

# REGGAE: Monte Carlo generator of momenta obeying energy and momentum conservation

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## 1. Motivation

**The task: to generate a sample of hadrons obeying energy and momentum conservation.**

The problem also appears when one needs to integrate phase space using Monte Carlo methods. Momenta must be generated so that the phase-space **is filled uniformly** or appropriate importance sampling must be performed.

Many algorithms are available on the market [1,2,3,4]. They usually do not fill the phase-space uniformly, but assign **weights** to the generated configurations and perform importance sampling. These algorithms are designed for the use in phase-space integration in high energy physics, i.e. with a small number of particles and typically masses of particles much smaller than their momenta.

Another possibility is to generate momenta according to statistical distribution with largest entropy and **calculate** the last one or two momenta so that total energy and momentum are conserved [5,6]. These algorithms may produce slight deviations from uniform filling of the phase-space. They also may not always work and need to be repeated.

All the available algorithms turn out to be ineffective or even unusable when tackling typical problem in heavy ion collisions and high multiplicity events: **generate large number of hadrons with typical momentum scale of the order of the involved masses**. This is the task for REGGAE.

## 2. Our approach

Recall that collisions among particles conserve total energy and momentum and lead to the most likely statistical distribution of the momenta. Thus the idea is:

**First generate the momenta with any algorithm which conserves energy and momentum.**

**Then modify the momenta pairwise as if the particles of the sample would collide with each other.**

The first step does not necessarily fill the phase space uniformly. We choose GENBOD [2] for this task.

The second step leads to uniform filling of the phase space. (We have proved this!) The microscopic details of the collisions between particles are unimportant, so one can choose any differential cross section.

## References

- [1] G. Kopylov, Zhurnal. Exp. I Teor. Fiz **39** (1960) 1091
- [2] F. James, Report CERN 68-15.
- [3] R. Kleiss, W.J. Stirling, S.D. Ellis, Comput. Phys. Commun. **40** (1986) 359
- [4] M.M. Block, Comput. Phys. Commun. **69** (1992) 459
- [5] V.D. Toneev, private communication
- [6] L. Ferroni, *The microcanonical ensemble of the relativistic hadron gas*, PhD thesis, University of Florence, 2006

## 3. Details of the application

Step 1: GENBOD (Fred James, [2])

Particles of masses  $m_1, m_2, m_3, \dots, m_n$  are generated as in a sequential decay of very heavy resonances. The angular distribution in each decay is uniform.

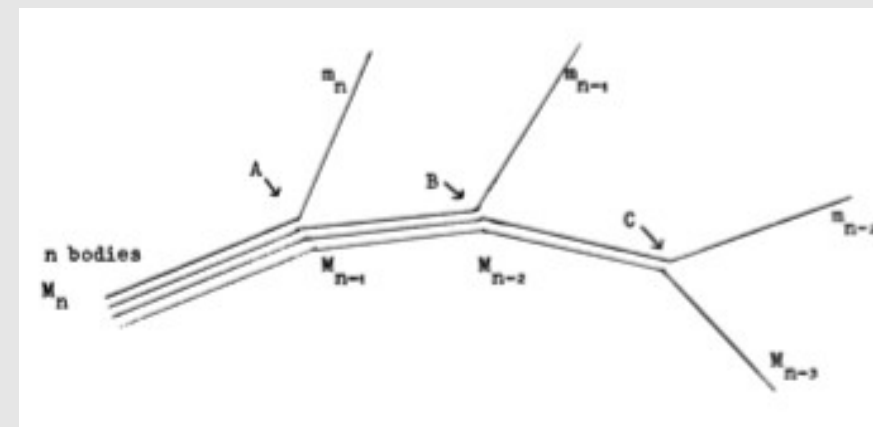


Figure (F. James [2]) particle momenta are generated as in a sequential decay of heavy resonances.

Masses of the resonances  $M_i$  must be such that

$$m_i + M_{i-1} \leq M_i \leq M_{i+1} - m_{i+1}$$

Step 2: collisions

Each particle collides a few times so that momenta thermalise. We use the s-wave scattering: momenta of a pair of particles are rotated randomly in the pair CMS.

Typically about 10 collisions per particle are sufficient, as shown in this figure:

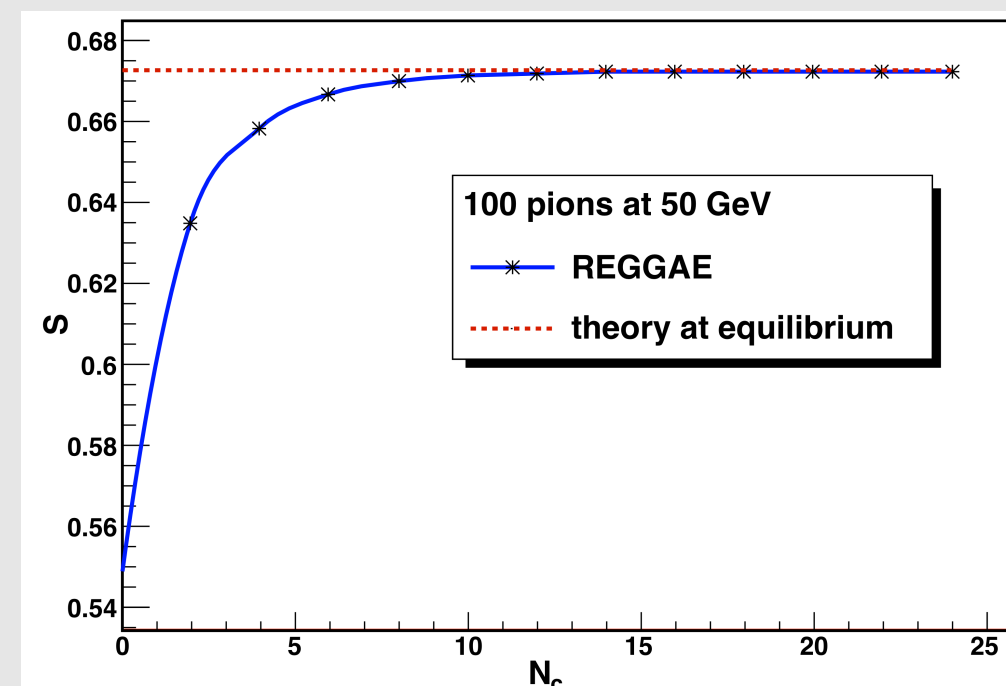


Figure: mean information entropy of the distribution of momenta as a function of the number of collisions which each particle suffers. The system consists of 100 pions with total energy 50 GeV. Red dotted line shows the value for thermal distribution. The simulated distributions converge to this value.

## 4. Comparison to other algorithms

We evaluated the integral over whole phase space of the function

$$f_5(p_1, p_2, p_3, p_4, p_5) = \frac{p_1 \cdot p_2 - p_3 \cdot p_4}{M^2 + (p_4 + p_5)^2}$$

With  $M^2=4 \text{ GeV}^2$ , total energy 100 GeV and masses of all particles 1 GeV/c.

We compared it with (weighted) GENBOD [2], RAMBO [3] and NUPHAZ [4].

For calculation of  $10^7$  configurations with 5 particles the computing times (AMD Athlon(tm) 64 X2 Dual Core Processor, speed: 3 GHz, RAM: 2 GB, OS: Ubuntu Linux, kernel: 2.6.31-22) were: REGGAE 25 min, NUPHAZ 5 min, RAMBO 1 min, wGNBOD 7 min

For events with 60 particles, the times for generation of  $10^5$  configurations were: REGGAE **5 min**, NUPHAZ 65 min and no other algorithm completed the task.

**REGGAE is an effective Monte Carlo generator obeying energy and momentum conservation and filling the phase space uniformly. It is designed for the use in heavy ion collisions and high multiplicity events. As input, it needs the masses of all particles and returns the momenta.**