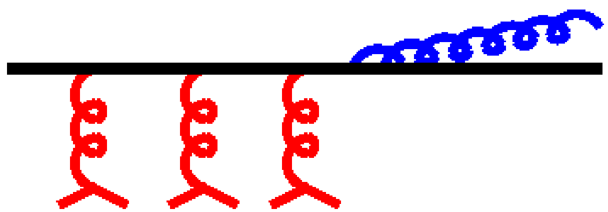
Coupling dependence of “jet” stopping in strongly-coupled gauge theories

Peter Arnold, Philip Szepietowski, and Diana Vaman

arXiv:1203.6658



$\alpha_s(Q_\perp)$

$\alpha_s(T)$ to $\alpha_s(Q_\perp)$

$$Q_\perp \sim (\hat{q}E)^{1/4}$$

typical transverse momentum transfer during formation time

How stopping length scales with energy (massless case)

weak coupling: $\alpha_s \sim \alpha_s$ small $\qquad \ell_{\rm stop} \propto E^{1/2}$ (up to logs)

[this scaling a corollary of BDMPS and Z (1996)]

mixed coupling:

α_s BIG, α_s small $\Big\}$ $\qquad \ell_{\rm stop} \propto E^{1/2}$ (believed)

$\qquad \ell_{\rm stop} \sim \alpha_s^{-1} (E/\hat{q})^{1/2}$

[e.g. related to motivation behind Liu, Rajagopal, Wiedeman (2006)]

all strong coupling: $\alpha_s = \alpha_s$ BIG $\qquad \ell_{\rm stop} \propto E^{1/3}$

($\mathcal{N}=4$ SYM, etc.)

[Gubser, Gollota, Pufu, Rocha; Hatta, Iancu, Mueller; Chesler, Jensen, Karch, Yaffe (2008)]

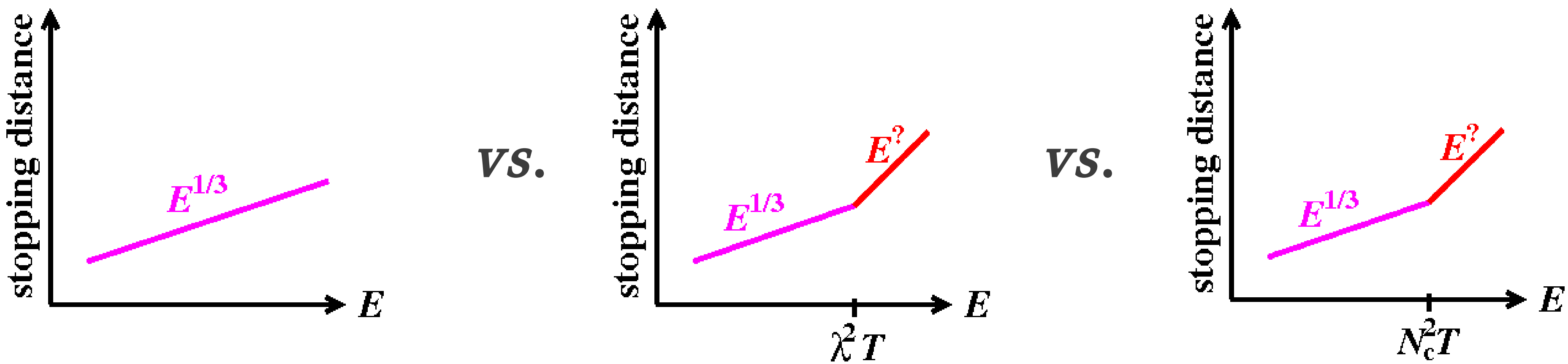
Interesting: Exponent in $\ell_{\rm stop} \propto E^{\nu}$ can depend on α_s .

Caveat: "$\alpha_s = \alpha_s$ BIG" result $\ell_{stop} \propto E^{1/3}$ has only been derived for $N_c = \infty$ and $\lambda = N_c\alpha = \infty$.

What could we learn by also studying N_c and λ BIG but $< \infty$?

Answer: Is the high-energy behavior <u>really</u> $E^{1/3}$?

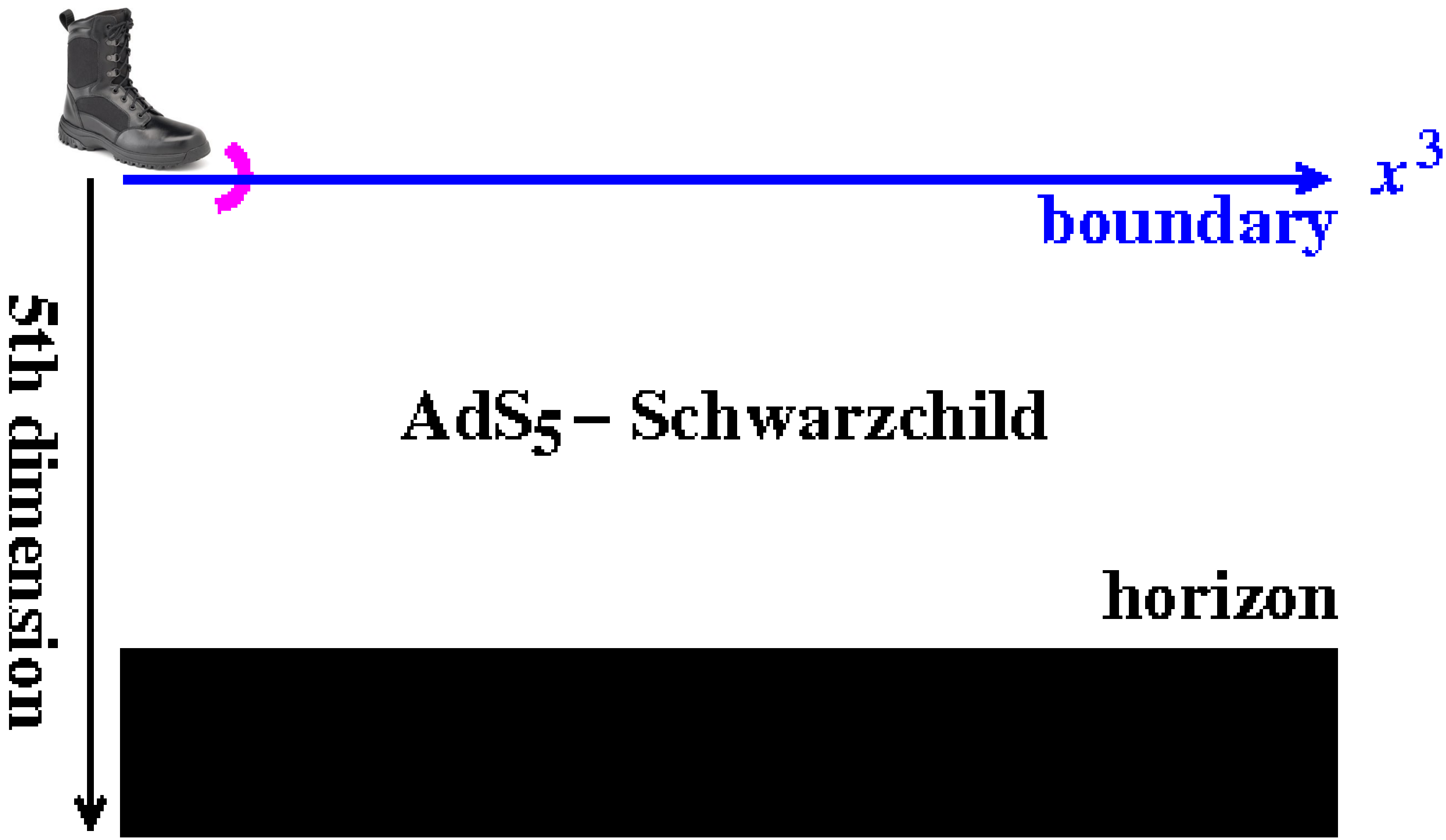
e.g.



<u>This talk</u>: $N_c = \infty$ but large $\lambda < \infty$.

Stopping Distance using AdS/CFT

BIG $\alpha_s = \alpha_s$: Large-N_c $\mathcal{N}$=4 SYM, etc. with $N_c \alpha_s \to \infty$



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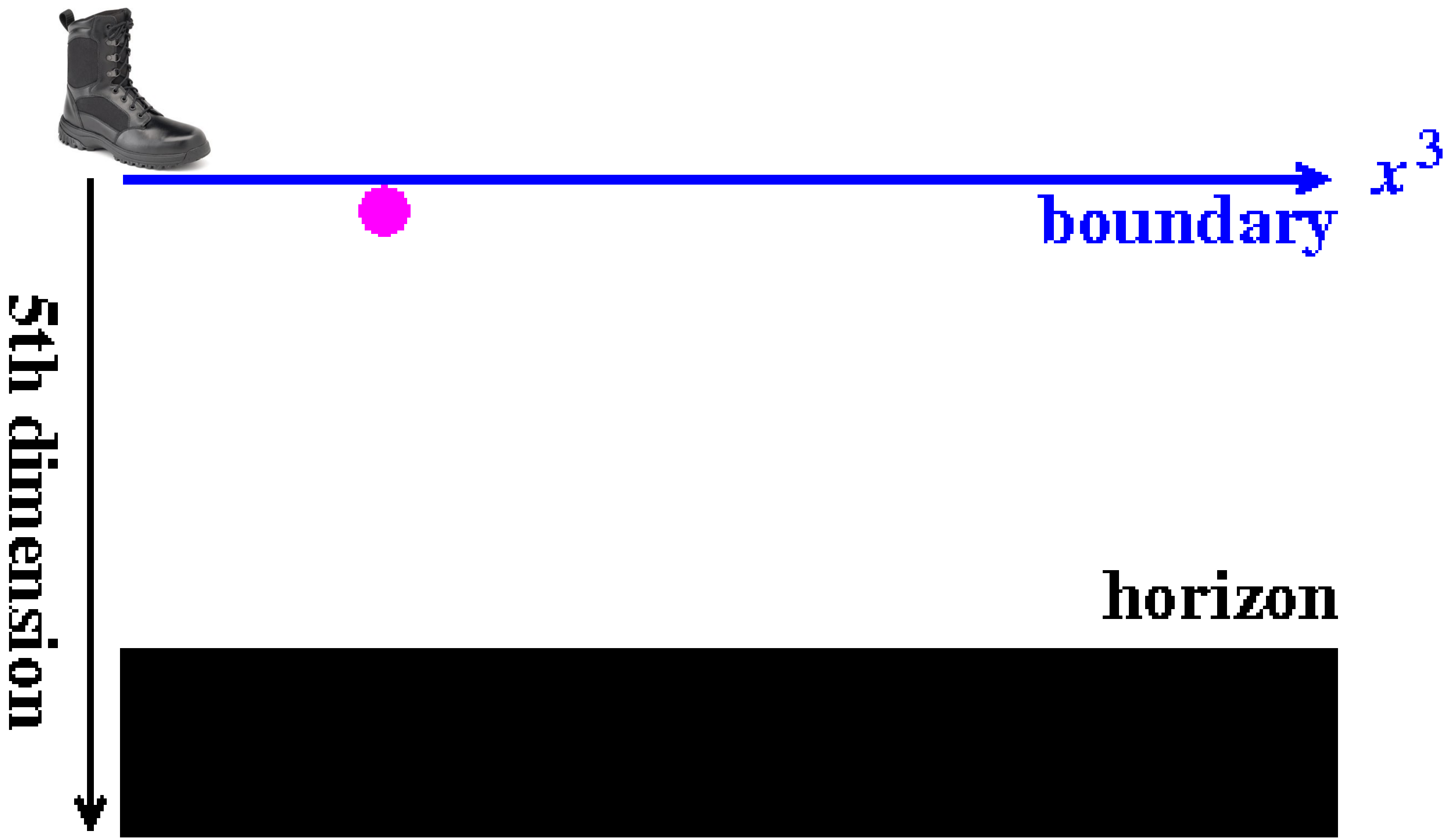

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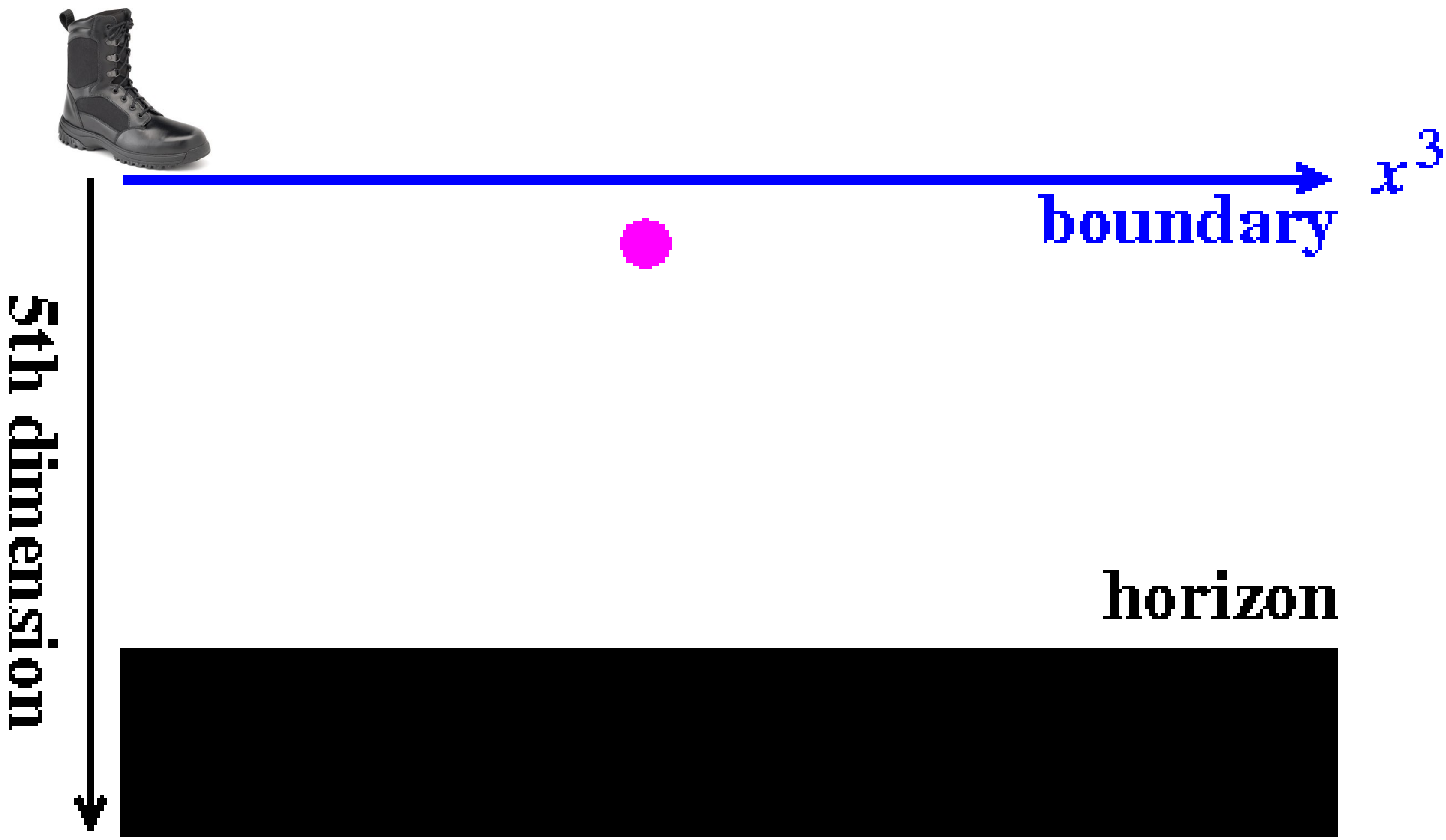

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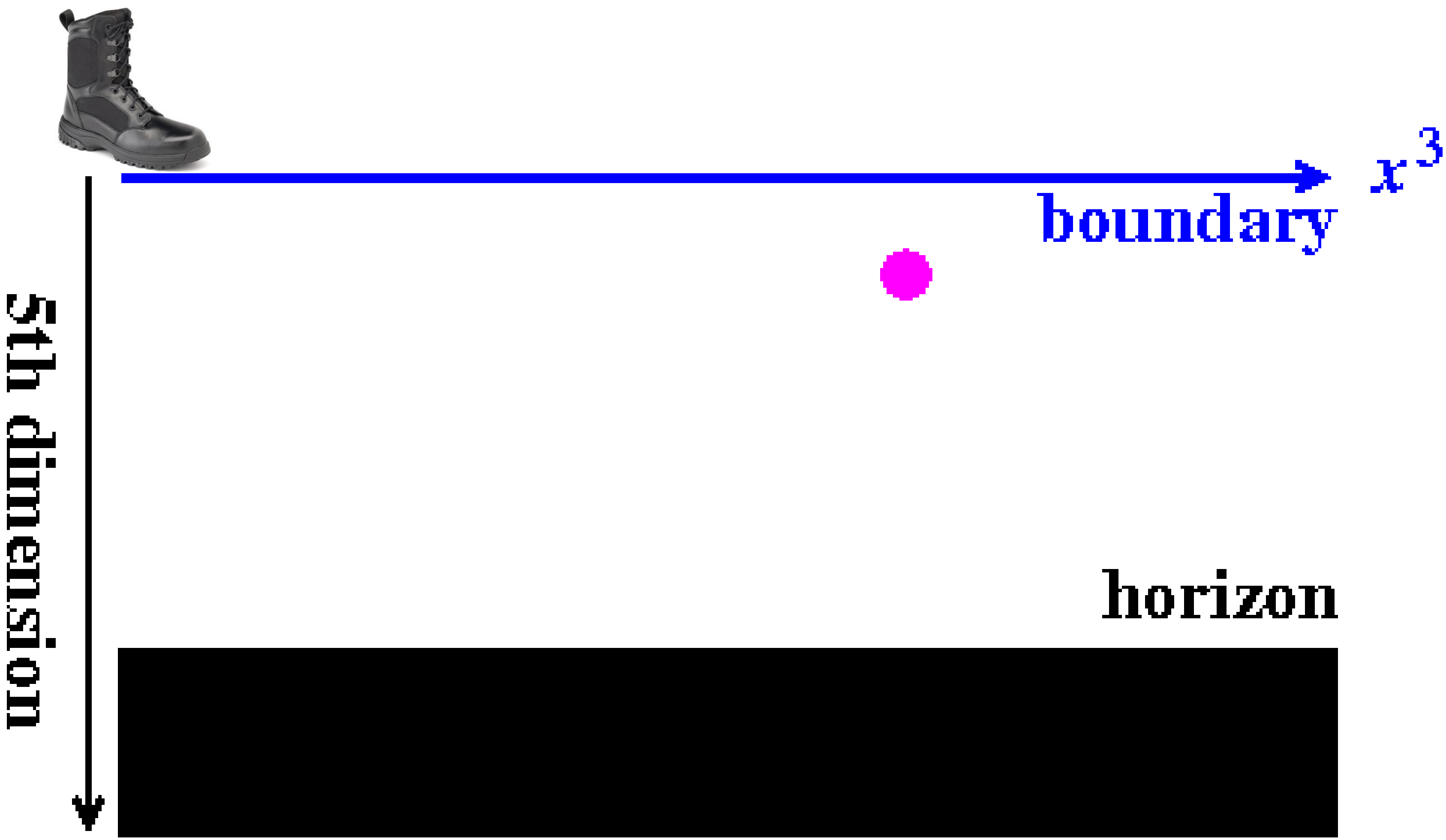

Stopping Distance using AdS/CFT

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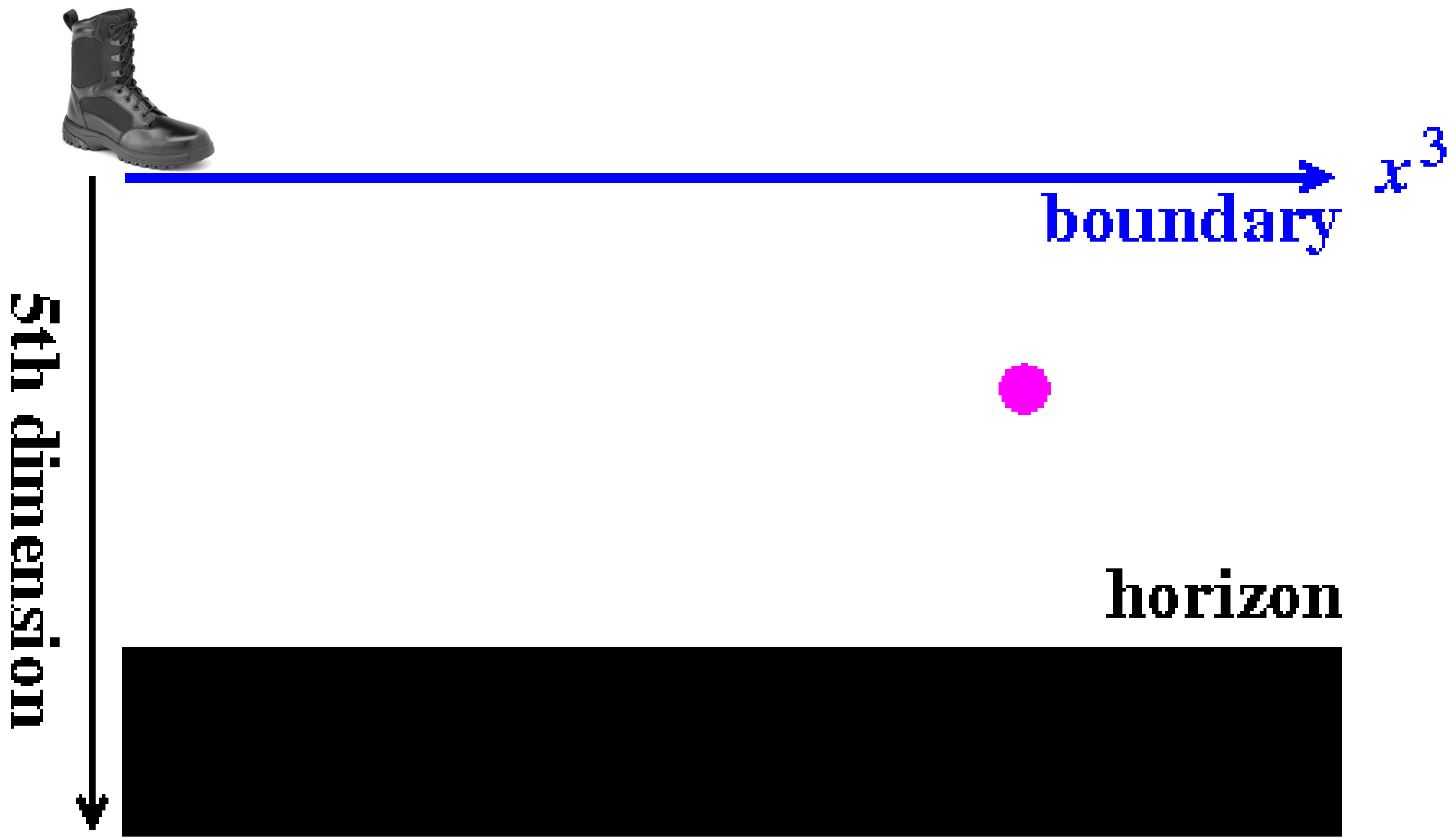

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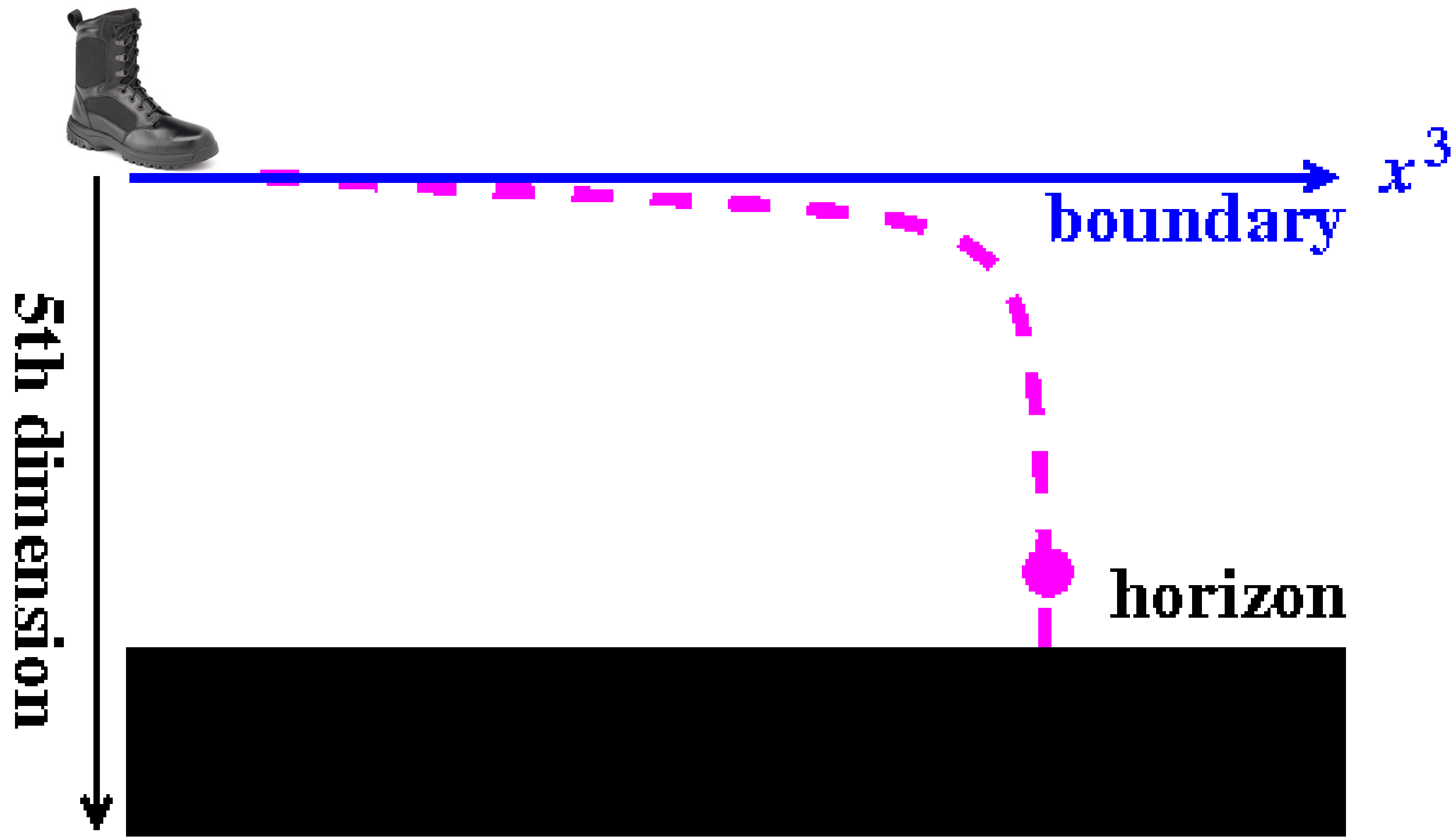

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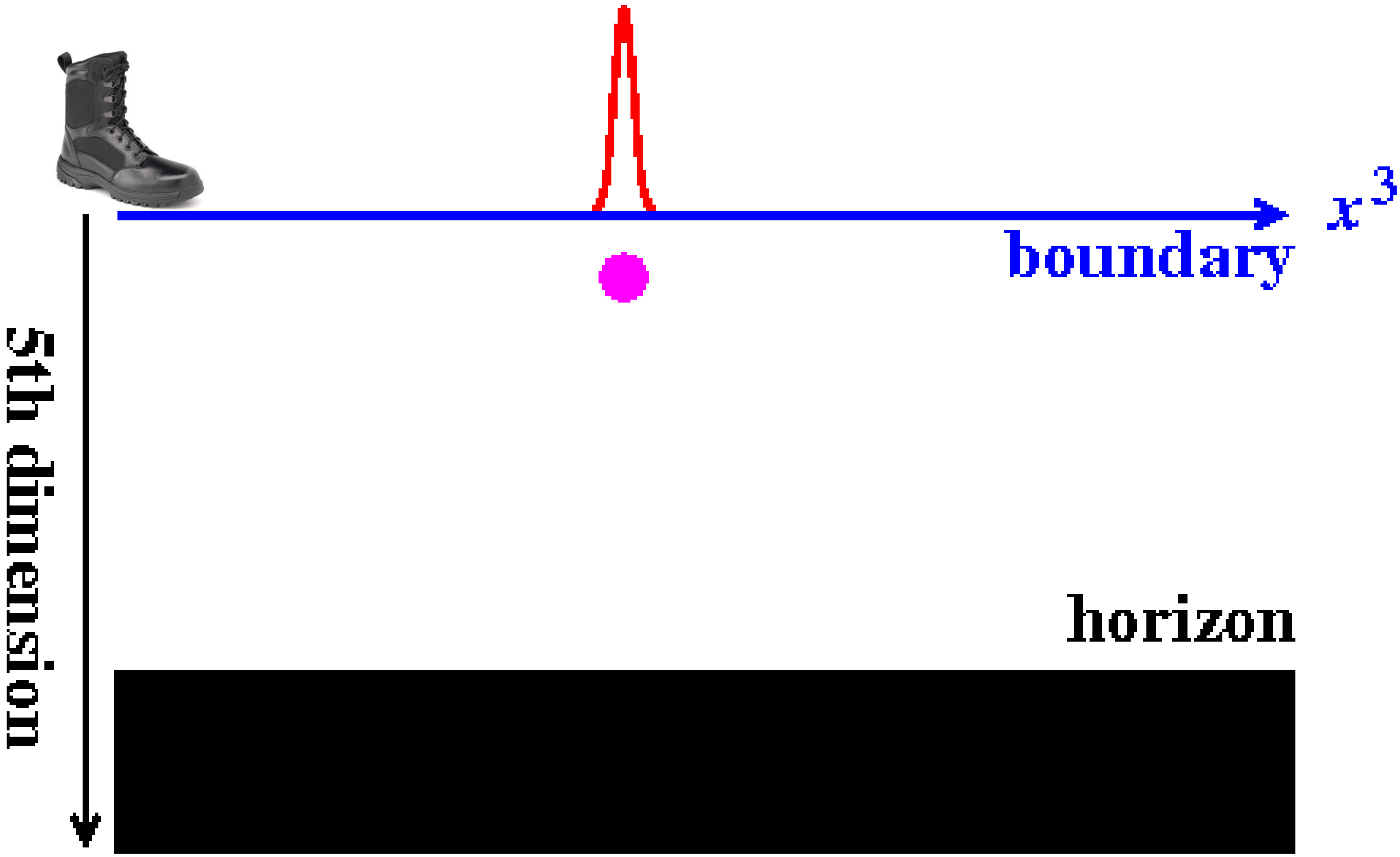

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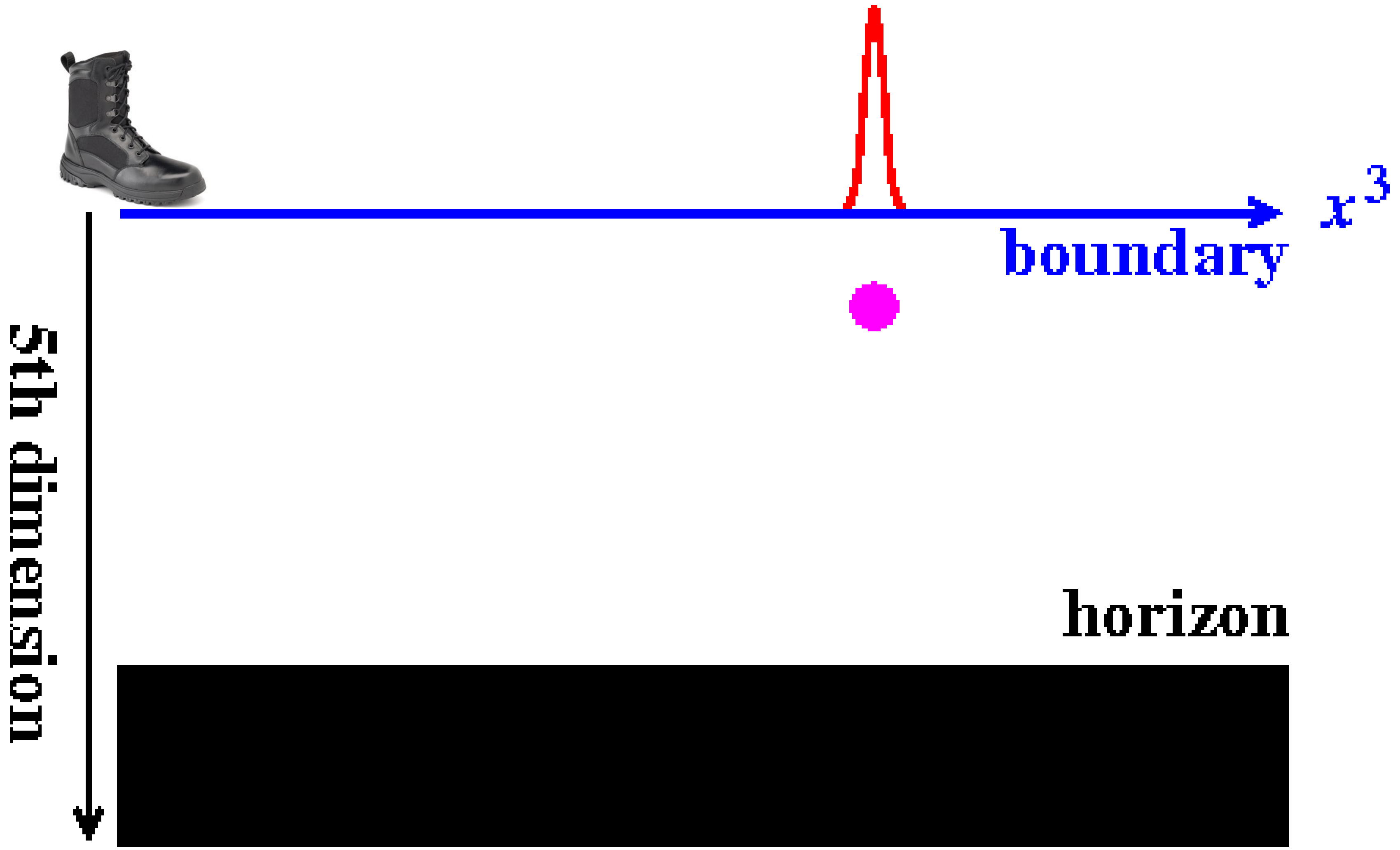

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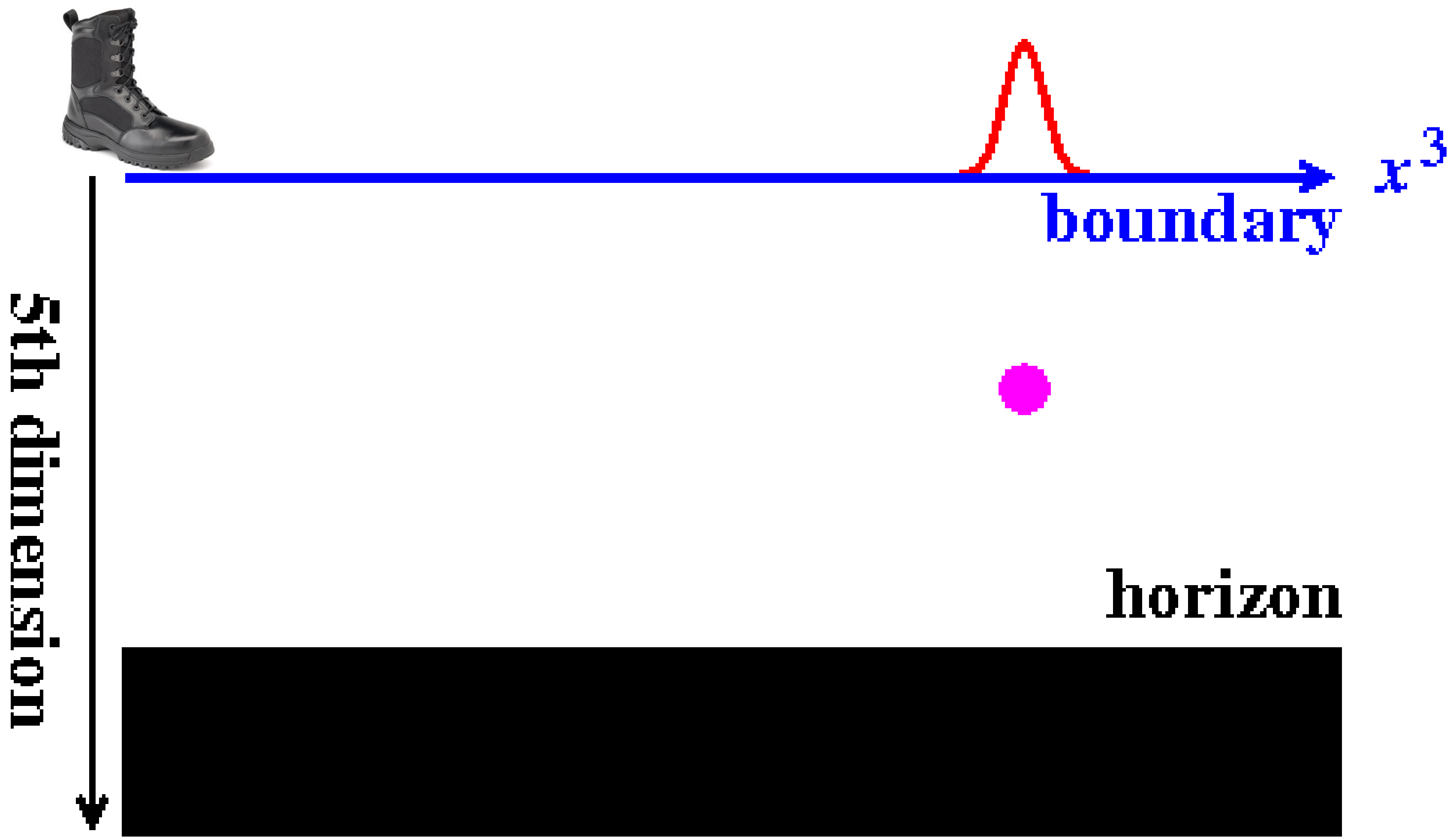

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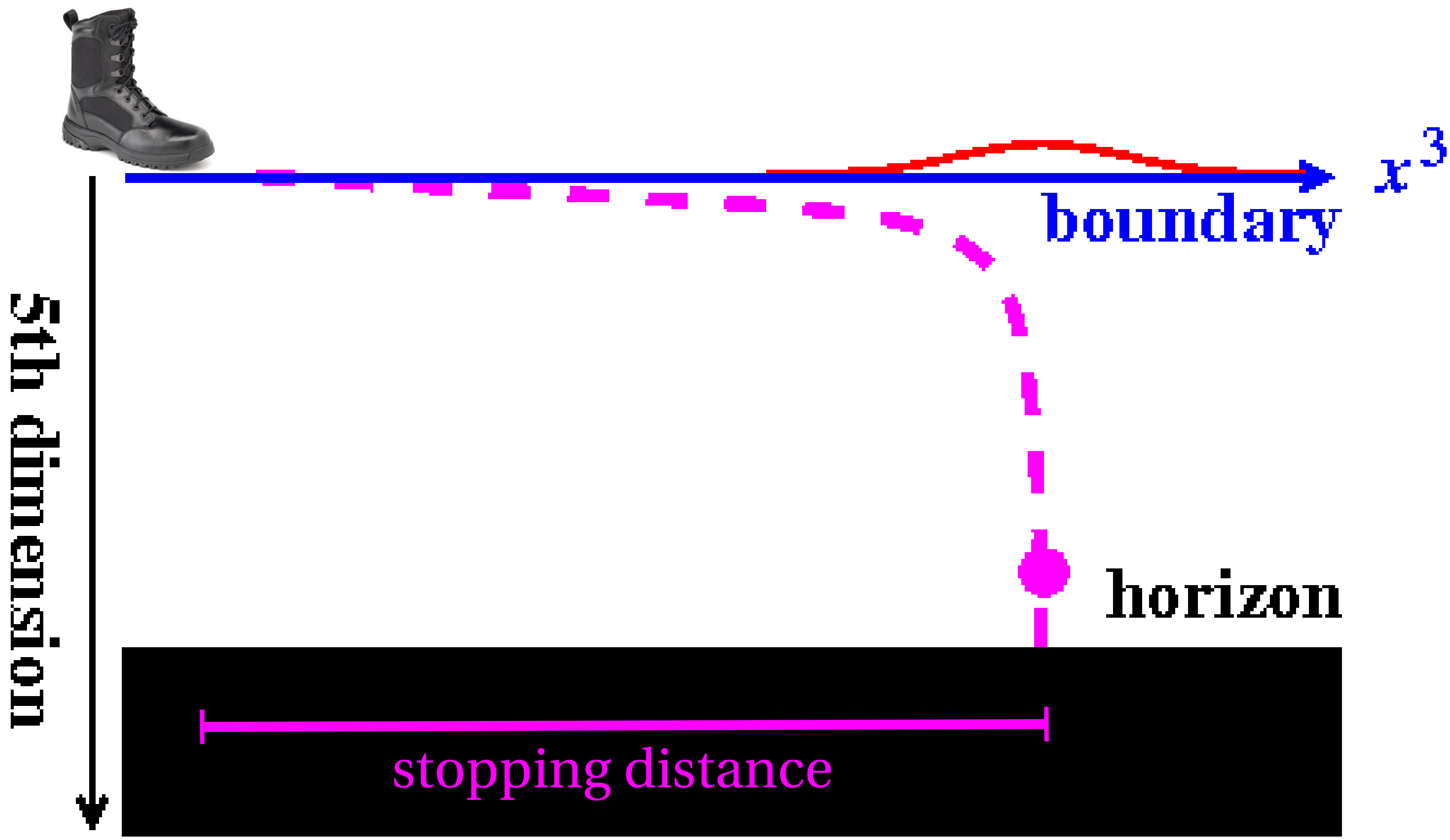


Choice of  in the QFT

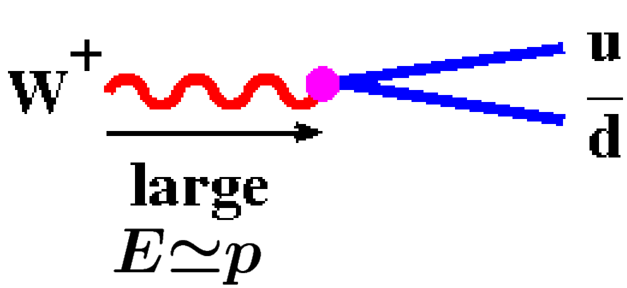
Gedanken experiments for creating localized, very high-momentum excitations in the plasma.

Synchrotron method: Drag a heavy test quark around in a circle
to make a beam of gluon synchrotron radiation.

[used by Chesler, Ho, Rajagopal (2011)]

Our method: Analogous to considering the hadronic decay of some
very high-momentum, unstable particle in a QCD plasma.

e.g.



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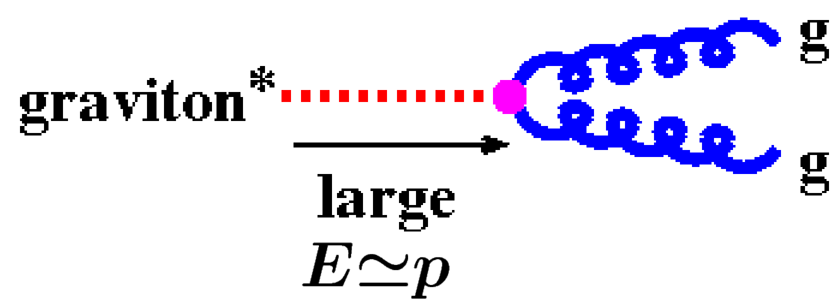
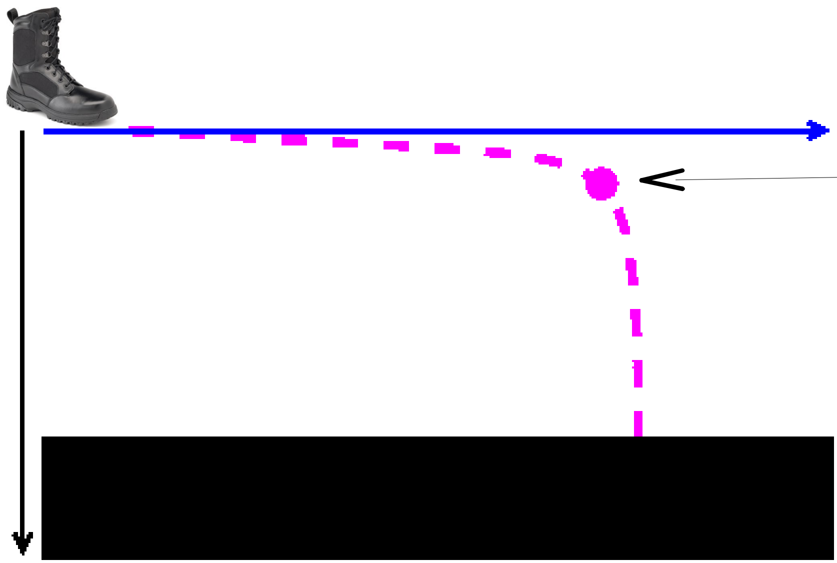


[Arnold & Vaman (2010)]

$\lambda = \infty$ result

$$\boldsymbol{\ell}_{\mathbf{stop}} \quad \lesssim \quad \boldsymbol{\ell}_{\mathbf{max}} \propto \boldsymbol{E}^{\mathbf{1/3}}$$

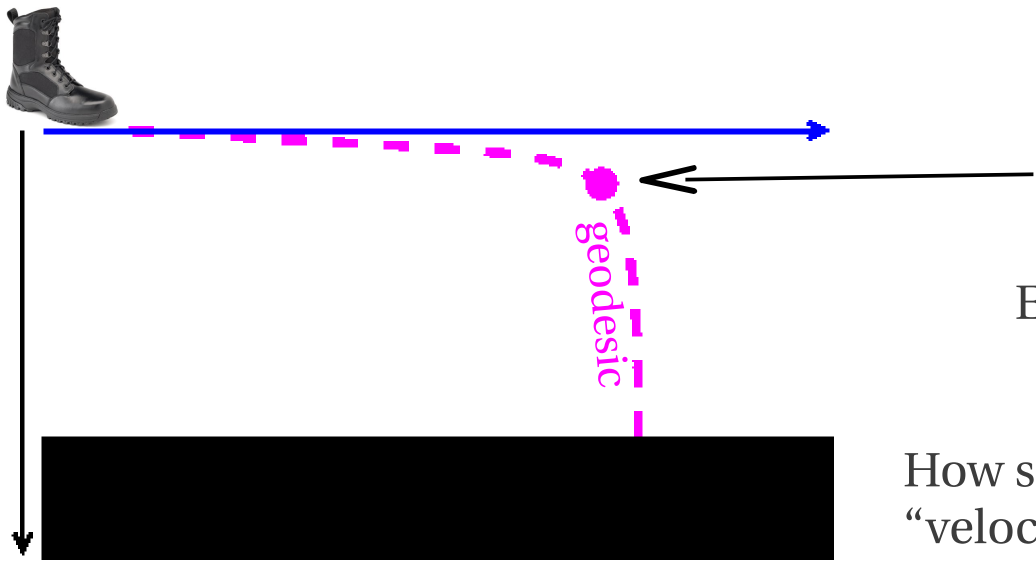
A simplified picture for $\ell_{\mathbf{stop}} \ll \ell_{\mathbf{max}}$



small size

Excitation moves like a 5-dim. particle!

A simplified picture for $\ell_{\mathbf{stop}} \ll \ell_{\mathbf{max}}$



small size

Excitation moves like a 5-dim. particle!

How soon it falls will depend on the initial "velocity" in the 5$^{\text{th}}$ dimension.

Q: What determines $\ell_{\mathbf{stop}}$?

A: the 4-virtuality $q^2 \equiv q_\mu \eta^{\mu\nu} q_\nu$ of the source 

Why?

Consider massless 5-dim. particle near the boundary:

$$0 = q_\mu q^\mu + q_5 q^5$$

Bigger $-q_\mu q^\mu$ $\rightarrow$ bigger q^5 $\rightarrow$ falls sooner !

The result:

$$\ell_{\text{stop}} \simeq \frac{\Gamma^2(\frac{1}{4})}{\sqrt{4\pi}} \left(\frac{E^2}{-q^2}\right)^{1/4}$$

Higher curvature corrections to gauge-gravity duality

AdS/CFT correspondence:

$\mathcal{N}$=4 SYM ⟺ string theory in $AdS_5 \times S^5$ background

Strong-coupling limit:

$\lambda \to \infty$ ⟺ "low energy" string theory
= supergravity in in $AdS_5 \times S^5$ background
(gravitons + other massless string modes)

$$\mathcal{L}_{\mathbf{grav}} \sim R$$

Higher curvature corrections to gauge-gravity duality

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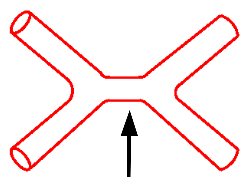
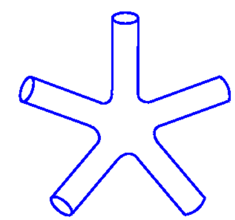
(gravitons + other massless string modes)

$$\mathcal{L}_{\text{grav}} \sim R \quad + \quad \alpha'^3 R^4 + \alpha'^5 D^4 R^4 + \alpha'^6 D^6 R^4 + \cdots \quad + \quad \alpha'^5 D^2 R^5 + \alpha'^6 D^4 R^5 + \cdots \quad + \quad \cdots$$



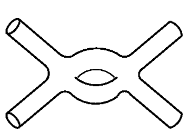
integrate out massive
intermediate string states



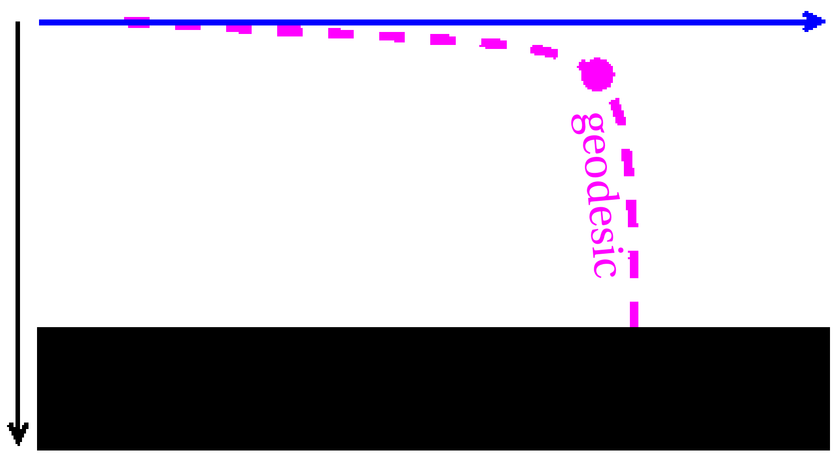
ditto

where

$\dfrac{1}{\sqrt{\lambda}}$ ⟺ proportional to $\alpha' \sim \dfrac{1}{\text{string tension}}$

Note: Loops  are suppressed by $g_{\text{string}} \propto \dfrac{1}{N_c}$.

How do the higher-derivative corrections affect λ



?

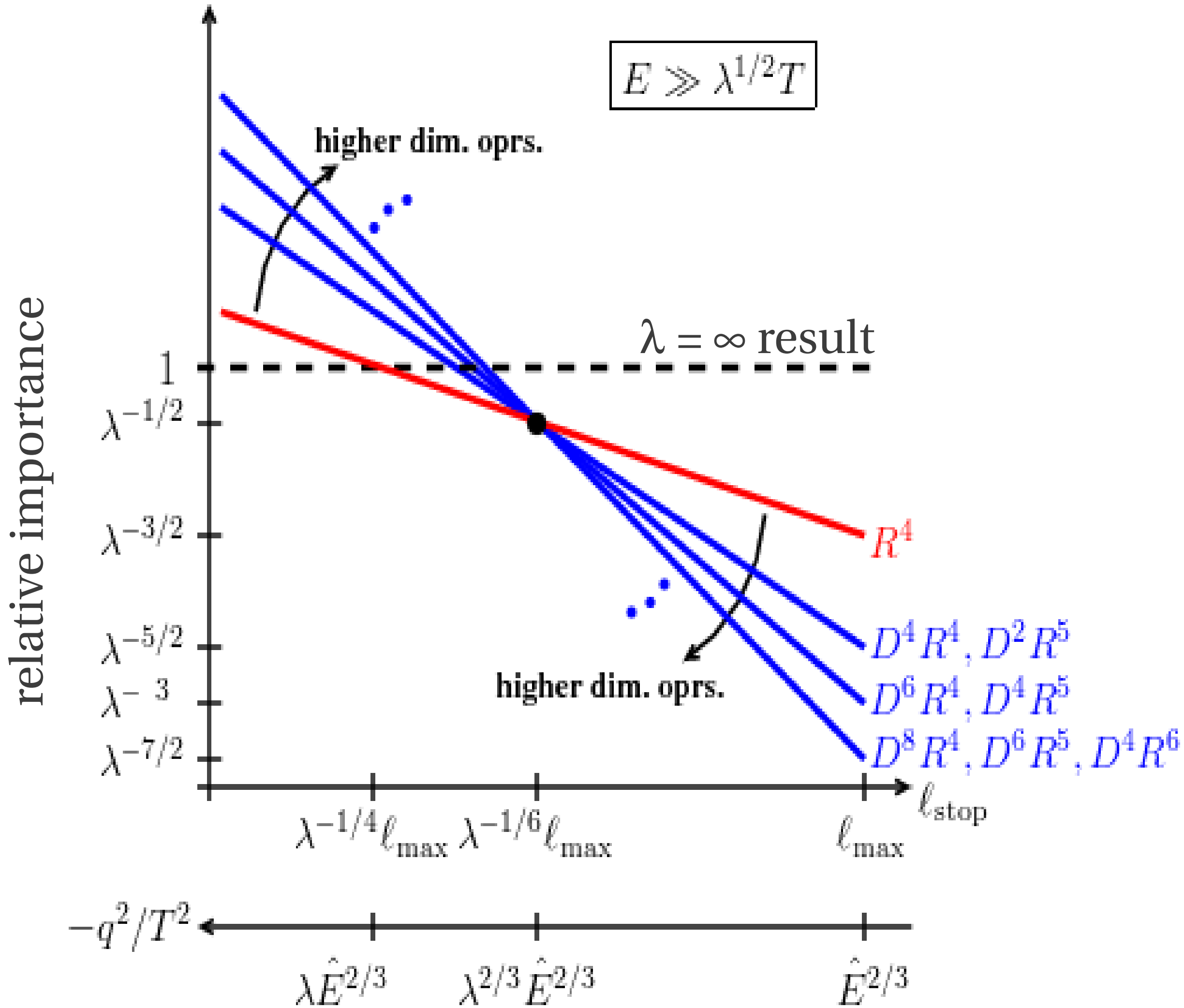
(1) Small corrections to AdS_5-Schwarzschild background (independent of E)
$\rightarrow$ small corrections to geodesics.

TINY

(2) Corrections to equation of motion
$\rightarrow$ wave packet no longer follows a geodesic.
(Corrections depend on E.)

POTENTIALLY LARGE

Importance to Jet Stopping



Reminder:
$\boldsymbol{\ell_{max} \propto E^{1/3}}$

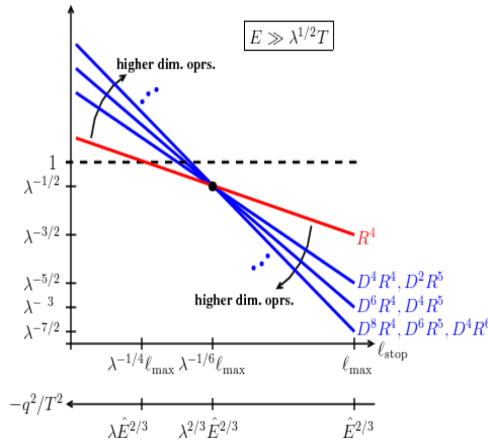
Moral: Expansion in $1/\lambda$ is well-behaved for $\boldsymbol{\lambda^{-1/6}\ell_{max} \ll \ell_{stop} \lesssim \ell_{max}}$

Expansion breaks down for $\boldsymbol{\ell_{stop} \lesssim \lambda^{-1/6}\ell_{max}}$

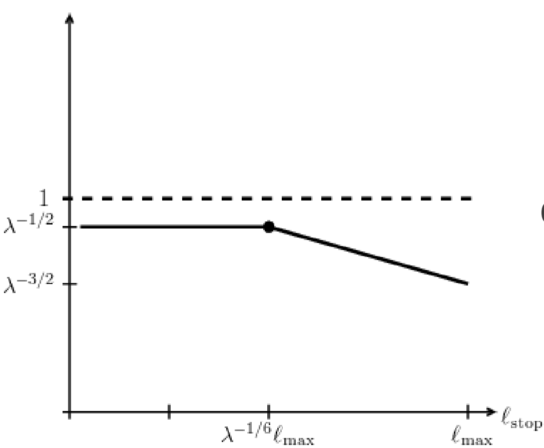
Note: Individual corrections all small ($\lambda^{-1/2}$) where expansion first breaks down.

Open Question

Does sum of corrections



add up to



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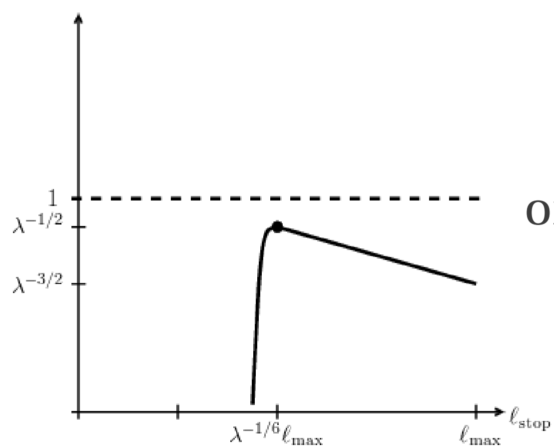

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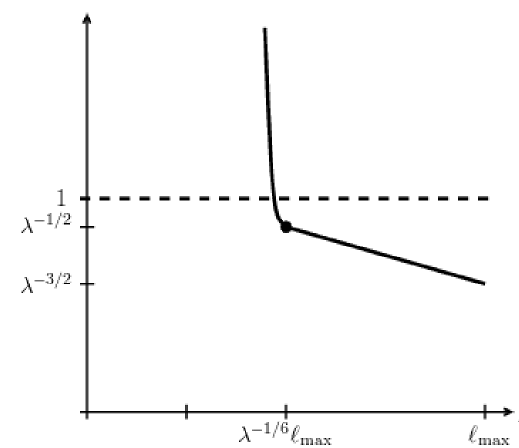


?

$\lambda = \infty$ results are okay everywhere!

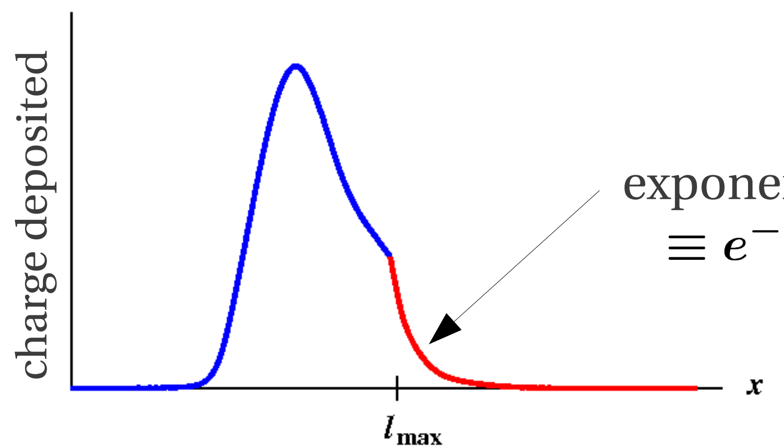
$\lambda = \infty$ works only for $\boldsymbol{\ell_{\rm stop} \gtrsim \lambda^{-1/6} \ell_{\rm max}}$

Moral: Fate of $\lambda = \infty$ results uncertain for $\boldsymbol{\ell_{\rm stop} \lesssim \lambda^{-1/6} \ell_{\rm max}}$

How big are corrections to $\ell_{\max}$?

Ill-posed: exact definition of $\ell_{\max}$ scale is fuzzy.

Suppose you try to get make a “jet” go far.



exponential fall-off for $x \gg \ell_{\max}$
$\equiv e^{-x/\ell_{\text{tail}}}$

Calculate ℓ_{tail} as a proxy for $\ell_{\max}$.

Example:  = decay of a high-momentum, slightly off-shell graviton

$$\ell_{\text{tail}} = \frac{0.3259\, E^{1/3}}{(2\pi T)^{4/3}} \left[1 + \frac{47.162}{\lambda^{3/2}} + \cdots \right]$$

Form of this result highlights an outstanding mystery...

An outstanding mystery

How does $\ell_{\max} \sim E^{f(\lambda)}$ interpolate from $f(\infty) = \frac{1}{3}$ (strong coupling)
to $f(0) = \frac{1}{2}$ (weak coupling)?

Naively, we might guess something like

$$f(\lambda) = \frac{1}{3} + \frac{\#}{\lambda^{3/2}} + \frac{\#}{\lambda^{5/2}} + \cdots$$

which would give

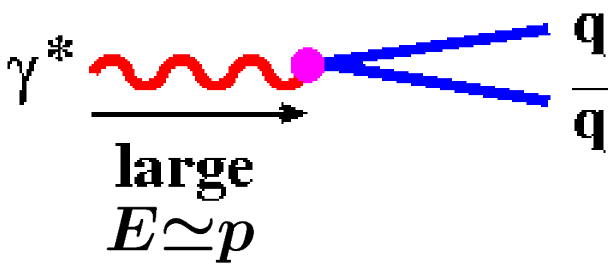
$$\ell_{\max} \sim E^{f(\lambda)} \sim E^{1/3} \left[1 + \frac{\#}{\lambda^{3/2}} \ln E + \cdots \right]$$

But there was no "ln E" in the result for our proxy ℓ_{tail}!

Backup

Our Method

In the field theory, think impressionistically of



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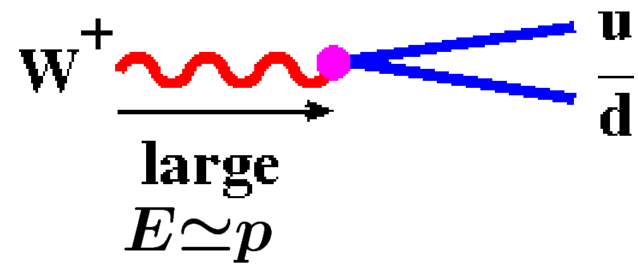
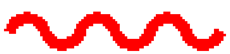


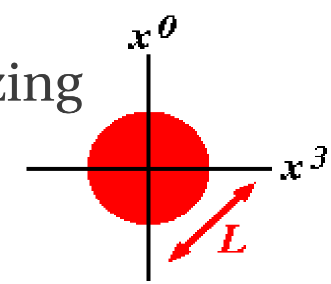
Treat  as a localized external field:

our 

$$\mathcal{L}_{\rm QFT} \to \mathcal{L}_{\rm QFT} + \boxed{\mathcal{O}(x)\,\Lambda_L(x)\,e^{i\bar{k}\cdot x}} \qquad \text{with } \bar{k}^\mu \simeq (E,0,0,E)$$

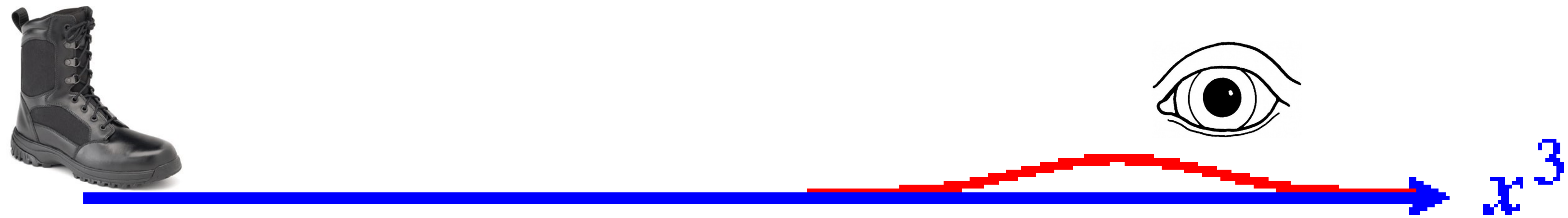
some source operator
e.g. $j_\mu(x)$

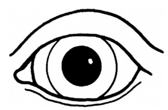
smooth envelope function localizing
source in space and time



Definition for purposes of this talk:

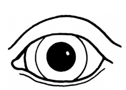
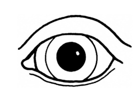
"jet" = localized, high-p excitation moving through the plasma.



The response  is measured by a **<u>3-point correlator</u>**.
A crude way to understand this:

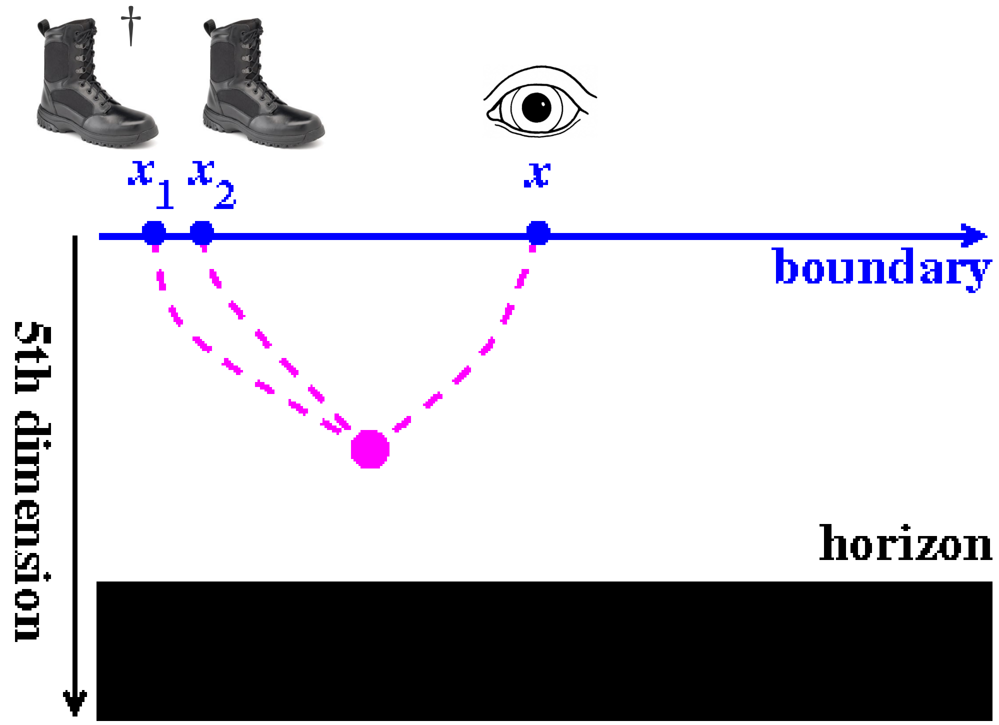
$$|\mathbf{jet}\rangle \quad = \quad$$  $$|\mathbf{plasma}\rangle .$$

So we want

$$\langle \mathbf{jet}|$$  $$|\mathbf{jet}\rangle \quad = \quad \langle$$  †   $$\rangle .$$

For *finite-temperature* AdS/CFT calculations:

- lots in literature on computing 2-point correlators
- almost nothing on 3-point correlators

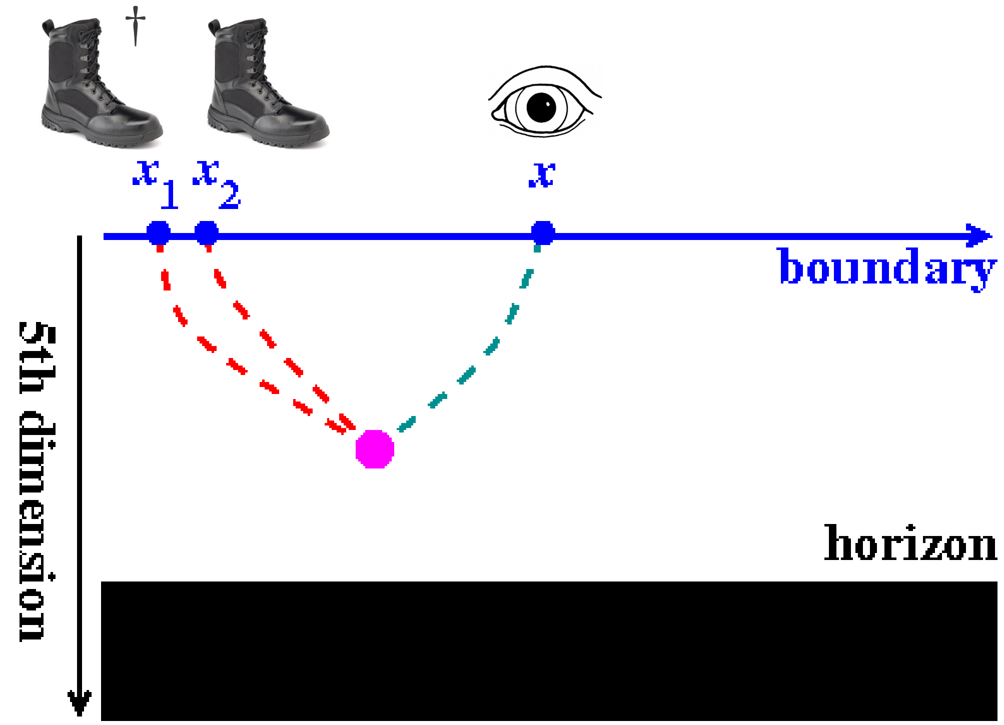


= 5-dim. SUGRA vertex

= a *Heun* function →

hard to make any analytic or numeric progress!

Fortunately, in our problem,

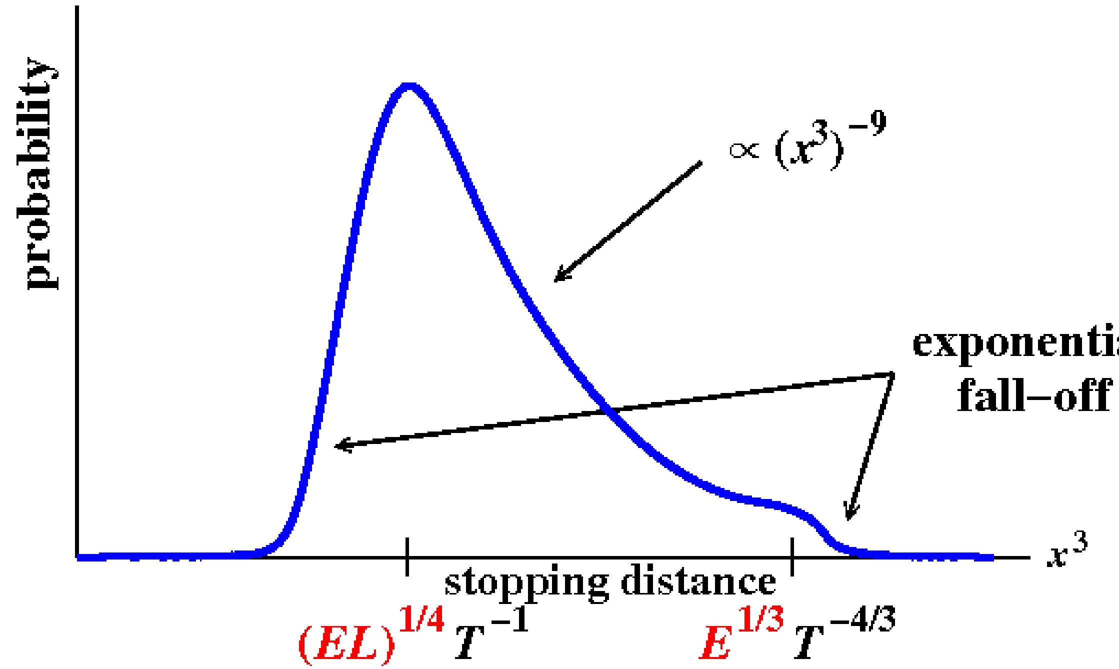


- - - high-energy source → high-k approximation (WKB / geometric optics)

- - - want to observe late-time diffusion → low-k approximation

Can do calculation!

Our Result



The farthest a jet will ever go is indeed $\propto E^{1/3}$.

<u>**But**</u> almost all jets will instead stop sooner at $\propto (EL)^{1/4}$

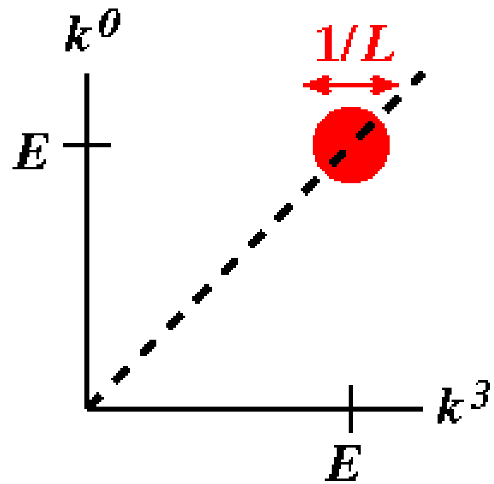
where L is the size of the space-time region in which the jet was initially created.

Q: What does the size L of the source have to do with it?

A: It determines how off-shell the source is.

 $\propto \quad \Lambda_L(x)\, e^{i\bar{k}\cdot x}$ with $\bar{k}^\mu = (E, 0, 0, E)$

implies that  has Fourier components



Typical stopping distance $(EL)^{1/4}$ really means $(E^2/q^2)^{1/4}$ where

q^2 = typical virtuality of the source