Shear Viscosity in NJL-type Models

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Motivation

Heavy-ion collisions at RHIC and LHC are one possible approach to explore the QCD phase diagram experimentally. In such collisions one probes the phase transition between confined hadronic matter (Polyakov loop \( (\Phi) \neq 0 \)) and deconfined quarkonic matter \( (\Phi) = 0 \). At the same time this a transition between phases where chiral symmetry is spontaneously broken \( (\langle \psi \bar{\psi} \rangle) \neq 0 \) and restored \( (\langle \psi \bar{\psi} \rangle \to 0) \), respectively.

![Figure: Sketch of the QCD phase diagram](image)

Comparisons between hydrodynamic simulations and the measured elliptic flow \( \eta \) indicate that the matter produced in such collisions behaves as an almost-perfect fluid. Its dissipative character can be treated perturbatively. In particular, for many different physical systems the ratio \( \eta/s \) has been found to be minimal at their phase transitions. The matter produced at RHIC features the lowest value for \( \eta/s \) measured so far.

![Figure: Temperature-dependent ratio \( \eta/s \) for different systems (left) [1]. Comparison between hydrodynamic simulations and RHIC data for the elliptic flow \( \eta_2 \) (right) [2].](image)

Viscous parameters and Kubo formula

For systems close to thermal equilibrium, the energy momentum tensor reads

\[
T^{\mu\nu} = u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu} + \tau^{\mu\nu},
\]

where the dissipative tensor \( \tau^{\mu\nu} \) is parametrized in this local equilibrium by two dissipation coefficients, the shear viscosity \( \eta \) and the bulk viscosity \( \zeta \).

\[
\tau^{\mu\nu} = \eta \left( \partial_\mu u_\nu + \partial_\nu u_\mu - \frac{2}{3} \delta^{\mu\nu} (\partial_\lambda u_\lambda) \right) + \zeta \Delta^{\mu\nu} (\partial_\lambda u_\lambda).
\]

The heat conductivity, \( \kappa \), compared to \( \eta \) and \( \zeta \), is suppressed for small chemical potentials and can be neglected for \( \mu/T < 1 \). A general treatment of these dissipation coefficients leads to the Kubo-type formulas [3] which relate them to retarded correlators, calculable in thermal quantum field theory [4]:

\[
\eta(E, \ell) = -\frac{1}{10} \int d^3 \ell' \int d^4 x \partial_{x'} \left( \langle \tau^{\mu\nu}(x', \ell') \tau_{\mu\nu}(x', \ell) \rangle \right)_{\text{ret}},
\]

with the viscous-stress tensor \( \tau_{\mu\nu} = (\Delta_{\alpha\nu} u_\alpha - \Delta_{\alpha\mu} u_\alpha) T^{\mu\nu} \), where \( \Delta_{\alpha\nu} = g_{\alpha\nu} - u_\alpha u_\nu \) is a projection operator, orthogonal to the four velocity: \( \Delta_{\alpha\nu} u^\alpha = 0 \).

![Figure: Phase space boundaries](image)

Skeleton expansion

The calculation of \( \eta \) for a given field theory requires the evaluation of the retarded \( \tau_{\mu\nu} \)-correlator which is essentially a four-point function. The naive one-loop approximation, a product of two Matsubara propagators, has been shown to be incomplete, even in the small-coupling limit. A consistent treatment requires in scalar \( \phi^4 \) theory a resummation of ladder diagrams [5]. However, at one-loop level in the fermionic, non-perturbative NJL model, the evaluation of the \( \tau_{\mu\nu} \)-correlator leads to [6]:

\[
\eta = \frac{G N_c}{16 \pi^2 T} \int_0^\infty \frac{dp}{2\pi} \frac{np_\perp^2}{2T} \left( 2E_p^4 G(p) + 4m_q^2 G(p) \right).
\]

![Figure: Parameter study of the shear viscosity \( \eta \) for different values of the momentum-independent spectral width \( \Gamma \) and its dependence on temperature (left) and on chemical potential (right).](image)

The NJL model in the light of a large-\( N_c \) expansion

All NJL-type models imitate the symmetry pattern of QCD [7], in particular the chiral symmetry and its symmetry breaking effects. We consider the NJL Lagrangian with two quark-flavors \( (\psi, \bar{\psi}) \) denoting the isospin Pauli matrices:

\[
\mathcal{L}_{Lagr} = \bar{\psi} \left( i \gamma_5 \gamma_\mu \partial^\mu - m_\psi \right) \psi + \frac{G}{2} \left( \langle \bar{\psi} \psi \rangle^2 + \langle (\bar{\psi} \psi)^2 \rangle \right).
\]

The four-fermion coupling \( G \) is supposed to collect all the gluon dynamics within one constant and therefore scales as \( G \approx 1/N_c \). A scaling analysis [8] leads at leading order in large-\( 1/N_c \) expansion to the gap equation (of quarks) in Hartree approximation:

\[
\langle \bar{\psi} \psi \rangle = \frac{m_\psi}{2G} + \frac{m_\psi}{2G} \mathcal{O}(1),
\]

The mesonic degrees of freedom arise from a non-perturbative iteration of quark-antiquark loops, resulting in the Bethe-Salpeter equation:

\[
\langle \bar{\psi} \psi \rangle = \frac{m_\psi}{2G} + \frac{m_\psi}{2G} \mathcal{O}(1).
\]

Solving these equations in the isospin limit one derives the \( (T, \mu) \)-dependence of the quark mass and meson (pion) mass.

![Figure: Results at \( T = 0 \) and \( T = 0 \)](image)

Dissipative recombination processes in the NJL model

At leading order in the large-\( N_c \) expansion the spectral width \( \Gamma(p) \) is caused by recombination processes in the heat bath: a quark (antiquark) recombines with an antiquark (quark) to a pion. This is described by cutting the pion-insertion diagram shown adjacent and leads to the following expression:

\[
\Gamma(p) = \frac{1}{2} \left( \Gamma(E_\pi) + \Gamma(E_\pi) \right) \int_{E_{\text{min}}}^{E_{\text{max}}} dE \left[ n_p(E_p) + n_\pi(E_p) \right] \frac{n_\pi(E_p)}{2},
\]

with the phase space boundaries \( E_{\text{max, min}} \) which force the kinematics to \( m_\pi(T) > 2m_q(T) \):

\[
E_{\text{max, min}} = \frac{1}{2} \left( m_\pi^2 - 2m_q^2 \right) \left( m_\pi^2 + p^2 \pm m_q \sqrt{m_\pi^4 - 4m_q^2} \right).
\]

![Figure: Spectral width \( \Gamma(p) \)](image)

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