Theta vacuum and entanglement interaction in the three-flavor Polyakov-loop extended Nambu-Jona-Lasinio model

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Introduction

The QCD vacuum is a superposition of vacua characterized by winding number n.

\[ |\theta\rangle = \sum_n e^{i\theta n} |n\rangle \]

Then QCD effective Lagrangian is required an extra term.

\[ \mathcal{L} = \sum_q \bar{\psi}(i\gamma_\mu D_\mu + m_\psi)\psi - g_S \sum_{q_\alpha} \left( \bar{q} \gamma_\mu \gamma_5 q_\alpha + \bar{q}_\alpha \gamma_\mu \gamma_5 q \right) + \frac{1}{6} F_{\mu\nu}^a F^{a\mu\nu} - \theta i \frac{g}{64\pi^2} F_{\mu\nu}^a F^{a\mu\nu} \]

At zero temperature, experimental measurement of neutron dipole moment gives the upper limit: \[ |\theta| < 10^{-10} \]
However there is no theoretical interpretation for this property (Strong CP problem).

Then, at finite temperature, the behavior of theta is nontrivial.

Explore the phase diagram with finite theta angle

Related physics

- (Effective) Theta vacuum affect to some interesting physics as parity violating effect.
- Chiral magnetic effects
- Cosmic evolution (QCD transition)

EPNJL model

The three-flavor PNJL Lagrangian with the 0-dependent anomaly term is obtained in Euclidean spacetime by

\[ \mathcal{L} = \bar{q} i \gamma_\mu \partial_\mu q + \bar{q} \gamma_5 \lambda q - g_S \sum_{q_\alpha} \left( \bar{q} \gamma_\mu \gamma_5 q_\alpha + \bar{q}_\alpha \gamma_\mu \gamma_5 q \right) + \frac{1}{6} F_{\mu\nu}^a F^{a\mu\nu} - \theta i \frac{g}{64\pi^2} F_{\mu\nu}^a F^{a\mu\nu} \]

Entanglement coupling

We introduce the correlation of chiral condensate and Polyakov loop phenomenologically

\[ G_\theta \to G_S \left[ 1 - \alpha_1 \bar{\Phi} \Phi^* - \alpha_2 (\bar{\Phi}^3 + \Phi^{3*}) \right] \]

The PNJL model with entanglement coupling is the EPNJL model.

Order parameters

- \( \sigma_\theta = \langle q \gamma_\theta q \rangle \) Chiral symmetry
- \( \eta_\theta = \langle q \gamma_\theta^i q \rangle \) Chiral symmetry
- \( \Phi = \frac{1}{N_c} \text{tr} \left( e^{A_\theta/T} \right) \) Confinement/Deconfinement

Properties of PNJL and EPNJL model

EPNJL model has good consistencies with lattice simulations.

<table>
<thead>
<tr>
<th></th>
<th>PNJL model</th>
<th>EPNJL model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transition temperature ( T_c )</td>
<td>(Deconfinement)</td>
<td></td>
</tr>
<tr>
<td>Equation of state</td>
<td>(Chiral)</td>
<td></td>
</tr>
<tr>
<td>Roberge-Weiss periodicity (for imaginary ( \mu ))</td>
<td>(Qualitatively)</td>
<td>(Qualitatively)</td>
</tr>
<tr>
<td>Quark mass dependence of RW end point [2,3]</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

Phase structure

T dependence of order parameters at \( \mu=0 \) and \( \theta=\pi \)

T dependence of the phase diagram

Circumvent the sign problem

With the \( SU(3) \times SU(1) \) transformation:

\[ q = e^{\gamma_\mu i/3} q' e^{i\gamma_5 \theta/4} q_d = e^{i\gamma_\mu \theta/4} q_d = q'_d \]

the model Lagrangian become

\[ \mathcal{L} = \bar{q} \gamma_\mu D_\mu + m_0 - \gamma_5 \lambda q - g_S \sum_{q_\alpha} \left( \bar{q} \gamma_\mu \gamma_5 q_\alpha + \bar{q}_\alpha \gamma_\mu \gamma_5 q \right) + \frac{1}{6} F_{\mu\nu}^a F^{a\mu\nu} - \theta i \frac{g}{64\pi^2} F_{\mu\nu}^a F^{a\mu\nu} \]

In this representation, \( \theta \)-dependence appears only in the mass term.

The P-odd mass is much smaller than the QCD scale.

In the lattice simulation, the system in which the P-odd mass is neglected is a good reference system in the reweighting method.

Summary

We have investigated effects of the theta vacuum on the QCD phase diagram, using the 2+1 flavor PNJL and EPNJL models.

EPNJL model with theta vacuum

There is a possibility that the chiral transition become the 1st order at large theta.

Sign problem vs. theta vacuum

We propose the new reweighting technique in which the theory with no P-odd mass is used as a reference theory.