THERMAL MESON PROPERTIES AND CHIRAL SYMMETRY

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Recent and ongoing work on thermal meson properties relevant for the hadron gas regime include transport coefficients, chemical nonequilibrium, susceptibilities, isospin breaking and different aspects of chiral symmetry restoration. Chiral Perturbation Theory ensures the model independency of the low-T regime. This is combined with unitarity when an accurate description of particle scattering is needed, as in the case of resonance studies and transport coefficients. Results agree with several phenomenological, theoretical and lattice analysis.

TRANSPORT COEFFICIENTS FOR A PION GAS IN CHPT

- **Kubo formula** for current-current spectral function:

$$\mathrm{Im}\ (\omega\to 0^+,\vec 0) \longrightarrow G^R(\omega',|\vec p|)G^A(\omega',|\vec p|)\approx \frac{\pi}{2E_p\Gamma_p}\delta(\omega'^2-E_p^2)$$

$0\neq\Gamma_p << E_p$ $\qquad E_p^2=|\vec p|^2+m_\pi^2$

Chiral Power Counting for TC modified:

- **Lines with same 4-momentum (doubled-lined) prop to inverse pion width:**

$$Y\sim\frac{n_B(E)}{E_p\Gamma_p} \quad (\text{instead of ChPT } O(p^2))$$

- **Diagrams** potentially dominant:

Naively:

Bubbles $\sim O(Y^{k+1})$

$$\alpha_{bub}^{(k)}=O\left(\frac{\omega}{\Gamma}+p^2\right)^k \overset{\omega\to 0^+}{=} O(Yp^{2k})$$

Ladders $\sim O(Y^{k+1}p^{2k})$

$T\ll m_\pi$: relevant three momentum $p\approx\sqrt{m_\pi T}$

$\tilde n_\pi\sim(\sqrt{m_\pi T})^3 e^{-m_\pi/T}\ll 1$

$\Gamma_p\sim\sigma_{\pi\pi}v_\pi\tilde n_\pi\sim\sqrt{T}\tilde n_\pi$

$Y\sim\sqrt{\frac{m_\pi}{T}}$ **potentially large**

loop spectral function $\sim\frac{p_{CM}}{m_\pi}\sim\frac{\sqrt{m_\pi T}}{m_\pi}=\frac{1}{Y}$

Effective ladder vertex

***T* dependence factorizes**

$$\alpha_{lad}^{(k)}=\alpha^{(0)}(T)O(p^{2k})=O(Yp^{2k})$$

Ladders ChPT perturbative

$T\simeq m_\pi$: relevant $p\approx T\Rightarrow$ unitarized scattering in Γ relevant

$\sigma_U^{\pi\pi}$ softer in $\Gamma_U\Rightarrow$ larger T effects in TC

Ladders still perturbative $Y\sim O(1)$

Higher T behaviour signals ChPT break-up and ladder resummation

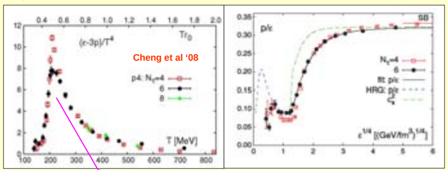
Bulk Viscosity

$J\to -T_k^k/3-c_s^2T_{00}\qquad c_s^2=\partial P/\partial\epsilon$

- Related to QCD trace anomaly $\langle\theta\rangle\equiv\langle T^\mu_\mu\rangle=\epsilon-3P=T^5\frac{d}{dT}\left(\frac{P}{T^4}\right)$

suggested by finite-*T* sum rules (plus spectral function assumptions). Shushpanov,Kapusta,Ellis '99; Karsch, Kharzeev, Tuchin '08; Romatschke,Son '09

Expected to grow near phase trans. (conformal sym.breaking)



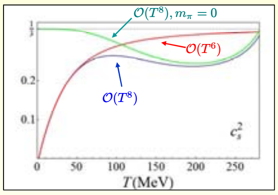
Gluon dominated: $(T^\mu_\mu)_{QCD}=\frac{\beta(g)}{2g}G^a_{\mu\nu}G^{\mu\nu}_a+(1+\gamma_m(g))\bar qMq$

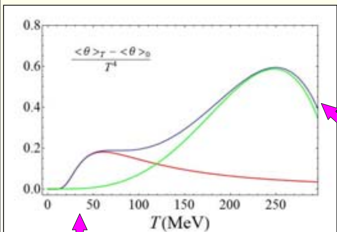
- **In ChPT:**

$$\zeta(T)=\int_0^\infty dp\frac{3p^2(p^2/3-c_s^2E_p^2)^2}{4\pi^2TE_p^2\Gamma_p}n_B(E_p)[1+n_B(E_p)]$$

- **Pion gas in chemical nonequilibrium between CFO and TFO (elastic scattering only)**
- **Chiral limit:** $\zeta=15(1/3-c_s^2)^2\eta\overset{T\gg m_\pi}{\longrightarrow}0$ **(conformal limit w/o anomaly)**

But ChPT to $\mathcal{O}(T^8)$ sensitive to c_s^2 anomalous minimum

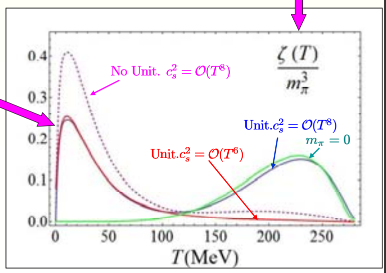




Conformal peak unchanged in chiral limit (not related to χSR, fermions suppressed)

Low-*T* peak from fermion mass conformal breaking

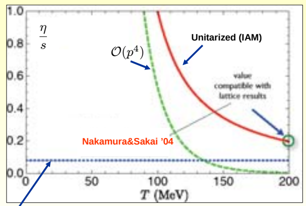
$\delta\langle\theta\rangle_T\sim 2m_q\delta\langle\bar qq\rangle_T$



Strong correlation between bulk visco and trace anomaly established within ChPT (no further assumptions)

- **A.Gómez Nicola, J.R.Peláez, J.Ruiz de Elvira,** *Scalar susceptibilities and four-quark condensates in the meson gas*, **in prep 2012.**
- **A.Gómez Nicola, R.Torres Andrés,** *Isospin breaking and Chiral Symmetry Restoration*, **Phys.Rev.D83:076005, 2011.**
- **D.Fernández-Fraile, A.Gómez Nicola,** *Transport coefficients and resonances for a meson gas in Chiral Perturbation Theory*, **Eur.Phys.J. C62:37-54, 2009.**
- **D.Fernández-Fraile, A.Gómez Nicola,** *Bulk viscosity and the conformal anomaly in the pion gas*, **Phys.Rev.Lett.102:121601,2009.**
- **D.Fernández-Fraile, A.Gómez Nicola,** *The electrical conductivity of a pion gas*, **Phys.Rev.D73:045025,2006.**

Shear Viscosity



$\frac{1}{4\pi}$ AdS/CFT bound (Kotvun,Son,Starinets 05)

$J\to T_{ij}-g_{ij}T^k_k/3$

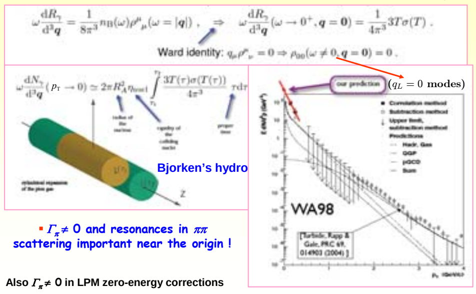
- **Elliptic flow data require "almost perfect" fluid with** $\eta/s\lesssim 0.5$

Romatschke& Romatschke '07; Dusling&Teaney '08; Song&Heinz '08

- **Compatible with analysis based on KT** (Dobado et al '04, Prakash et al '93)
- **Minimum of η/s expected at phase trans:** $\mathcal{O}(N_c^2)$ and decreasing at low T; $\mathcal{O}(1)$ and increasing at high T — Arnold et al

Electrical conductivity and photons

- **DC cond. related to soft-photon emitted by the meson (pion) gas:** $J\to J_i^{EM}$



- **$\Gamma_\pi\neq 0$ and resonances in $\pi\pi$ scattering important near the origin !**

Also $\Gamma_\pi\neq 0$ in LPM zero-energy corrections — W.Liu, R.Rapp '07

$T\ll m_\pi$ limit for TC compatible with NR Kinetic Theory:

- **Electrical Conductivity:**

$$\sigma(T)\cong 15e^2\frac{f_\pi^4}{m_\pi^{5/2}\sqrt{T}}\approx e^2\frac{N_\pi\lambda}{m_\pi}\ \text{with mean free path } \lambda\approx 1/\Gamma$$

$$\Gamma\approx n_\pi v_\pi\sigma^{\pi\pi}\approx n_\pi\left(\frac{\sqrt{m_\pi T}}{m_\pi}\right)\left(\frac{m_\pi^2}{f_\pi^4}\right)$$

- **Shear and bulk viscosities:**

$$\zeta(T)\simeq 13.3\frac{f_\pi^4\sqrt{T}}{m_\pi^{3/2}}\sim 0.36\eta(T)\approx m_\pi v_\pi n_\pi\lambda$$

SUSCEPTIBILITIES, ISOSPIN AND CHIRAL SYMMETRY

- Conn/Disc separation of **Scalar susceptibility** for n_f light flavors:

$$\chi_S\equiv-\frac{\partial}{\partial\hat m}\langle\bar qq\rangle_T=\frac{1}{\beta V}\frac{\partial^2}{\partial\hat m^2}\log Z_{QCD}=n_f^2\ \chi_{dis}+n_f\ \chi_{con}\quad(IL)$$

$\chi_{dis}=\langle(\mathrm{Tr}D^{-1})^2\rangle_A-\langle\mathrm{Tr}D^{-1}\rangle_A^2 \qquad \chi_{con}=-\langle\mathrm{Tr}D^{-2}\rangle_A$

Fluctuations of $\langle\bar qq\rangle$. More sensitive to chiral restoration, maximum near critical region

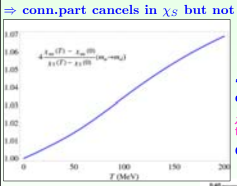
- $m_u\neq m_d$ allows to separate χ_{con}/χ_{dis} in the effective theory.
- ChPT provides their **model-independent** T, m_q behaviour, relevant e.g. for scaling lattice results near $(T,m_q)\sim(T_c,0^+)$.
- For $n_f=2$ and $m_u\neq m_d$: $\hat m=\frac{m_u+m_d}{2},\ m_\delta=\frac{m_u-m_d}{2}$

$$\begin{aligned}\chi_S&=\chi_{uu}+\chi_{dd}+2\chi_{ud}=-\partial_{\hat m}\langle\bar uu+\bar dd\rangle_T &&\text{Isosinglet }(\sigma)\\ \chi_{con}&=\frac{1}{2}(\chi_{uu}+\chi_{dd})-\chi_{ud}=-\frac{1}{2}\partial_{m_\delta}\langle\bar uu-\bar dd\rangle_T &&\text{Isotriplet }(a_0)\\ \chi_{dis}&=\chi_{ud}\end{aligned}$$

Sensitive to $\mathcal{O}(m_\delta)$ corrections $\Rightarrow\chi_{con}\neq 0$ in the isospin limit contrary to naive expectation $\chi_{uu}=\chi_{dd}=\chi_{ud}$.

Actually, taking the IL *after* diff $\rightarrow\begin{cases}\chi_{uu}\simeq\chi_{dd}\simeq\chi_S/4-\chi_{con}/2\\ \chi_{ud}\simeq\chi_S/4+\chi_{con}/2\end{cases}$

$\Rightarrow$ conn.part cancels in χ_S but not in the one-flavour equivalent $4\chi_{uu}$.

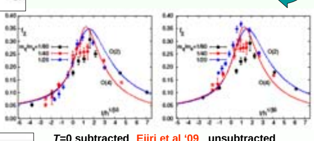


Up to 30% corrections for $4\chi_{uu}/\chi_S$ at $T=0$ coming from χ_{con}

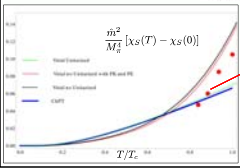
"Connected contamination" in the definition may explain (partly) *O(N)* lattice scaling violations

χ_{con} gets contributions from "false" GB-loops from extra tastes in staggered formalism

Condensates OK with scaling



***T*=0 subtracted** Ejiri et al '09 **unsubtracted**



Lattice points — Aoki et al '09

OK with lattice when normalizing to respective T_c

$\chi_{con}(T)$, $\chi_{dis}(T)$ **explicit expressions found in LO ChPT for $\hat m\neq 0$, $N_f=2+1$, including $m_u\neq m_d$ and EM corrections (virtual photons)**

Relevant T behaviour can be approximated by the IR/Chiral + Isospin limit:

$M_\pi\ll T\ll M_K,M_\eta$ and $m_u\to m_d$ *after* differentation

$$\frac{4\hat m^2}{M_\pi^4}\chi_{dis}^{IR}(T=0)=-\frac{3}{32\pi^2}\log\frac{M_\pi^2}{\mu^2}+32L_6^r(\mu)+\frac{1}{288\pi^2}\left(-28+5\log\frac{M_\eta^2}{\mu^2}\right)$$

$$\frac{4\hat m^2}{M_\pi^4}\chi_{con}^{IR}(T=0)=8\left[H_2^r(\mu)+2L_8^r(\mu)\right]-\frac{1}{16\pi^2}\left(1+\log\frac{M_K^2}{\mu^2}+\frac{2}{3}\log\frac{M_\eta^2}{\mu^2}\right)$$

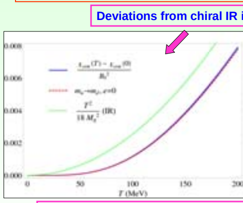
$$\frac{4\hat m^2}{M_\pi^4}[\chi_{dis}(T)-\chi_{dis}(0)]^{IR}=\frac{3T}{16\pi M_\pi}\quad\textbf{Critical}$$

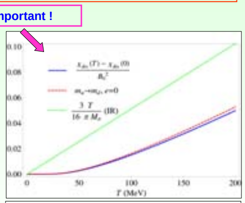
Divergent part in χ_{dis} OK with Smilga, Verbaarschot '96

$$\frac{4\hat m^2}{M_\pi^4}[\chi_{con}(T)-\chi_{con}(0)]^{IR}=\frac{T^2}{18M_\eta^2}\quad\textbf{Regular}$$

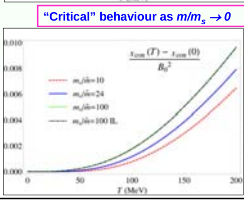
Larger scale, comes from IB $\pi^0\eta$ mixing

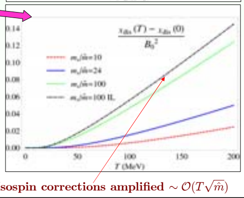
Deviations from chiral IR important !





"Critical" behaviour as *m/m_s* → 0





Isospin corrections amplified $\sim\mathcal{O}(T\sqrt{\hat m})$

Pseudoscalar/Scalar Degeneration @ Chiral Symmetry Restoration

- Pseudoscalar Susceptibility $\rightarrow\int_0^\beta\int d^3\vec x\ \langle\mathcal{T}(\bar q\gamma_5\tau^a q)(x)\,(\bar q\gamma_5\tau^b q)(0)\rangle=\chi_P\delta^{ab}$
- From Current Algebra (LO ChPT, $T=0$):

$$\chi_P=4B_0^2\frac{f_\pi^2}{M_\pi^2}=\frac{\langle\bar qq\rangle}{m_q}\quad\text{whereas, e.g. from L}\sigma\text{M, }\chi_S=4B_0^2\frac{f_\pi^2}{M_\sigma^2}$$

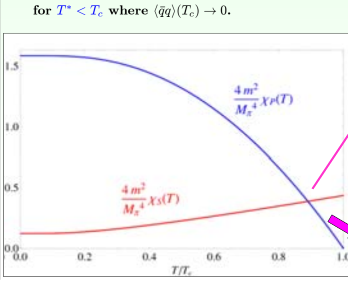
Pion pole $\qquad$ Sigma pole

- NLO (incl T corrections) ChPT Model-independent result: (coupling external p^a sources)

$$\frac{\chi_P(T)}{\chi_P(0)}=\frac{\langle\bar qq\rangle(T)}{\langle\bar qq\rangle(0)}$$

$\Rightarrow$ Drops as the condensate.
Same behaviour obtained in nuclear matter $\chi_P(\rho)\sim\langle\bar qq\rangle(\rho)$ (Chanfray&Ericson 2003).

$\Rightarrow$ Since $\chi_S(T)$ grows, chiral S/P partner degeneration $\chi_S(T^*)=\chi_P(T^*)$ for $T^*<T_c$ where $\langle\bar qq\rangle(T_c)\to 0$.



$T^*\sim 0.9T_c$ for phys.pions

$T^*\simeq T_c-\frac{3M_\pi}{4\pi}$ (for $M_\pi\ll T$)

$T^*\to T_c$ in chiral (IR) limit (exact chiral sym.rest.)

Possible lattice signal ? (clearer than $\langle\bar qq\rangle$ e.g. for crossover)