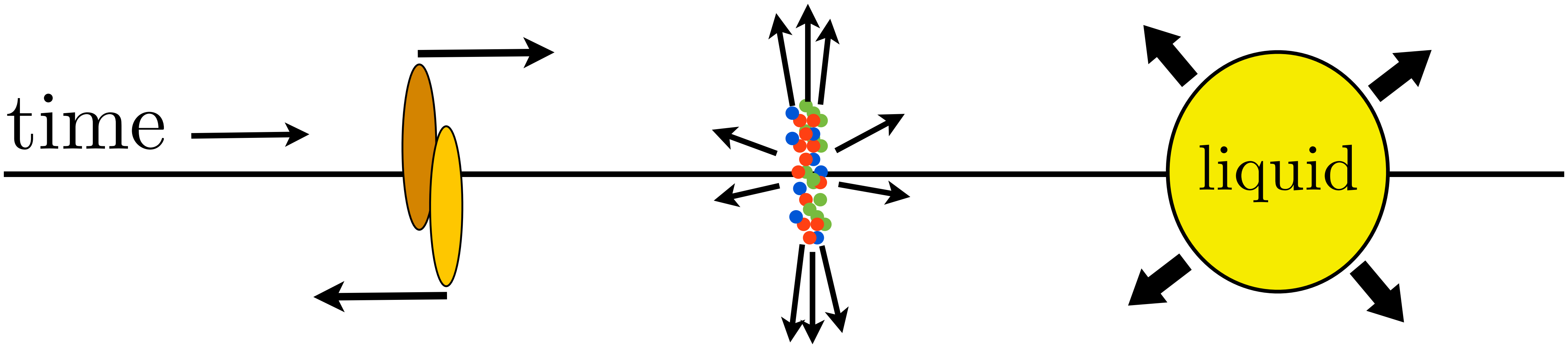
Gravitational collapse and holographic thermalization

Paul Chesler

Massachusetts Institute of Technology

Work done with Derek Teaney

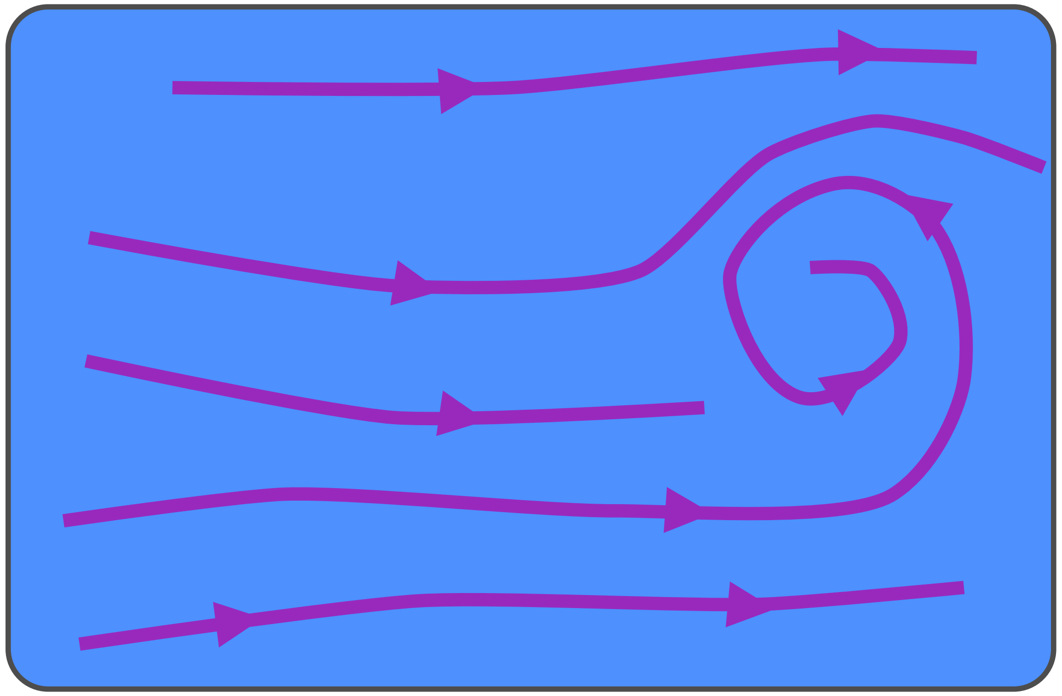
Hydrodynamics & thermalization in heavy ion collisions



Basic theoretical questions:

- How long is t_{hydro}?
- How long is t_{therm}?
- How is t_{therm} correlated with t_{hydro}?

What does it mean for a system to behave like a liquid?



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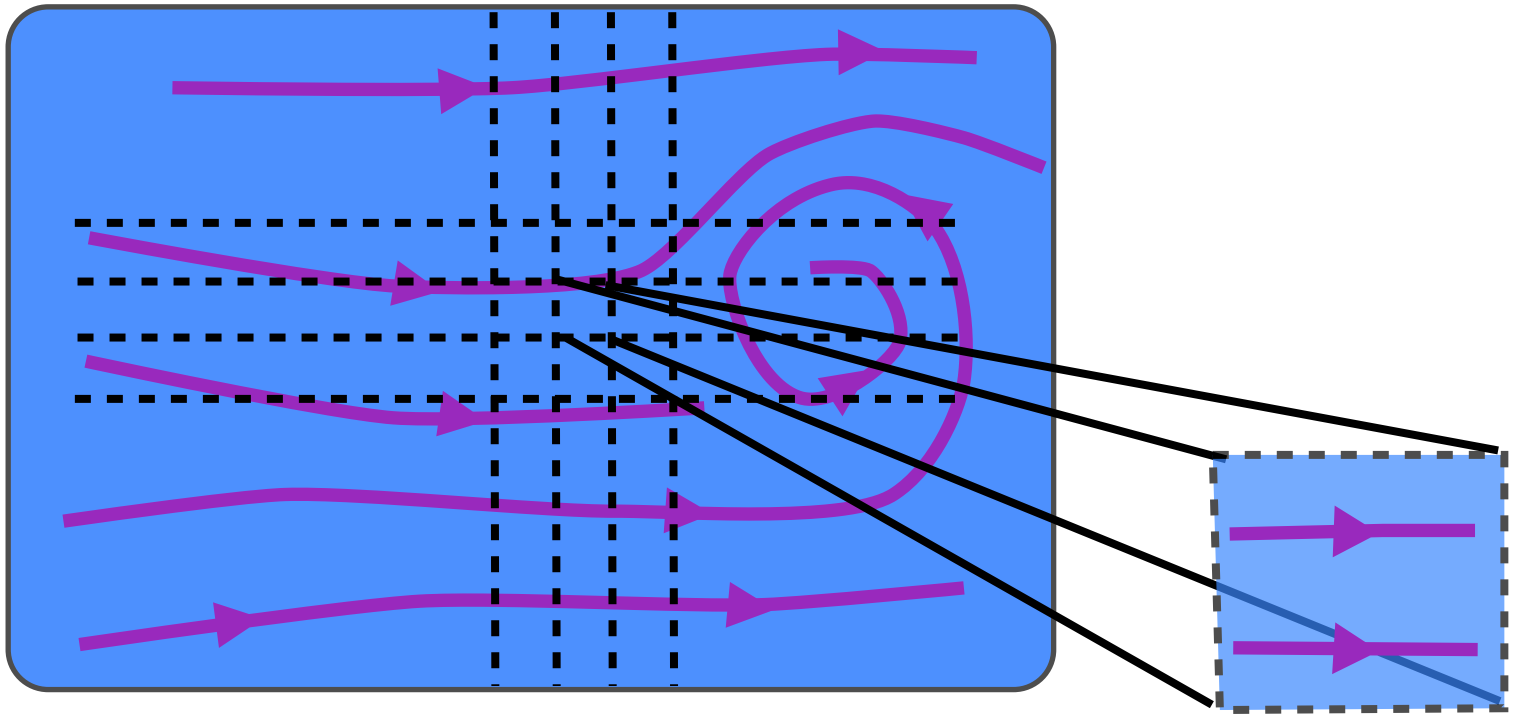

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- Proper energy density ϵ.
- Equation of state $p = p(\epsilon)$.

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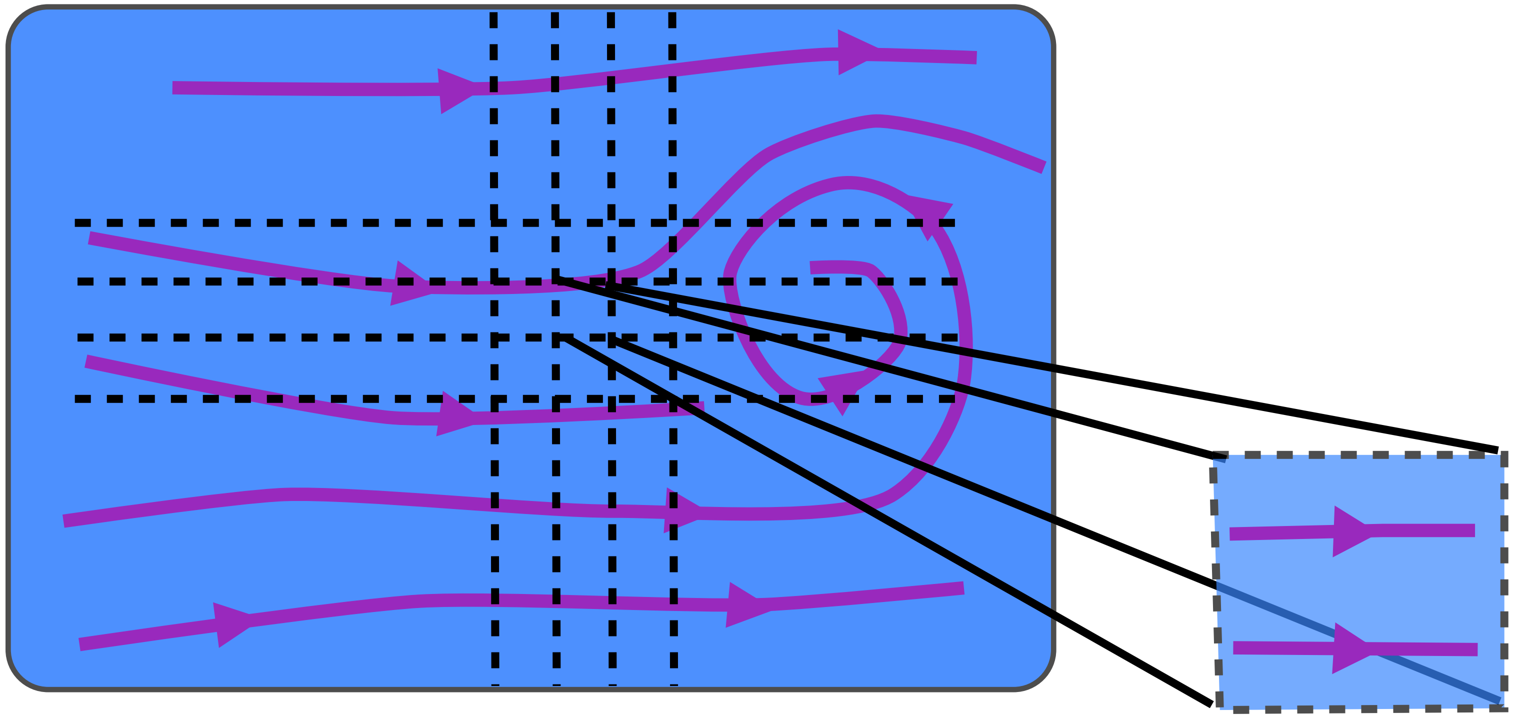

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Constitutive relations (in local fluid rest frame):

$$T^{00}_{\text{hydro}} = \epsilon,$$
$$T^{0i}_{\text{hydro}} = 0,$$
$$T^{ij}_{\text{hydro}} = p\,\delta_{ij} - \eta\left[\nabla_i u_j + \nabla_j u_i - \tfrac{2}{3}\delta_{ij}\nabla\cdot\mathbf{u}\right] + \ldots.$$

Dynamics: $\partial_\mu T^{\mu\nu}_{\text{hydro}} = 0.$

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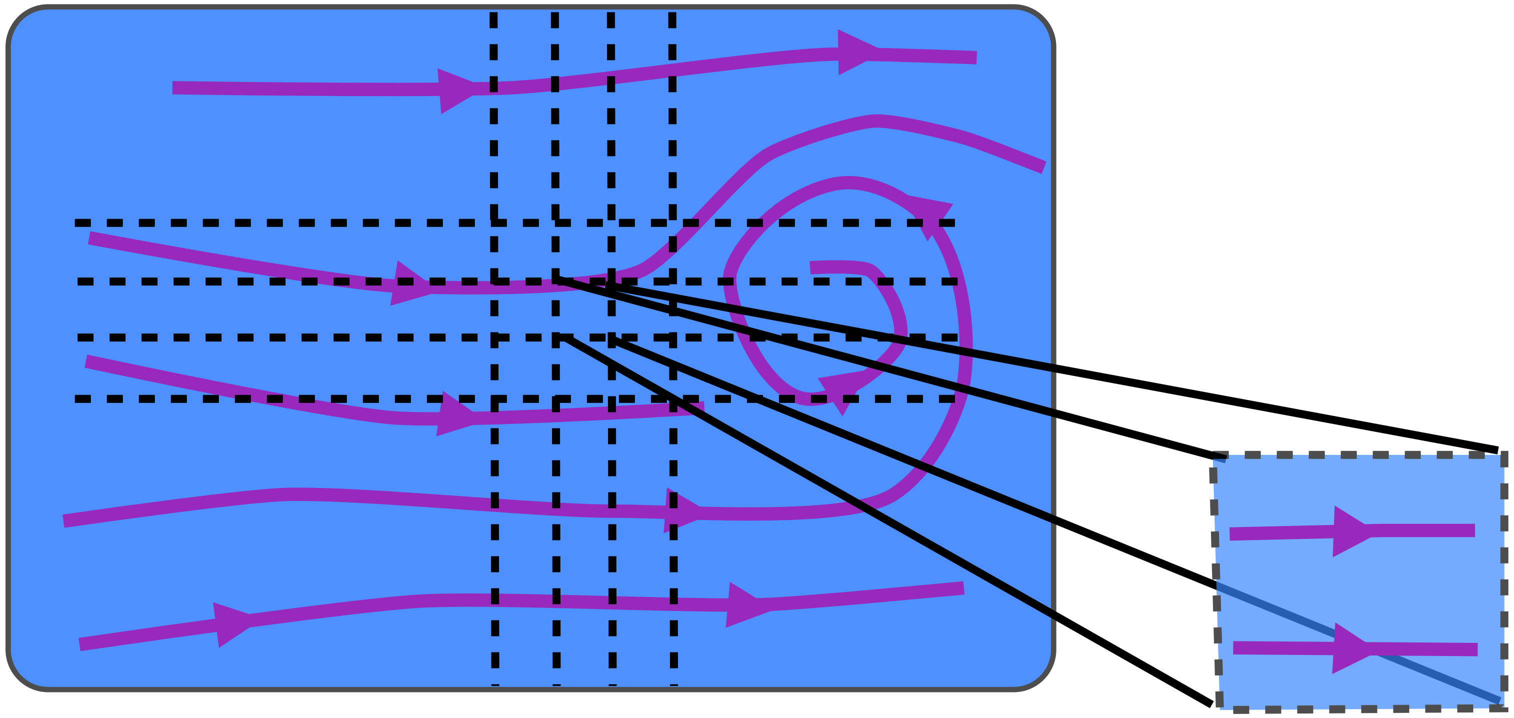

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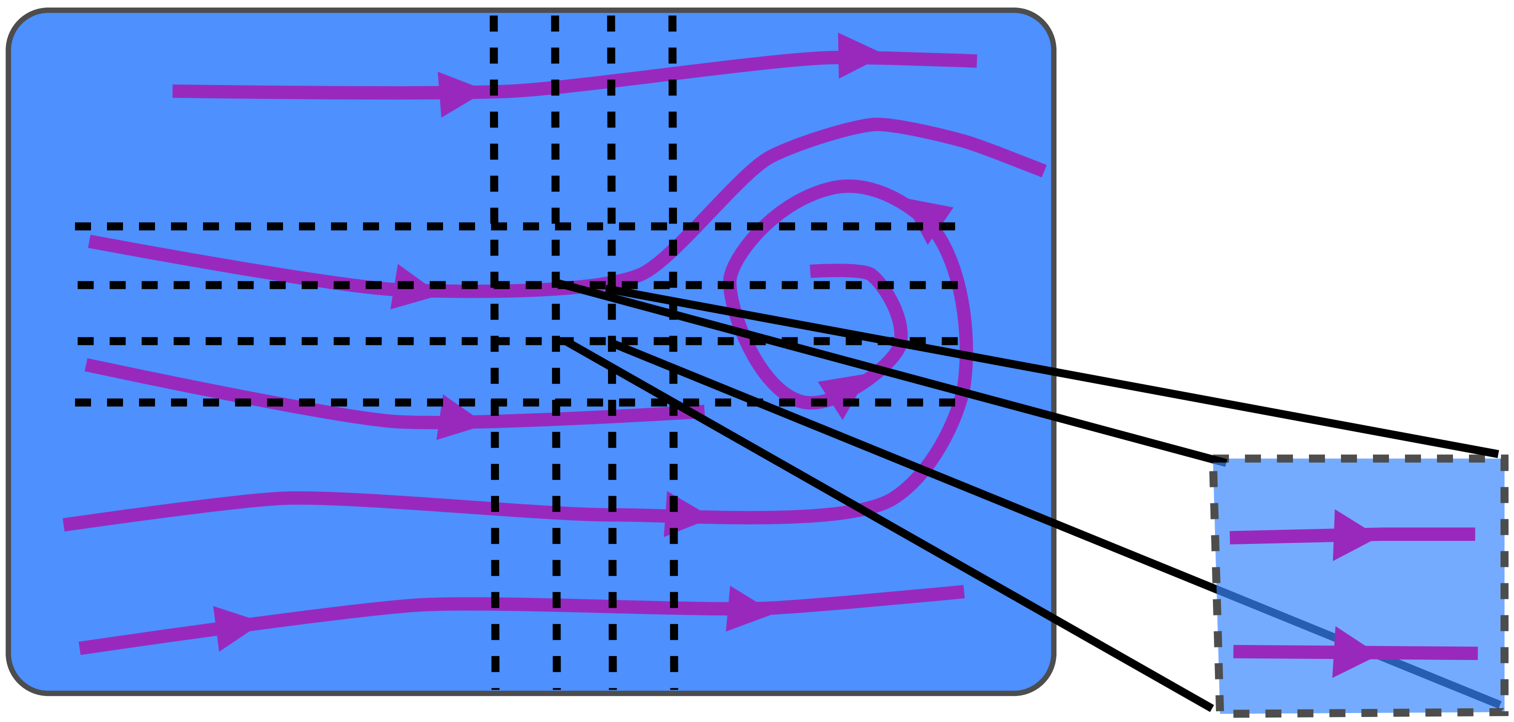
Dynamics: $\partial_\mu T^{\mu\nu}_{\text{hydro}} = 0.$

Liquid behavior: $\langle T^{\mu\nu}(x)\rangle \approx T^{\mu\nu}_{\text{hydro}}(x)$

What does it mean for a system to be locally thermalized?

Minimalist answer: Well defined notion of $T(x)$.



Fluctuation-Dissipation Theorem:

$$g_<(x,q) = e^{q\cdot u/T(x)} g_>(x,q)$$

emission rate **absorption rate**

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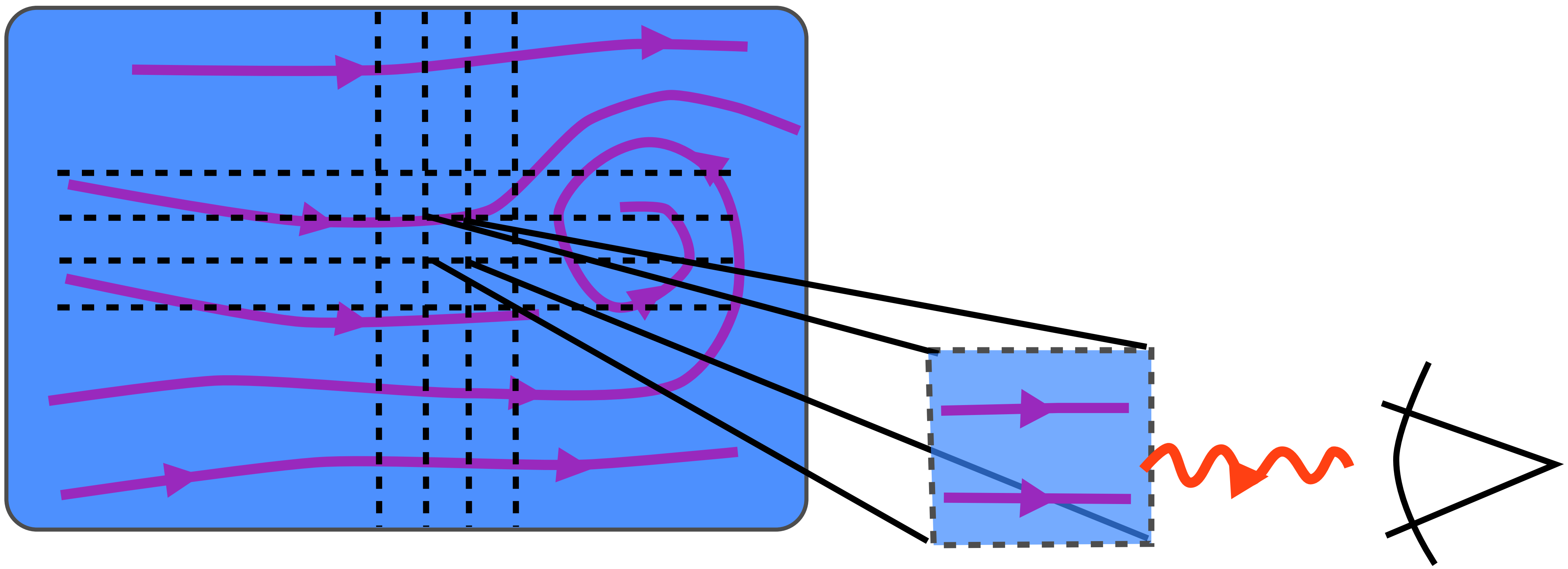

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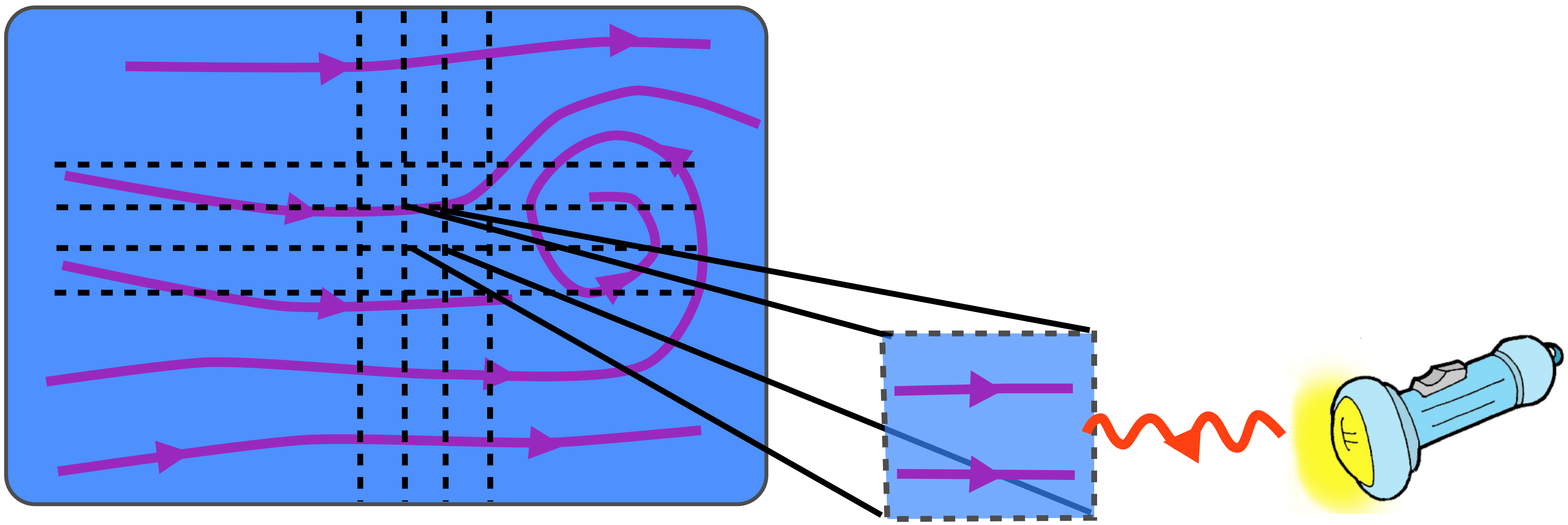

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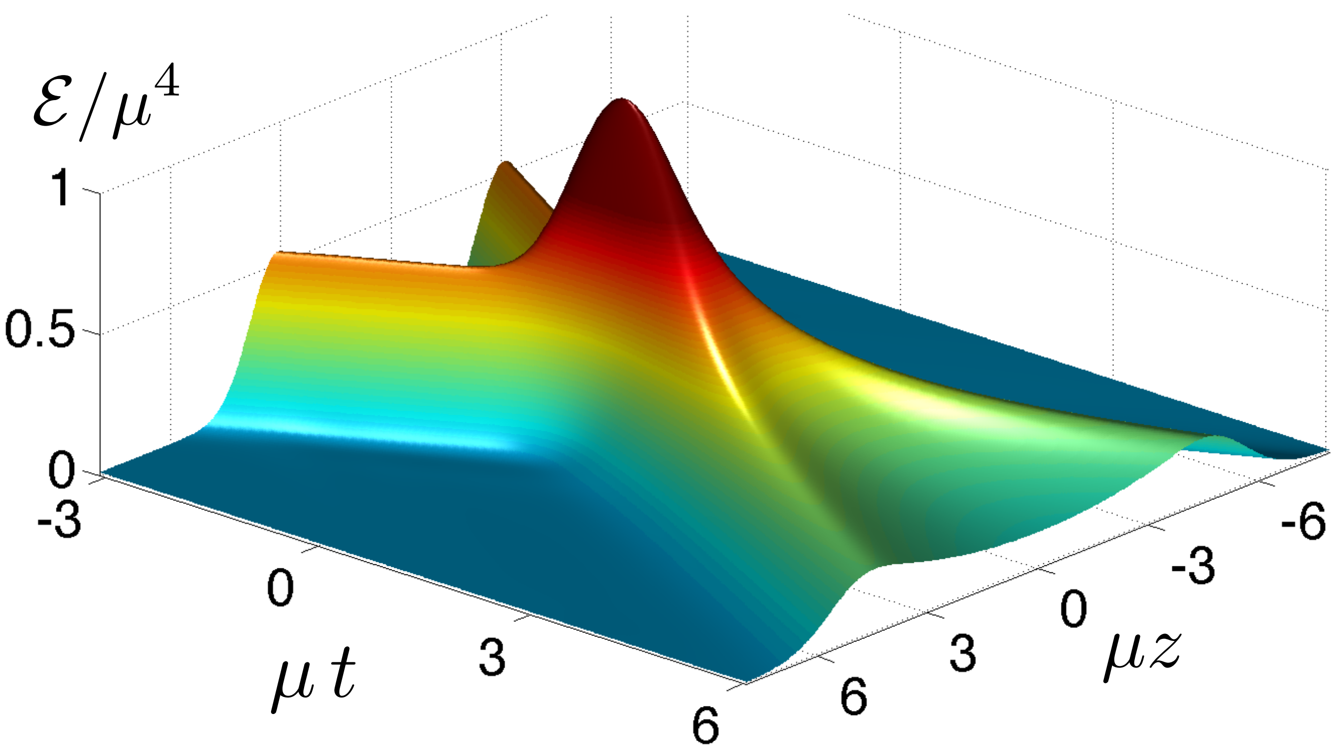
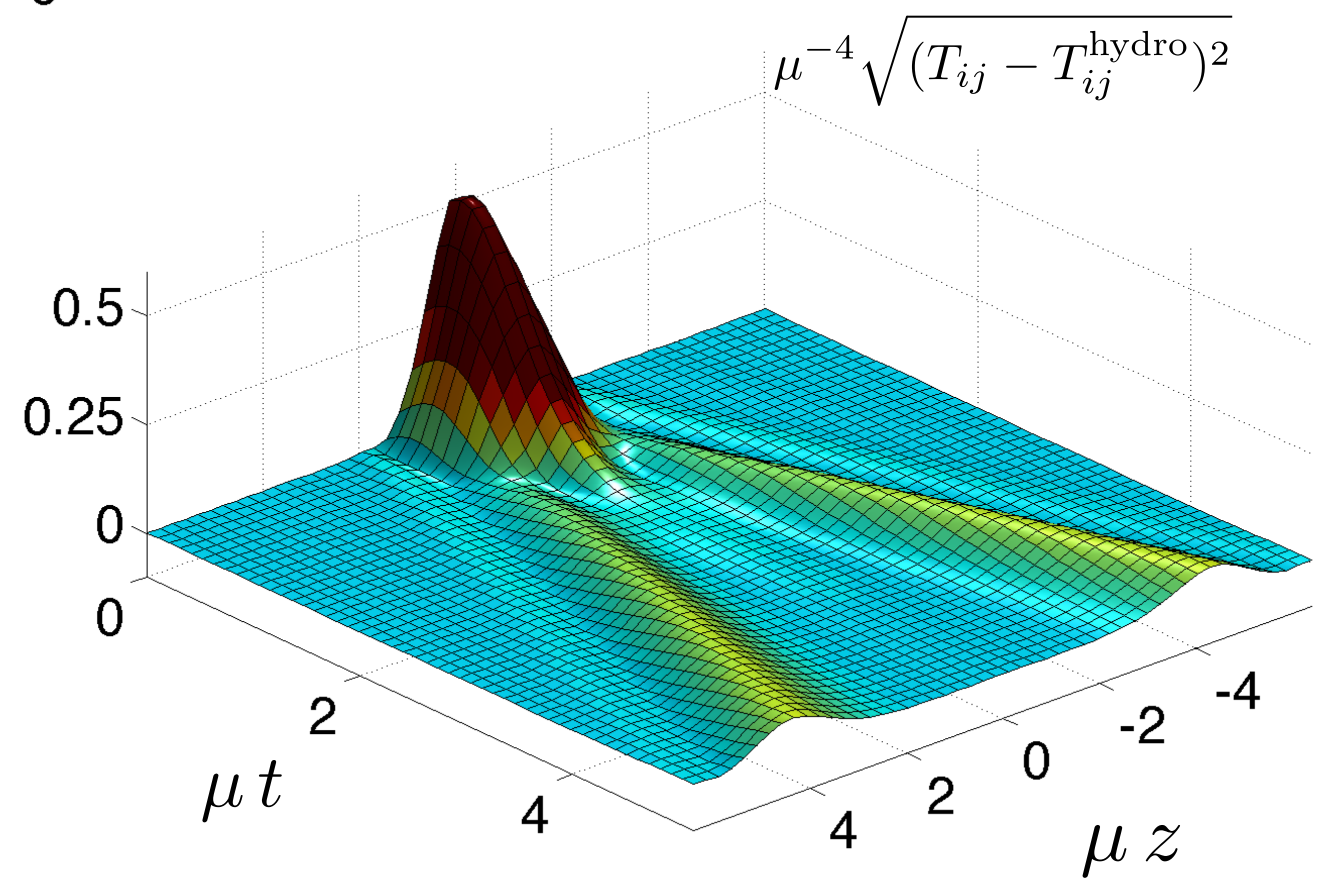
Holographic "Hydrodynamization"



energy density

[PC & Yaffe: 1011.3562]



- Expanding bubble of liquid.
- Estimate for RHIC:
  $t_{\text{hydro}} \approx 0.35$ fm/c.

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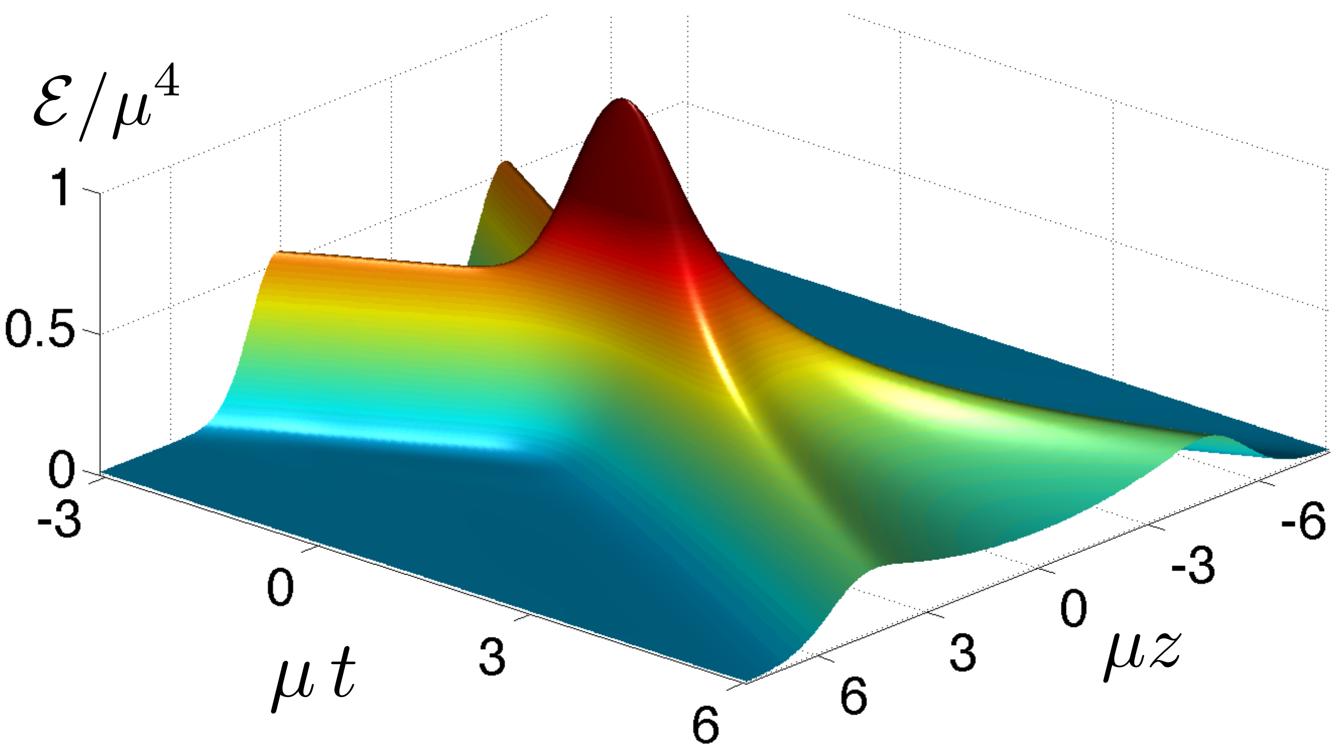

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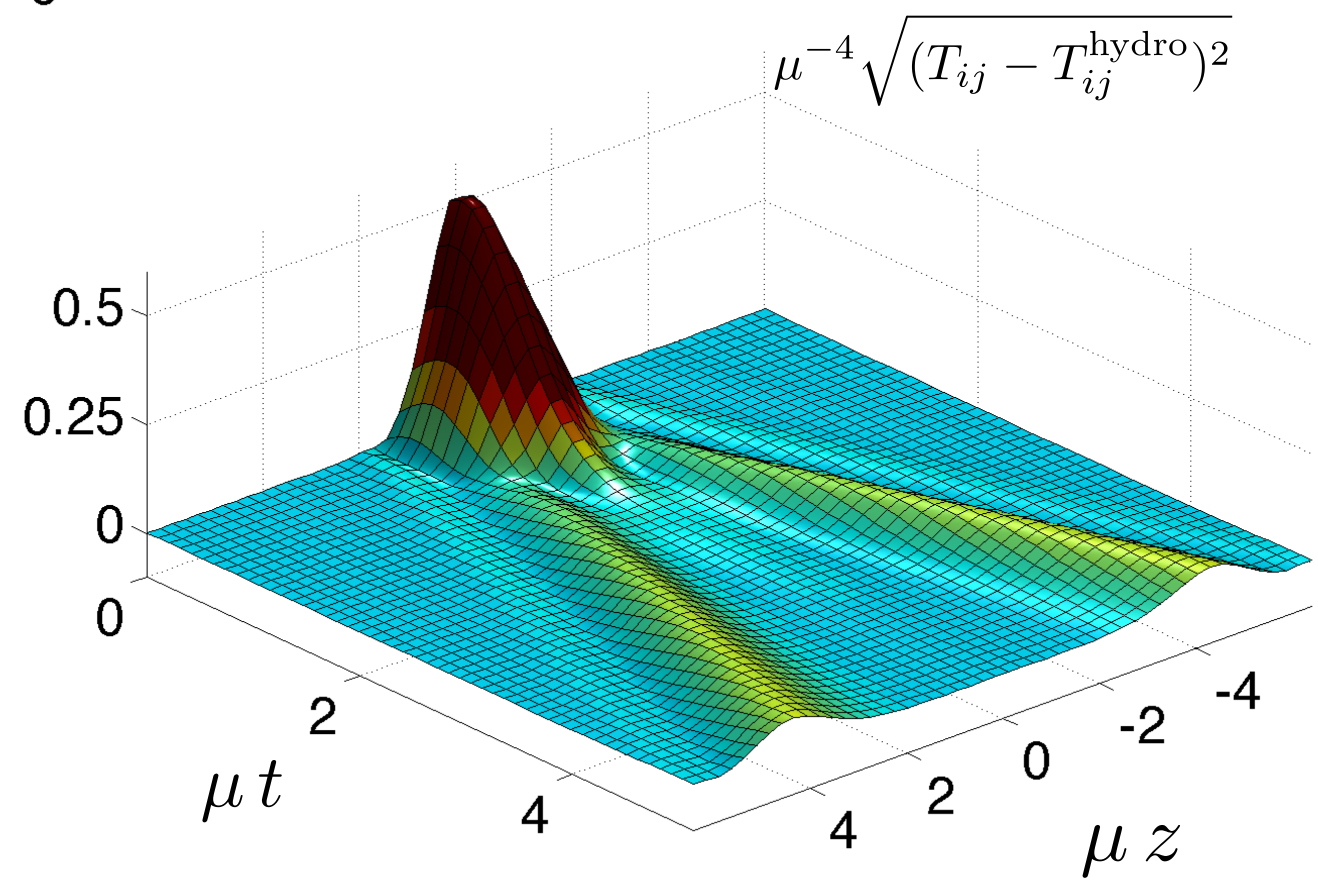

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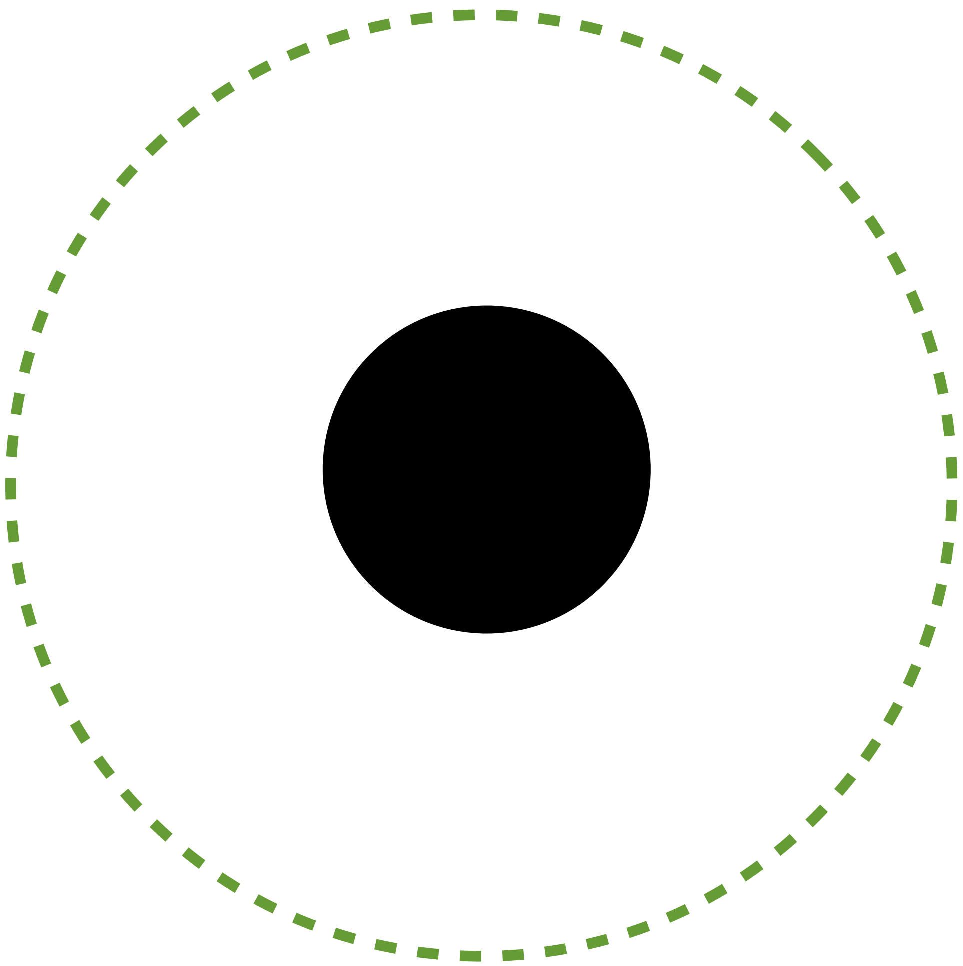
When does the produced QGP thermalize?

Holographic thermalization

Holographic dictionary: Black hole in a "box" $\Leftrightarrow$ QGP.

- QGP formation & thermalization $\Leftrightarrow$ gravitational collapse and black hole thermalization.

How does one tell when BH has locally thermalized?



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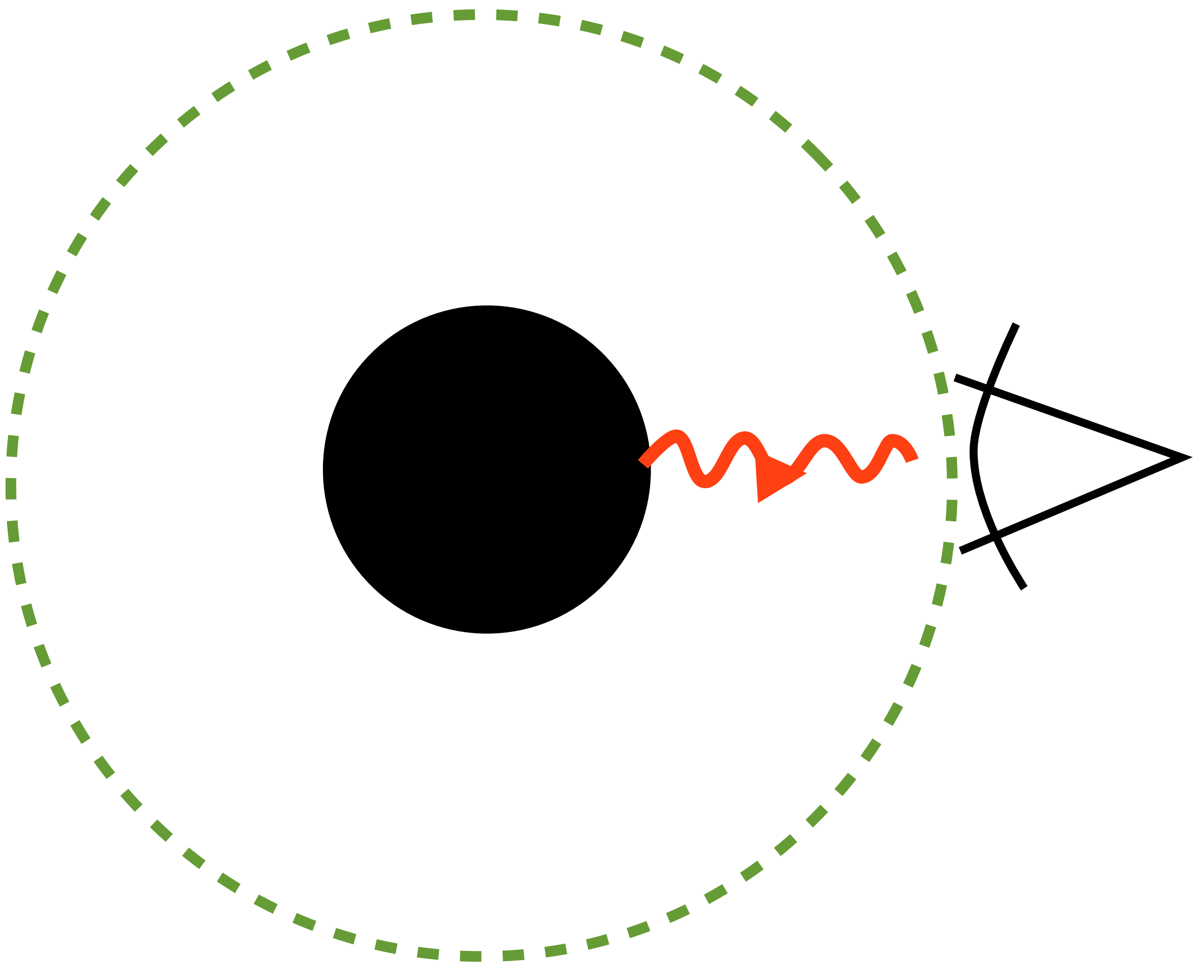

Emission of Hawking Radiation

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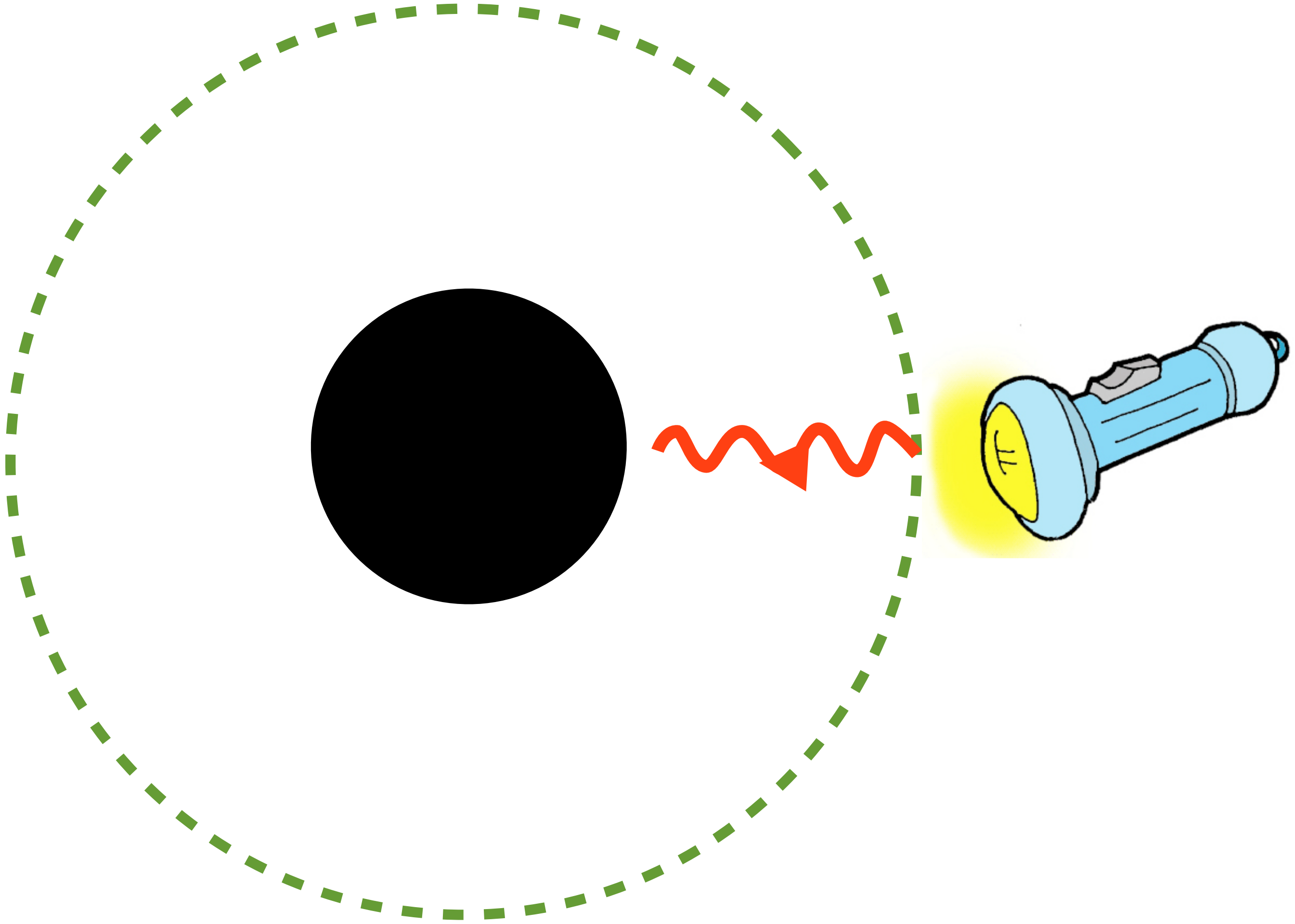

Absorption of radiation

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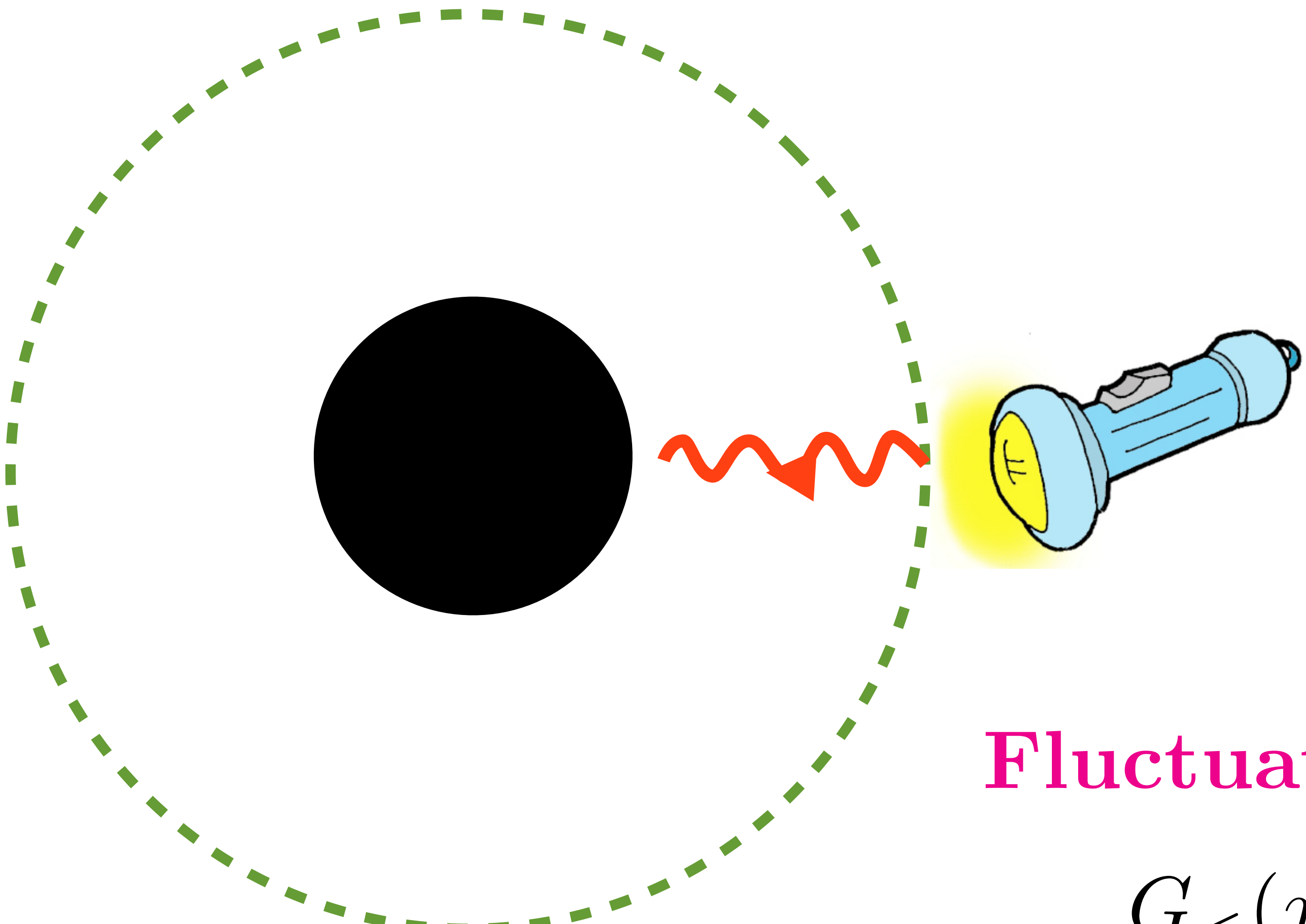


Absorption of radiation

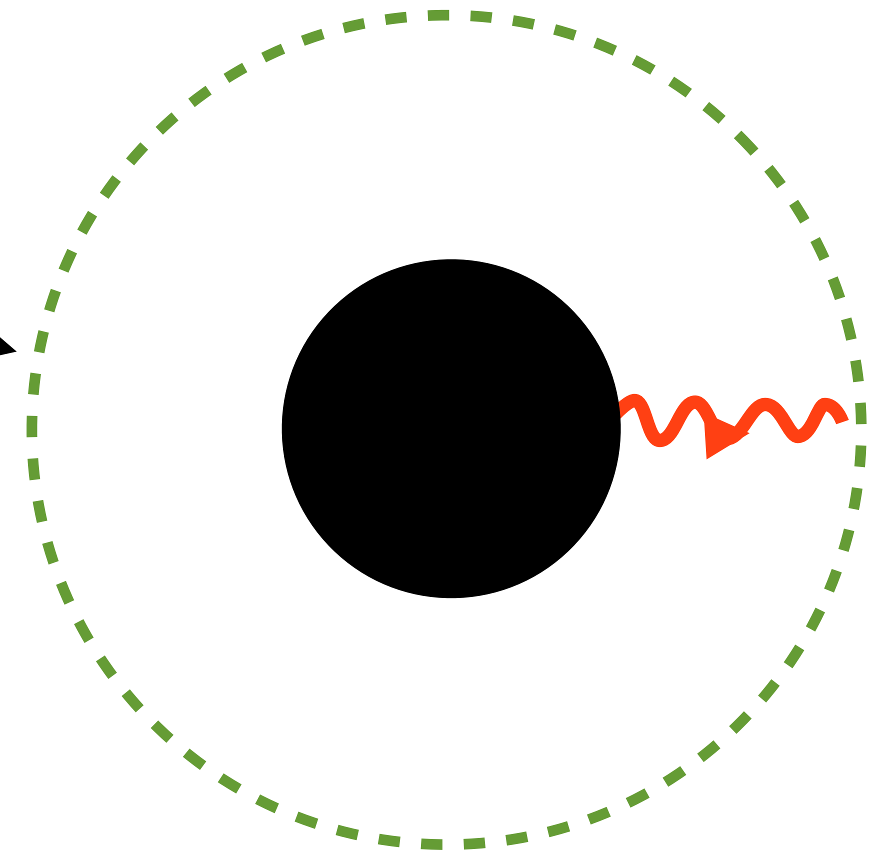
Fluctuation-Dissipation Theorem

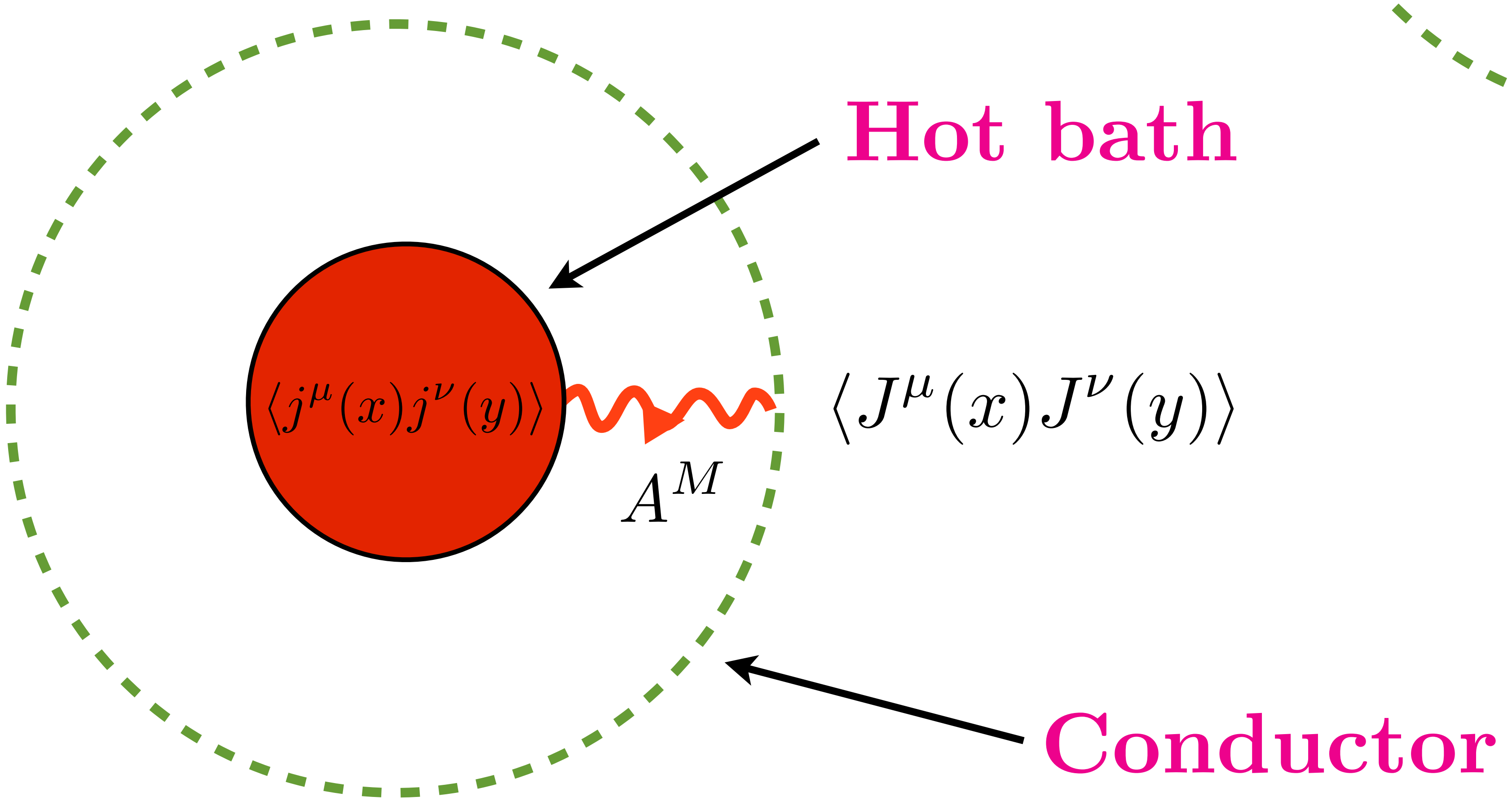
$$G_{<}(x,q) = e^{-\omega/T_{\text{Hawking}}(x)} G_{>}(x,q)$$

Connecting Hawking radiation to QFT photo emission

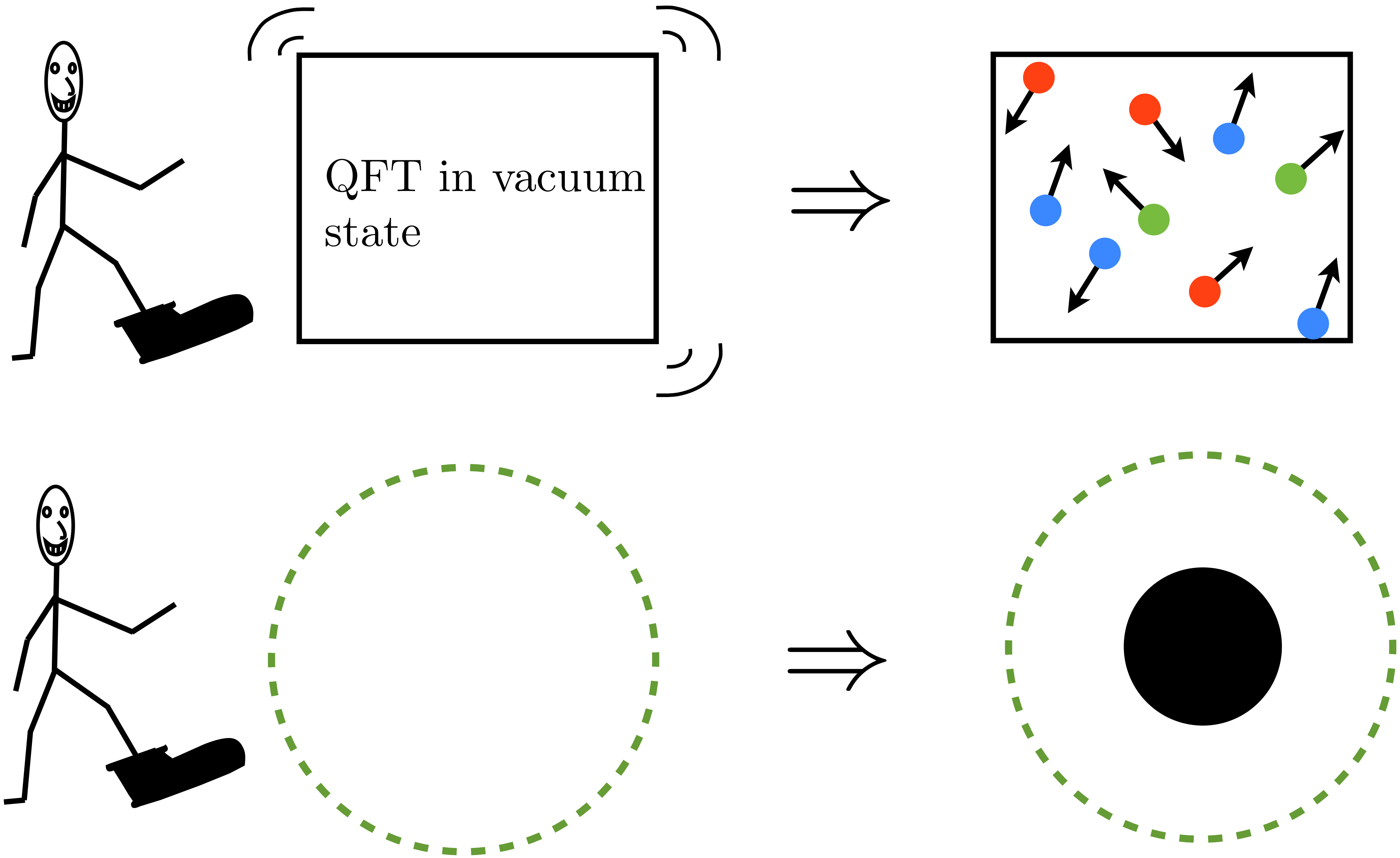
Boundary of "box"

- Where dual QFT lives.





The "experimental" setup

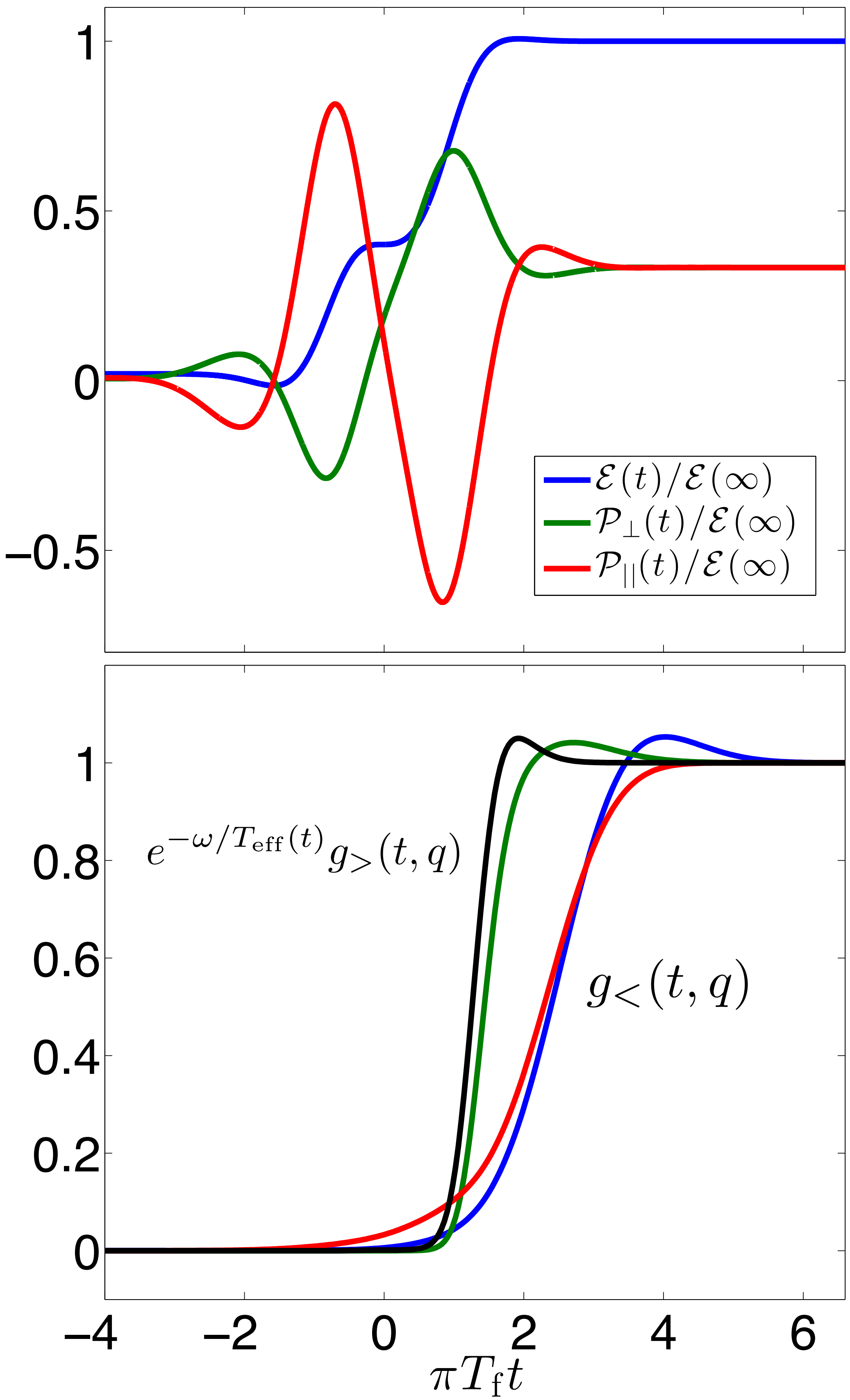


1. Take kick to be homogenous but anisotropic.

2. Solve Einstein $\Rightarrow$ extract $\langle T^{\mu\nu}(t) \rangle$.

3. Compute Hawking $\Rightarrow$ extract QFT emission & absorption rates.

Isotropization & thermalization of scalar radiation



Stress isotropization

- $$\langle T^\mu{}_\nu(t)\rangle = \mathrm{diag}[-\mathcal{E}(t), \mathcal{P}_\perp(t), \mathcal{P}_\perp(t), \mathcal{P}_{||}(t)]$$
  $$\to \mathrm{diag}[-\epsilon, p, p, p], \;\; \epsilon = 3p.$$
- Effective "temperature" $T_{\rm eff}(t) \propto |\mathcal{E}(t)|^{1/4}$.
- Final temperature: $T_{\rm f} = \lim_{t\to\infty} T_{\rm eff}(t)$

Thermalization litmus test:

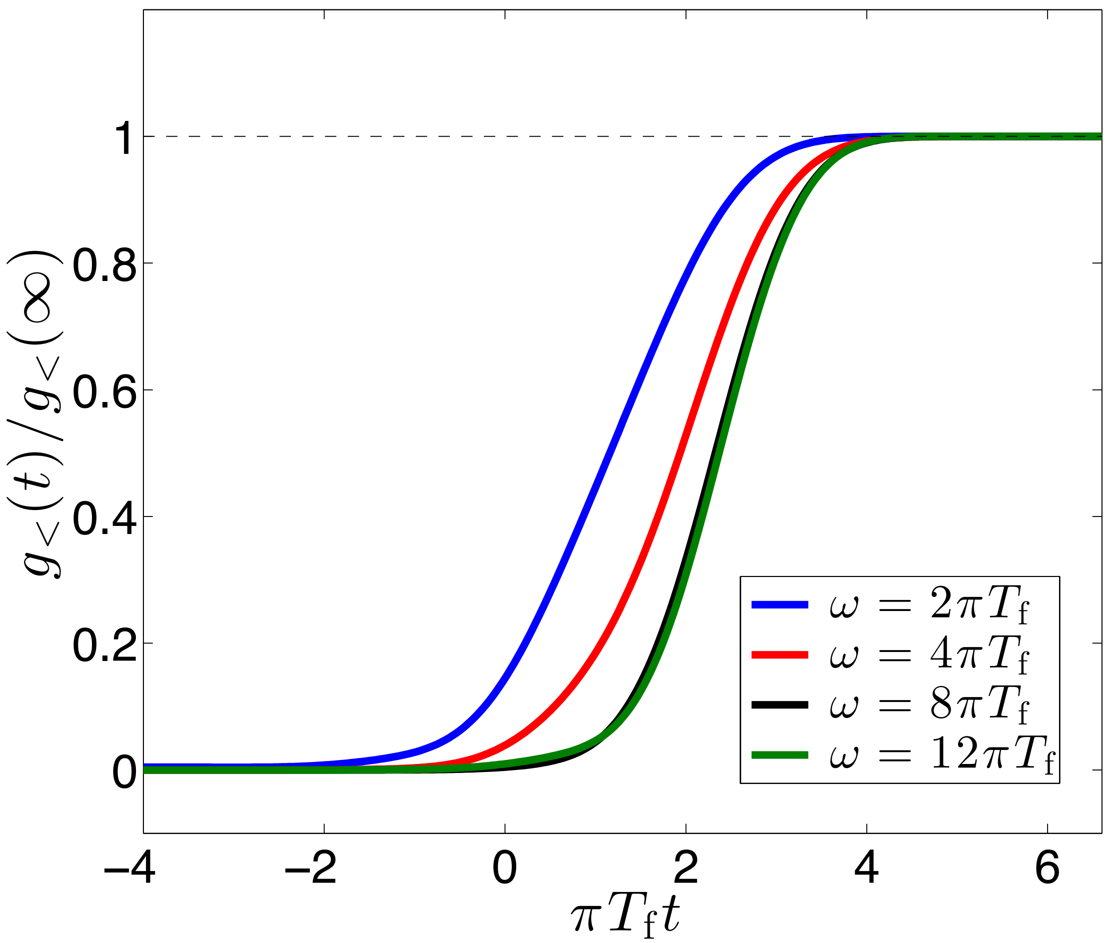
$$g_<(t,q) = e^{-\omega/T_{\rm eff}(t)} g_>(t,q).$$

Key points:

- $g_>(t,q)$ equilibrates **before** $g_<(t,q)$.
- Thermalization happens **after** isotropization.

$$t_{\rm therm} \approx t_{\rm iso} + \frac{2}{\pi T_{\rm f}}.$$

Time-like and space-like scalar emission

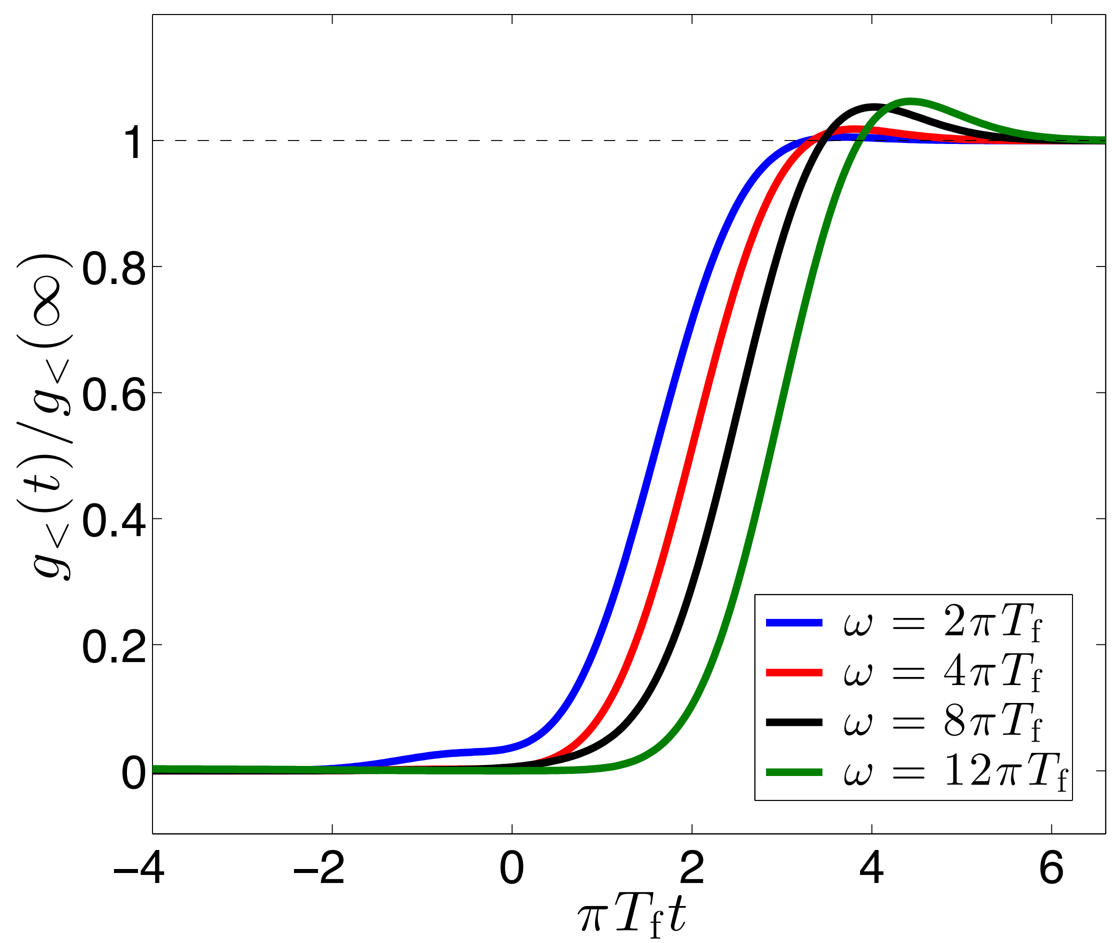


Time-like emission $\omega \to \infty$, $|\mathbf{q}|$ fixed.

$$t_{\text{therm}}(\omega, |\mathbf{q}|) = t_{\text{iso}} + \frac{O(1)}{\pi T_f}.$$

Light-like emission $\omega = |\mathbf{q}|$

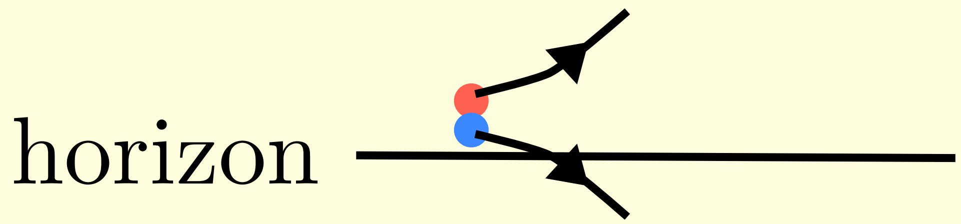
$$t_{\text{therm}}(\omega, |\mathbf{q}|) \sim \frac{1}{T_f}\left(\frac{|\mathbf{q}|}{T_f}\right)^{1/3}.$$



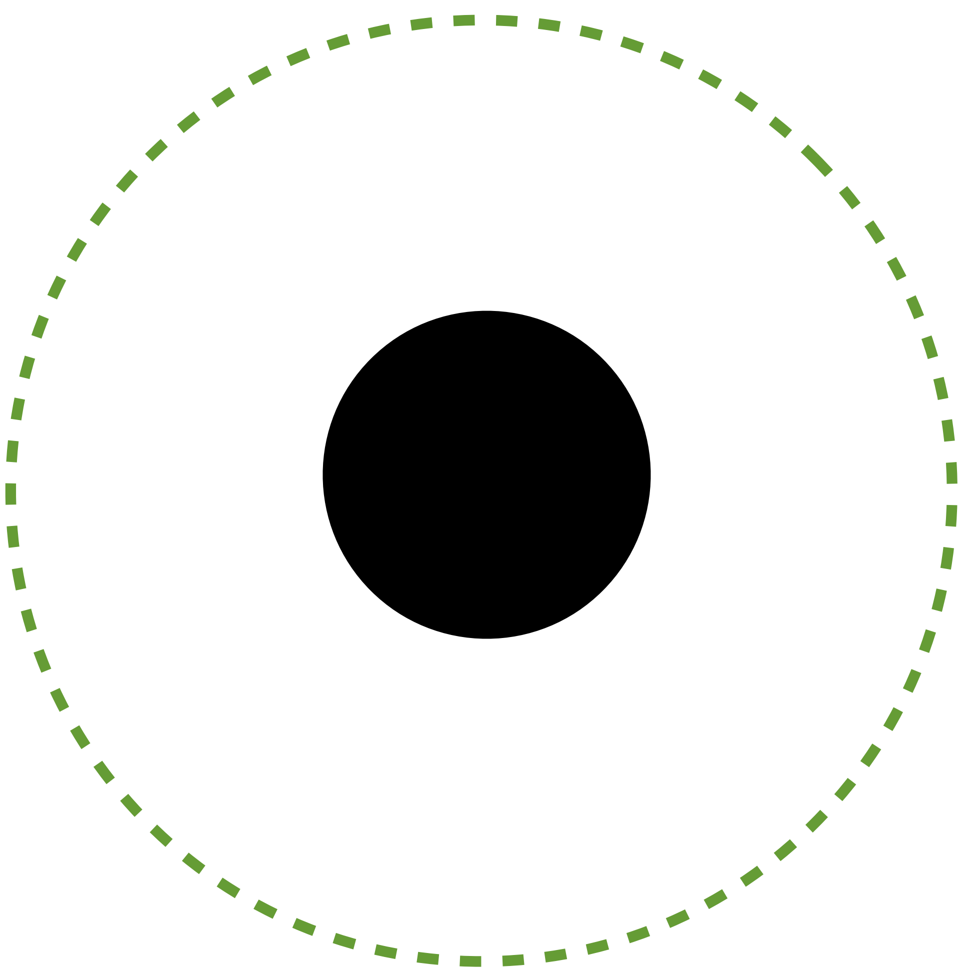
Gravitational origin of $t_{\mathrm{therm}}(\omega, |\mathbf{q}|)$

- **Isotropization** of $\langle T^{\mu\nu}\rangle$ requires isotropization of near-boundary geometry.

  $\Rightarrow$ **Horizon equilibrates** $t_{\mathrm{horizon}} \sim t_{\mathrm{iso}} + t_{\mathrm{infall}} \sim t_{\mathrm{iso}} + \frac{1}{\pi T_{\mathrm{f}}}$



- **Causality:** $t_{\mathrm{therm}}(\omega, |\mathbf{q}|) \gtrsim t_{\mathrm{iso}} + \frac{2}{\pi T_{\mathrm{f}}}$.



Geometric optics

- **First modes to thermalize:** $\omega \to \infty$, $|\mathbf{q}|$ fixed.

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- **Last modes to thermalize:** $\omega = |\mathbf{q}| \to \infty$, $|\mathbf{q}|$

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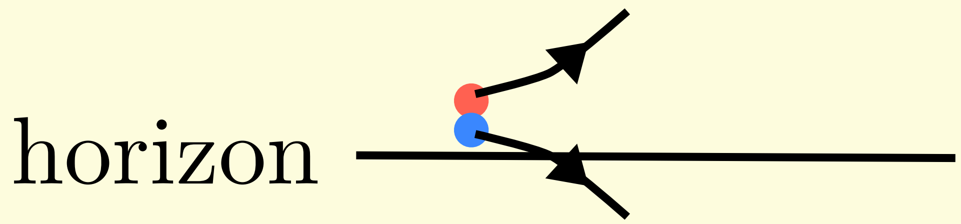

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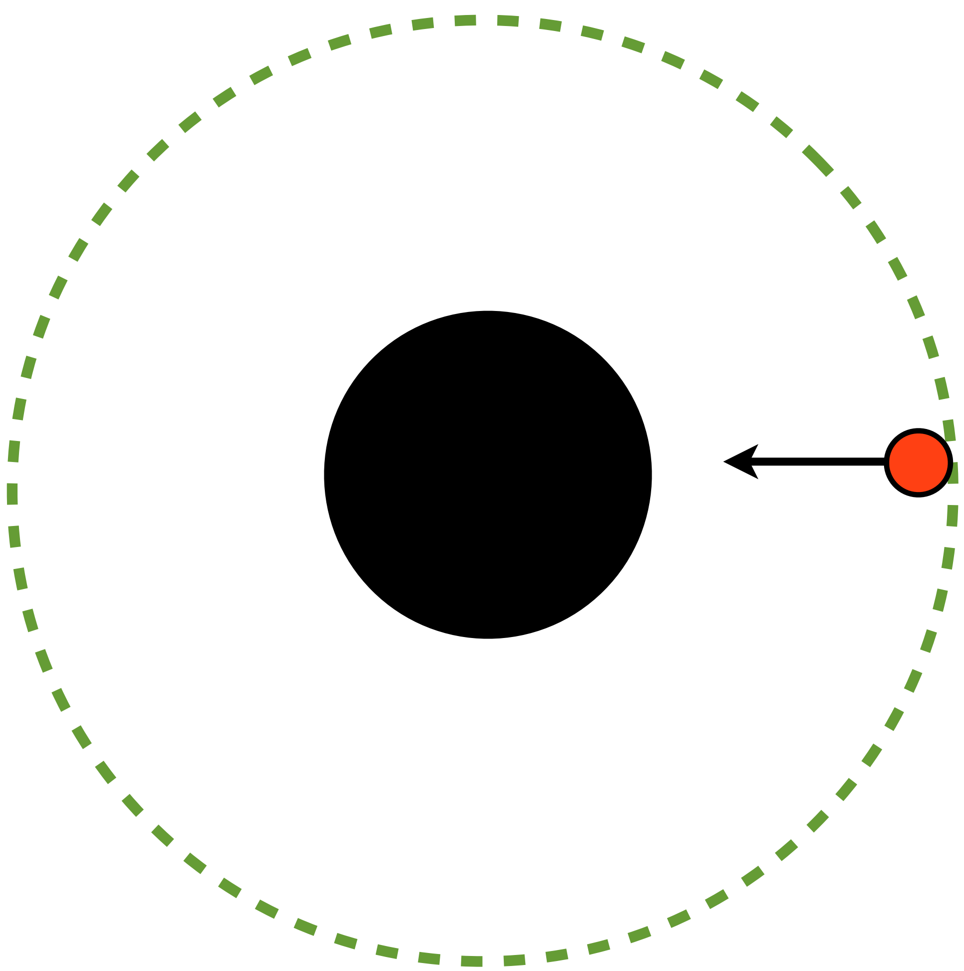

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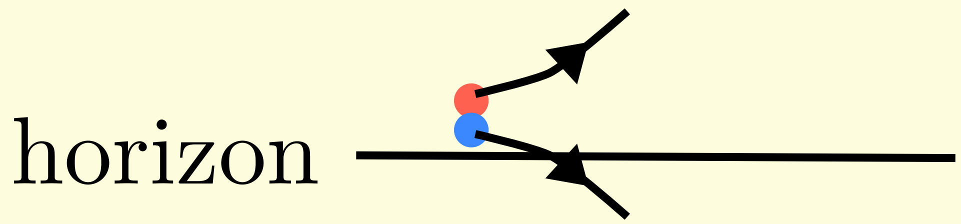

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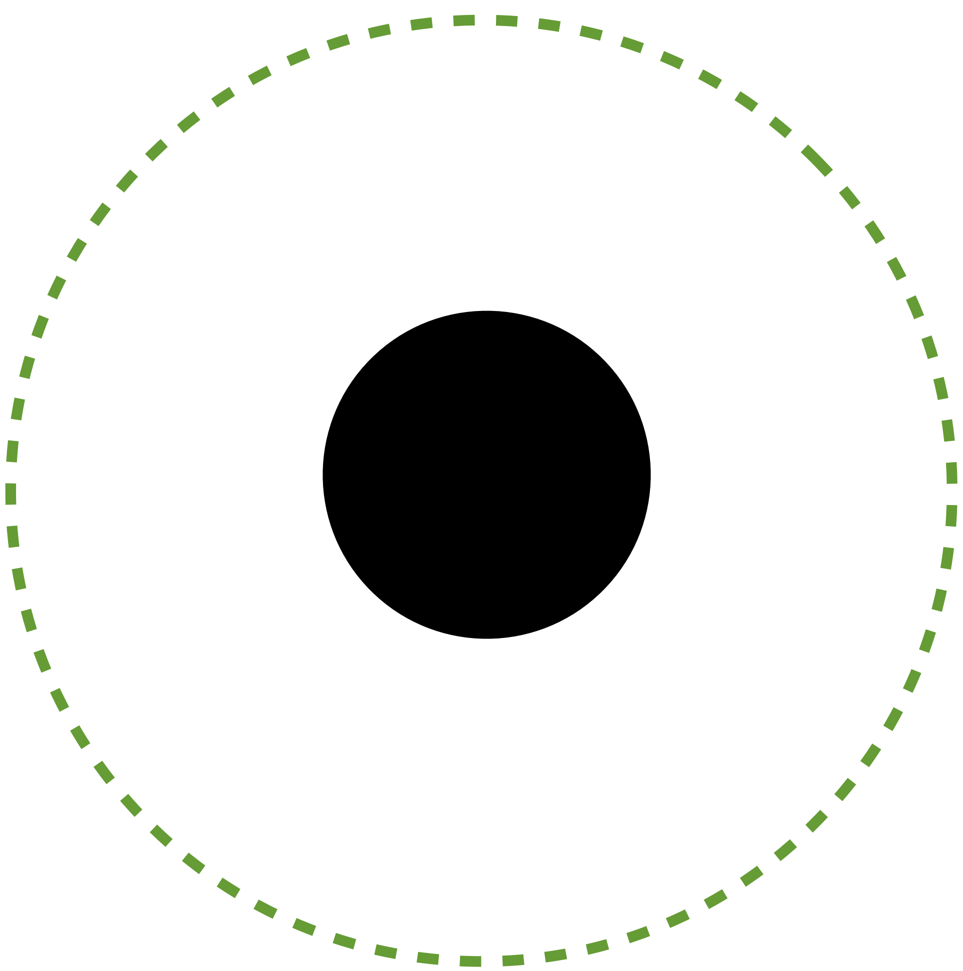

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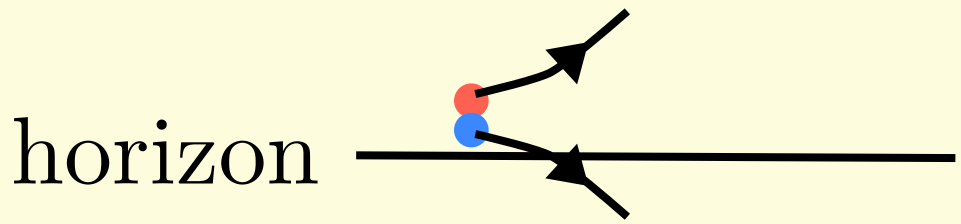

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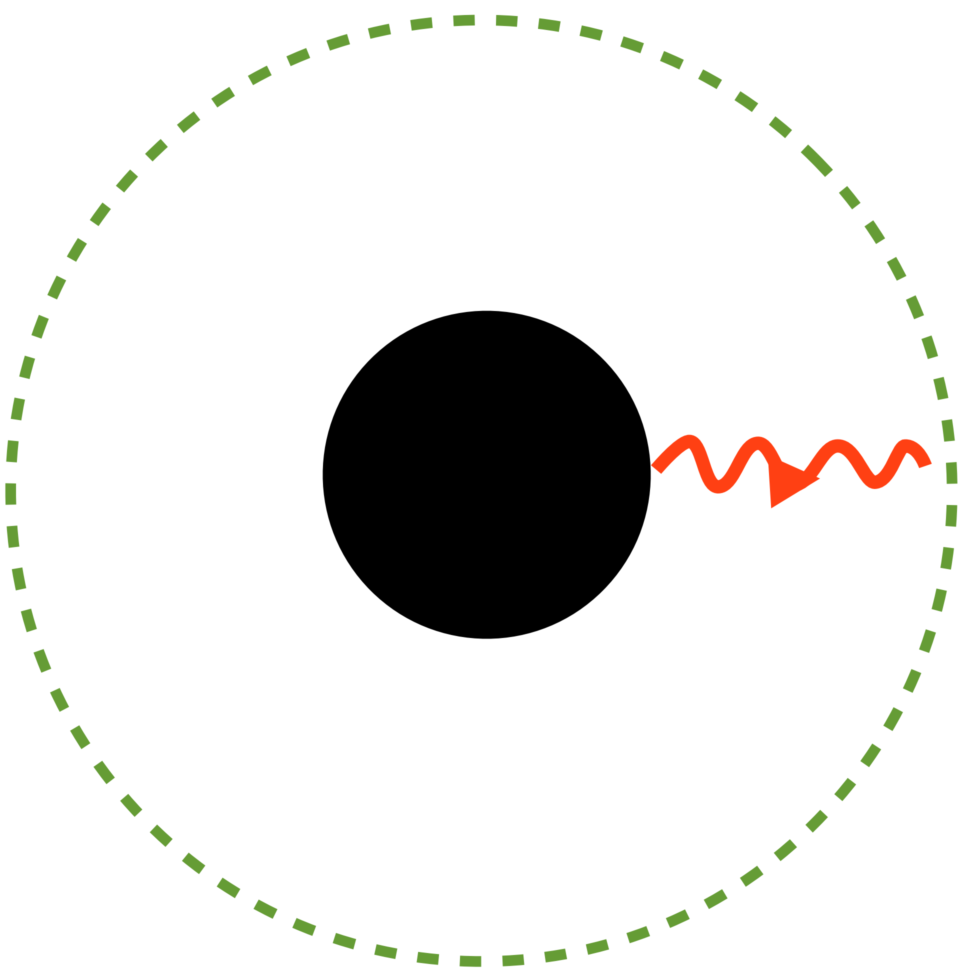

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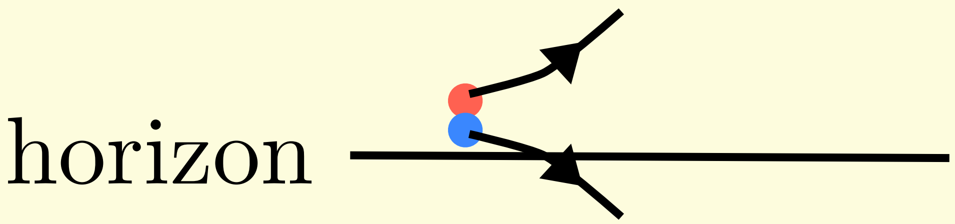

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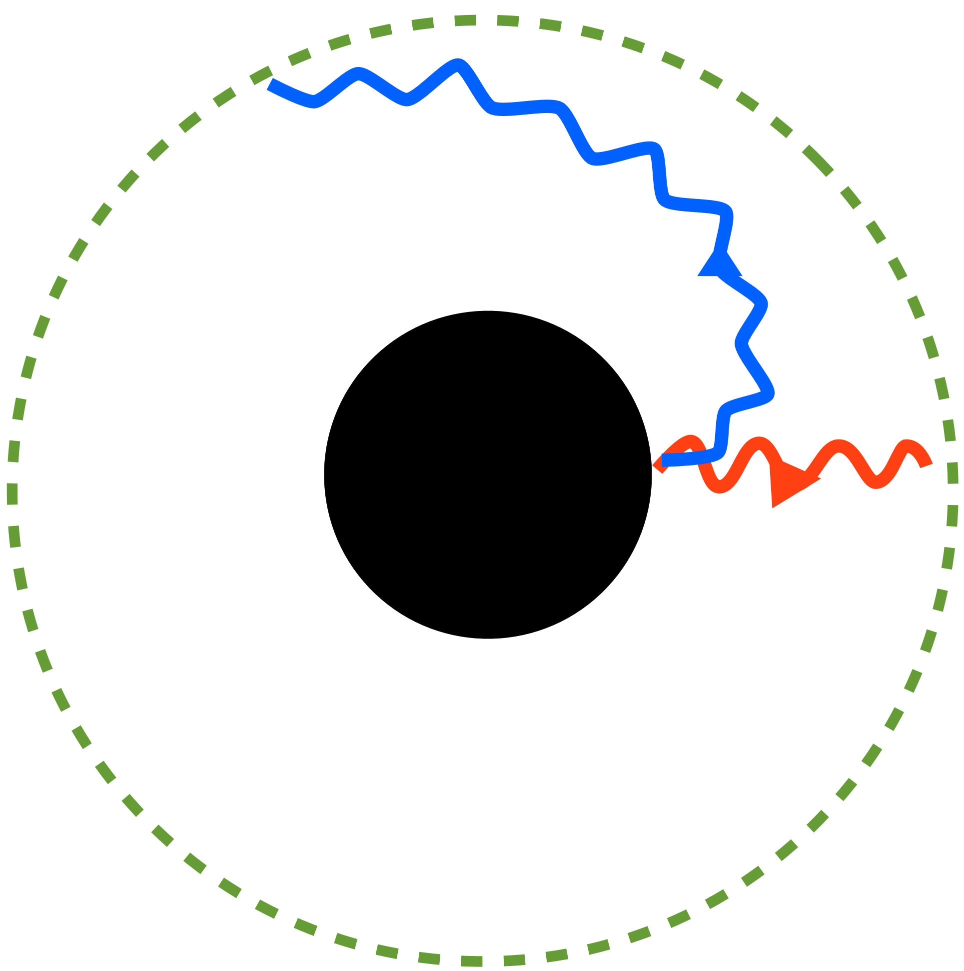

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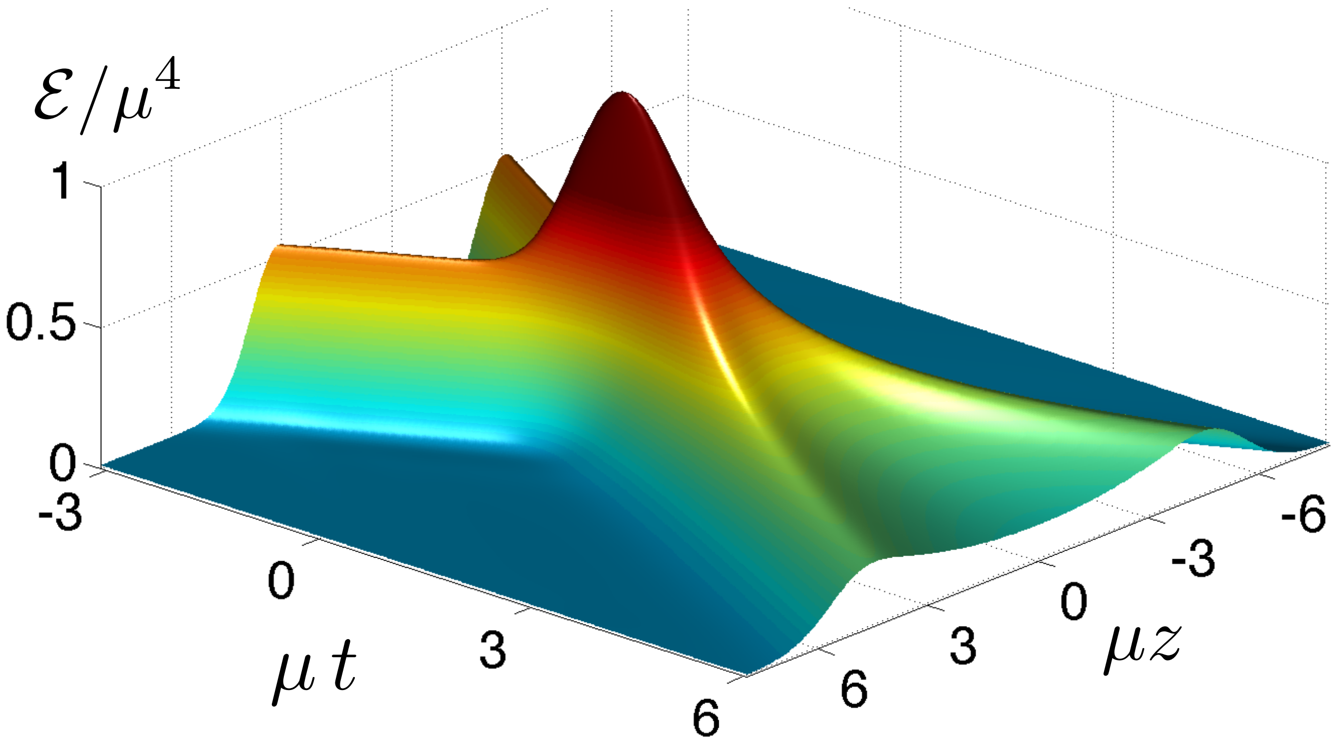
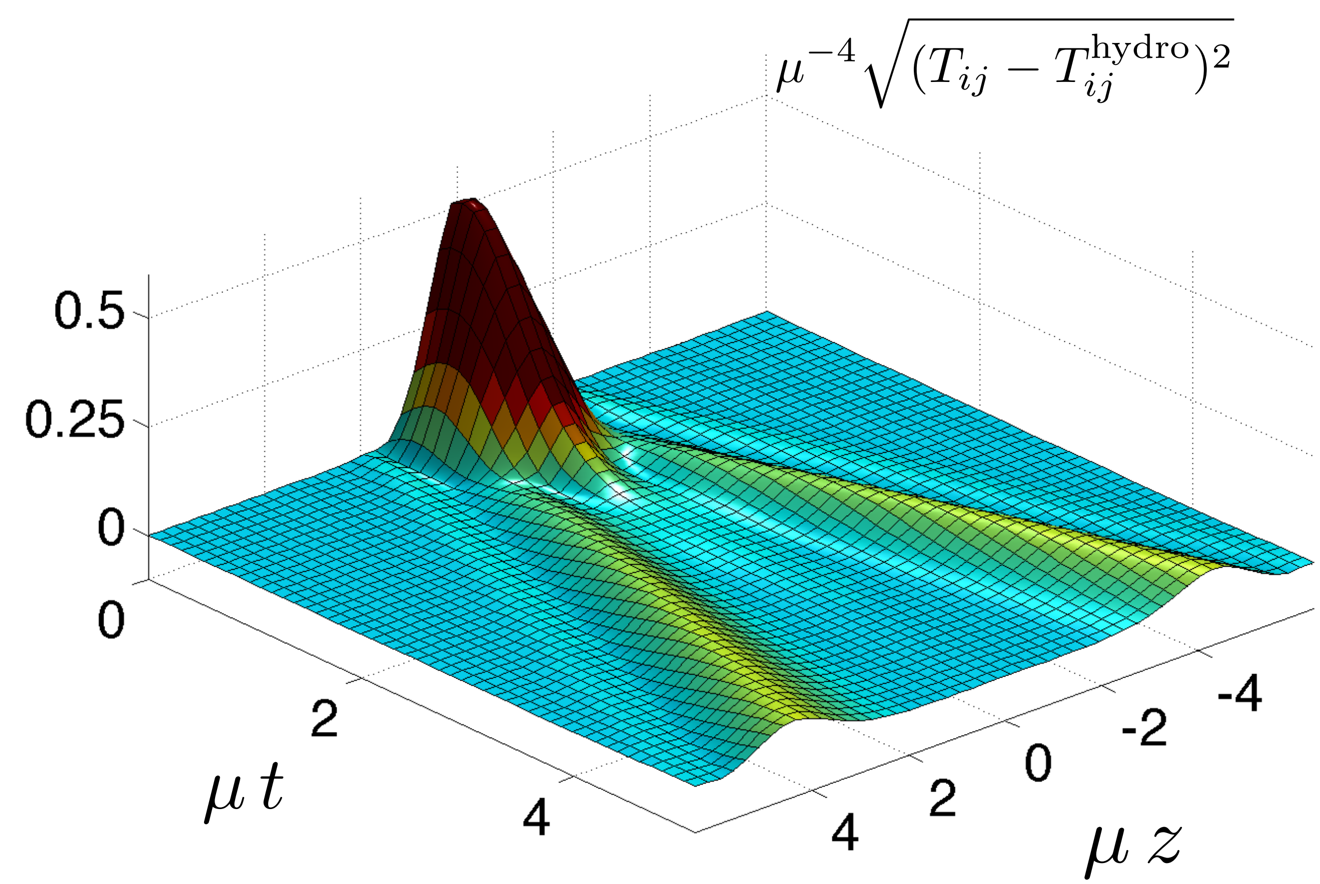
What does this mean for collisions?



An analytic calculation shows

- viscous effects don't violate FDT.

$$t_{\rm therm}(\omega, |\mathbf{q}|) \gtrsim t_{\rm hydro} + \frac{2}{\pi T}, \sim (2 \text{ to } 3) \times t_{\rm hydro}.$$



Doesn't equation of state $p = p(\epsilon)$ require thermal equilibrium?

Counterexamples: QCD at $T \to \infty$ or conformal field theories $T^{\mu}_{\ \mu} = 0$.

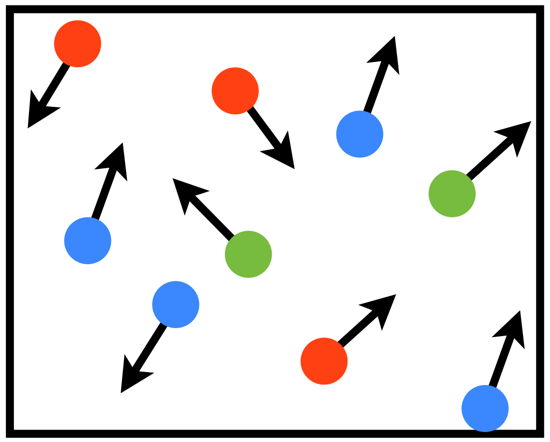
Weakly coupled explanation:

- If $f_s(x^\mu, \mathbf{p}) = f_s(x^\mu, |\mathbf{p}|)$, then

$$\langle T^{\mu\nu} \rangle = \mathrm{diag}[\epsilon, p, p, p],$$

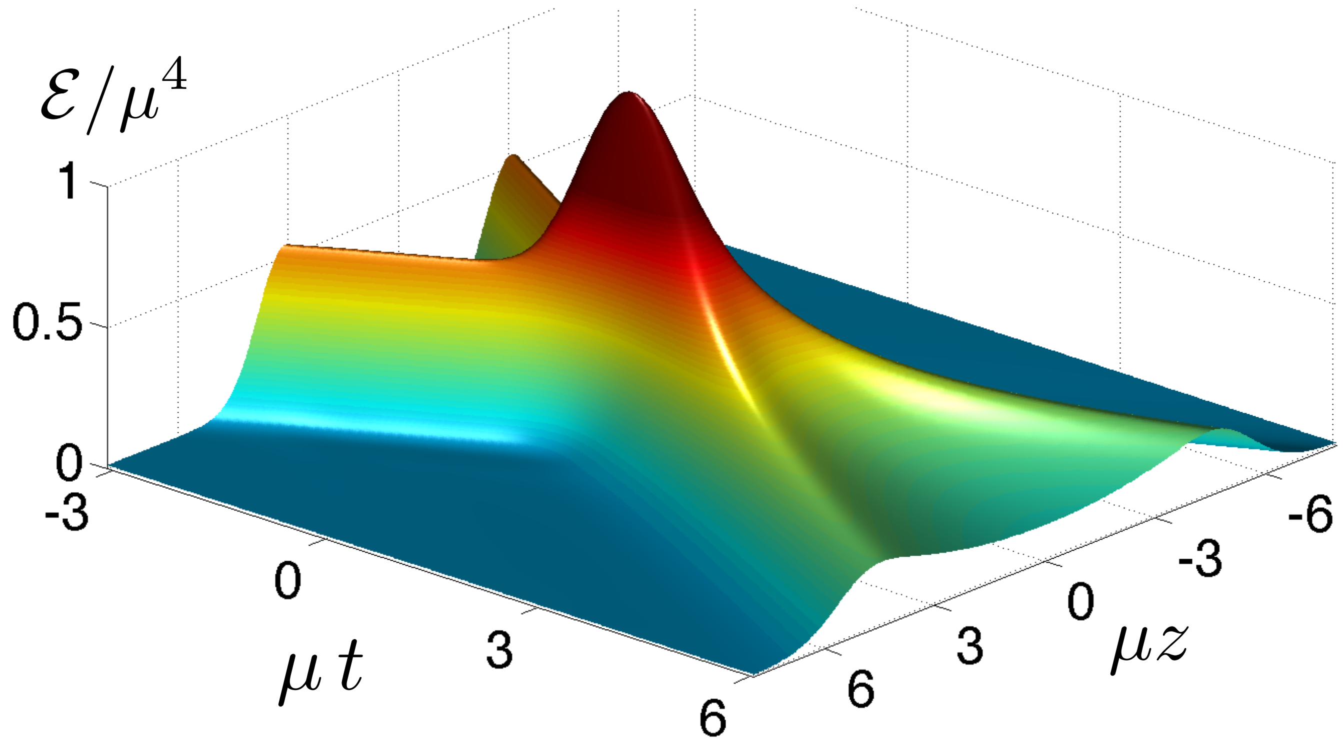
where $p = \frac{1}{3}\epsilon$ even if

$$f_s(x^\mu, \mathbf{p}) \neq \frac{1}{e^{E(p)/T} \pm 1}.$$

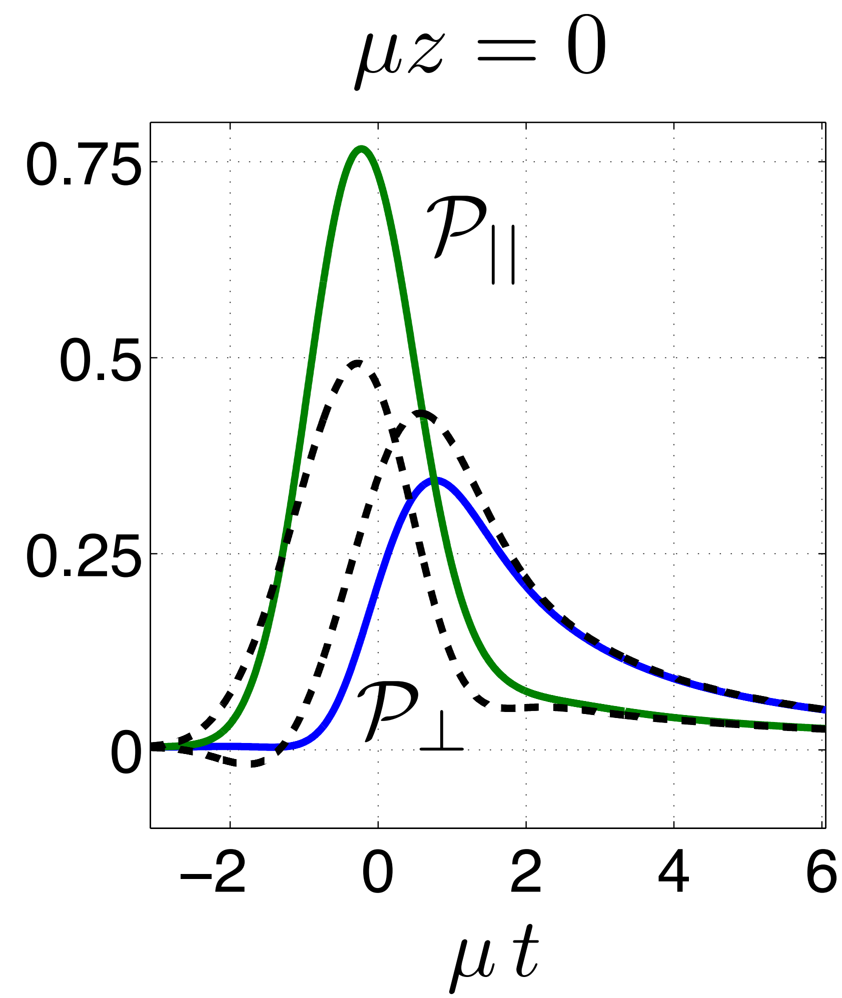


What have we learned?

- Holography is a powerful tool to explore real-time dynamics!
  - Difficult QFT dynamics on a laptop!
- Relaxation to hydro is quick!
  - RHIC estimate: $t_{\text{hydro}} \sim 0.35$ fm/c.
  - $t_{\text{hydro}} \sim 1$ fm/c need not be thought of as unnaturally rapid!
- Natural order: thermalization after "hydrodynamization."
  - $(\text{a few}) \times t_{\text{hydro}} \lesssim t_{\text{therm}}(\omega, |\mathbf{q}|) \lesssim \frac{1}{T}\left(\frac{|\mathbf{q}|}{T}\right)^{1/3}.$

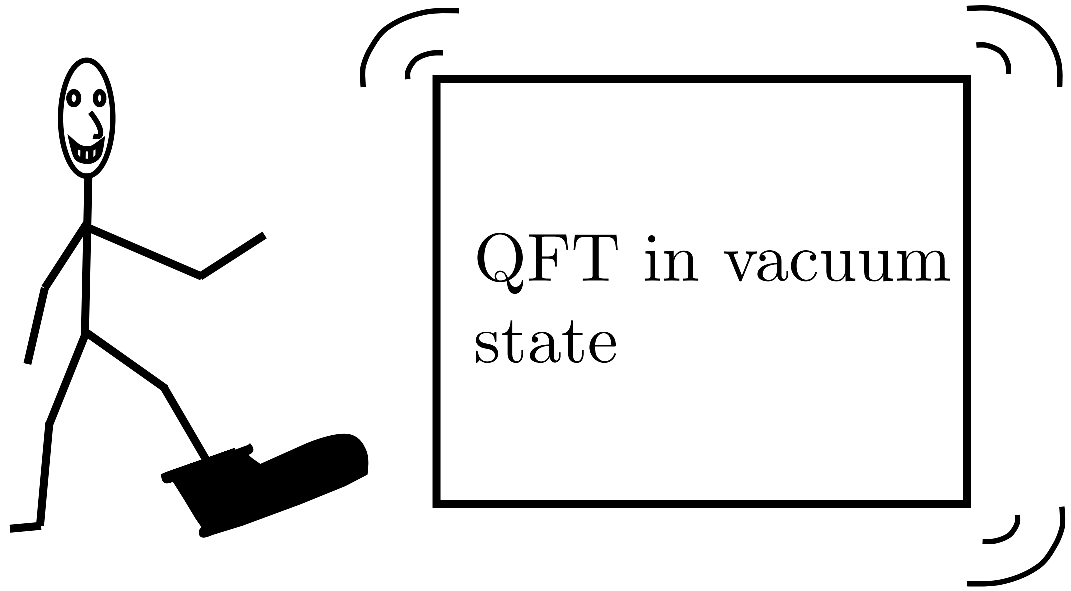


$\mathcal{E}/\mu^4$
1
0.5
0
-3
0
3
6
$\mu\, t$
-6
-3
0
3
6
μz



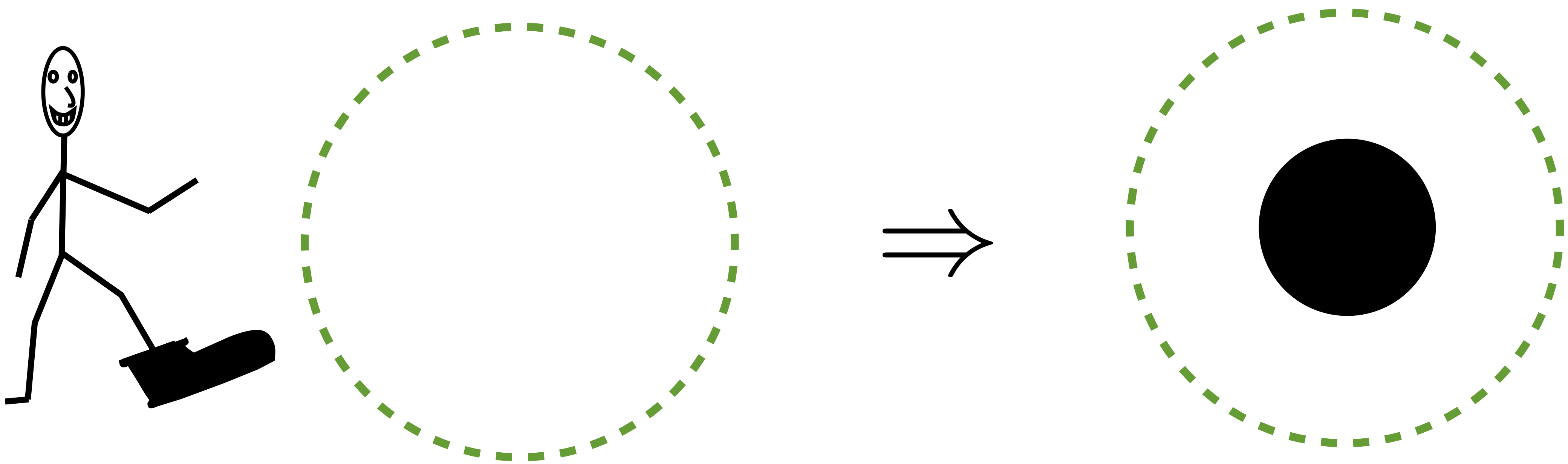
$\mu z = 0$
0.75
0.5
0.25
0
–2
0
2
4
6
$\mu\, t$
$\mathcal{P}_{||}$
$\mathcal{P}_{\perp}$

Creating homogenous anisotropic plasma with gravity



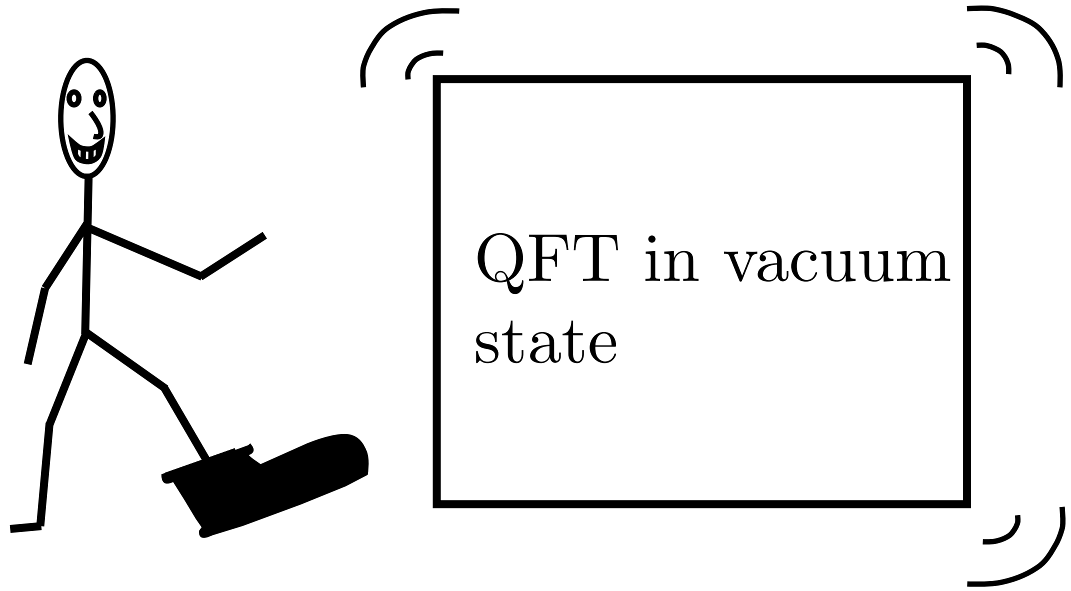
Kick: QFT in curved space.

$$ds^2 = -dt^2 + e^{B(t)} d\mathbf{x}_\perp^2 + e^{-2B(t)} dx_{||}^2$$



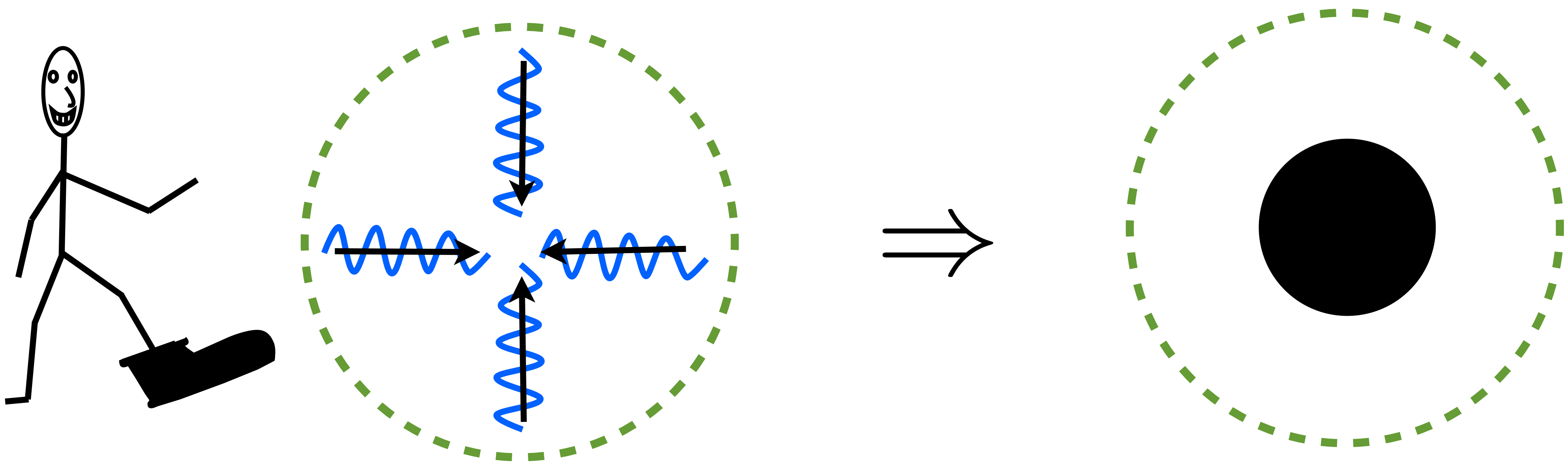
$\therefore$ Must solve Einstein subject to time dependent BCs.

Creating homogenous anisotropic plasma with gravity

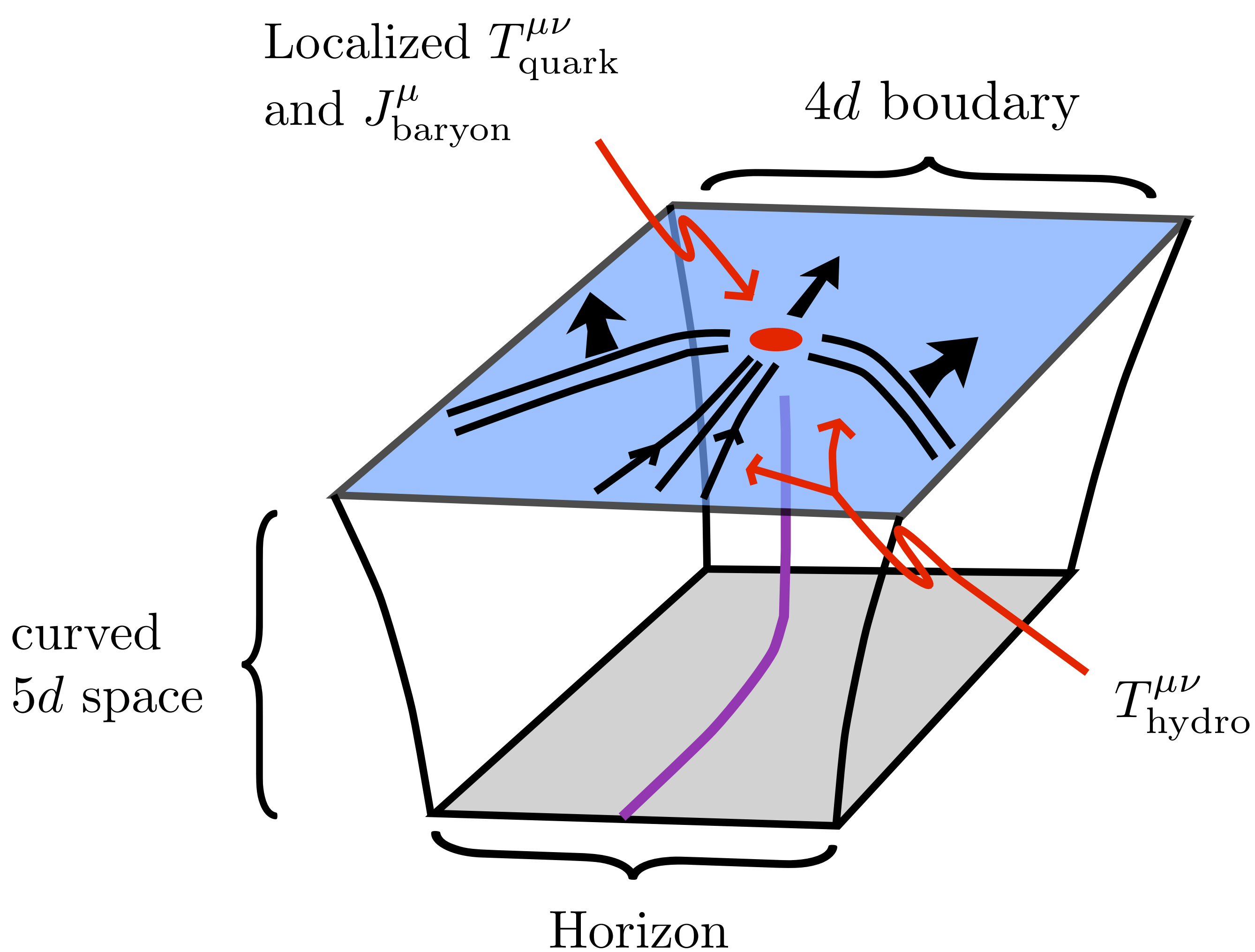


Kick: QFT in curved space.

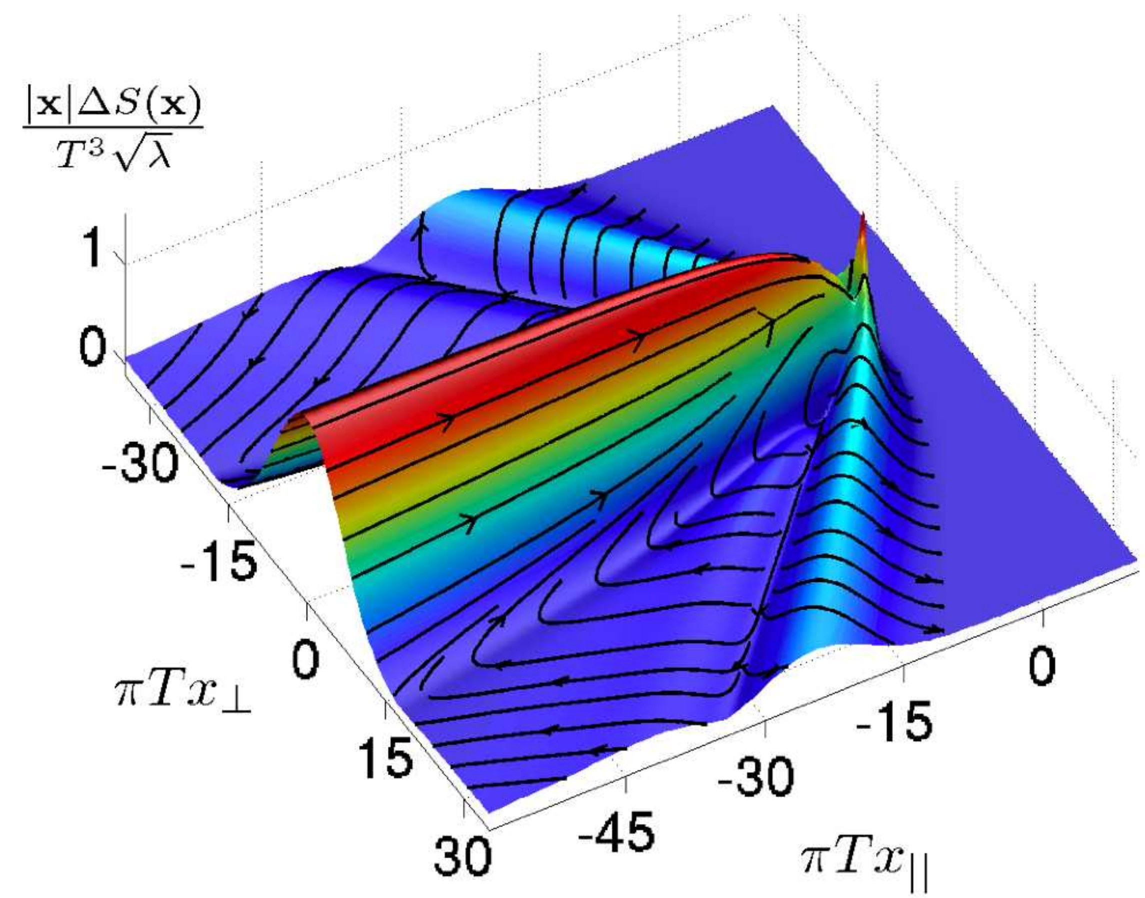
$$ds^2 = -dt^2 + e^{B(t)} d\mathbf{x}_\perp^2 + e^{-2B(t)} dx_{||}^2$$



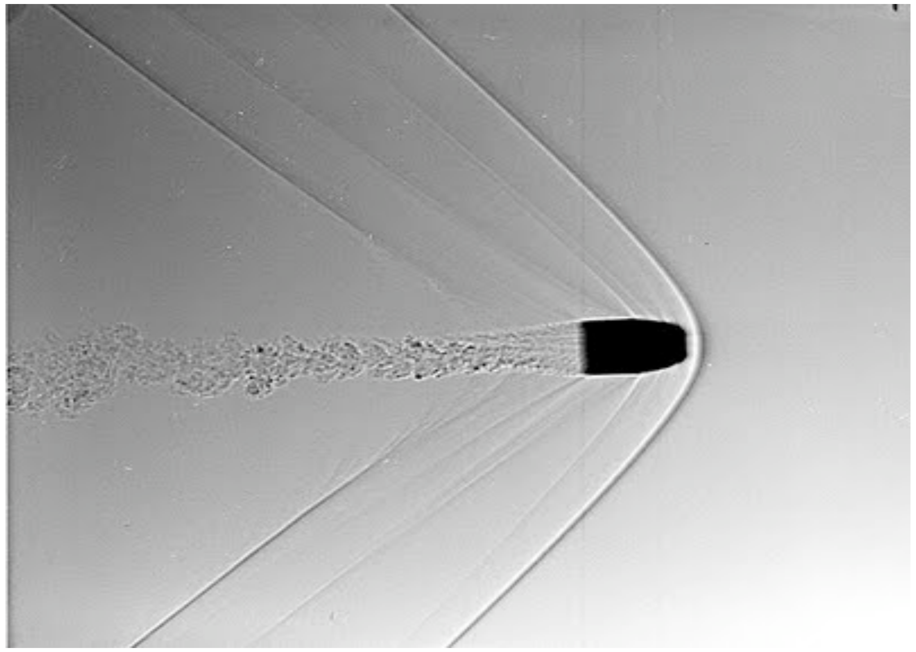
$\therefore$ Must solve Einstein subject to time dependent BCs.

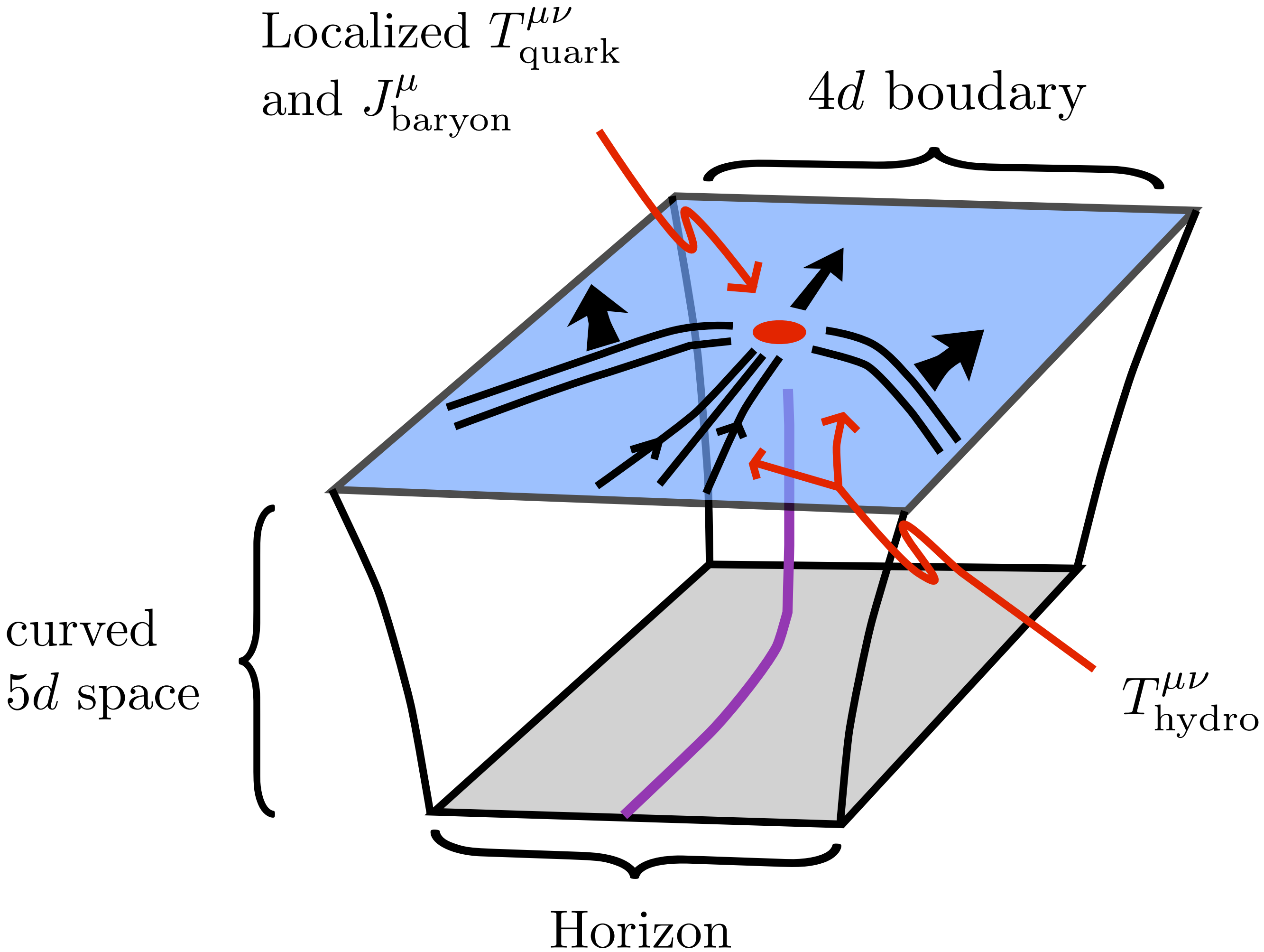


Localized $T^{\mu\nu}_{\mathrm{quark}}$ and $J^{\mu}_{\mathrm{baryon}}$
$4d$ boudary
curved $5d$ space
$T^{\mu\nu}_{\mathrm{hydro}}$
Horizon

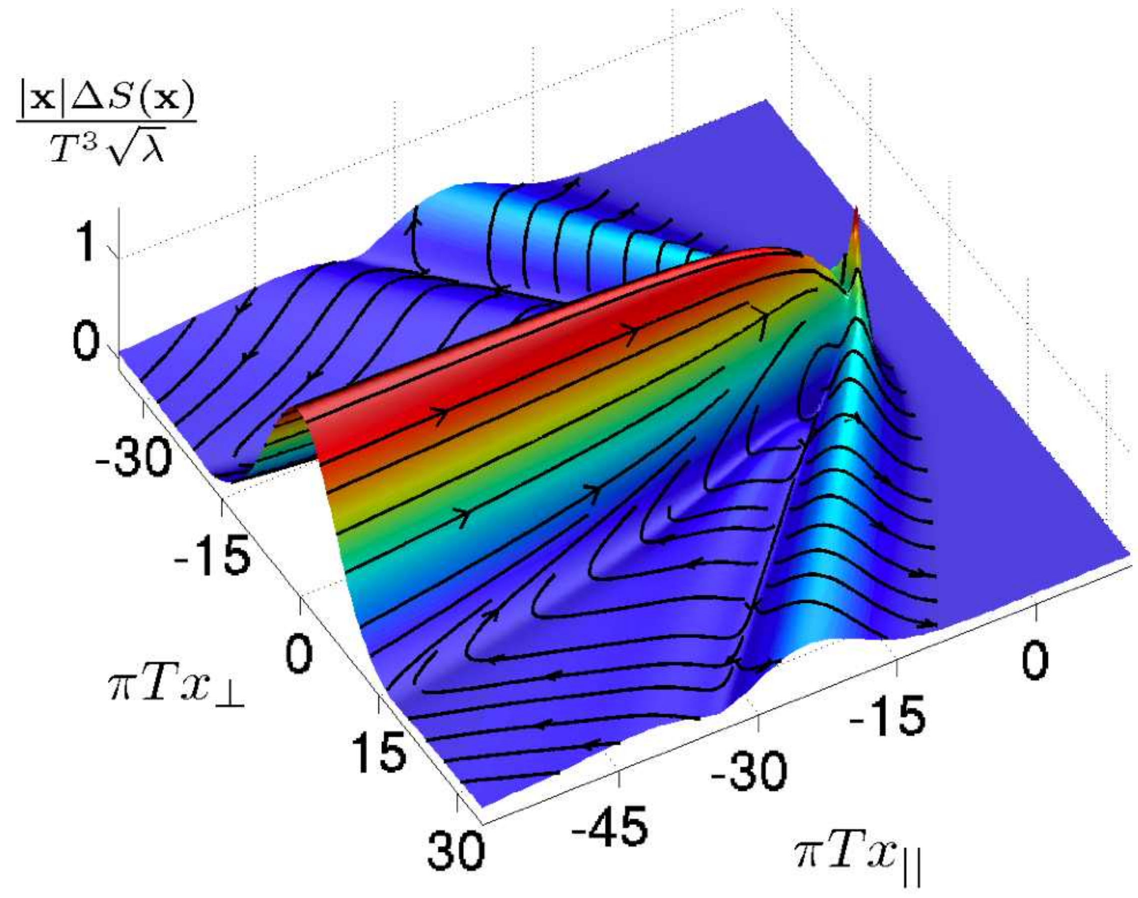


$\frac{|\mathbf{x}|\Delta S(\mathbf{x})}{T^3\sqrt{\lambda}}$
1
0
-30
-15
0
15
30
$\pi T x_\perp$
-45
-30
-15
0
$\pi T x_{||}$





Localized $T^{\mu\nu}_{\text{quark}}$
and J^{μ}_{baryon}
$4d$ boudary
curved
$5d$ space
$T^{\mu\nu}_{\text{hydro}}$
Horizon



$\frac{|\mathbf{x}|\Delta S(\mathbf{x})}{T^3\sqrt{\lambda}}$
1
0
-30
-15
0
15
30
$\pi T x_\perp$
-45
-30
-15
0
$\pi T x_{||}$