Shear Viscosity in NJL-type Models

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Motivation

Heavy-ion collisions at RHIC and LHC are one possible approach to explore the QCD phase diagram experimentally. In such collisions one probes the phase transition between confined hadronic matter (Polyakov loop $\langle\Phi\rangle = 0$) and deconfined quarkonic matter ($\langle\Phi\rangle \neq 0$). At the same time this a transition between phases where chiral symmetry is spontanously broken ($\langle\bar\psi\psi\rangle \neq 0$) and restored ($\langle\bar\psi\psi\rangle \to 0$), respectively.

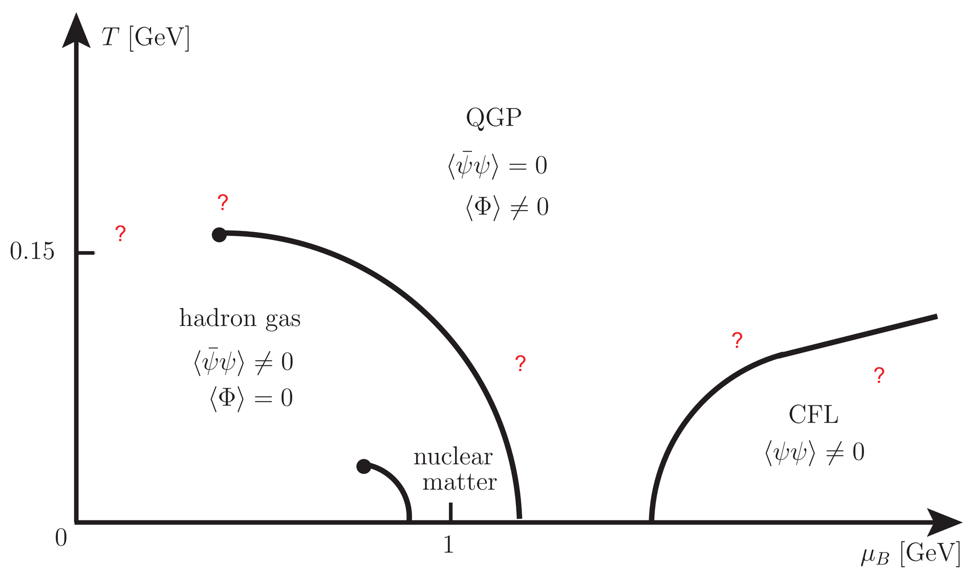


Figure: Sketch of the QCD phase diagram

Comparisons between hydrodynamic simulations and the measured elliptic flow v_2 indicate that the matter produced in such collisions behaves as an almost-perfect fluid. Its dissipative character can be treated perturbatively. In particular, for many different physical systems the ratio η/s has been found to be minimal at their phase transitions. The matter produced at RHIC features the lowest value for η/s measured so far.

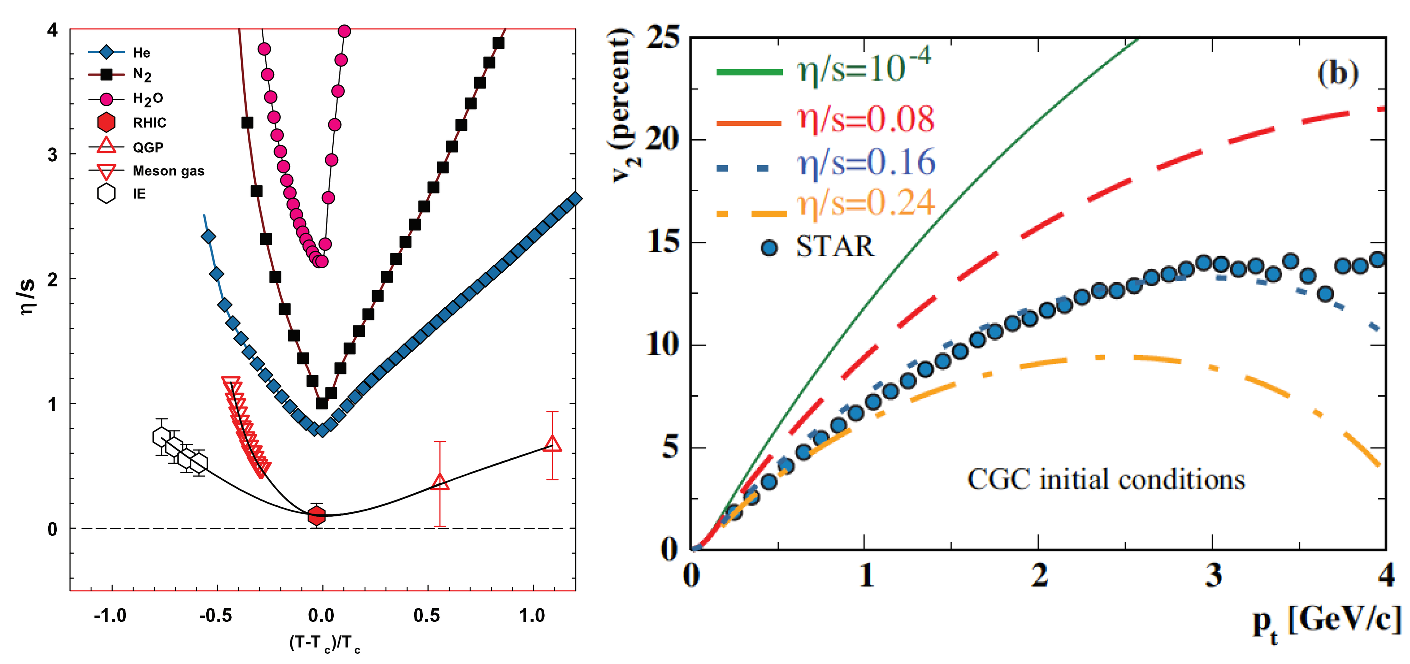


Figure: Temperature-dependent ratio η/s for different systems (left) [1]. Comparison between hydrodynamic simulations and RHIC data for the elliptic flow v_2 (right) [2].

Viscous parameters and Kubo formula

For systems close to thermal equilibrium, the energy momentum tensor reads

$$T^{\mu\nu} = u^\mu u^\nu(\epsilon + P) - P g^{\mu\nu} + \tau^{\mu\nu} \, ,$$

where the dissipative tensor $\tau^{\mu\nu}$ is parametrized in this local equilibrium by two dissipation coefficients, the shear viscosity, η, and the bulk viscosity, ζ:

$$\tau^{\mu\nu} = \eta\left[\partial^\mu_\perp u^\nu + \partial^\nu_\perp u^\mu - \frac{2}{3}\Delta^{\mu\nu}(\partial_\perp \cdot u)\right] + \zeta\Delta^{\mu\nu}(\partial_\perp\cdot u) \, .$$

The heat conductivity, κ, compared to η and ζ, is suppressed for small chemical potentials and can be neglected for $\mu/T \ll 1$. A general treatment of these dissipation coefficients leads to the Kubo-type formulas [3] which relate them to retarded correlators, calculable in thermal quantum field theory [4]:

$$\eta(\vec{x},t) = -\frac{1}{10}\int \mathrm{d}^3\vec{x}\,'\int_{-\infty}^{t}\mathrm{d}t'\int_{-\infty}^{t'}\mathrm{d}\tilde{t}\,\langle\pi^{\mu\nu}(\vec{x},t),\pi_{\mu\nu}(\vec{x}\,',\tilde{t})\rangle_{\text{ret}} \, ,$$

with the viscous-stress tensor $\pi_{\mu\nu} = \left(\Delta_{\mu\nu}\Delta_{\rho\sigma} - \frac{1}{3}\Delta_{\mu\rho}\Delta_{\nu\sigma}\right)T^{\rho\sigma}$, where $\Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$ is a projection operator, orthogonal to the four velocity: $\Delta_{\mu\nu}u^\mu = 0$.

Skeleton expansion

The calculation of η for a given field theory requires the evaluation of the retarded $\pi\pi$-correlator which is essentially a four-point function. The naive one-loop approximation, a product of two Matsubara propagators, has been shown to be incomplete, even in the small-coupling limit. A consistent treatment requires in scalar ϕ^4 theory a resummation of ladder diagrams [5]. However, at one-loop level in the fermionic, non-perturbative NJL model, the evaluation of the $\pi\pi$-correlator leads to [6]:

$$\eta = \frac{64N_c N_f}{15\pi T}\int_{-\infty}^{\infty}\frac{\mathrm{d}\epsilon}{2\pi}\int_0^\infty\frac{\mathrm{d}p}{2\pi}\frac{p^6 m_q^2\,\Gamma^2(p)\,n_F(\epsilon)[1-n_F(\epsilon)]}{\left[\left(\epsilon^2 - p^2 - m_q^2 + \Gamma^2(p)\right)^2 + 4m_q^2\Gamma^2(p)\right]^2}$$

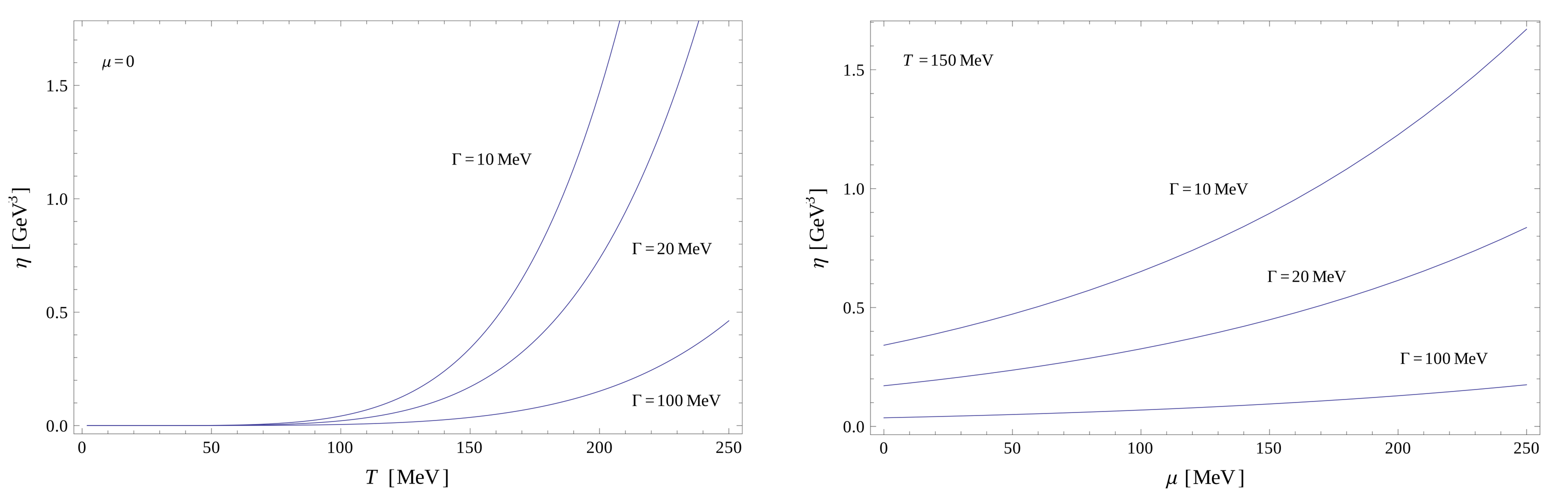


Figure: Parameter study of the shear viscosity η for different values of the momentum-independent spectral width Γ and its dependence on temperature (left) and on chemical potential (right).

The NJL model in the light of a large-N_c expansion

All NJL-type models imitate the symmetry pattern of QCD [7], in particular the chiral symmetry and its symmetry breaking effects. We consider the NJL Lagrangian with two quark-flavors ($\vec\tau$ denotes the isospin Pauli matrices):

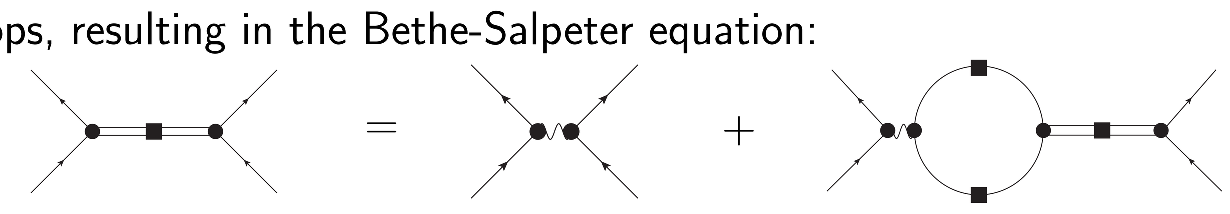
$$\mathcal{L}^{2f}_{\text{NJL}} = \bar\psi\left(i\partial\!\!\!/ - m_0\right)\psi + \frac{G}{2}\left[(\bar\psi\psi)^2 + (\bar\psi i\gamma_5\vec\tau\psi)^2\right] .$$

The four-fermion coupling G is supposed to collect all the gluon dynamics within one constant and therefore scales as $G \sim 1/N_c$. A scaling analysis [8] leads at leading order in large-$1/N_c$ expansion to the gap equation (of quarks) in Hartree approximation:

$\mathcal{O}(1)$:

The mesonic degrees of freedom arise from a non-perturbative iteration of quark-antiquark loops, resulting in the Bethe-Salpeter equation:

$\mathcal{O}(\frac{1}{N_c})$:



Solving these equations in the isospin limit one derives the (T,μ)-dependence of the quark mass and meson (pion) mass.

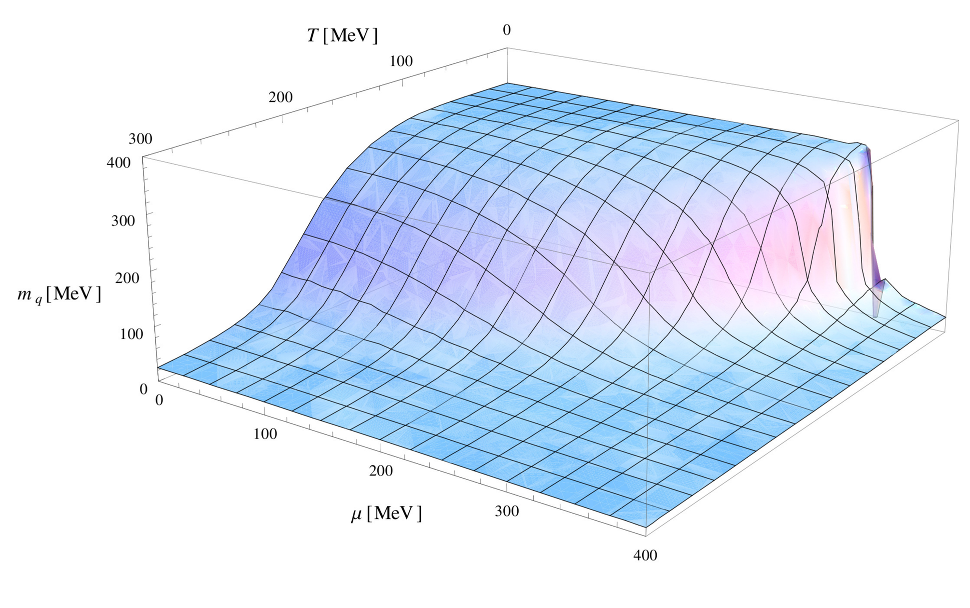


Figure: Quark mass as function of temperature T and chemical potential μ

Input parameters

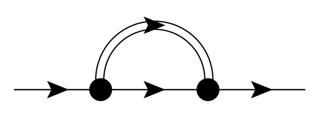
$\Lambda = 651$ MeV
$m_0 = 5.5$ MeV
$G = 10.08\ \text{GeV}^{-2}$

Results at $(T,m) = (0,0)$

$\langle\bar\psi\psi\rangle = (351\ \text{MeV})^3$
$m_q = 326$ MeV
$m_\pi = 144$ MeV
$f_\pi = 94.1$ MeV

Dissipative recombination processes in the NJL model

At leading order in the large-N_c expansion the spectral width $\Gamma(p)$ is caused by recombination processes in the heat beath: a quark (antiquark) recombines with an antiquark (quark) to a pion. This is described by cutting the pion-insertion diagram shown adjacent and leads to the following expression:



$$\Gamma(p) = \frac{1}{2}\left(\Gamma_q(p) + \Gamma_{\bar q}(p)\right) = \frac{m_q g^2_{\pi qq} N_c N_f}{8\pi p}\int_{E_{\min}}^{E_{\max}}\mathrm{d}E_f\left[n_B(E_b) + \frac{n_F^-(E_f) + n_F^+(E_f)}{2}\right]$$

with the phase space boundaries $E_{\max,\min}$ which force the kinematics to $m_\pi(T) > 2m_q(T)$:

$$E_{\max,\min}(p) = \frac{1}{2m_q^2}\left[\left(m_\pi^2 - 2m_q^2\right)\sqrt{m_q^2 + p^2} \pm m_\pi p\sqrt{m_\pi^2 - 4m_q^2}\right] .$$

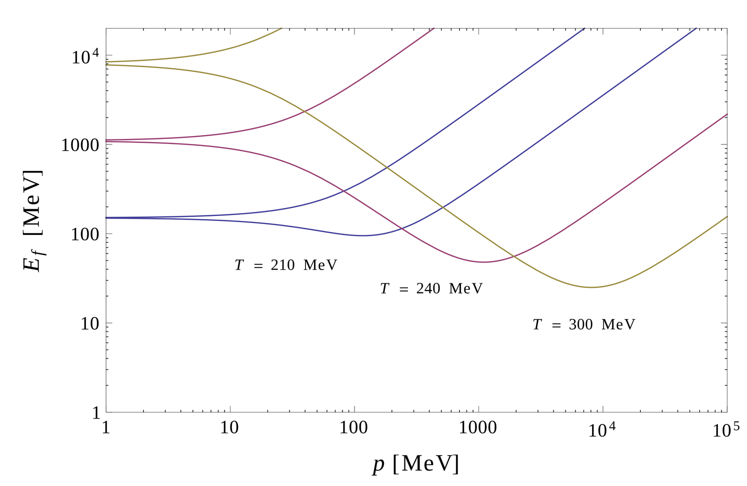


Figure: Phase space boundaries

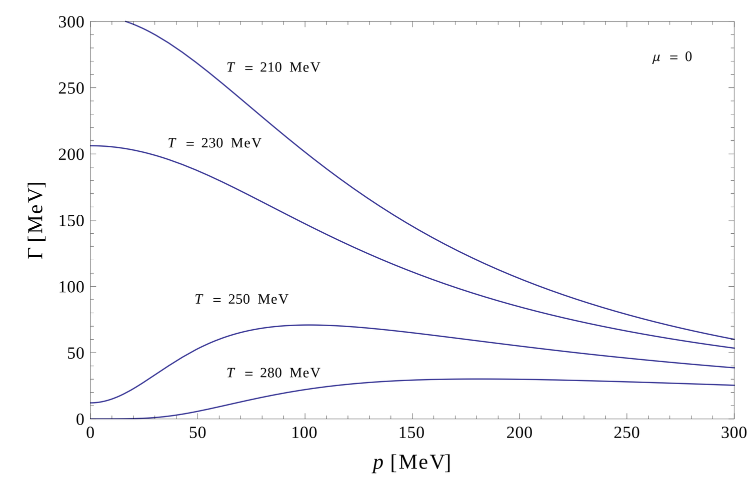


Figure: Spectral width $\Gamma(p)$

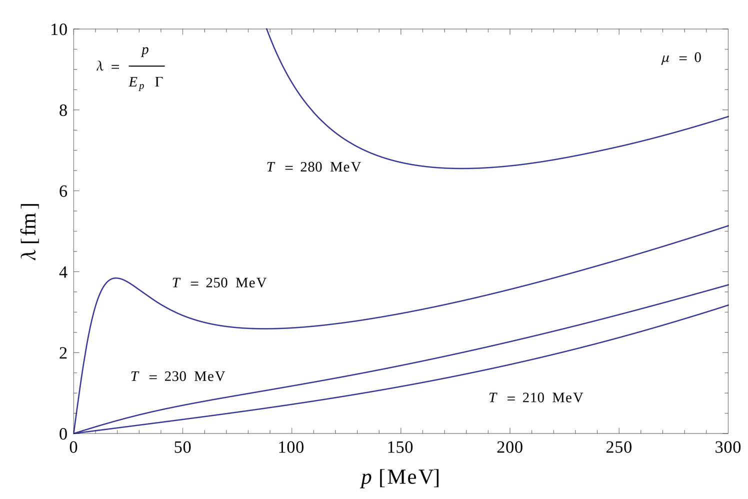


Figure: Resulting mean free path for quarks (or antiquarks) in the heat bath. The temperature-dependent quark mass causes the non-trivial max/min-structures.

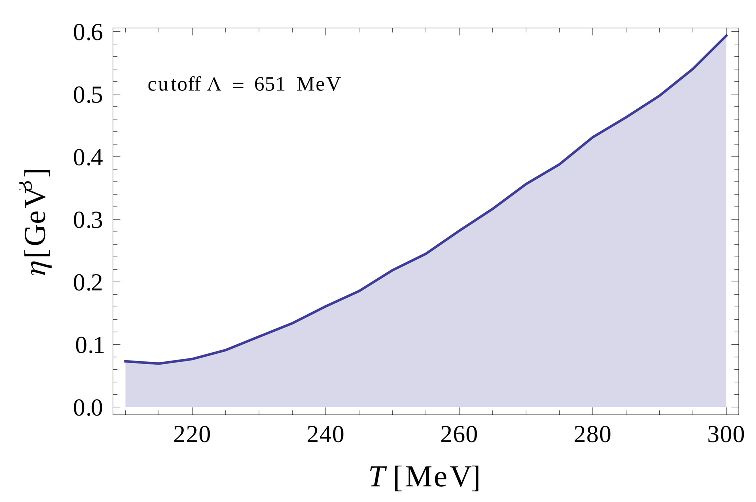


Figure: Temperature dependence of the shear viscosity in the NJL model including the quark-antiquark recombination process described above.

References and Acknowledgement

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We thank the organization committee of the "Quark Matter 2012 Conference" for providing this comfortable environment to discuss and exchange ideas. This work is partially supported by the German Bundesministerium für Bildung und Forschung (BMBF), the TUM Graduate School (TUM-GS), and the DFG Cluster of Excellence "Origin and Structure of the Universe".