Derivation of the medium-induced splitting kernels from Soft Collinear Effective Theory with Glauber gluons

Grigory Ovanesyan

arXiv:1109.5619 JHEP

arXiv:1103.1074 Phys.Lett.B

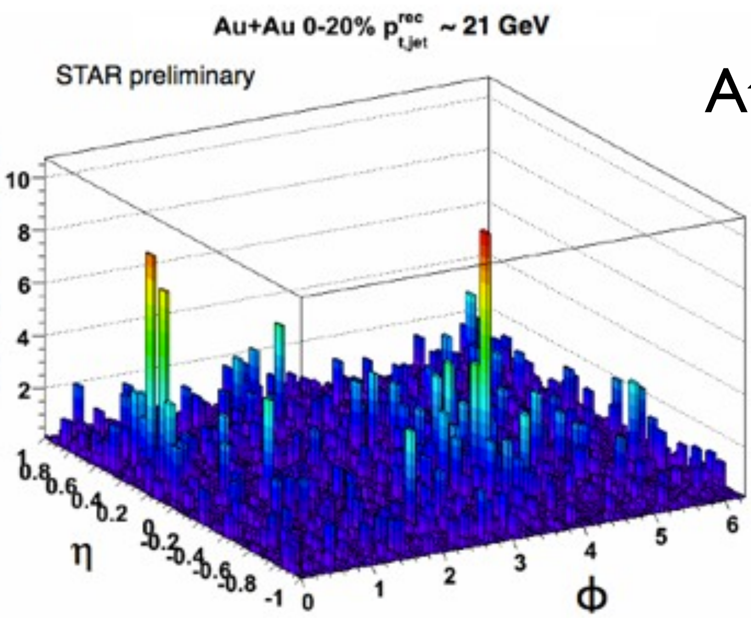
(in collaboration with Ivan Vitev)

The XXIII International Conference on Ultrarelativistic Nucleus-Nucleus Collisions, August 13-18, 2012, Washington D.C. (Quark Matter 2012)

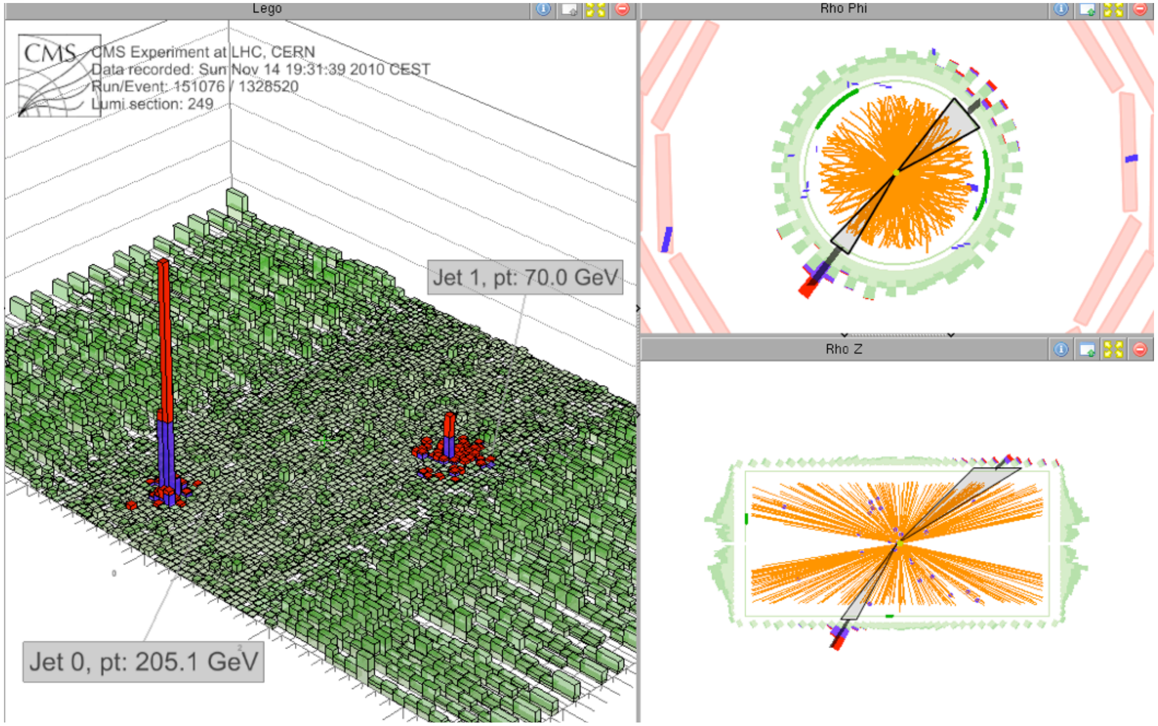
Outline

- Motivation
- An effective theory for jets in the medium
- Medium-Induced splitting kernels
- Future Outlook
- Conclusions

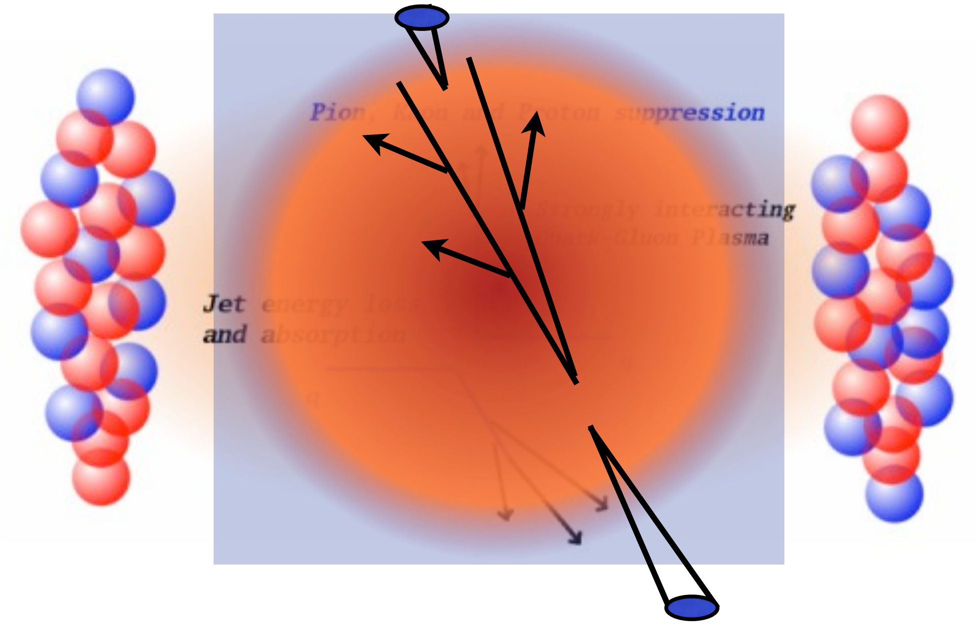
Jets at RHIC vs LHC



At LHC energies the jets have much less soft background and are cleaner probes



Theoretical Approaches

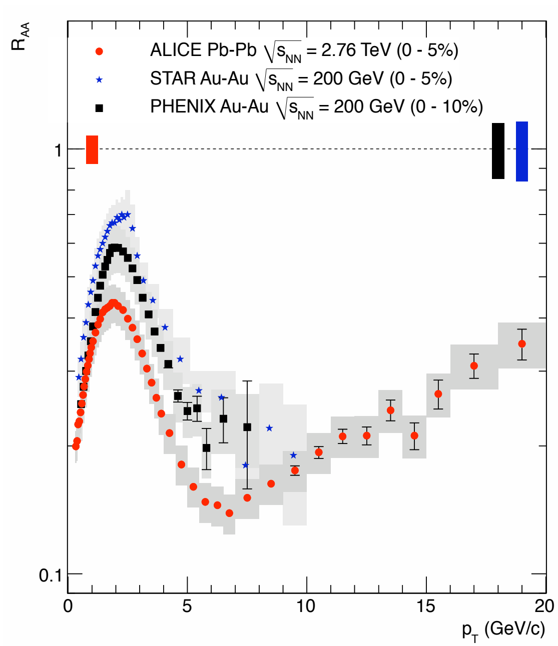


- PQCD
- Thermal Field Theory
- Lattice QCD
- Hydrodynamics
- AdS/CFT correspondence

We will construct effective theory for perturbative calculation of jet energy loss

Motivation

ALICE collaboration, 11-12/2010



$$R_{AA}(p_T) = \frac{(1/N_{evt}^{AA})\, d^2N_{\rm ch}^{AA}/d\eta\, dp_T}{\langle N_{\rm coll}\rangle\, (1/N_{evt}^{pp})\, d^2N_{\rm ch}^{pp}/d\eta\, dp_T},$$

the number of binary nucleon-nucleon collisions

- Improved jet quenching phenomenology
- Effective theory valid for calculations of radiative and collisional energy losses
- Factorization of medium-induced splittings
- Gauge invariance

An effective theory for jets in the medium

Soft Collinear Effective Theory

ψ, A ξ_n, A_c, A_s

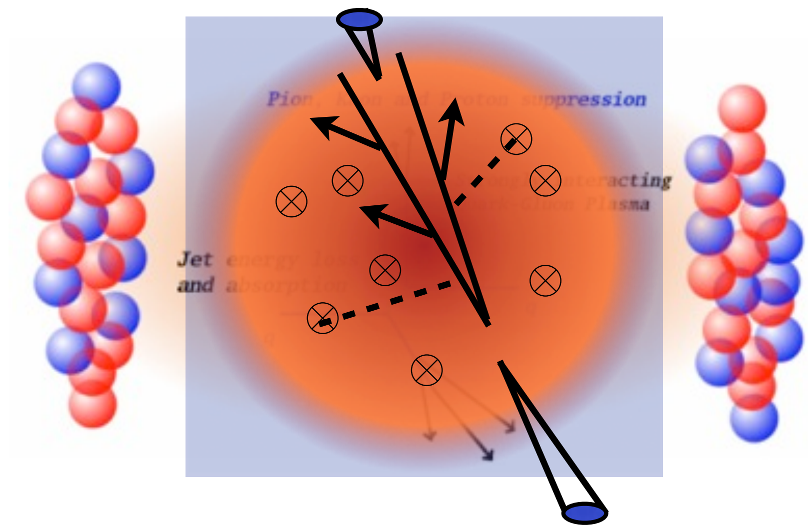
Bauer, Fleming, Luke, Pijrol, Stewart, 00

QCD → SCET $p_s \sim (\lambda^2, \lambda^2, \lambda^2)$ $p_c \sim (1, \lambda^2, \lambda)$

- Soft Collinear Effective Theory (SCET) is an effective theory of QCD that describes the dynamics of highly energetic quarks and gluons
- Ideally suited for highly energetic jets
- In QCD one uses the full theory Lagrangian and performs high energy expansion diagram by diagram
- SCET Lagrangian captures the leading power of QCD and so, no further expansion needed
- In addition one gets all the benefits of effective field theory: higher symmetries, straightforward way to incorporate power corrections, power to resum large logarithms
- 1000+ papers exist on using SCET for hadron and lepton collisions
- ~5 papers on using SCET in the medium

Gyulassy-Wang model

Gyulassy, Wang, 94



- A simple model of medium has a finite number of scattering centers with a static Debye-screened potential
- It helps us understand that SCET needs to be expanded in order to correctly describe jet properties in the medium

$$H = \sum_{n=1}^{N} H(q; x_n) = 2\pi\delta(q^0)\, v(q) \sum_{n=1}^{N} \mathrm{e}^{iqx_n}\, T^a(R) \otimes T^a(n)$$

$$v(q) = \frac{4\pi\alpha_s}{q_z^2 + \mathbf{q^2} + \mu^2}$$

- The momentum scaling of the exchange gluon is that of the Glauber gluon: $q(\lambda^2, \lambda^2, \lambda)$

What we want from effective theory

- The goal is to construct an effective theory for highly energetic quarks and gluons in the medium
- Lagrangian that describes elastic scattering of quarks and gluons from the medium scattering centers
- SCET is a good start
- Need to add the Glauber gluons to the SCET Lagrangian: SCET_G

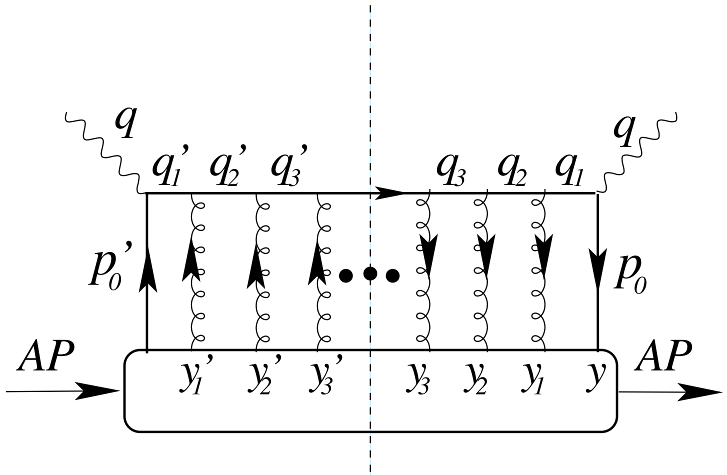
$$q \propto (\lambda^2, \lambda^2, \lambda)$$

Glauber gluons have to be integrated out of the theory: effective potential.

Other papers on $SCET_G$

Idilbi, Majumder(08)

SIDIS



$\mathcal{L}_G(\xi_n, A_G)$

- n-collinear jet
- $\bar{\text{n}}$-collinear source of Glauber gluons
- covariant, light-cone gauge

D'Eramo, Liu, Rajagopal(10)

$$P(k_\perp) = \int d^2x_\perp \, e^{-ik_\perp \cdot x_\perp} \, \mathcal{W}_\mathcal{R}(x_\perp)$$

$$\mathcal{W}_\mathcal{R}(x_\perp) = \frac{1}{d(\mathcal{R})} \left\langle \mathrm{Tr}\left[W_\mathcal{R}^\dagger[0, x_\perp] \, W_\mathcal{R}[0,0] \right] \right\rangle$$

$$W_F\left[y^+, y_\perp\right] \equiv P\left\{ \exp\left[ig \int_0^{L^-} dy^- \, A^+(y^+, y^-, y_\perp) \right] \right\}$$

- Probability density of the scattered jet is equal to exp.value of two Wilson Lines
- Derived in the covariant gauge

Lagrangian of SCET$_G$

The SCET Lagrangian contains everything :)

$$\mathcal{L}_{\rm SCET}(\xi_n, A_n, A_s) = \bar{\xi}_n \left[in\cdot D + i\not{D}^{\perp} \frac{1}{i\bar{n}\cdot D}\, i\not{D}^{\perp} \right] \frac{\not{\bar{n}}}{2} \xi_n + \mathcal{L}_{\rm YM}(A_n, A_s) ,$$

$$\mathcal{L}_{\rm YM}(A_n, A_s) = \frac{1}{2g^2} {\rm tr} \left\{ \left[iD_s^{\mu} + gA^{\mu}_{n,q}, iD_s^{\nu} + gA^{\nu}_{n,q'} \right] \right\}^2 + \mathcal{L}_{\rm G.F.} ,$$

$$\mathcal{L}_{\rm G.F.}(R_{\xi}) = \frac{1}{\xi} {\rm tr} \left\{ \left[iD_{s\mu}, A^{\mu}_{n,q} \right] \right\}^2 ,$$

$$\mathcal{L}_{\rm G.F.}({\rm LCG}(b)) = \frac{1}{\xi} {\rm tr} \left\{ b_{\mu} A^{\mu}_{n,q} \right\}^2 .$$

$$iD^{\mu} = i\partial^{\mu} + g(A^{\mu}_s + A^{\mu}_c + A^{\mu}_G)$$

All we need in order to derive all interactions between collinear(and soft) particles with Glaubers is the scaling rule for the vector potential

covariant gauge $\quad A^{\mu}_G \propto (\lambda^4, \lambda^2, \lambda^3)$

Anti-collinear source of Glaubers

light-cone gauge $\quad A^{\mu}_G \propto (\lambda^2, 0, \lambda)$

Idilbi, Majumder, 08

Lagrangian of $\mathrm{SCET_G}$

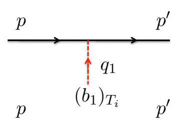
$$\mathcal{L}_{\rm G}\left(\xi_n, A_n, \eta\right) = \sum_{p,p',q} \mathrm{e}^{-i(p-p'+q)x} \left(\bar{\xi}_{n,p'} \Gamma^{\mu,a}_{\rm qqA_G} \frac{\bar{\not{n}}}{2} \xi_{n,p} - i \Gamma^{\mu\nu\lambda,abc}_{\rm ggA_G} \left(A^b_{n,p'}\right)_\nu \left(A^c_{n,p}\right)_\lambda \right) \bar{\eta}\, \Gamma^{\nu,a}_{\rm s}\, \eta\, \Delta_{\mu\nu}(q)$$

- Our Glauber Lagrangian is invariant under the gauge symmetries of SCET $\mathcal{L}_{\rm G}\left(\xi_n, A_n, \eta\right) \to \mathcal{L}_{\rm G}\left(W_n^\dagger \xi_n, \mathcal{B}_n(A_n), \eta\right) \equiv \mathcal{L}_{\rm G}\left(\chi_n, \mathcal{B}_n, \eta\right)$
- We will use the static source and three gauge choices:
- covariant(A_G, A_c)
- light-cone(A_G, A_c) and
- hybrid($A_c^+=0$, covariant(A_G))

Feynman rules of SCET$_G$

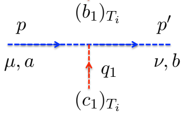
from Lagrangian

R_ξ



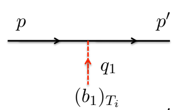
$$= i\, v(q_{1\perp})\,(b_1)_R\,(b_1)_{T_i}\,\frac{\bar{\not n}}{2}$$

$$A^\mu_G \propto (\lambda^2, \lambda^2, \lambda^3)$$



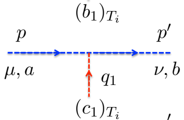
$$= v(q_{1\perp})\, f^{abc_1}\,(c_1)_{T_i}\left[g^{\mu\nu}\,\bar n\cdot p + \bar n^\mu\, q^\nu_{1\perp} - \bar n^\nu\, q^\mu_{1\perp} - \frac{1-\frac{1}{\xi}}{2}\,(\bar n^\nu p^\mu + \bar n^\mu p'^\nu)\right]$$

A^+

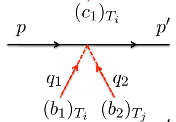


$$= i\, v(q_{1\perp})\,(a)_R\,(b_1)_{T_i}\left(1 + \frac{p^2-p'^2}{p^+[q_1^+]}\right)\frac{\bar{\not n}}{2}$$

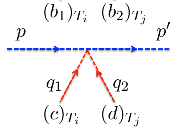
$$A^\mu_G \propto (0, \lambda^2, \lambda)$$



$$= v(q_{1\perp})\, f^{abc_1}\,(c_1)_{T_i}\left[g^{\mu\nu}_\perp\,\bar n\cdot p\left(1 + \frac{p^2-p'^2}{p^+[q_1^+]}\right) + \frac{q^\mu_{1\perp}p'^\nu + q^\nu_{1\perp}p^\mu}{[q_1^+]}\right]$$

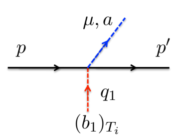


$$= i\, v(q_{1\perp})v(q_{2\perp})\,(b_1b_2)_R\,(b_1)_{T_i}(b_2)_{T_j}\,\frac{2\,q_{1\perp}\cdot q_{2\perp}}{p^+[q_1^+][q_2^+]}\,\frac{\bar{\not n}}{2}$$



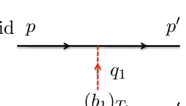
$$= \frac{(-i)v(q_{1\perp})v(q_{2\perp})}{[q_1^+][q_2^+]}\,(c)_{T_i}\,(d)_{T_j}\Big[f^{cax}f^{xdb}\left(q_{1\perp}\cdot q_{2\perp}\,g^{\mu\nu}_\perp - q^\nu_{1\perp}q^\mu_{2\perp}\right) +$$

$$+\ f^{cbx}f^{xad}\left(q^\mu_{1\perp}q^\nu_{2\perp} - q_{1\perp}\cdot q_{2\perp}\,g^{\mu\nu}_\perp\right) + f^{cdx}f^{xab}\left(q^\mu_{1\perp}q^\nu_{2\perp} - q^\nu_{1\perp}q^\mu_{2\perp}\right)\Big]$$



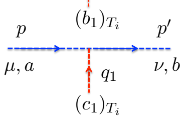
$$= \frac{(-i)\,v(q_{1\perp})}{[q_1^+]}\left[\frac{(ab_1)_R\,(b_1)_{T_i}}{p^+}\,\gamma^\mu_\perp \not q_{1\perp} + \frac{(b_1)_{T_i}\,(b_1a)_R}{p'^+}\,\not q_{1\perp}\gamma^\mu_\perp\right]\frac{\bar{\not n}}{2}$$

Hybrid



$$= i\, v(q_{1\perp})\,(b_1)_R\,(b_1)_{T_i}\,\frac{\bar{\not n}}{2}$$

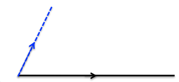
$$A^\mu_G \propto (\lambda^2, \lambda^2, \lambda^3)$$



$$= v(q_{1\perp})\, f^{abc_1}\,(c_1)_{T_i}\, g^{\mu\nu}_\perp\,\bar n\cdot p$$

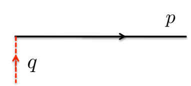
from Wilson lines

W_n

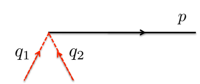


Idilbi, Scimemi, 10

T_n

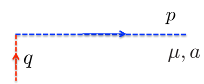


$$= (b)_r(b)_{T_i}\, v(q_\perp)C^{(\text{Pres})}_\infty\left(\frac{1}{q^+ + i\varepsilon} - \frac{1}{q^+ - i\varepsilon}\right)$$

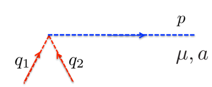


$$= -\frac{1}{2}\,\frac{l_\perp q_{1\perp}(b_1b_2)_r + l_\perp q_{2\perp}(b_2b_1)_r}{l_\perp q_{1\perp} + l_\perp q_{2\perp}}\,(b_1)_{T_i}(b_2)_{T_j}\, v(q_{1\perp})v(q_{2\perp})$$

$$\times\left[C^{(\text{Pres})}_\infty\right]^2\left(\frac{1}{q_1^+ + i\varepsilon} - \frac{1}{q_1^+ - i\varepsilon}\right)\left(\frac{1}{q_2^+ + i\varepsilon} - \frac{1}{q_2^+ - i\varepsilon}\right)$$



$$= -i f^{abc}\,(-g^{\mu\nu}_\perp)\,(c)_r(b)_{T_i}\, v(q_\perp)C^{(\text{Pres})}_\infty\left(\frac{1}{q^+ + i\varepsilon} - \frac{1}{q^+ - i\varepsilon}\right)$$



$$= -\frac{1}{2}\,(-g^{\mu\nu}_\perp)\,\frac{l_\perp q_{1\perp}\, f^{ab_1c_1}f^{c_1b_2e} + l_\perp q_{2\perp}\, f^{ab_2c_1}f^{c_1b_1e}}{l_\perp q_{1\perp} + l_\perp q_{2\perp}}\,(e)_r\,(b_1)_{T_i}(b_2)_{T_j}$$

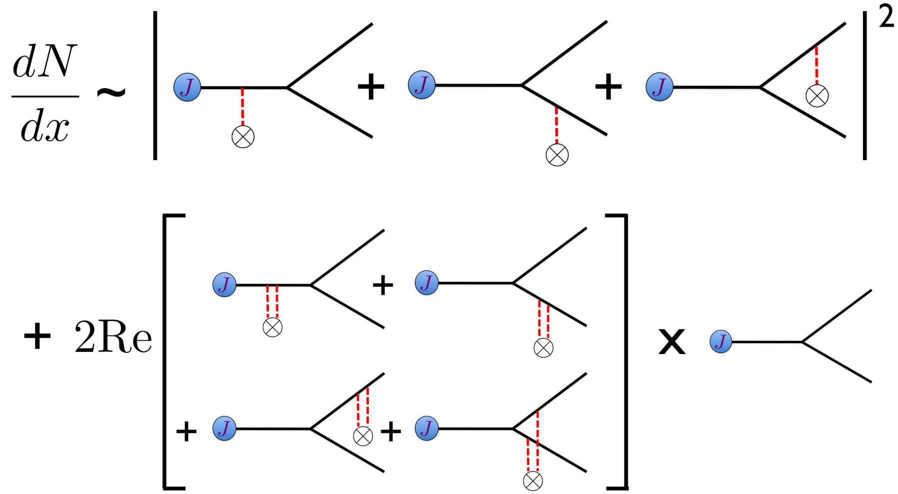
$$\times\left[C^{(\text{Pres})}_\infty\right]^2\left(\frac{1}{q_1^+ + i\varepsilon} - \frac{1}{q_1^+ - i\varepsilon}\right)\left(\frac{1}{q_2^+ + i\varepsilon} - \frac{1}{q_2^+ - i\varepsilon}\right)$$

Medium-induced splitting kernels

Using effective theory SCET_G it is a straightforward task to calculate the medium-induced splittings

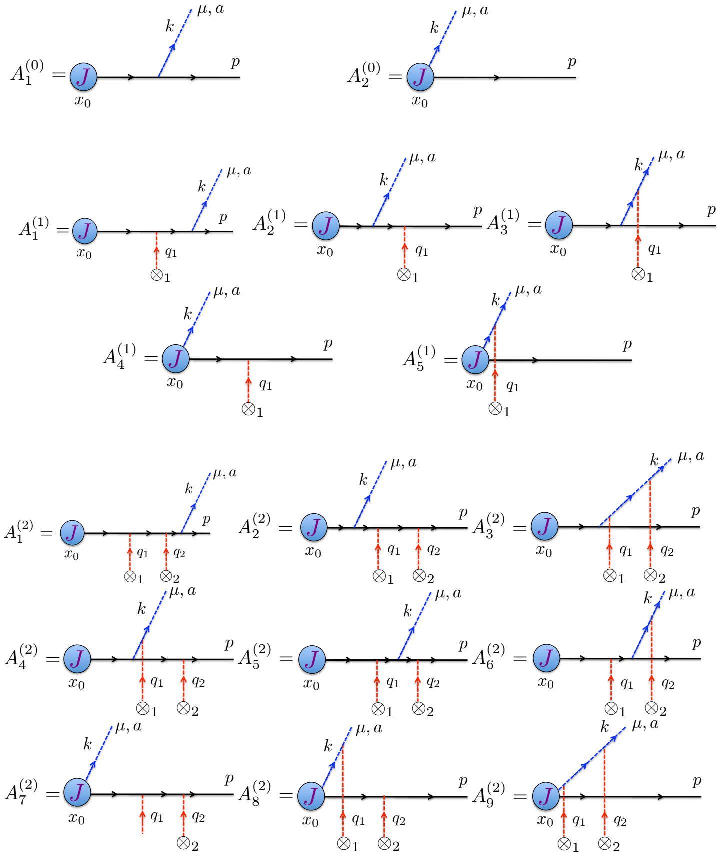
We pay special attention to proving the factorization of the splitting from the production probability

GO, Vitev, 2011



Radiative energy loss at first order in opacity

$$A_{\rm brem} = \langle J | T\, \bar{\chi}_n(x_0)\, {\rm e}^{i\int d^4x \left(\mathcal{L}^{\rm QCD} + \mathcal{L}^{\rm SCET_G}\right)} |\mathbf{p}, \mathbf{k}\rangle$$



An approximate formula in small x approximation for LPM effect has been derived in GLV, 00

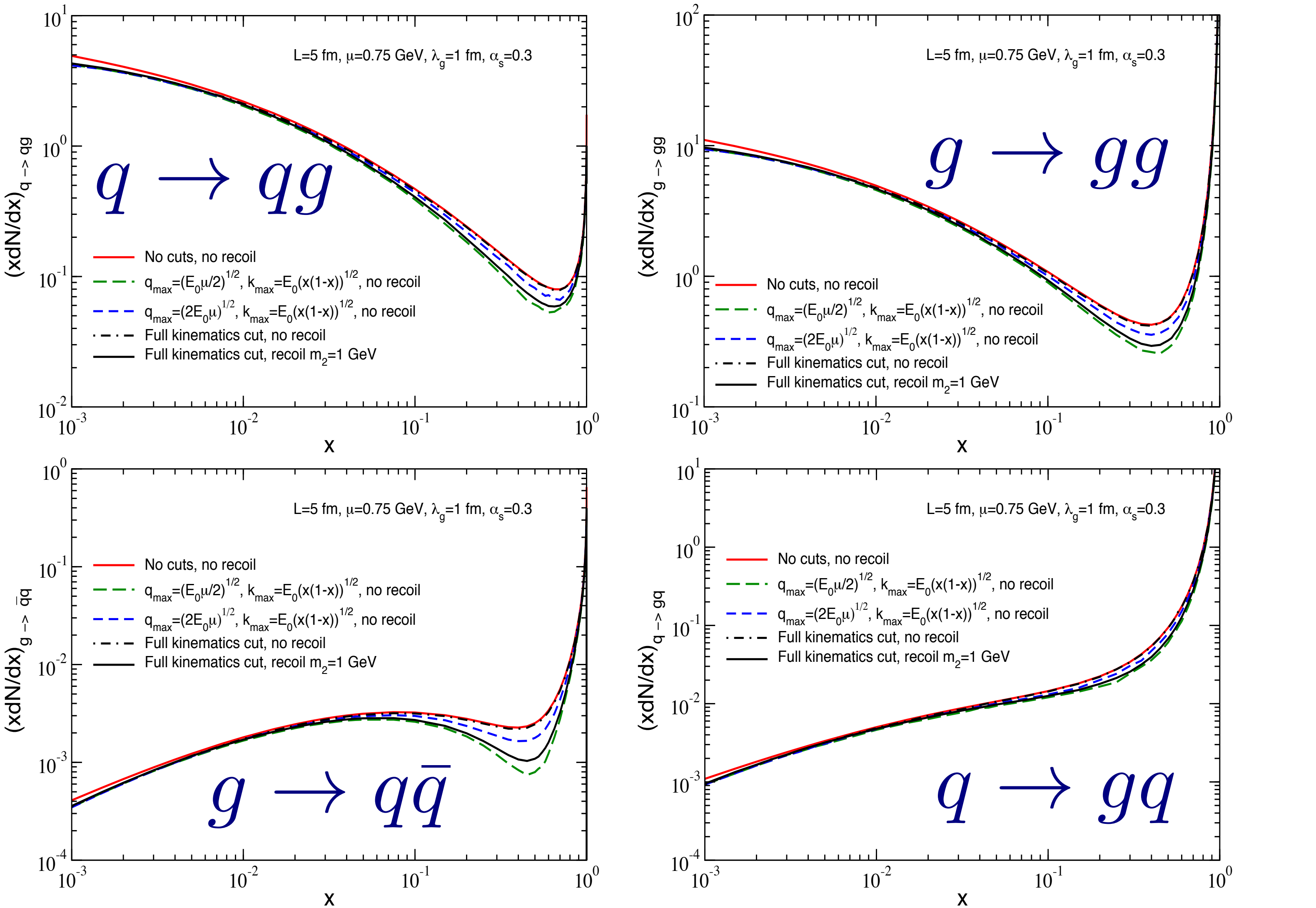
Later extended to initial state interactions in Vitev, 07

$$x\frac{dN^g}{dx d^2\boldsymbol{k}_\perp} = C_F \frac{\alpha_s}{\pi^2}\left(1 - x + \frac{x^2}{2}\right)\int \frac{d\Delta z}{\lambda_g(z)} \int d^2\mathbf{q}_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{g\ \rm medium}}{d^2\mathbf{q}_\perp}\left[-\left(\frac{\boldsymbol{A}}{\boldsymbol{A}^2}\right)^2 + 2\left(\frac{\boldsymbol{C}}{\boldsymbol{C}^2}\right)^2 - \frac{\boldsymbol{A}}{\boldsymbol{A}^2}\cdot\frac{\boldsymbol{C}}{\boldsymbol{C}^2} \right.$$
$$- \frac{\boldsymbol{B}}{\boldsymbol{B}^2}\cdot\frac{\boldsymbol{C}}{\boldsymbol{C}^2}\left(1 - \cos[(\Omega_1 - \Omega_2)\Delta z] + \cos[(\Omega_2 - \Omega_3)\Delta z]\right)$$
$$+ \frac{\boldsymbol{C}}{\boldsymbol{C}^2}\cdot\left(\frac{\boldsymbol{A}}{\boldsymbol{A}^2} + \frac{\boldsymbol{B}}{\boldsymbol{B}^2} - 2\frac{\boldsymbol{C}}{\boldsymbol{C}^2}\right)\cos[(\Omega_1 - \Omega_3)\Delta z] + \frac{\boldsymbol{A}}{\boldsymbol{A}^2}\cdot\left(\frac{\boldsymbol{A}}{\boldsymbol{A}^2} - \frac{\boldsymbol{D}}{\boldsymbol{D}^2}\right)\cos[\Omega_4 \Delta z]$$
$$\left. + \frac{\boldsymbol{A}}{\boldsymbol{A}^2}\cdot\frac{\boldsymbol{D}}{\boldsymbol{D}^2}\cos[\Omega_5\Delta z] + \left(\frac{N_c^2 - 1}{N_c^2}\left(\frac{\boldsymbol{B}}{\boldsymbol{B}^2}\right)^2 + \frac{1}{N_c^2}\frac{\boldsymbol{A}}{\boldsymbol{A}^2}\cdot\frac{\boldsymbol{B}}{\boldsymbol{B}^2}\right)\left(1 - \cos[(\Omega_1 - \Omega_2)\Delta z]\right)\right]. \quad (9.43)$$

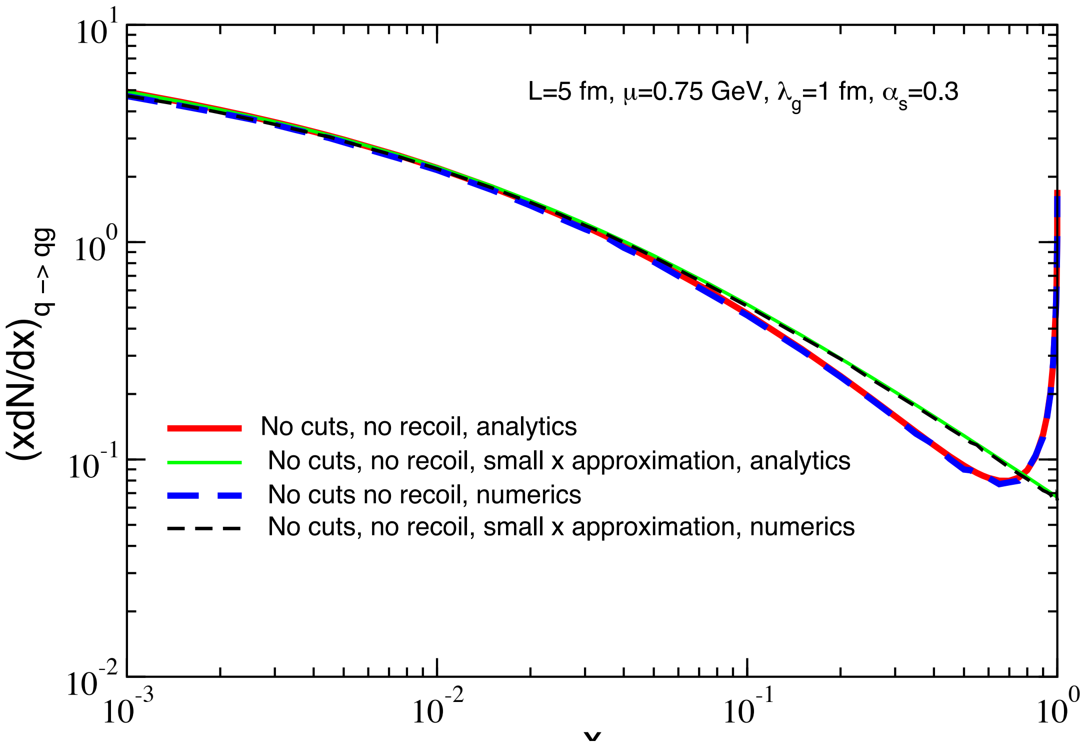
GO, Vitev, 11

Medium-induced splitting kernels

GO, Vitev, 2011



Taking the small x limit



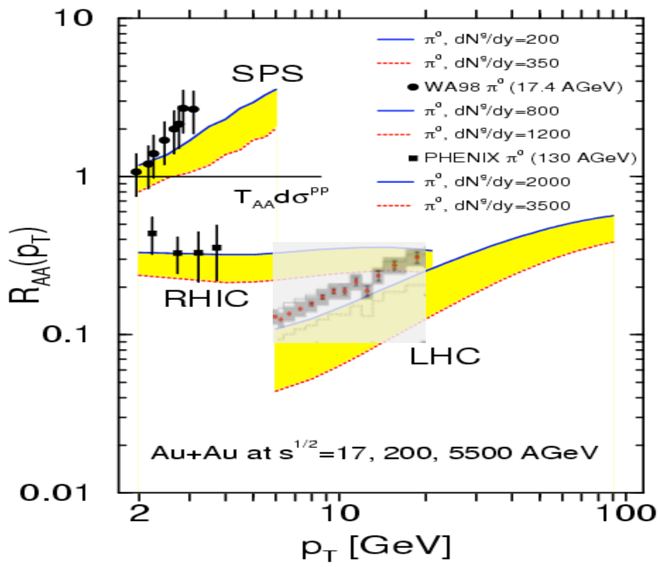
$$x\left(\frac{dN}{dx}\right)_{\begin{Bmatrix} q\to qg \\ g\to gg \\ g\to q\bar{q} \\ q\to gq \end{Bmatrix}} = \frac{\alpha_s}{\pi^2}\begin{Bmatrix} C_F[1+\mathcal{O}(x)] \\ C_A[1+\mathcal{O}(x)] \\ T_R[0+\frac{x}{2}+\mathcal{O}(x^2)] \\ C_F[0+\frac{x}{2}+\mathcal{O}(x^2)] \end{Bmatrix}$$

$$\times\int d\Delta z\begin{Bmatrix} \frac{1}{\lambda_g(z)} \\ \frac{1}{\lambda_g(z)} \\ \frac{1}{\lambda_q(z)} \\ \frac{1}{\lambda_q(z)} \end{Bmatrix}\int d^2\mathbf{k}_\perp d^2\mathbf{q}_\perp \frac{1}{\sigma_{el}}\frac{d\sigma_{el}^{\text{medium}}}{d^2\mathbf{q}_\perp}$$

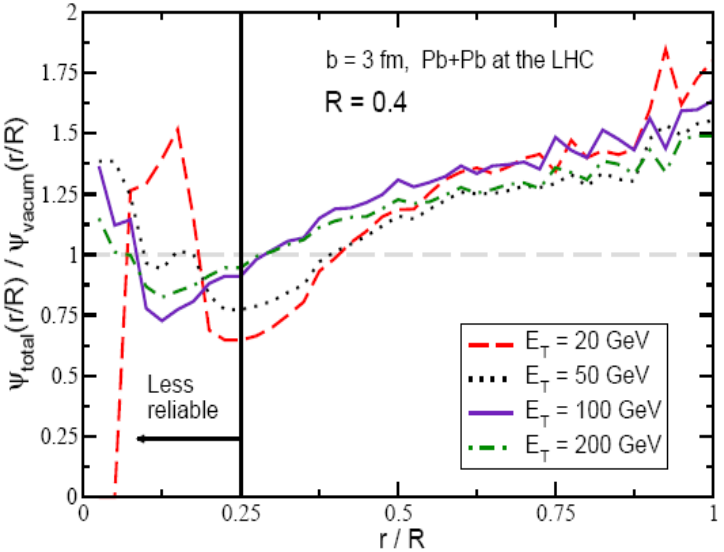
$$\times\frac{2\boldsymbol{k}_\perp\cdot\boldsymbol{q}_\perp}{\boldsymbol{k}_\perp^2(\boldsymbol{k}_\perp-\boldsymbol{q}_\perp)^2}\left[1-\cos\frac{(\boldsymbol{k}_\perp-\boldsymbol{q}_\perp)^2}{xp_0^+}\Delta z\right]. \quad (18)$$

In the small x approximation the 4 splittings reduce to only 2, which coincide with the results derived in GLV, 00

Future Outlook I



Vitev, Gyulassy (2002)

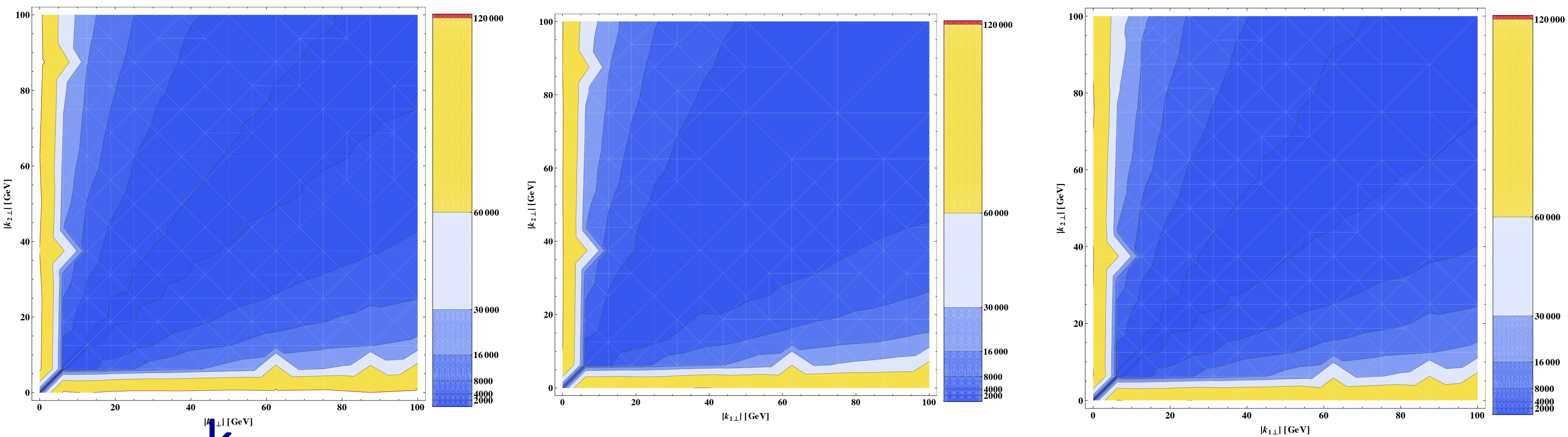


Vitev, Wicks, Zhang, 08

- GLV prediction from 2002 describes the trends of Alice data remarkably well
- Vitev, Wicks, Zhang 08 prediction qualitatively agrees with new CMS data on jet shapes in the medium
- Currently we are working on implementing full x medium-splittings (need beyond energy loss)

Future Outlook II

k_{T2} 1→3 | Cascade 1→2 | Cascade 1→2(AO)



k_{T1}

- Angular ordering is a well-known effect used in parton showers like Pythia and Herwig
- Using SCET we derived 1→3(Catani, Grazzini 99) splittings and compared to cascade of 1→2 splittings
- Angular ordering can be viewed as a practical prescription to improve the precision of parton shower
- Similar calculation in the medium using $SCET_G$ is coming soon (Fickinger, G.O., Vitev)

Conclusions

- We constructed effective theory applicable for jets(quark and gluon) in the medium
- We derived all medium-induced kernels beyond the small x approximation
- We derived the factorization of splitting from the hard production
- We explicitly checked the gauge invariance of jet broadening and radiative energy loss
- We showed how phase-space cuts and nuclear recoil can be incorporated
- These advances put jet quenching phenomenology on stronger grounds and will lead to higher accuracy of theoretical predictions