2}(x_\perp) R^{(2)}_{-1,i\nu}(Q_{\nu l 2}\tau) \\ b^\eta_{\nu l 2}(x_\perp) R^{(2)}_{+1,i\nu}(Q_{\nu l 2}\tau) \end{pmatrix} e^{i\nu\eta}$$

$$\begin{aligned} b^x_{\nu l 2} &= i\nu \mathcal{D}_{(0)x} b^\eta_{\nu l 2} \\ b^y_{\nu l 2} &= i\nu \mathcal{D}_{(0)y} b^\eta_{\nu l 2} \\ -\mathcal{P}_{(0)ii} b^\eta_{\nu l 2}(x_\perp) &= Q^2_{\nu l 2} b^\eta_{\nu l 2}(x_\perp) \end{aligned}$$

$$R^{(a)}_{b,\alpha}(k_\perp\tau) \equiv \int^\tau dx\, x^b H^{(a)}_\alpha(k_\perp x)$$

Invert very large matrices:
2 (N_c^2-1) N_T^2 × 2 (N_c^2-1) N_T^2

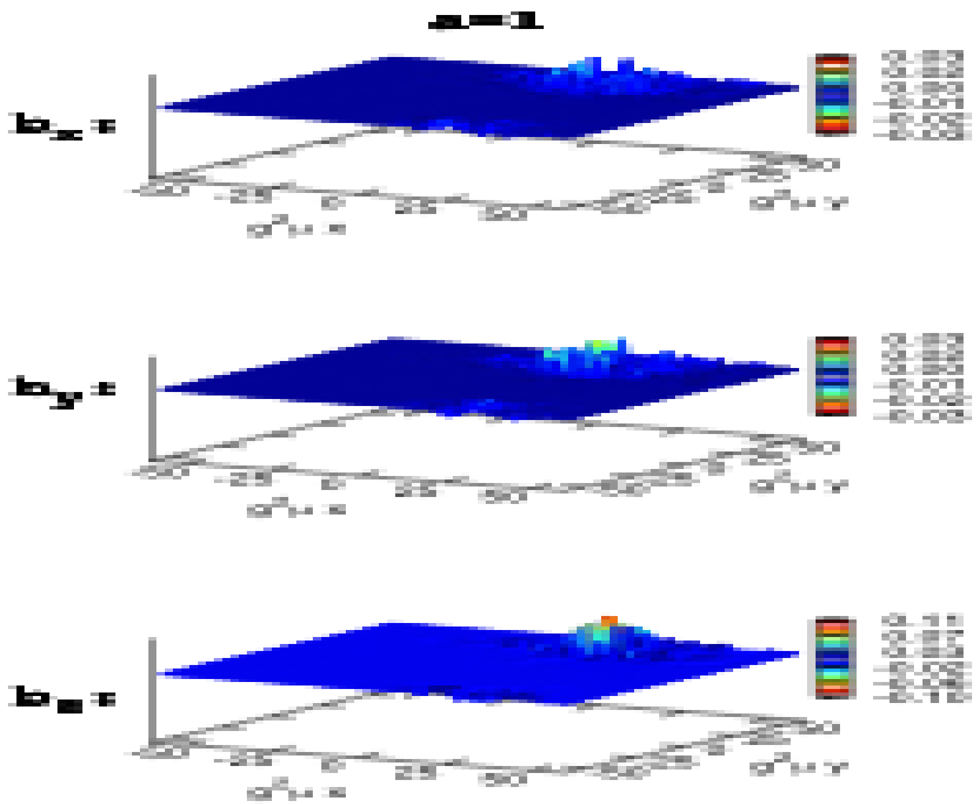


ii) Construct gauge field configurations at initial time step:

$$A^\mu(x_\perp,\eta,\tau_0) = \mathcal{A}^\mu(x_\perp,\tau_0) + \int d\mu_K \left(c_K\, a^\mu_K + c^\star_K\, a^{\mu\,\star}_K\right)$$

iii) Solve 3+1-D Yang-Mills equations for each element of random Gaussian ensemble c_K to determine energy density and pressure tensor

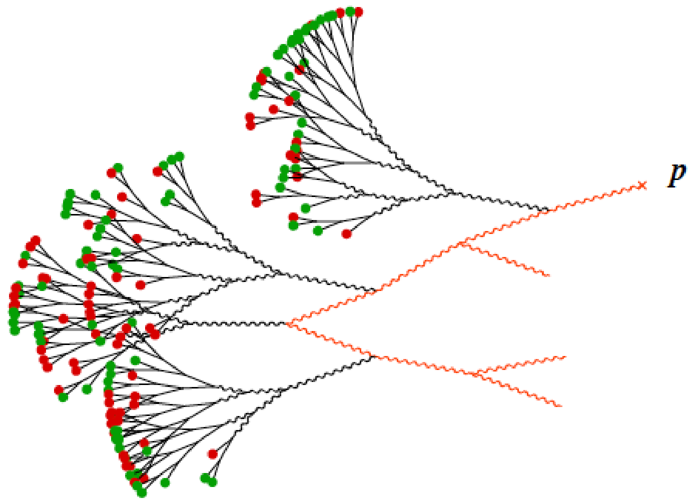
QCD: Real time evolution of quantum fluctuations



Preliminary results for small fluctuation spectrum…

Dusling,Epelbaum,Gelis,RV: in progress

The initial shower in a classical background



The resummed gluon spectrum corresponds to a "parton shower" which is qualitatively different from the usual pQCD vacuum shower – what's its contribution to jet quenching ?

The rapid growth also generates sphaleron transitions -- providing an ab initio mechanism for the chiral magnetic effect at early times

Autocorrelations of energy-momentum tensor enable extraction of "anomalous" transport coefficients

Summary

- Presented *ab initio* picture of multi-particle production and evolution in heavy ion collisions

- The paradigm is classical but quantum fluctuations play an essential role.

- Rapid decoherence of classical fields occurs after the collision. For a conformal theory, this generates an EOS, making hydrodynamics applicable at early times

- Efficient flow requires isotropy (or scaling solutions with fixed anisotropy), not thermalization

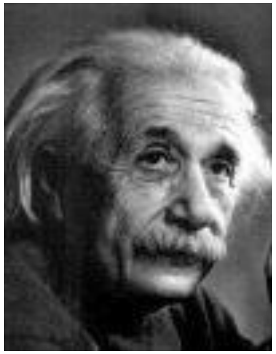
- The separation of scales (electric and magnetic screening) required for thermalization can be generated on much longer time scales.

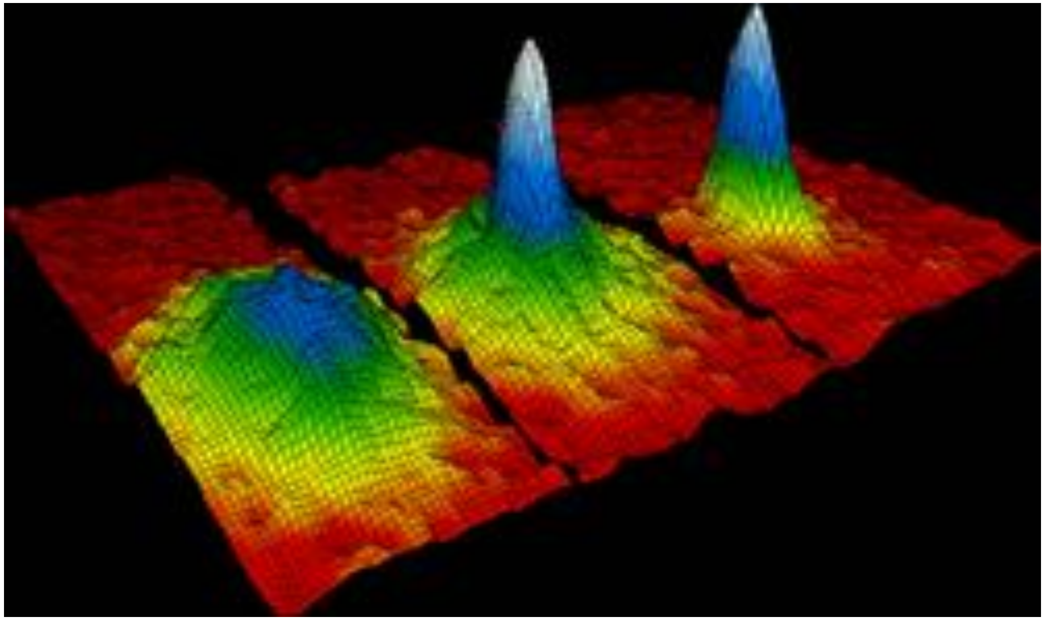
(See Liao's talk. Also work by Kurkela and Moore, and by Schlicting)

THE END

Bose-Einstein Condensation in HI Collisions ?

Blaizot,Gelis,Liao,McLerran,RV: arXiv:1107.5295v2





Cold rubidium atoms in a magnetic trap

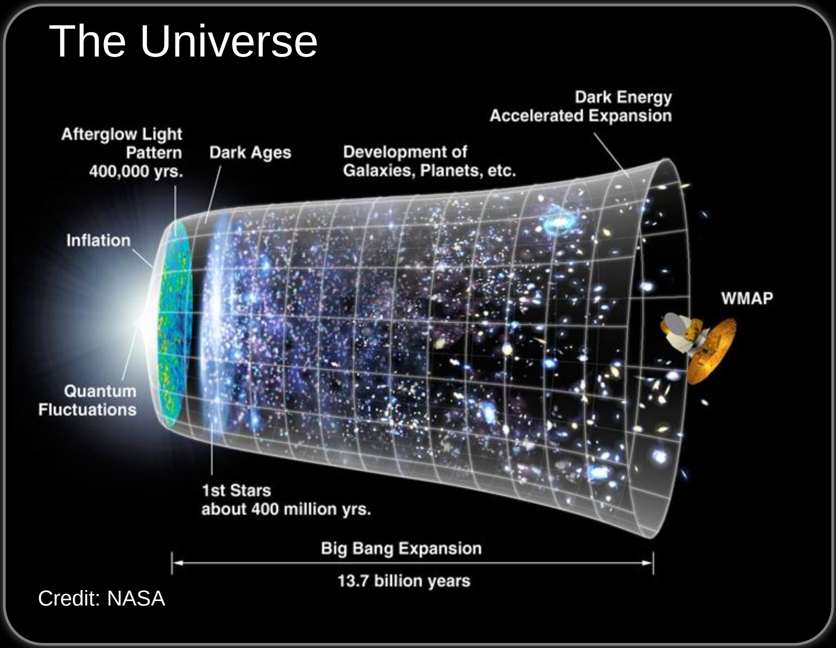
Gell-Mann's Totalitarian Principle of Quantum Mechanics:
Everything that is not forbidden is Compulsory

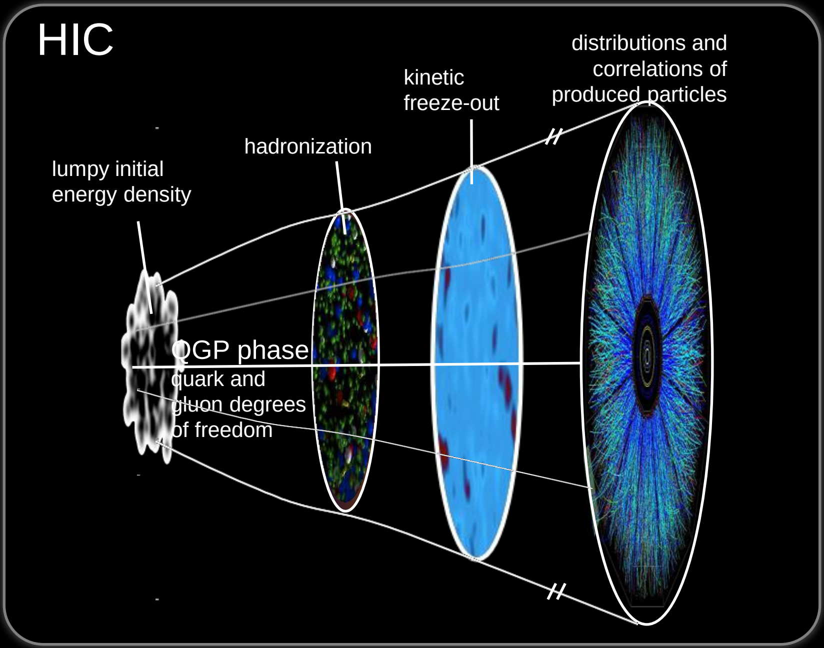
✧ ***Possible phenomenological consequences…***

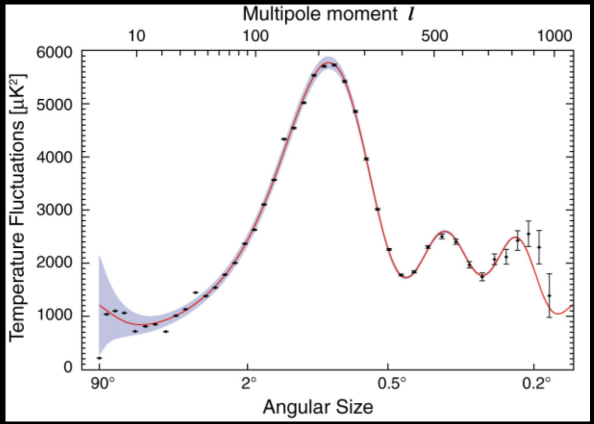
Mickey Chiu et al., 1202.3679

An Analogy with the Early Universe

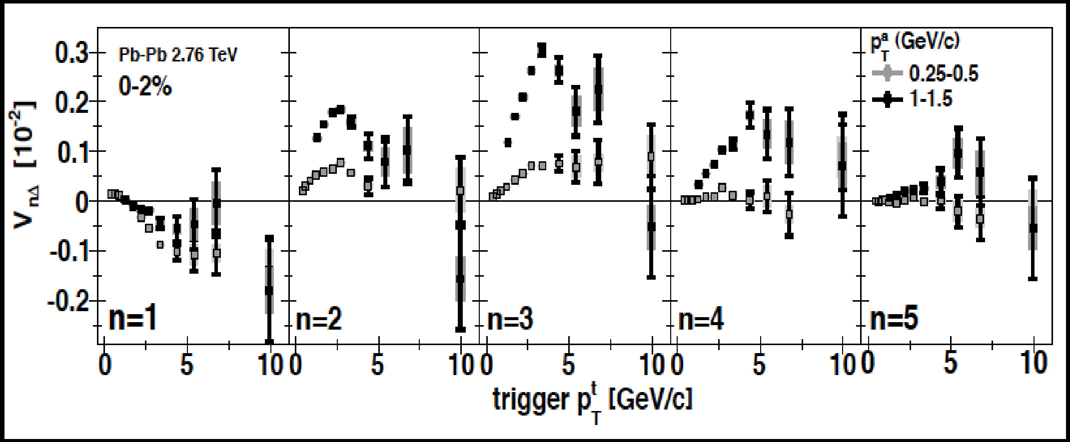
Mishra et al; Mocsy- Sorensen







WMAP

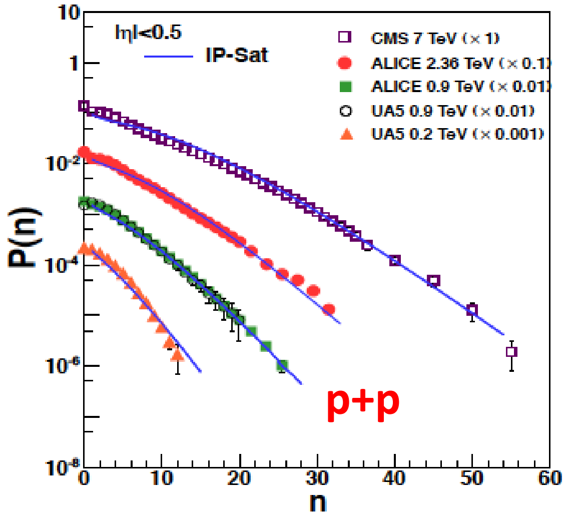


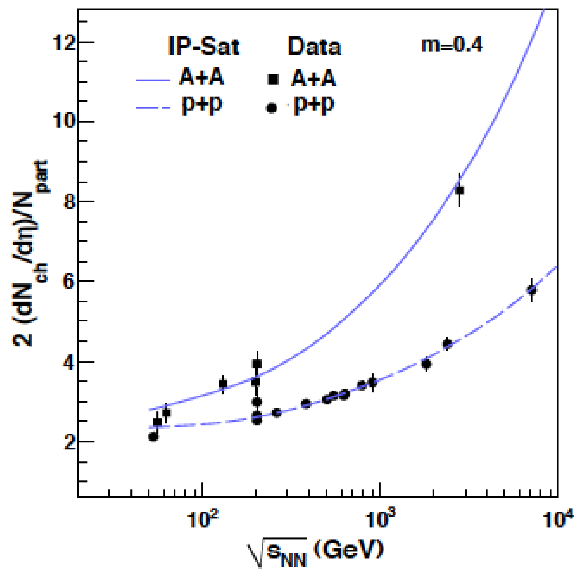
HIC-ALICE

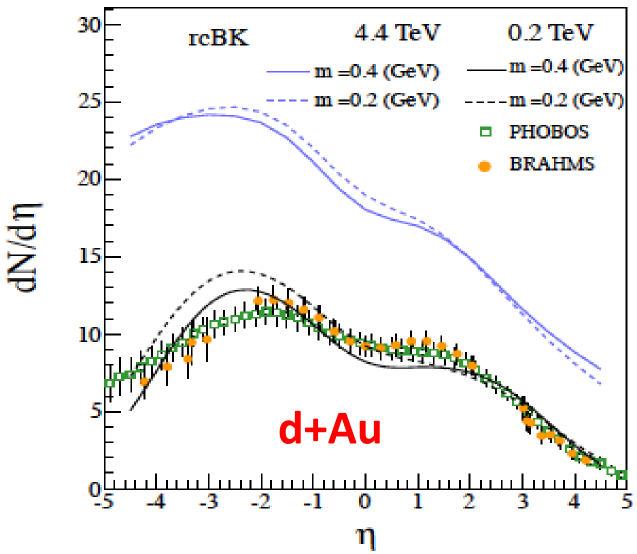
CGC based models and bulk distributions

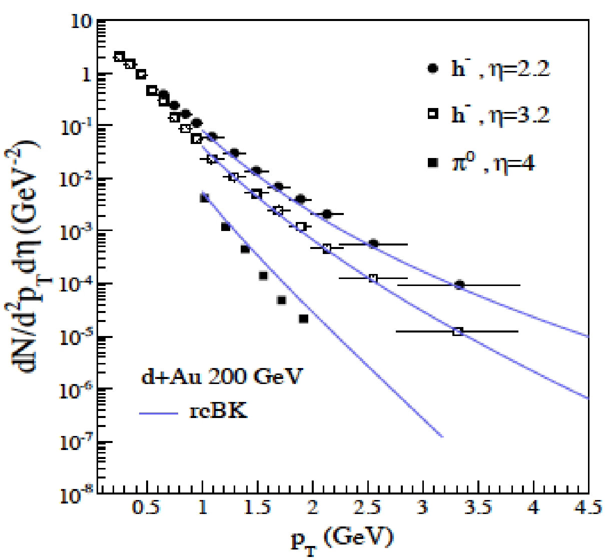
Kowalski, Motyka, Watt
Tribedy, RV: 1112.2445

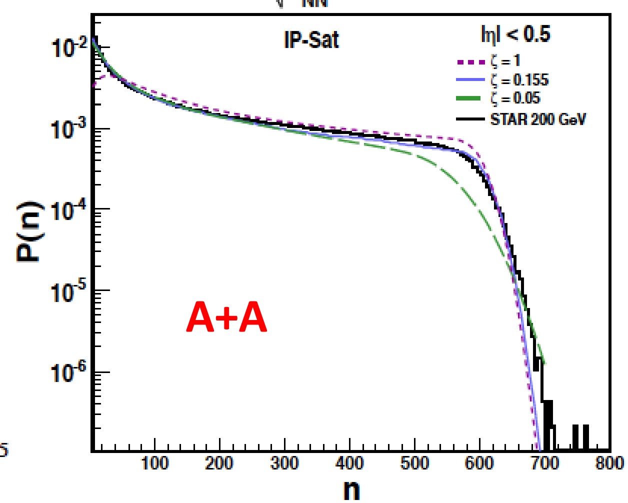
e+p constrained fits give good description of hadron data











Also:

Collimated long range Rapidity correlations "the ridge"

Di-hadron d+A correlations