Thermal dileptons in high-energy heavy ion collisions with 3+1D relativistic hydrodynamics

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Outline

- Overview of Dilepton sources

Low Mass Dileptons
- Thermal Sources of Dileptons
  1) QGP (w/ viscous corrections)
  2) In-medium vector mesons (w/ viscous corrections)
- 3+1D Viscous Hydrodynamics
- Thermal Dilepton Yields & $v_2$

Intermediate Mass Dileptons
- Charmed Hadrons: Yield & $v_2$
- Conclusion and outlook
Evolution of a nuclear collision

**Space-time diagram**

- **Thermal** dilepton sources: HG+QGP
  - a) QGP: $q+q-\bar{q}\rightarrow \gamma^* \rightarrow e^+e^-$
  - b) HG: In medium vector mesons $V= (\rho, \omega, \phi)$
    - $V \rightarrow \gamma^* \rightarrow e^+e^-$

- **Kinetic freeze-out:**
  - c) Cocktail ($\pi^0$, $\eta$, $\eta'$, etc.)

**Other** dilepton sources: Formation phase
- d) Charmed hadrons: e.g. $D^{+/-} \rightarrow K^0 + e^+/\nu_e$
- e) Beauty hadrons: e.g. $B^{+/-} \rightarrow D^0 + e^+/\nu_e$
- f) Other vector mesons: Charmonium, Bottomonium
- g) Drell-Yan Processes

Sub-dominant
the intermediate
mass region
Dilepton rates from the QGP

- An important source of dileptons in the QGP

The rate in kinetic theory (Born Approx)

\[
\frac{d^4 R}{d^4 q} = \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} n(E_1)n(E_2) \nu_{12} \sigma \delta^4(q - k_1 - k_2)
\]

\[
\nu_{12} = \frac{M^2}{2E_1 2E_2}
\]

\[
\sigma = \frac{16 \pi^2 \alpha^2 N_c \Sigma_q e_q^2}{3M^2}
\]

\[
\frac{d^4 R}{d^4 q} = \left( \frac{\alpha^2 N_c}{12\pi^4} \sum_q e_q^2 \right) \frac{1}{\exp \left( \frac{q^0}{T} \right) - 1} \left\{ 1 - \frac{2T}{q} \ln \left[ \frac{1 + \exp \left( -\frac{q^0 - q}{2T} \right)}{1 + \exp \left( -\frac{q^0 + q}{2T} \right)} \right] \right\}
\]

- More sophisticated calculations: HTL, Lattice QCD.
Thermal Dilepton Rates from HG


\[ \Pi^\text{Total}_V = \Pi^\text{Vac}_V(M) + \sum_a \Pi_{Va}(q, T) \]

\[ \Pi^\text{Vac}_V(M) \text{ is described by effective Lagrangians} \]

\[ \Pi_{Va}(q, T) = -\int_{m_a}^{\infty} dk^0 \ln \left[ \frac{1 + \exp \left( -\frac{\omega_-}{T} \right)}{1 + \exp \left( -\frac{\omega_+}{T} \right)} \right] f_{va} \left( \frac{m_V}{m_a} k^0 \right) \]

\[ \omega_\pm = \frac{(q^0k^0 \pm qk)}{m_V} \]


- The dilepton production rate is:

\[ \frac{d^4R}{dq^4} = \frac{\alpha^2 L(M) m_V^4}{\pi^3 M^2 g_V^2} \left\{ -\frac{1}{3} [ImD_V^R]_\mu \right\} n_{BE}(q^0) ; \quad L(M) = \left( 1 + \frac{2m_i^2}{M^2} \right) \sqrt{1 - \frac{4m_i^2}{M^2}} \]

- Resonances contributing to \( \rho \)'s scatt. amp. & similarly for \( \omega, \phi \):

- \( N(1700) \Delta(1905) \)
- \( N(1720) \Delta(1940) \)
- \( N(1900) \Delta(2000) \)
- \( N(2000) \phi(1020) \)
- \( N(2080) \eta_1(1170) \)
- \( N(2090) a_1(1260) \)
- \( N(2100) \pi(1300) \)
- \( N(2190) a_2(1320) \)
- \( \Delta(1700) \omega(1420) \)
- \( \Delta(1900) \)
3+1D Hydrodynamics

- Viscous hydrodynamics equations for heavy ions:

\[
\begin{align*}
\partial_\mu T^{\mu \nu} &= 0. \quad \text{Energy-momentum conservation} \\
\partial_\mu J_\mu^B &= 0. \quad \text{Charge conservation}
\end{align*}
\]

\[
T^{\mu \nu} = T_0^{\mu \nu} + \pi^{\mu \nu}
\]

\[
T_0^{\mu \nu} = (\varepsilon + \mathcal{P})u^\mu u_\nu - \mathcal{P}g^{\mu \nu} \quad \mathcal{P} = \mathcal{P}(\varepsilon, \rho_B)
\]

\[
\Delta_\alpha^\mu \Delta_\beta^\nu u_\sigma \pi^{\alpha \beta} = -\frac{1}{\tau_\pi} (\pi^{\mu \nu} - S^{\mu \nu}) - \frac{4}{3} \pi^{\mu \nu} (\partial_\alpha u^\alpha) \quad \eta/s = 1/4\pi
\]

\[
S^{\mu \nu} = \eta \left( \nabla_\mu u_\nu + \nabla_\nu u_\mu - \frac{2}{3} \Delta^{\mu \nu} \nabla_\alpha u^\alpha \right) \quad t = \tau \cosh \eta_s \quad z = \tau \sinh \eta_s
\]

- Initial conditions are set by the Glauber model.

- Solving the hydro equations numerically done via the Kurganov-Tadmor method using a Lattice QCD EoS [P. Huovinen and P. Petreczky, Nucl. Phys. A 837, 26 (2010).]

- The hydro evolution is run until the kinetic freeze-out. [For details: B. Schenke, et al., Phys. Rev. C 85, 024901(2012)]
Viscous Corrections: QGP

- Viscous correction to the rate in kinetic theory rate

\[
\frac{d^4R}{dq^4} = \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} n_{FD}(k_1^0)n_{FD}(k_2^0)v_{12}\sigma\delta^4(q - k_1 - k_2)
\]

- Using the quadratic ansatz to modify F.-D. distribution

\[
n_{FD}(k^0) \rightarrow n_{FD}(k^0) + \frac{\eta C_q}{s 2T^3} n_{FD}(k^0)[1 - n_{FD}(k^0)]k^\mu k^\nu \frac{\pi_{\mu\nu}}{\eta}
\]

\[
\frac{d^4R}{dq^4} = \frac{d^4R_{\text{ideal}}}{dq^4} + \frac{d^4\delta R}{dq^4} ; \quad \frac{d^4\delta R}{dq^4} = \frac{\eta C_q}{s 2T^3} b_2(q^0, |\vec{q}|)q^\mu q^\nu \frac{\pi_{\mu\nu}}{\eta}
\]

\[
b_2(q^0, |\vec{q}|) = \frac{1}{|\vec{q}|^5} \int_{E_-}^{E_+} n(E_1)n(q^0 - E_1)(1 - n(E_1)) \times \left[ (3q_0^2 - |\vec{q}|^2)E_1^2 - 3q^0E_1M^2 + \frac{3}{4}M^4 \right] dE_1
\]

Viscous corrections to HG?

- Two modifications are plausible: $-\frac{1}{3} \text{Im} D_{\nu}^{R \mu}_\nu$ and $n_{BE}(q^0)$

- Self-Energy

$$\Pi_{Va}(p,T) = -\frac{m_a m_V T}{\pi p} \int \frac{d^3 k}{(2\pi)^3} \frac{\sqrt{s}}{k^0} f_{Va}(s) n_a(k^0); \quad \Pi^{Total}_{Va} = \Pi^{Ideal}_{Va} + \delta \Pi_{Va}$$

$$n_a(k^0) \rightarrow n_a(k^0) + \frac{\eta \; C_a}{s \; 2T^3} n_a(k^0)[1 \pm n_a(k^0)] k^\mu k^\nu \frac{\pi_{\mu\nu}}{\eta}; \quad \delta \Pi_{Va} = \frac{\eta \; C_a}{s \; 2T^3} B_2(q^0, q) q^\mu q^\nu \frac{\pi_{\mu\nu}}{\eta}$$

$$B_2(q^0, q) = -\frac{m_a}{2\pi q^2} \int_{m_a}^\infty dk^0 f_{Va}(\frac{m_V}{m_a} k^0)$$

$$\left\{ \begin{aligned} &- \frac{m_V T}{q k} \left[ m_a^2 + \frac{3(q^0 k + q k^0)^2 - (q^0 k^0 + p k)^2}{m_V^2} \right] \frac{1}{\exp \left( \frac{\omega_+}{T} \right) \pm 1} \\
&+ \frac{m_V T}{q k} \left[ m_a^2 + \frac{3(q^0 k - q k^0)^2 - (q^0 k^0 - p k)^2}{m_V^2} \right] \frac{1}{\exp \left( \frac{\omega_-}{T} \right) \pm 1} \\
&\pm 2 \left( \frac{m_V T}{q k} \right)^2 \left[ (3(q^0)^2 - q^2) \frac{k^2}{m_V^2} + \frac{2q^0 k^0 q k}{m_V^2} \right] \ln \left( 1 \pm \exp \left( \frac{\omega_+}{T} \right) \right) \\
&\pm 2 \left( \frac{m_V T}{q k} \right)^2 \left[ (3(q^0)^2 - q^2) \frac{k^2}{m_V^2} - \frac{2q^0 k^0 q k}{m_V^2} \right] \ln \left( 1 \pm \exp \left( \frac{\omega_-}{T} \right) \right) \\
&\pm 2 \left( \frac{m_V T}{q k} \right)^3 \left[ (3(q^0)^2 - q^2) \frac{k^2}{m_V^2} \right] [Li_2(\mp \exp \left( \frac{\omega_+}{T} \right)) - Li_2(\mp \exp \left( \frac{\omega_-}{T} \right))] \end{aligned} \right\}$$

- Performing the calculation => these corrections don’t matter!
Proof of concept: our yields are similar to yields obtained through R. Rapp’s rates. Both cases: ideal hydro.
How important are viscous corrections to HG rate?

- Fluid rest frame, viscous corrections to HG rates: $\propto q^i q^j \pi_{ij} / \eta$
- HG gas exists from $\tau \sim 4$ fm/c $\Rightarrow \pi_{ij} / \eta$ is small
- Direct computation shows that viscous correction to HG rates doesn’t affect the end result!

Since viscous corrections to HG rates don’t matter, only viscous flow is responsible for the modification of the $p_T$ distribution.

Also observed viscous photons HG [M. Dion et al., Phys. Rev. C 84, 064901 (2011)]

For QGP rates, both corrections matter (see yields).
A measure of elliptic flow ($v_2$)

- Elliptic Flow

- A nucleus-nucleus collision is typically not head on; an almond-shape region of matter is created.
- This shape and its pressure profile gives rise to elliptic flow.

To describe the evolution of the shape use a Fourier decomposition, i.e. flow coefficients $v_n$

$$\frac{dN}{dM p_T dp_T d\phi dy} = \frac{1}{2\pi} \frac{dN}{dM p_T dp_T dy} \left[ 1 + \sum_{n=1}^{\infty} 2v_n \cos (n\phi - n\psi) \right]$$

- Important note: when computing $v_n$'s from several sources, one must perform a **weighted average**.
$v_2$ from ideal and viscous HG+QGP (1)

- Similar elliptic flow when comparing with R. Rapp’s rates.
- Viscosity lowers elliptic flow.
- Viscosity slightly broadens the $v_2$ spectrum with $M$. 
M is extremely useful to isolate HG from QGP. At low M HG dominates and vice-versa for high M.


We can clearly see two effects of viscosity in the $v_2(p_T)$.

- Viscosity stops the growth of $v_2$ at large $p_T$ for the HG (dot-dashed curves)
- Viscosity shifts the peak of $v_2$ from to higher momenta (right, solid curves).
Charmed Hadron contribution

- Since $M_q >> T$ (or $\Lambda_{QCD}$), heavy quarks must be produced perturbatively; very soon after the nucleus-nucleus collision.

- For heavy quarks, many scatterings are needed for $p$ to change appreciably.

- In this limit, Langevin dynamics applies [Moore & Teaney, Phys. Rev. C 71, 064904 (2005)]

\[
\frac{dp^i}{dt} = -\eta DP^i + \xi^i(t) \quad \langle \xi^i(t)\xi^j(0) \rangle = \kappa \delta^{ij} \delta(t) \quad \eta_D = \frac{\kappa}{2M_qT}
\]

- Charmed Hadron production:
  - PYTHIA -> (p+p) Charm spectrum (rescale from p+p to Au+Au).
  - PYTHIA Charm -> Modify mom. dist. via Langevin (& hydro).
  - Hadronization -> Charmed Hadrons
  - Charmed Hadrons -> Dileptons
Charmed Hadrons yield and $v_2$

- Heavy-quark energy loss (via Langevin) affects the invariant mass yield of Charmed Hadrons (vs rescaled p+p), by increasing it in the low $M$ region and decreasing it at high $M$.
- Charmed Hadrons develop a $v_2$ through energy loss (Langevin dynamics) so there is a non-zero $v_2$ in the intermediate mass region.
- $v_2$ is a weighted average => it is reduced when Charmed Hadrons are included.
Conclusion & Future work

Conclusions

- First calculation of dilepton yield and $v_2$ via viscous 3+1D hydrodynamical simulation.

- $v_2(p_T)$ for different invariant masses allows to separate QGP and HG contributions.

- Slight modification to dilepton yields owing to viscosity.

- $v_2(M)$ is reduced by viscosity and the shape is slightly broadened.

- Studying yield and $v_2$ of leptons coming from charmed hadrons allows to investigate heavy quark energy loss.

Future work

- Include cocktail’s yield and $v_2$ with viscous hydro evolution.

- Study the effects of Fluctuating Initial Conditions (see Björn Schenke’s talk).

- Results for LHC are on the way.
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$V_2$ including charm at Min Bias

![Graph showing $V_2$ vs. M (GeV) for different scenarios including Ideal HG + QGP (Min Bias), Viscous HG + QGP (Min Bias), Ideal HG + QGP + Charm (Min Bias), Viscous HG + QGP + Charm (Min Bias), and Charmed Hadrons (Min Bias).]
Yields Ideal vs Viscous Hydro

\[ \frac{dN}{dM} \text{ at } y=0 \text{ (GeV)}^{-1} \]

- Ideal Hydro: HG (Min Bias)
- Viscous Hydro: HG (Min Bias)
- Ideal Hydro: QGP (Min Bias)
- Viscous Hydro: QGP (Min Bias)
- Ideal Hydro: HG + QGP (Min Bias)
- Viscous Hydro: HG+QGP (Min Bias)
Freeze-out Vector mesons

- $\omega$ and $\phi$ mesons

$$\frac{d^4N}{dq^4} = \frac{\Gamma_{V\to e^+e^-}}{\Gamma_{V}^{total}} \frac{L(M)}{M^2} \left[-\text{Im}D_V^R\right] \times \text{Cooper-Frye inc. resonance decays}$$

- $\rho$ meson has a large width: mixing of $\rho$’s thermal and “freeze-out” dilepton emission. To deal with that assume equal time freeze-out $t=t_{fo}$. Then,

$$\frac{dN_{after\ fo}}{d^3xd^4q} \approx \Delta t \left[\frac{dN}{d^4xd^4q}\right]_{th}^{at\ fo}$$

$$\Delta t = \frac{q^0}{M\Gamma_{\rho}^{fo}} = \frac{q^0}{-\text{Im}\Pi_{\rho}^{fo}}$$

$$\frac{dN}{d^4q} \approx \frac{\alpha^2 L(M) m_V^4}{\pi^3 M^2 g_V^2} \left[-\text{Im}D_\rho^R\right] \frac{q^0 d^3x}{\exp(\beta q \cdot u) - 1} \frac{1}{-\text{Im}\Pi_{\rho}^{fo}}$$

- Finally lifting the condition that hydro freezes out at the time $t_{fo}$ implies $q^0 d^3x \to q^\mu d^3\Sigma_\mu$

Lastly, include rho mesons coming from resonance decays.

\[
E \frac{dN_\rho}{d^3p} \text{ (res. decays)} = E \frac{dN_\rho}{d^3p} \text{ (Total)} - E \frac{dN_\rho}{d^3p} \text{ (Cooper – Frye)}
\]

\[
E \frac{dN_\rho}{d^3p} \text{ (res. decays)} \rightarrow E \frac{dN_{\rho \rightarrow e^+e^-}}{d^3p}
\]

Summary of cocktail contributions:

1. Dalitz decays (p,h,w,\(\eta’,\phi\)) including resonance decays in the original meson distributions.
2. Dileptons from \(\omega\) and \(\phi\) vector meson after freeze-out.
3. Dileptons from the \(\rho\) meson using modified Cooper-Frye distribution.
4. Lepton pairs from the \(\rho\) meson originating from resonance decays

\[
E \frac{dN_\rho}{d^3p} \text{ (res. decays)} \rightarrow E \frac{dN_{\rho \rightarrow e^+e^-}}{d^3p}
\]
STAR acceptance

- Assume perfect azimuthal coverage of the detector.
- $M, p_t, y$ of the virtual photon
  - 4-momentum of the two leptons in the CM frame
  - The boost from CM to lab frame
- Integrate over all possible angles $(\theta, \phi)$ of the leptons in the CM to get the geometric weight

$$W = \frac{\int P_{Geom} \sin(\theta) \, d\theta \, d\phi}{4\pi}$$

$$P_{Geom} = \begin{cases} 1 & |\eta^e| < 1 \text{ and } p_t^e > 0.2 \frac{GeV}{c} \text{ and } |y^{ee}| < 1 \\ 0 & \text{otherwise} \end{cases}$$

- Detector acceptances are function of $M, p_t, y$
- Detector efficiency factor 0.7
- Apply to theoretical curves