Search for the QCD Critical Point by Higher Moments of Net-proton Multiplicity Distributions at STAR

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Main Goals of RHIC Beam Energy Scan (BES):

- Signals for phase transition/phase boundary.
- Search for Critical Point (CP).
- Bulk properties of QCD matter.

Lattice QCD:

- Crossover at $\mu_B = 0$, 1\textsuperscript{st} order phase transition at large $\mu_B$.


- QCD Critical Point (CP): The end point of first order phase transition boundary.

Experiment: Access the QCD Phase Diagram

STAR Detector

Time Projection Chamber (TPC)

- Large Uniform Acceptance
- Excellent Particle Identification

Varying beam energy varies Temperature and Baryon Chemical Potential.

(RHIC BES-Phase I: Au+Au collisions at \(\sqrt{s}=7.7, 11.5, 19.6, 27, 39, 62.4, 200 \text{ GeV}\))

<table>
<thead>
<tr>
<th>(\sqrt{s} (\text{GeV}))</th>
<th>(\mu_B (\text{MeV}))</th>
<th>(T (\text{MeV}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.7</td>
<td>422</td>
<td>140</td>
</tr>
<tr>
<td>11.5</td>
<td>316</td>
<td>152</td>
</tr>
<tr>
<td>19.6</td>
<td>206</td>
<td>160</td>
</tr>
<tr>
<td>27</td>
<td>156</td>
<td>163</td>
</tr>
<tr>
<td>39</td>
<td>112</td>
<td>164</td>
</tr>
<tr>
<td>62.4</td>
<td>73</td>
<td>165</td>
</tr>
<tr>
<td>200</td>
<td>24</td>
<td>166</td>
</tr>
</tbody>
</table>


- Access a broad region of QCD phase diagram by RHIC BES program.
- STAR is an ideal detector to perform correlation and fluctuation analysis to study the QCD phase diagram.

Observable: Higher Moments of Net-proton Distributions.

Moments sensitive to correlation length ($\xi$):
(Study phase transition and Critical Point.)

\[ < (\delta N)^2 > \sim \xi^2 \quad < (\delta N)^3 > \sim \xi^{4.5} \]
\[ < (\delta N)^4 > - 3 < (\delta N)^2 >^2 \sim \xi^7 \]

Moments products relates to baryon number susceptibility ratio:
(Study Bulk properties of QCD matter.)

\[ \kappa \sigma^2 \sim \chi^{(4)}/\chi^{(3)} \quad S \sigma \sim \chi^{(3)}/\chi^{(2)} \quad \sigma^2 / M \sim \chi^{(2)}/\chi^{(1)} \]

Moments/Cumulants: Describe shape and fluctuations.

\[ S \sigma = \frac{C_3}{C_2} = \frac{< (\delta N)^3 >}{< (\delta N)^2 >} \quad \kappa \sigma^2 = \frac{C_4}{C_2} = \frac{< (\delta N)^4 >}{< (\delta N)^2 >^2 - 3} \]

S: skewness, $\kappa$: kurtosis, $C_n$: $n^{th}$ order cumulants


If proton and anti-proton are independent Poissonian distributions, the distributions of net-protons is **Skellam distributions**, which is the case in Hadron Resonance Gas Model.

\[
P(N) = \left( \frac{N_{\bar{p}}}{N_p} \right)^{N/2} I_N \left( 2\sqrt{N_{\bar{p}} N_p} \right) e^{-(N_{\bar{p}} + N_p)}
\]

\(N_{\bar{p}}\) : Mean number of anti-protons
\(N_p\) : Mean number of protons

The Poisson baselines (skellam distributions) are only determined by measured average number of protons and anti-protons. This baseline will be used in our data analysis.

Then we have the Poisson baseline for various moments/cumulants measurements:

\[
C_{2n} = N_p + N_{\bar{p}}
\]
\[
C_{2n-1} = N_p - N_{\bar{p}}, (n = 1,2,3,...)
\]

\[S\sigma = \frac{C_3}{C_2} = \frac{N_p - N_{\bar{p}}}{N_p + N_{\bar{p}}}, \kappa\sigma^2 = \frac{C_4}{C_2} = 1\]

The Poisson expectations may have energy and centrality dependence.
Data Analysis Details

**Techniques Used in the Moments Analysis:**

1. **Centrality Bin Width Correction:**
   (To suppress the volume fluctuations):
   Moments are corrected for centrality bin-width effects by using the weighted average of the moments inside each centrality bin.  

   (Items 2 and 3 are updated techniques since QM 2011.)

2. **Statistical Error Estimations: Delta theorem method.**

3. **Centrality (To avoid auto-corrections)**
   Determine the centrality using charged particles within \(|\eta|<0.5\) but excluding proton/anti-proton used in the analysis.

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**PID:** Energy loss (dE/dx) in Time Projection Chamber of STAR detector is used to identify protons with high purity within \(0.4<p_T<0.8\) (GeV/c) and at mid-rapidity \(|y|<0.5\).

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<table>
<thead>
<tr>
<th>Energy (GeV)</th>
<th>7.7</th>
<th>11.5</th>
<th>19.6</th>
<th>27</th>
<th>39</th>
<th>62.4</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics ( Million)</td>
<td>~3</td>
<td>~6.6</td>
<td>~15</td>
<td>~30</td>
<td>~87</td>
<td>~47</td>
<td>~242</td>
</tr>
</tbody>
</table>
Skellam distributions (dash lines)

\[ P(N) = \left( \frac{N_p}{N_p} \right)^{N/2} I_N(2\sqrt{N_p N_p}) e^{-\left(N_p + N_p\right)} \]

Input parameters: measured average number of protons and anti-protons.

The shape of the net-proton distributions vary with the centrality and energy.

These are uncorrected event-by-event distributions of net-protons and the moments beyond mean are obtained by correcting for the finite centrality bin width effect.
Centrality Dependence of Various Order Cumulants

1\textsuperscript{st} order polynomial fit: Central Limit Theorem (CLT) expectations for Cumulants.

\[ C_n \propto V \]

V: Volume of the system.

- All cumulants show general linear dependence on N\textsubscript{part}.
- C\textsubscript{1} \sim C\textsubscript{3} (odd order) and C\textsubscript{2} \sim C\textsubscript{4} (even order).
- The differences between odd and even order cumulants decrease when the energy decrease.

(The produced number of anti-protons decrease with decreasing energy.)
For most of the energies and centralities, deviations below Poisson expectations are observed and the higher order cumulants, the larger deviation from Poisson baselines.

Below 19.6 GeV, cumulants are larger than Poisson expectations for the peripheral collisions.
Moment products are related to the susceptibility ratios:
\[ \kappa \sigma^2 \sim \chi^{(4)}/\chi^{(3)} \]
\[ S\sigma \sim \chi^{(3)}/\chi^{(2)} \]

Deviations below Poisson expectations are observed in most of the energies and centralities.

Below 19.6 GeV, moment products are larger than Poisson expectations in peripheral collisions.

Poisson baseline: \[ S\sigma(Poisson) = \frac{C_3}{C_2} = \frac{N_p - N_p}{N_p + N_p} \]
\[ \kappa \sigma^2(Poisson) = \frac{C_4}{C_2} = 1 \]


- $\kappa\sigma^2$ and $S\sigma$/Poisson are below unity (Poisson baseline) and approach the baseline as the rapidity acceptance is decreased at 0-5% most central collisions. ($|y|<0.1, 0.2, 0.3, 0.4, 0.5$)
- Experimental values approach Poisson expectations as the rapidity acceptance is decreased.
Deviations below Poisson expectations are observed beyond statistical and systematic errors in 0-5% most central collisions for $\kappa\sigma^2$ and $S\sigma$ above 7.7 GeV.

For peripheral collisions, the deviations above Poisson expectations are observed below 19.6 GeV.

UrQMD model shows monotonic behavior for the moment products, in which non-CP physics, such as baryon conservation, hadronic scattering effects, are implemented.
Summary

Measurements:
- The centrality and energy dependence for the first four moments/cumulants of the net proton multiplicity distributions in Au+Au collisions at RHIC BES-Phase I energies (7.7, 11.5, 19.6, 27, 39, 62.4 and 200 GeV) have been presented.

Expectations from Central Limit Theorem:
- Various order cumulants shows a general linear increase with increase in number of participant nucleons (system volume), which is expected by Central Limit Theorem.

Comparisons with Poisson Baselines and Transport Model:
- Deviations below Poissonian expectation are observed in 0-5% most central Au+Au collisions beyond the statistics and systematics errors for the moment products $\kappa \sigma^2$ and $S \sigma$ above 7.7 GeV. For peripheral collisions, the deviations above Poission expectations are observed below 19.6 GeV. Monotonic behavior for the moment products is observed in the UrQMD model.
- Higher statistics are needed in order to draw physics conclusion at lower beam energies.
Backup Slides
Moment Products/Poisson Ratio: Energy Dependence

Au+Au Collisions
Net-proton
0.4<p_t<0.8 (GeV/c), |y|<0.5

STAR Preliminary

\[ \frac{(S \sigma)}{\text{Poisson}} \]

\[ \sqrt{s_{NN}} \text{ (GeV)} \]
Moment Products/Poisson Ratio: Energy Dependence

\[ (S/\sigma)_{\text{Poisson}} \]

Au+Au : 0-5%

- |y|<0.1
- |y|<0.3
- |y|<0.5

Net-proton

0.4<p_T<0.8 (GeV/c)

STAR Preliminary
The efficiency of proton and anti-proton as obtained for HIJING+GEANT simulations.

The detector effects (efficiency, acceptance etc) seems small based on the Hijing+Geant simulations.
Rapidity Window Dependence: UrQMD Cal.

(a) 7.7 GeV  (b) 11.5 GeV  (c) 19.6 GeV  (d) 27 GeV
(e) 39 GeV  (a) 62.4 GeV  (g) 200 GeV

Au+Au Collisions
Net-proton
0.4<p_T<0.8 (GeV/c)

UrQMD
△ 0-5%
● 30-40%
☆ 70-80%
Resonance Decay and Neutron Effect

**Model: Therminator-2.0 (arXiv:1102.0273)**

- **Effect of resonance decay on \( S_\sigma \) and \( \kappa \sigma^2 \)** is small. (based on the right two plots).
- **Effect of inclusion of neutrons is small:**
  - Indicates: Net-proton fluctuation can reflect the net-baryon fluctuation.
- **Error estimation:** X. Luo, arXiv:1109.0593

- **W/O Decay: Without Decay**
Process Final State Interaction (FSI) between hadrons or not can be controlled by “ART” program in the AMPT model.

Effects of Final State Interaction (FSI) on $S\sigma$ and $\kappa\sigma^2$ are small.
(based on the results in the right two plots).