Thermal dileptons in high-energy heavy ion collisions with 3+1D relativistic hydrodynamics

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Outline

- Overview of Dilepton sources

Low Mass Dileptons

- Thermal Sources of Dileptons

  1) QGP (w/ viscous corrections)

  2) In-medium vector mesons (w/ viscous corrections)

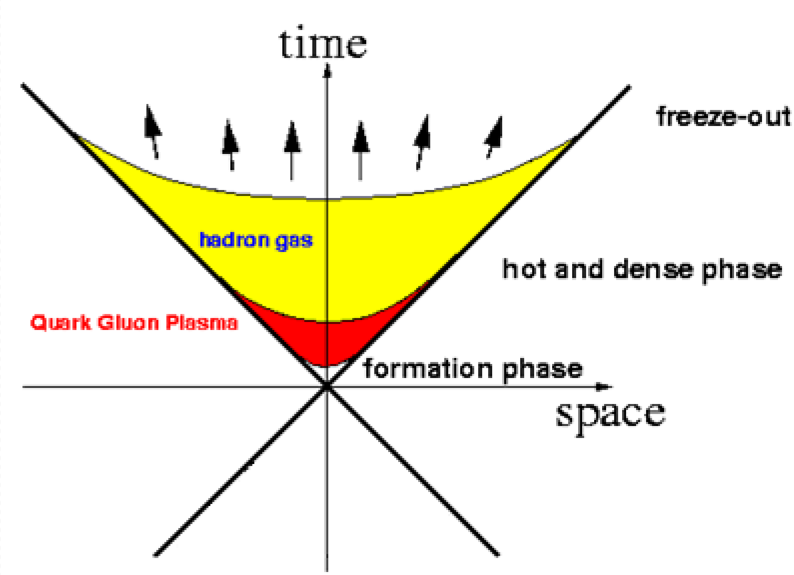
- 3+1D Viscous Hydrodynamics
- Thermal Dilepton Yields & v_2

Intermediate Mass Dileptons

- Charmed Hadrons: Yield & v_2
- Conclusion and outlook

Evolution of a nuclear collision

Space-time diagram



Thermal dilepton sources: HG+QGP

a) QGP: q+q-bar-> γ^* -> e^+e^-

b) HG: In medium vector mesons V=(ρ, ω, ϕ)

V-> γ^* -> e^+e^-

Kinetic freeze-out:

c) Cocktail (π^0, η, η', etc.)

Other dilepton sources: Formation phase

d) Charmed hadrons: e.g. $D^{+/-}$-> K^o + $e^{+/-}$ ν_e

e) Beauty hadrons: e.g. $B^{+/-}$->D^0 + $e^{+/-}\nu_e$

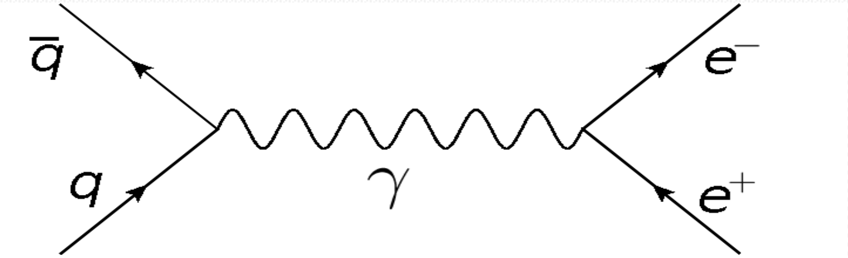
f) Other vector mesons: Charmonium, Bottomonium

g) Drell-Yan Processes

Sub-dominant the intermediate mass region

Dilepton rates from the QGP

- An important source of dileptons in the QGP



- The rate in kinetic theory (Born Approx)

$$\frac{d^4R}{d^4q} = \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3} n(E_1)n(E_2)v_{12}\sigma\delta^4(q-k_1-k_2)$$

$$v_{12} = \frac{M^2}{2E_1 2E_2} \qquad \sigma = \frac{16\,\pi^2\alpha^2 N_c \sum_q e_q^2}{3M^2}$$

$$\frac{d^4R}{d^4q} = \left(\frac{\alpha^2 N_c}{12\pi^4}\sum_q e_q^2\right)\frac{1}{\exp\left(\frac{q^0}{T}\right)-1}\left\{1-\frac{2T}{q}\ln\left[\frac{1+\exp\left(-\frac{q^0-q}{2T}\right)}{1+\exp\left(-\frac{q^0+q}{2T}\right)}\right]\right\}$$

- More sophisticated calculations: HTL, Lattice QCD.

Thermal Dilepton Rates from HG

- Model based on forward scattering amplitude [Eletsky, et. al., Phys. Rev. C, 64, 035202 (2001)]

$$\Pi_V^{Total} = \Pi_V^{Vac}(M) + \sum_a \Pi_{Va}(q,T)$$

$\Pi_V^{Vac}(M)$ is described by effective Lagrangians

$$\Pi_{Va}(q,T) = -\int_{m_a}^{\infty} dk^0 \ln\left[\frac{1 \pm exp\left(-\frac{\omega_-}{T}\right)}{1 \pm exp\left(-\frac{\omega_+}{T}\right)}\right] f_{Va}\left(\frac{m_V}{m_a}k^0\right)$$

$$\omega_\pm = \frac{(q^0k^0 \pm qk)}{m_V}$$

Resonances contributing to ρ's scatt. amp. & similarly for ω, ϕ

$$\left\{\begin{array}{ll} N(1700) & \Delta(1905) \\ N(1720) & \Delta(1940) \\ N(1900) & \Delta(2000) \\ N(2000) & \phi(1020) \\ N(2080) & h_1(1170) \\ N(2090) & a_1(1260) \\ N(2100) & \pi(1300) \\ N(2190) & a_2(1320) \\ \Delta(1700) & \omega(1420) \\ \Delta(1900) & \end{array}\right.$$

- Effective Lagrangian method by R. Rapp [Phys. Rev. C 63, 054907 (2001)]
- The dilepton production rate is :

$$\frac{d^4R}{dq^4} = \frac{\alpha^2}{\pi^3}\frac{L(M)}{M^2}\frac{m_V^4}{g_V^2}\left\{-\frac{1}{3}[ImD_V^R]_\mu^\mu\right\} n_{BE}(q^0)\,; \qquad L(M) = \left(1+\frac{2m_l^2}{M^2}\right)\sqrt{1-\frac{4m_l^2}{M^2}}$$

3+1D Hydrodynamics

- Viscous hydrodynamics equations for heavy ions:

$$\partial_\mu T^{\mu\nu} = 0 \longleftarrow \text{Energy-momentum conservation}$$

$$\partial_\mu J_B^\mu = 0 \longleftarrow \text{Charge conservation}$$

$$T^{\mu\nu} = T_0^{\mu\nu} + \pi^{\mu\nu} \quad T_0^{\mu\nu} = (\varepsilon + \mathcal{P})u^\mu u^\nu - \mathcal{P} g^{\mu\nu} \quad \mathcal{P} = \mathcal{P}(\varepsilon, \rho_B)$$

$$\Delta_\alpha^\mu \Delta_\beta^\nu u^\sigma \partial_\sigma \pi^{\alpha\beta} = -\frac{1}{\tau_\pi}(\pi^{\mu\nu} - S^{\mu\nu}) - \frac{4}{3}\pi^{\mu\nu}(\partial_\alpha u^\alpha) \qquad \eta/s=1/4\pi$$

$$S^{\mu\nu} = \eta\left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \tfrac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha\right) \qquad t = \tau\cosh\eta_s, \quad z = \tau\sinh\eta_s$$

- Initial conditions are set by the Glauber model.

- Solving the hydro equations numerically done via the Kurganov-Tadmor method using a Lattice QCD EoS [P. Huovinen and P. Petreczky, Nucl. Phys. A 837, 26 (2010).]

- The hydro evolution is run until the kinetic freeze-out. [For details: B. Schenke, et al., Phys. Rev. C 85, 024901(2012)]

Viscous Corrections: QGP

- Viscous correction to the rate in kinetic theory rate

$$\frac{d^4R}{dq^4} = \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3} n_{FD}(k_1^0) n_{FD}(k_2^0) v_{12}\sigma\delta^4(q-k_1-k_2)$$

- Using the quadratic ansatz to modify F.-D. distribution

$$n_{FD}(k^0) \to n_{FD}(k^0) + \frac{\eta}{s}\frac{C_q}{2T^3} n_{FD}(k^0)[1-n_{FD}(k^0)]k^\mu k^\nu \frac{\pi_{\mu\nu}}{\eta}$$

$$\frac{d^4R}{dq^4} = \frac{d^4R_{ideal}}{dq^4} + \frac{d^4\delta R}{dq^4}; \qquad \frac{d^4\delta R}{dq^4} = \frac{\eta}{s}\frac{C_q}{2T^3} b_2(q^0,|\vec{q}|) q^\mu q^\nu \frac{\pi_{\mu\nu}}{\eta}$$

$$b_2(q^0,|\vec{q}|) = \frac{1}{|\vec{q}|^5}\int_{E_-}^{E_+} n(E_1)n(q^0-E_1)\big(1-n(E_1)\big) \times \left[(3q_0^2-|\vec{q}|^2)E_1^2 - 3q^0E_1M^2 + \frac{3}{4}M^4\right] dE_1$$

- Dusling & Lin, Nucl. Phys. A 809, 246 (2008).

Viscous corrections to HG ?

- Two modifications are plausible: $-\frac{1}{3}[ImD_V^R]^{\mu}_{\mu}$ and $n_{BE}(q^0)$
- Self-Energy

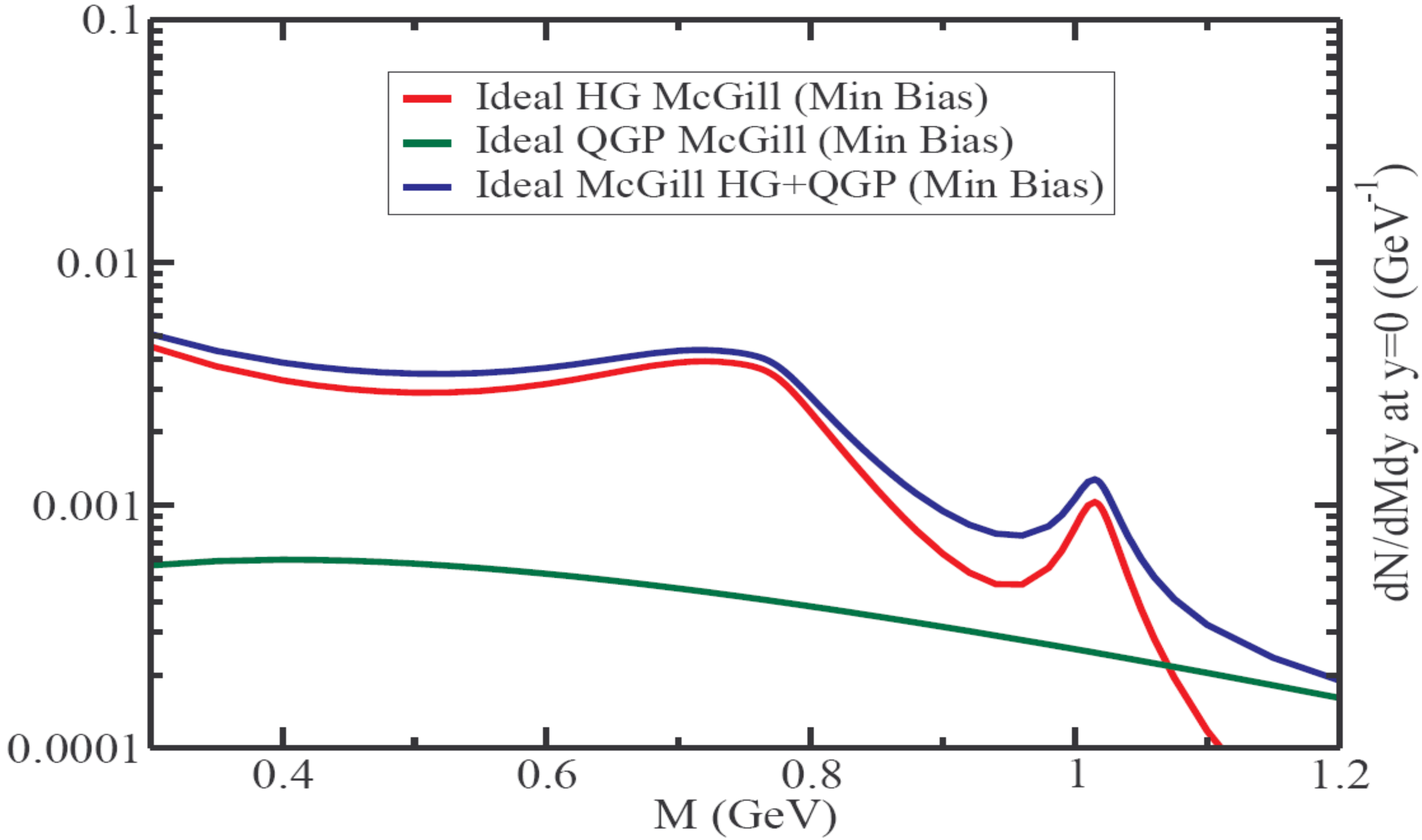
$$\Pi_{Va}(p,T) = -\frac{m_a m_V T}{\pi p}\int \frac{d^3k}{(2\pi)^3}\frac{\sqrt{s}}{k^0} f_{Va}(s) n_a(k^0); \qquad \Pi_{Va}^{Total} = \Pi_{Va}^{Ideal} + \delta\Pi_{Va}$$

$$n_a(k^0) \to n_a(k^0) + \frac{\eta}{s}\frac{C_a}{2T^3} n_a(k^0)[1 \pm n_a(k^0)]k^\mu k^\nu \frac{\pi_{\mu\nu}}{\eta}; \qquad \delta\Pi_{Va} = \frac{\eta}{s}\frac{C_a}{2T^3} B_2(q^0,q) q^\mu q^\nu \frac{\pi_{\mu\nu}}{\eta}$$

$$B_2(q^0,q) = -\frac{m_a}{2\pi q^2}\int_{m_a}^{\infty} dk^0 f_{Va}\left(\frac{m_V}{m_a}k^0\right)\left\{\begin{array}{l} -\frac{m_V T}{qk}\left[m_a^2 + \frac{3(q^0k+qk^0)^2 - (q^0k^0+pk)^2}{m_V^2}\right]\frac{1}{\exp\left(\frac{\omega_+}{T}\right) \pm 1} \\ +\frac{m_V T}{qk}\left[m_a^2 + \frac{3(q^0k-qk^0)^2 - (q^0k^0-pk)^2}{m_V^2}\right]\frac{1}{\exp\left(\frac{\omega_-}{T}\right) \pm 1} \\ \mp 2\left(\frac{m_V T}{qk}\right)^2\left[(3(q^0)^2 - q^2)\frac{k^2}{m_V^2} + \frac{2q^0k^0qk}{m_V^2}\right]\ln\left(1 \pm \exp\left(\frac{\omega_+}{T}\right)\right) \\ \mp 2\left(\frac{m_V T}{qk}\right)^2\left[(3(q^0)^2 - q^2)\frac{k^2}{m_V^2} - \frac{2q^0k^0qk}{m_V^2}\right]\ln\left(1 \pm \exp\left(\frac{\omega_-}{T}\right)\right) \\ \pm 2\left(\frac{m_V T}{qk}\right)^3\left[(3(q^0)^2 - q^2)\frac{k^2}{m_V^2}\right]\left[Li_2\left(\mp\exp\left(\frac{\omega_+}{T}\right)\right) - Li_2\left(\mp\exp\left(\frac{\omega_-}{T}\right)\right)\right] \end{array}\right\}$$

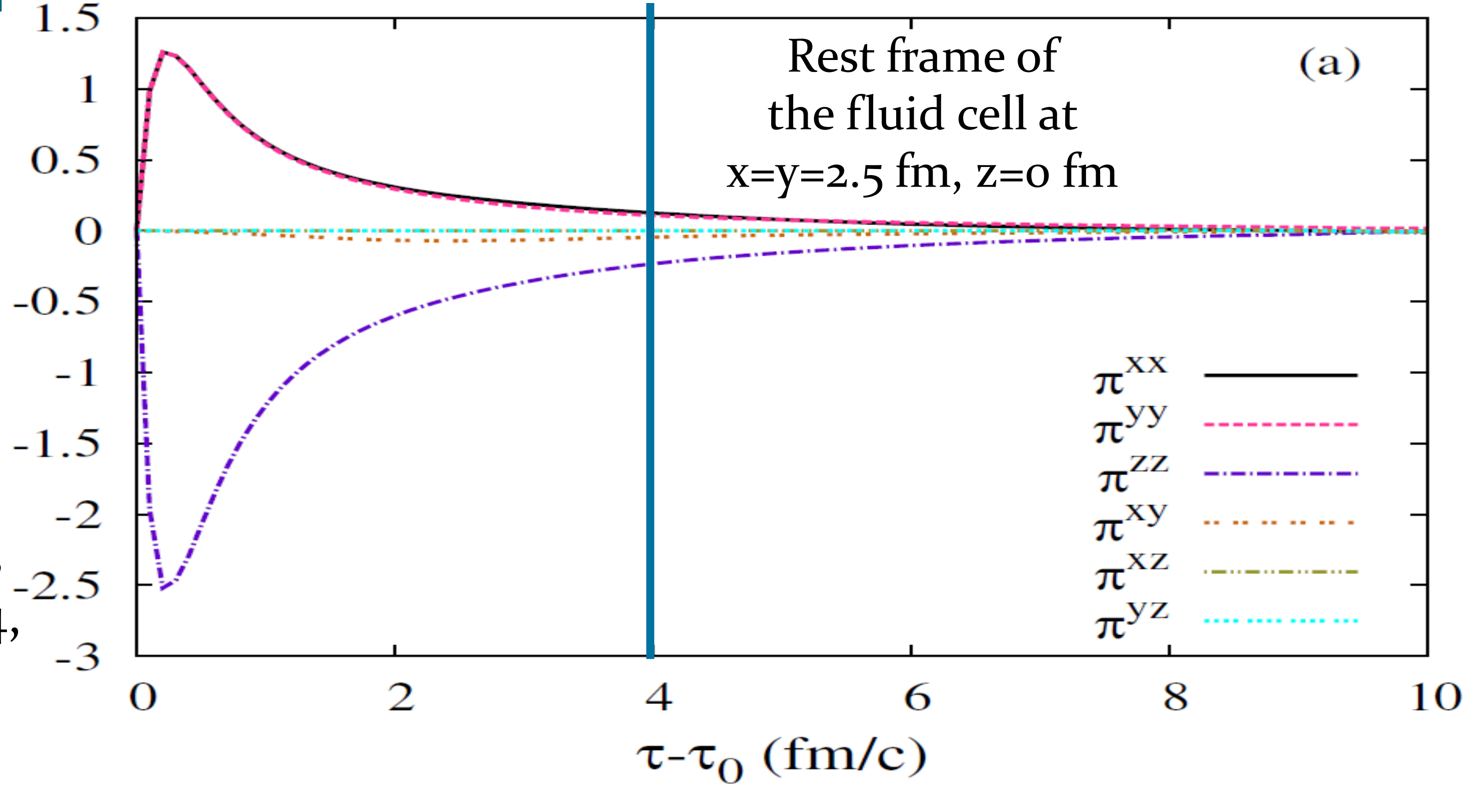
- Performing the calculation => these corrections don't matter!

Low Mass Dilepton Yields: HG+QGP



- Proof of concept: our yields are similar to yields obtained through R. Rapp's rates. Both cases: ideal hydro.

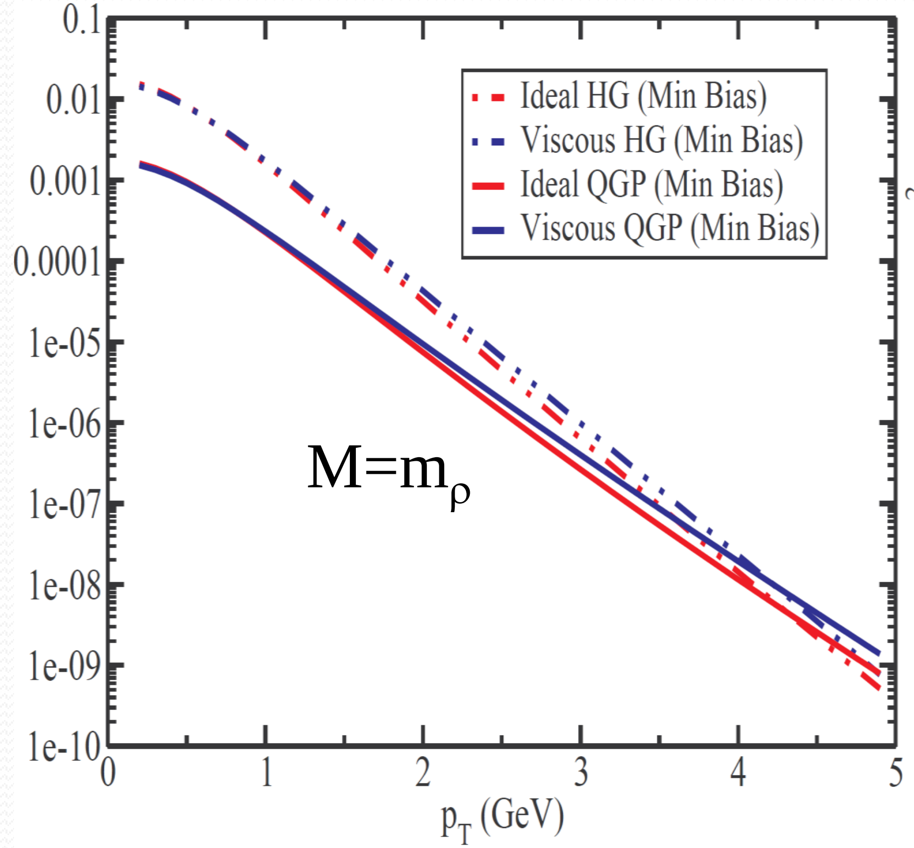
How important are viscous corrections to HG rate?

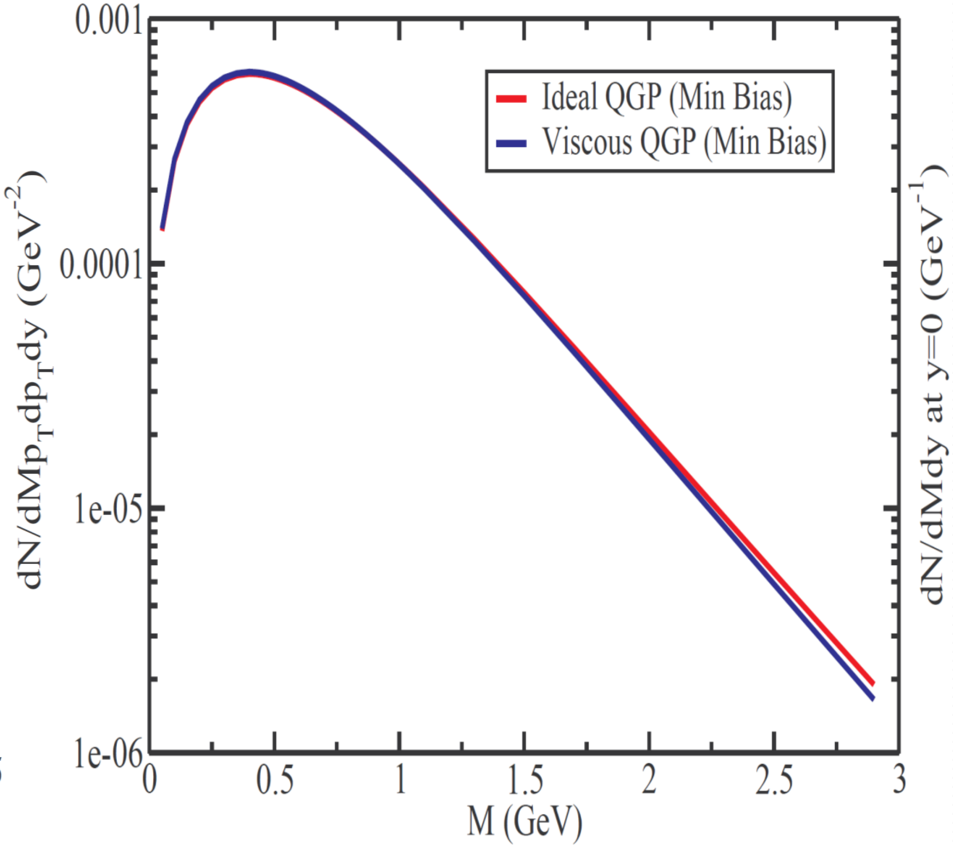


M. Dion et al., Phys. Rev. C 84, 064901 (2011)

- Fluid rest frame, viscous corrections to HG rates: $\propto q^i q^j \pi_{ij}/\eta$
- HG gas exists from τ~4 fm/c => π_{ij}/η is small
- Direct computation shows that viscous correction to HG rates doesn't affect the end result!

Dilepton yields Ideal vs Viscous Hydro

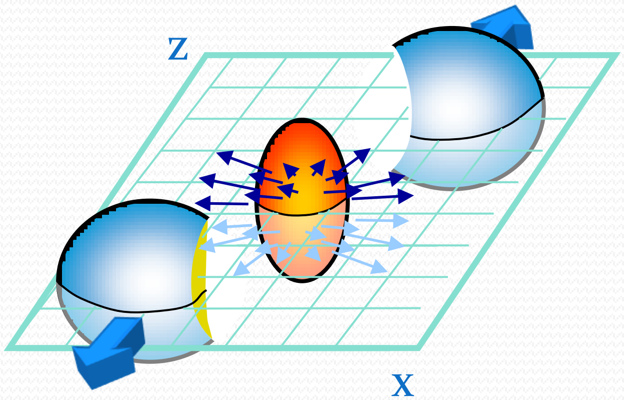




- Since viscous corrections to HG rates don't matter, only viscous flow is responsible for the modification of the p_T distribution.
- Also observed viscous photons HG [M. Dion et al., Phys. Rev. C 84, 064901 (2011)]
- For QGP rates, both corrections matter (see yields).

A measure of elliptic flow (v_2)

- Elliptic Flow



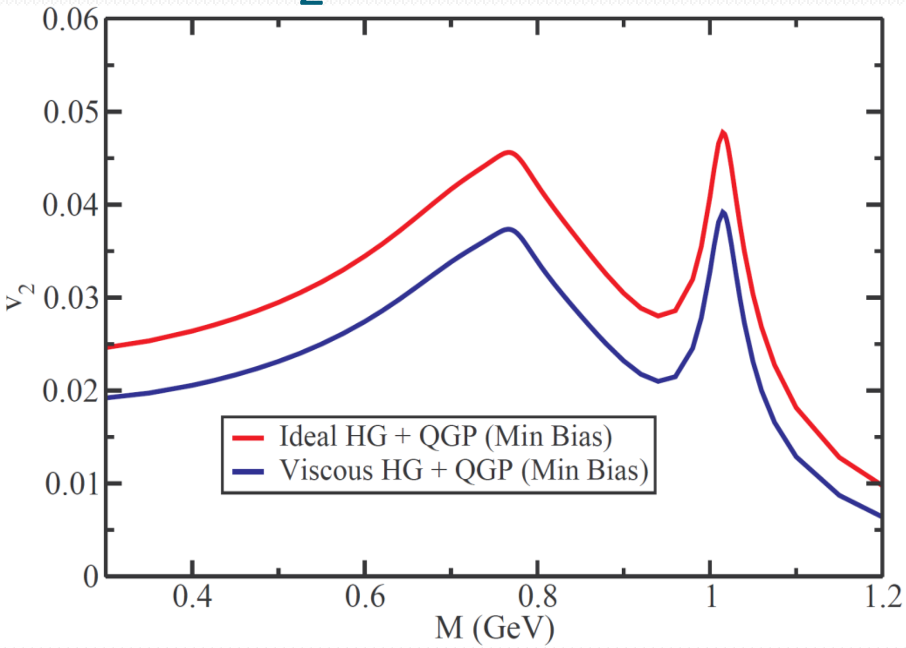
- A nucleus-nucleus collision is typically not head on; an almond-shape region of matter is created.
- This shape and its pressure profile gives rise to elliptic flow.

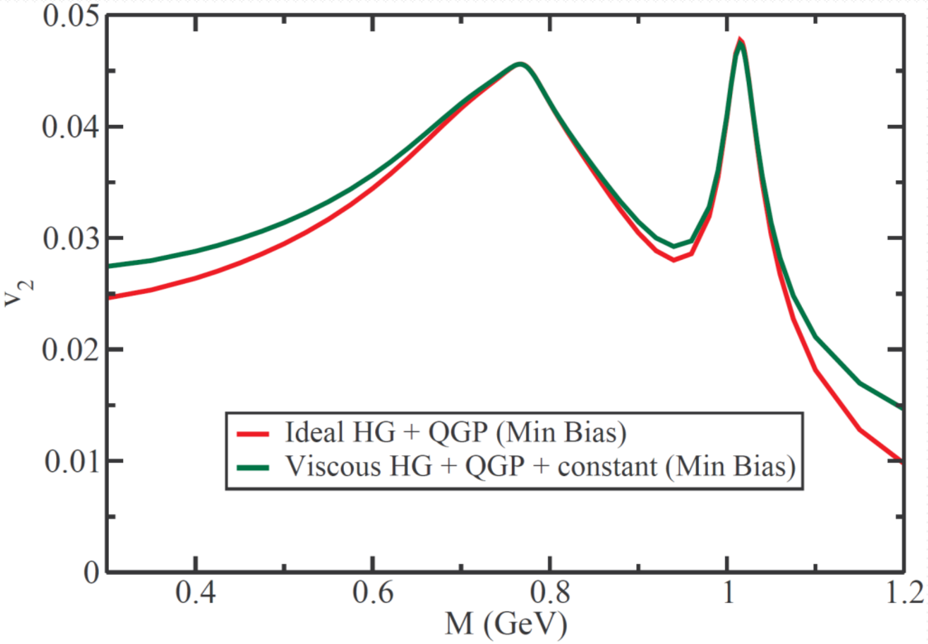
- To describe the evolution of the shape use a Fourier decomposition, i.e. flow coefficients v_n

$$\frac{dN}{dMp_T dp_T d\phi dy} = \frac{1}{2\pi}\frac{dN}{dMp_T dp_T dy}\left[1 + \sum_{n=1}^{\infty} 2v_n \cos\left(n\phi - n\psi_n\right)\right]$$

- Important note: when computing v_n's from several sources, one must perform a weighted average.

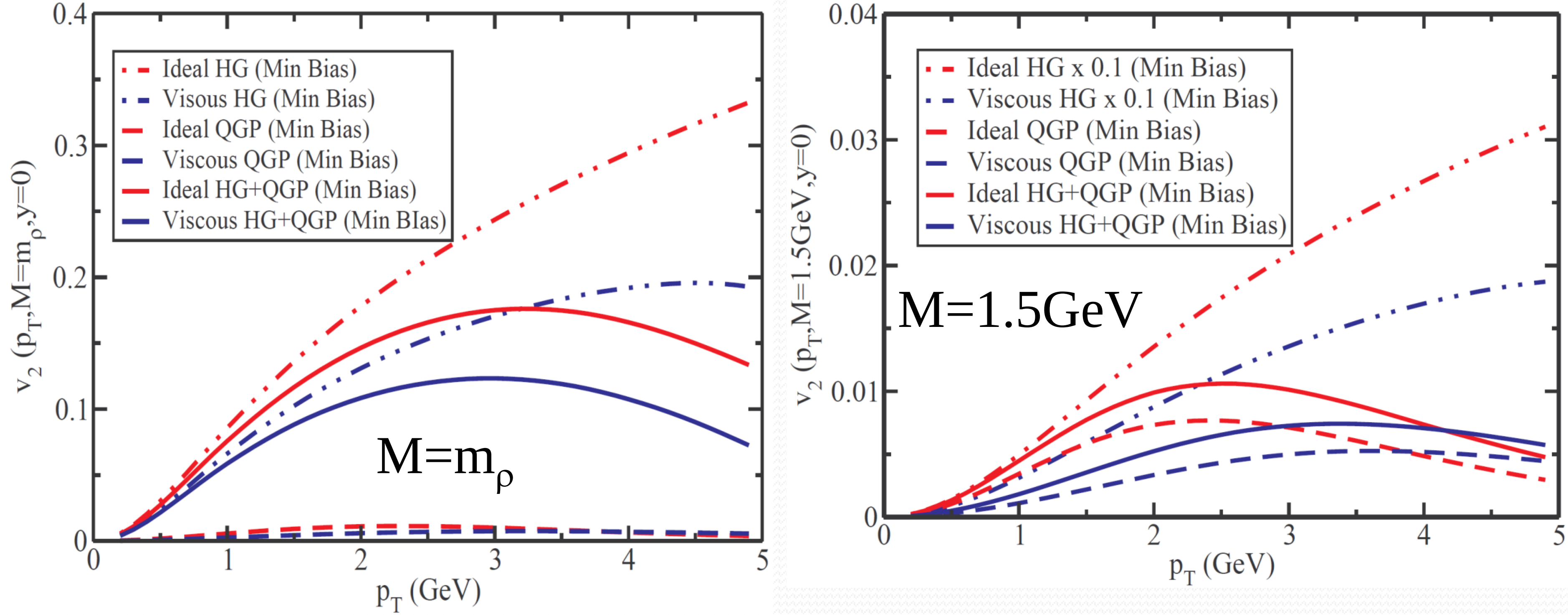
v_2 from ideal and viscous HG+QGP (1)





- Similar elliptic flow when comparing w/ R. Rapp's rates.
- Viscosity lowers elliptic flow.
- Viscosity slightly broadens the v_2 spectrum with M.

$v_2(p_T)$ from ideal and viscous HG+QGP (2)



- M is extremely useful to isolate HG from QGP. At low M HG dominates and vice-versa for high M.
- R. Chatterjee et al. Phys. Rev. C 75 054909 (2007).
- We can clearly see two effects of viscosity in the $v_2(p_T)$.
  - Viscosity stops the growth of v_2 at large p_T for the HG (dot-dashed curves)
  - Viscosity shifts the peak of v_2 from to higher momenta (right, solid curves).

Charmed Hadron contribution

- Since $M_q >> T$ (or Λ_{QCD}), heavy quarks must be produced perturbatively; very soon after the nucleus-nucleus collision.

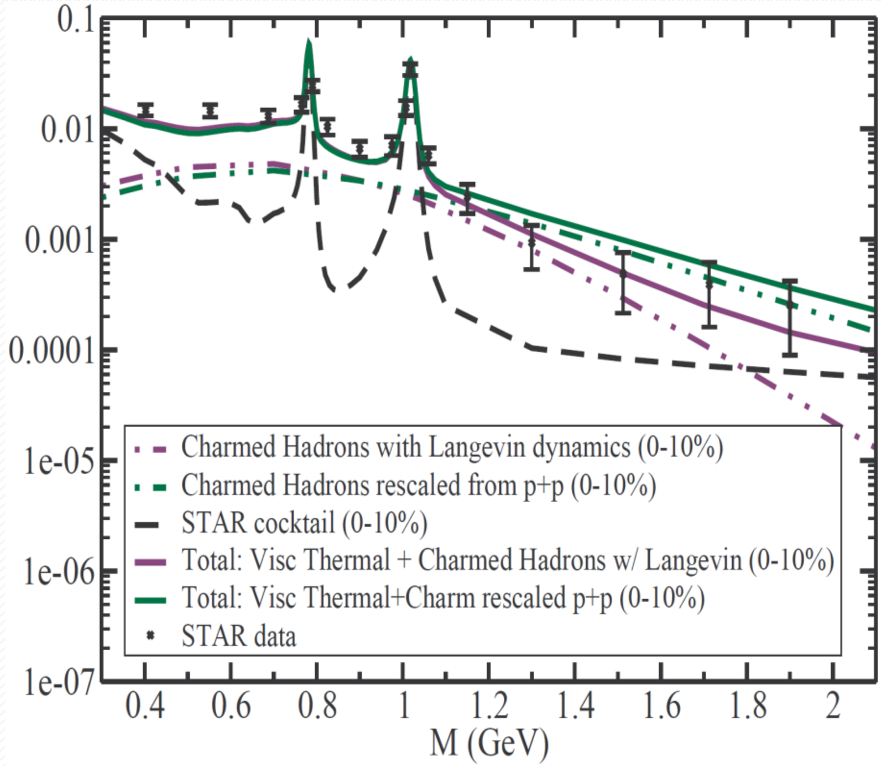
- For heavy quarks, many scatterings are needed for p to change appreciably.

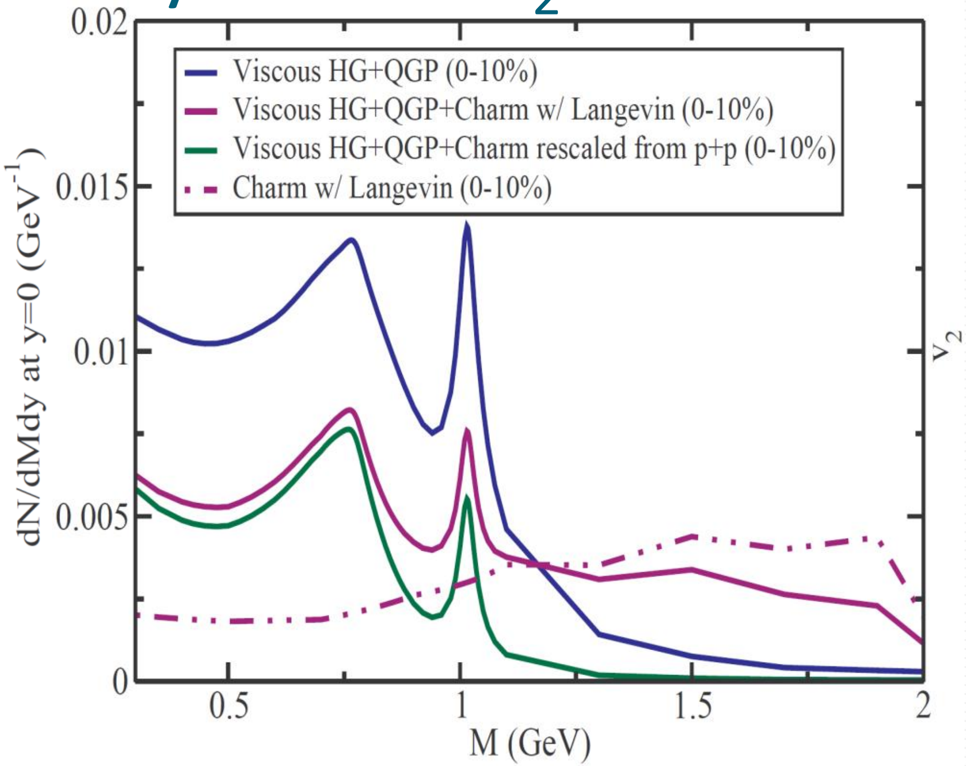
- In this limit, Langevin dynamics applies [Moore & Teaney, Phys. Rev. C 71, 064904 (2005)]

$$\frac{dp^i}{dt} = -\eta_D p^i + \xi^i(t) \quad \langle \xi^i(t)\xi^j(0)\rangle = \kappa\delta^{ij}\delta(t) \quad \eta_D = \frac{\kappa}{2M_q T}$$

- **Charmed Hadron production:**
  - PYTHIA -> (p+p) Charm spectrum (rescale from p+p to Au+Au).
  - PYTHIA Charm -> Modify mom. dist. via Langevin (& hydro).
  - Hadronization -> Charmed Hadrons
  - Charmed Hadrons -> Dileptons

Charmed Hadrons yield and v_2





- Heavy-quark energy loss (via Langevin) affects the invariant mass yield of Charmed Hadrons (vs rescaled p+p), by increasing it in the low M region and decreasing it at high M.
- Charmed Hadrons develop a v_2 through energy loss (Langevin dynamics) so there is a non-zero v_2 in the intermediate mass region.
- v_2 is a weighted average => it is reduced when Charmed Hadrons are included.

Conclusion & Future work

Conclusions

- First calculation of dilepton yield and v_2 via viscous 3+1D hydrodynamical simulation.
- $v_2(p_T)$ for different invariant masses allows to separate QGP and HG contributions.
- Slight modification to dilepton yields owing to viscosity.
- $v_2(M)$ is reduced by viscosity and the shape is slightly broadened.
- Studying yield and v_2 of leptons coming from charmed hadrons allows to investigate heavy quark energy loss.

Future work

- Include cocktail's yield and v_2 with viscous hydro evolution.
- Study the effects of Fluctuating Initial Conditions (see Björn Schenke's talk).
- Results for LHC are on the way.

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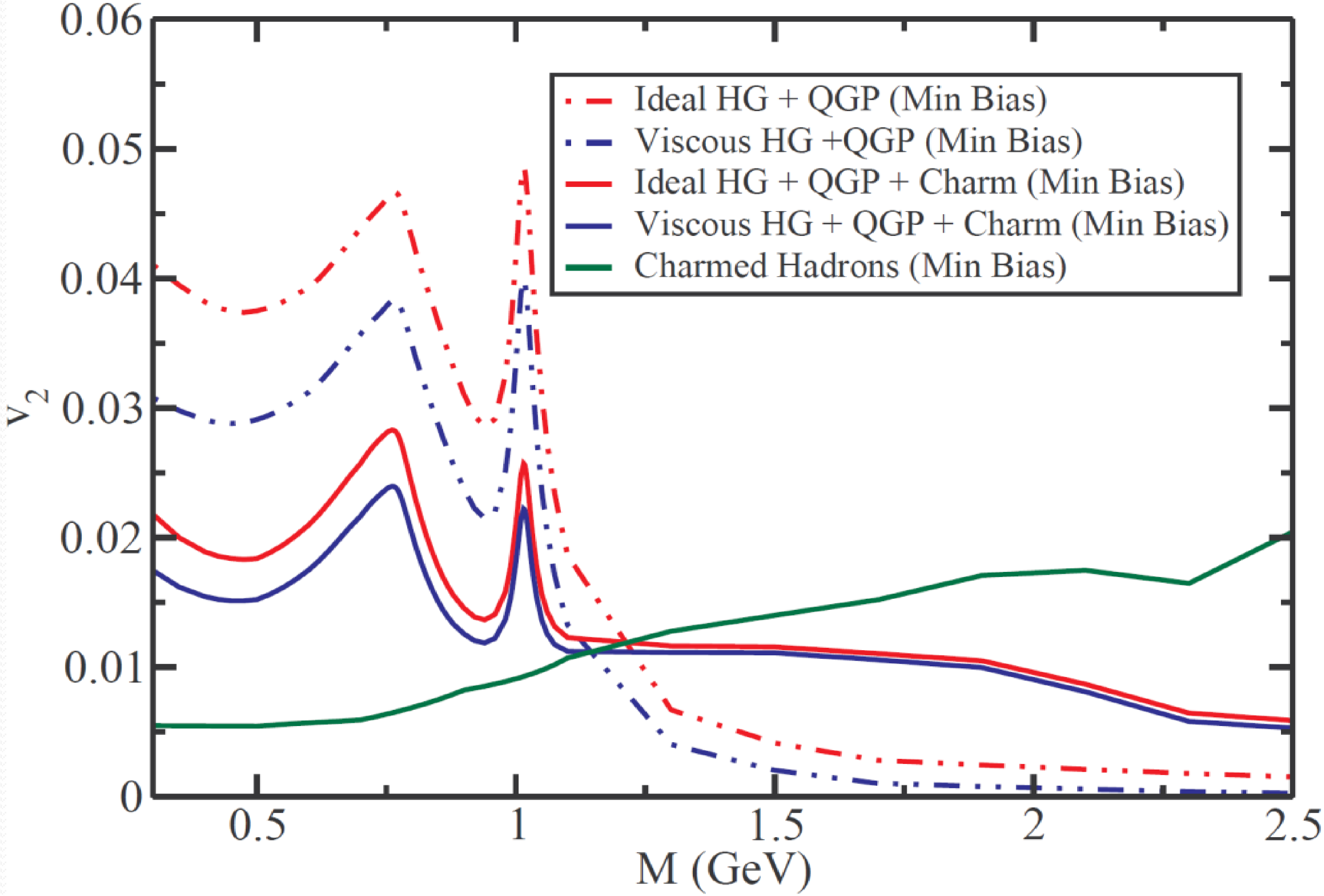
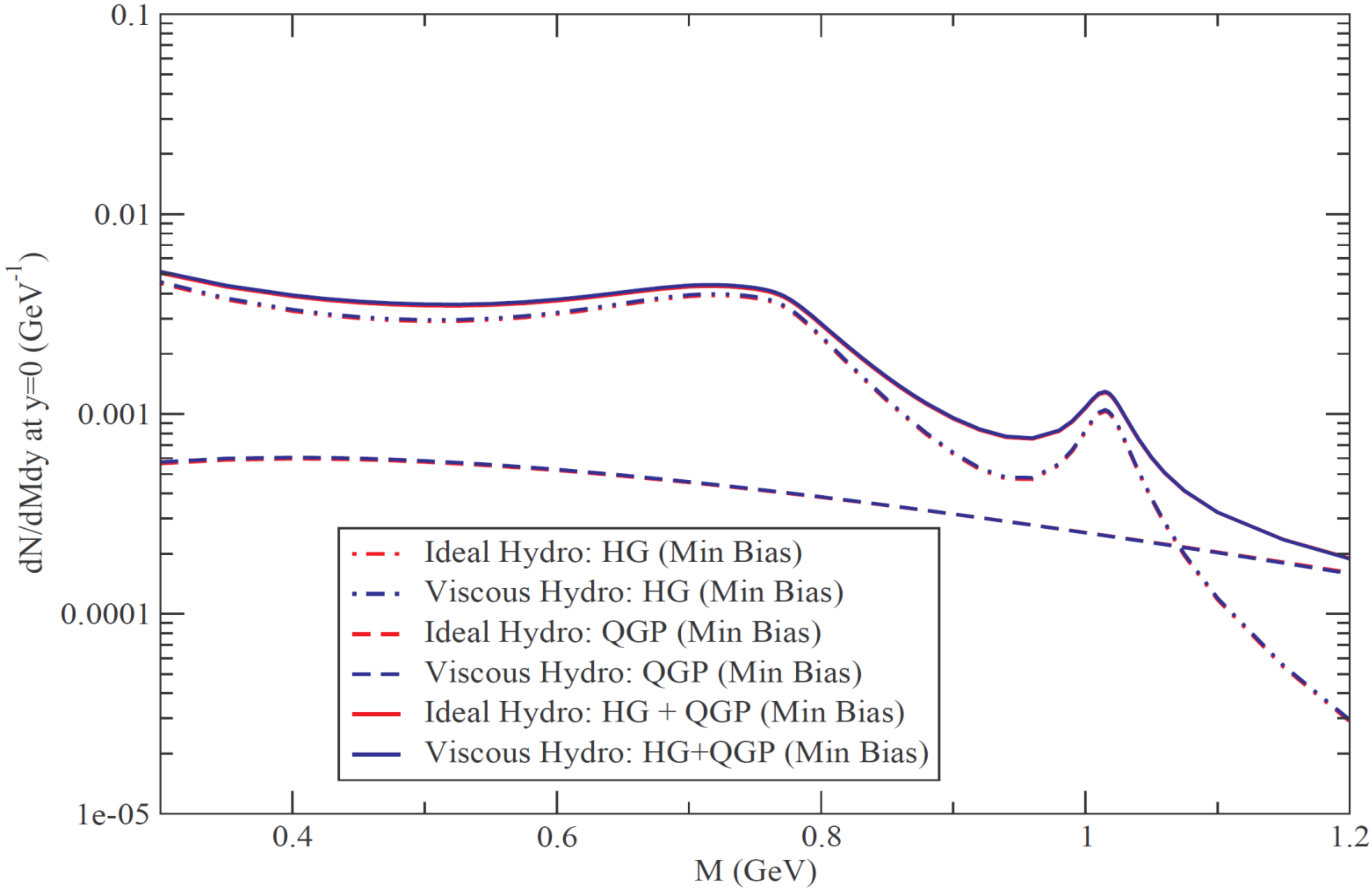
Jean-François Paquet

Igor Kozlov

V_2 including charm at Min Bias



Yields Ideal vs Viscous Hydro



Freeze-out Vector mesons

- ω and ϕ mesons

$$\frac{d^4N}{dq^4} = \frac{\Gamma_{V\to e^+e^-}}{\Gamma_V^{total}} \frac{L(M)}{M^2} [-ImD_V^R] \times \text{Cooper-Frye inc. resonance decays}$$

- ρ meson has a large width: mixing of ρ's thermal and "freeze-out" dileption emission. To deal with that assume equal time freeze-out t=t_{fo}. Then,

$$\frac{dN^{after\ fo}}{d^3x d^4q} \cong \Delta t \left[\frac{dN}{d^4x d^4q}\right]_{th}^{at\ fo} \qquad \Delta t = \frac{q^0}{M\Gamma_\rho^{fo}} = \frac{q^0}{-Im\Pi_\rho^{fo}}$$

$$\frac{dN}{d^4q} \cong \frac{\alpha^2}{\pi^3} \frac{L(M)}{M^2} \frac{m_V^4}{g_V^2} [-ImD_\rho^R] \frac{q^0 d^3x}{\exp(\beta q\cdot u)-1} \frac{1}{-Im\Pi_\rho^{fo}}$$

- Finally lifting the condition that hydro freezes out at the time t_{fo} implies $q^0 d^3x \to q^\mu d^3\Sigma_\mu$

R. Rapp [Nucl. Phys A 806 (2008) 339].

Freeze-out Vector mesons (2)

- Lastly, include rho mesons coming from resonance decays.

$$E\frac{dN_\rho}{d^3p}(res.decays) = E\frac{dN_\rho}{d^3p}(Total) - E\frac{dN_\rho}{d^3p}(\,Cooper - Frye)$$

$$E\frac{dN_\rho}{d^3p}(res.\ decays) \rightarrow E\frac{dN_{\rho\rightarrow e^+e^-}}{d^3p}$$

- Summary of cocktail contributions:

1. Dalitz decays (p,h,w,η',φ) including resonance decays in the original meson distributions.
2. Dileptons from ω and φ vector meson after freeze-out.
3. Dileptons from the ρ meson using modified Cooper-Frye distribution.
4. Lepton pairs from the ρ meson originating from resonance decays

$$E\frac{dN_\rho}{d^3p}(res.\ decays) \rightarrow E\frac{dN_{\rho\rightarrow e^+e^-}}{d^3p}$$

STAR acceptance

- Assume perfect azimuthal coverage of the detector.
- M, p_t, y of the virtual photon
  - 4-momentum of the two leptons in the CM frame
  - The boost from CM to lab frame
- Integrate over all possible angles (θ,ϕ) of the leptons in the CM to get the geometric weight

$$w = \frac{\int P_{Geom} \sin(\theta)\, d\theta d\phi}{4\pi} \quad P_{Geom} = \begin{cases} 1 & |\eta^e| < 1 \;\&\; p_t^e > 0.2\frac{GeV}{c} \;\&\; |y^{ee}| < 1 \\ 0 & otherwise \end{cases}$$

- Detector acceptances are function of M, p_t, y
- Detector efficiency factor 0.7
- Apply to theoretical curves