Theta vacuum and entanglement interaction in the three-flavor Polyakov-loop extended Nambu-Jona-Lasinio model

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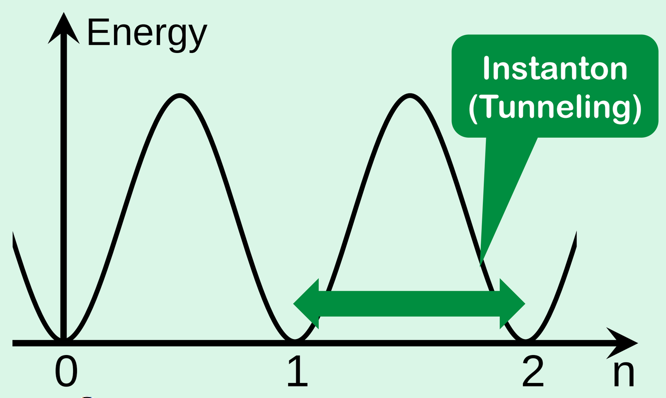
Introduction

The QCD vacuum is a superposition of vacua characterized by winding number n.

$$|\theta\rangle = \sum_n e^{in\theta}|n\rangle$$



Then QCD effective Lagrangian is required an extra term.

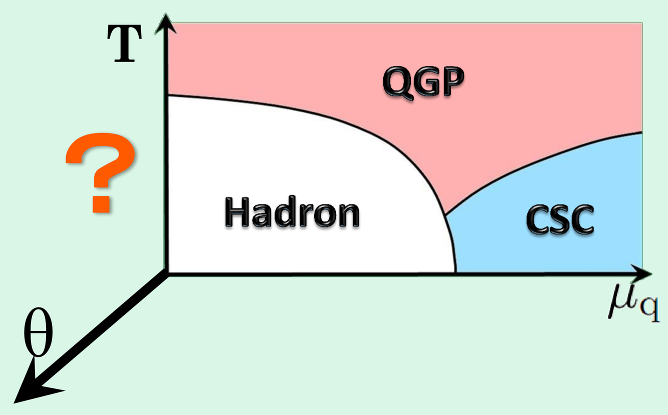
$$\mathcal{L} = \sum_f \bar{q}_f(\gamma_\nu D_\nu + m_f)q_f + \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - i\theta\frac{g^2}{64\pi^2}\epsilon_{\mu\nu\sigma\rho}F^a_{\mu\nu}F^a_{\sigma\rho}$$

At zero temperature, experimental measurement of neutron dipole moment gives the upper limit : $|\theta| < 10^{-10}$

However there is no theoretical interpretation for this property (Strong CP problem).

Then, at finite temperature, the behavior of theta is nontrivial.

➡ Explore the phase diagram with finite theta angle



Related physics

(Effective) Theta vacuum affect to some interestingh physics as parity violating effect.

- Chiral magnetic effects
- Cosmic evolution (QCD transition)

EPNJL model

Polyakov loop-extended Nambu-Jona-Lasinio model

The three-flavor PNJL Lagrangian with the θ-dependent anomaly term is obtained in Euclidean spacetime by

$$\mathcal{L} = \bar{q}(\gamma_\nu D_\nu + \hat{m}_0 - \gamma_4\hat{\mu})q - G_S\sum_{a=0}^{8}[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2]$$
$$+ G_D[e^{i\theta}\det_{ij}\bar{q}_i(1-\gamma_5)q_j + e^{-i\theta}\det_{ij}\bar{q}_i(1+\gamma_5)q_j] + \mathcal{U}(\Phi[A], \Phi^*[A], T)$$

Entanglement coupling

We introduce the correlation of chiral condensate and Polyakov loop phenomenologically

$$G_S \to G_S\left[1 - \alpha_1\Phi\Phi^* - \alpha_2(\Phi^3 + \Phi^{*3})\right]$$

The PNJL model with entanglement coupling is the EPNJL model.

Order parameters

$\sigma_f \equiv \langle\bar{q}_f q_f\rangle$	➡	Chiral symmetry
$\eta_f \equiv \langle\bar{q}_f i\gamma_5 q_f\rangle$	➡	Chiral symmetry Parity symmetry
$\Phi \equiv \frac{1}{N_c}\mathrm{tr}_c\left(e^{iA_4/T}\right)$	➡	Confinement/Deconfinement

Properties of PNJL and EPNJL model

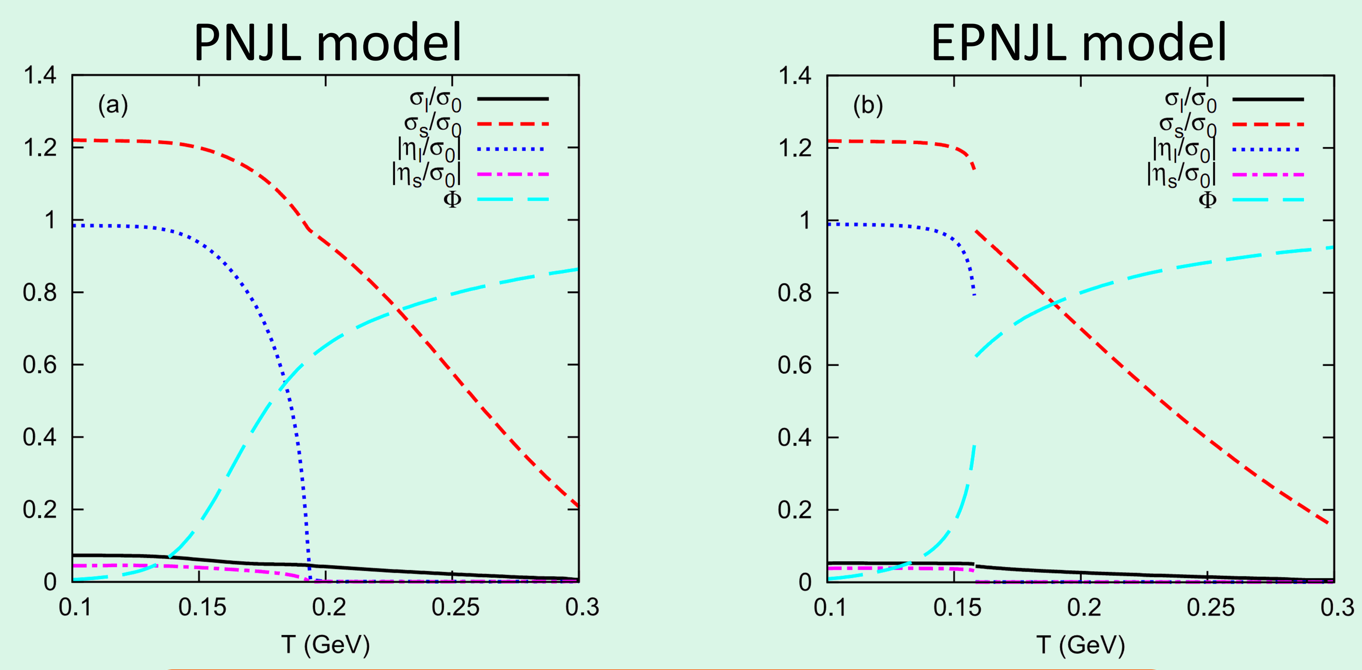
EPNJL model has good consistencies with lattice simulations.

	PNJL model	EPNJL model
Transition temperature T_c	○ (Deconfinement) △ (Chiral)	○
Equation of state	○ (Qualitatively)	○ (Qualitatively)
Roberge-Weiss periodicity (for imaginary μ)[1]	○	○
Quark mass dependence of RW end point [2,3]	×	○

[1] A. Roberge and N. Weiss, Nucl. Phys. B**275**, 734 (1986)
[2] P. de Forcrand and O. Philipsen, Phys. Rev. Lett. **105**, 152001 (2010); M. D'Elia and F. Sanfilippo, Phys. Rev. D**80,** 111501 (2009)
[3] T. S. ,Y. Sakai, H. Kouno, and M. Yahiro, Phys. Rev. D**84,** 091901 (2011)

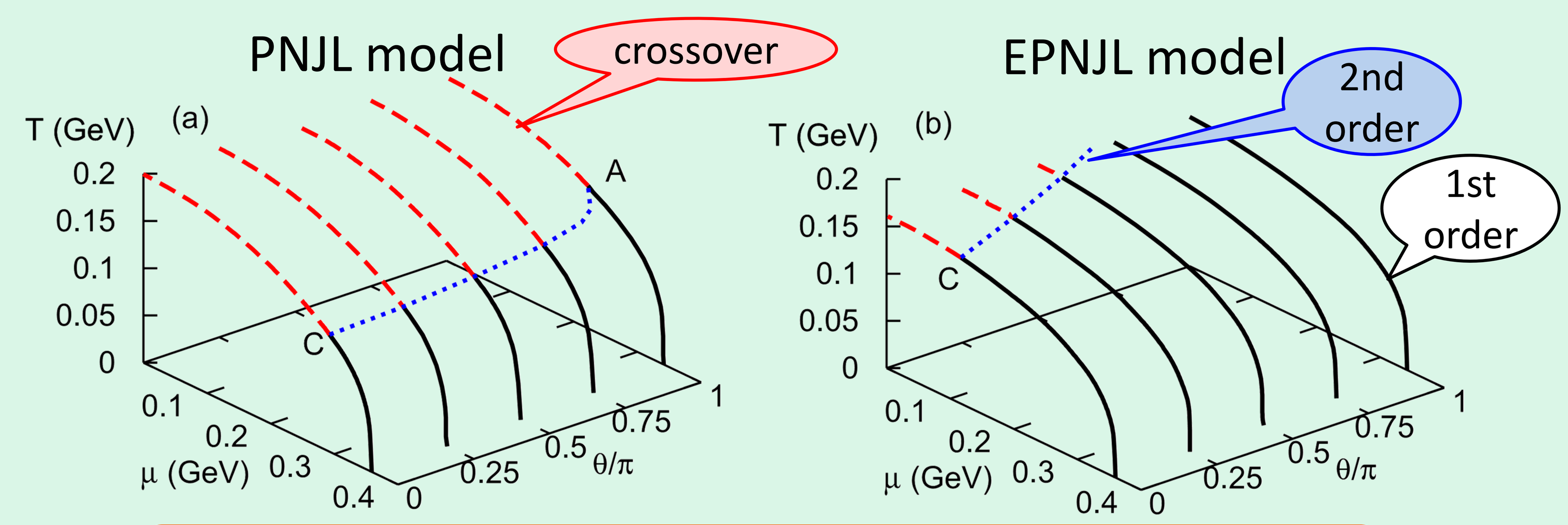
Phase structure

T dependence of order parameters at μ=0 and θ=π



Theta vacuum effect could make chiral and deconfinement transition 1st order.

T dependence of the phase diagram



CEP moves to large T as θ increases.
The 1st-order-transition region is wider in the EPNJL model.

Circumvent the sign problem

With the $SU_A(3)\otimes U_A(1)$ transformation;

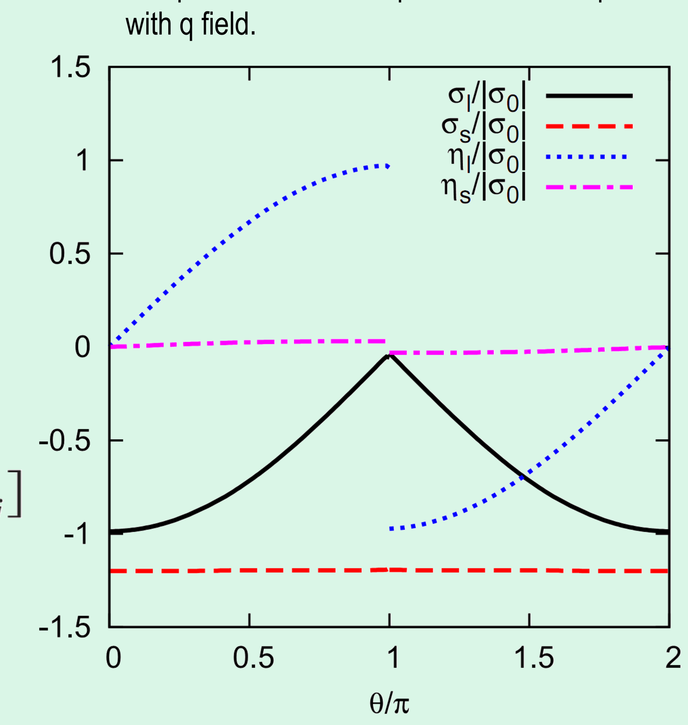
$$q_u = e^{i\gamma_5(\theta/4)}q'_u,\ q_d = e^{i\gamma_5(\theta/4)}q'_d,\ q_s = q'_s$$

the model Lagrangian become

$$\mathcal{L} = \bar{q}'(\gamma_\nu D_\nu + \hat{m}_{0+} + i\hat{m}_{0-}\gamma_5 - \gamma_4\hat{\mu})q'$$
$$- G_S\sum_{a=0}^{8}[(\bar{q}'\lambda_a q')^2 + (\bar{q}'i\gamma_5\lambda_a q')^2]$$
$$+ G_D[\det_{ij}\bar{q}'_i(1-\gamma_5)q'_j + \det_{ij}\bar{q}'_i(1+\gamma_5)q'_j]$$
$$+ \mathcal{U}(\Phi[A], \Phi^*[A], T)$$

$$\hat{m}_{0+} = \mathrm{diag}\left(\cos\left(\frac{\theta}{2}\right)m_u, \cos\left(\frac{\theta}{2}\right)m_d, m_s\right)$$
$$\hat{m}_{0-} = \mathrm{diag}\left(\sin\left(\frac{\theta}{2}\right)m_u, \sin\left(\frac{\theta}{2}\right)m_d, 0\right)$$

θ dependence of order parameters at T=μ=0 with q field.

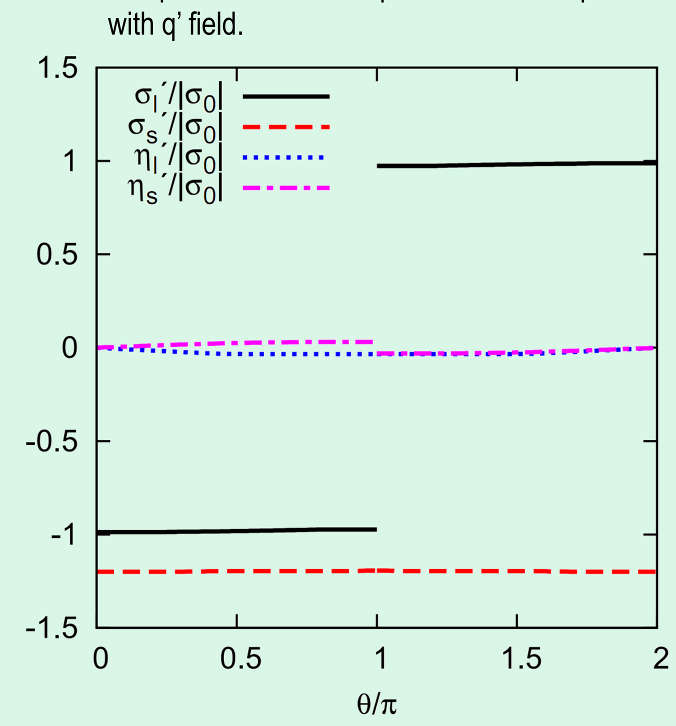


In this representation, θ-dependence appears only in the mass term.

➡ The P-odd mass is much smaller than the QCD scale.

➡ In the lattice simulation, the system in which the P-odd mass is neglected is a good reference system in the reweighting method.

θ dependence of order parameters at T=μ=0 with q' field.



(cf. T. Sasaki, H. Kouno, and M. Yahiro, arXiv:hep-ph/1208.0375)

Summary

We have investigated effects of the theta vacuum on the QCD phase diagram, using the 2+1 flavor PNJL and EPNJL models.

EPNJL model with theta vacuum

There is a possibility that the chiral transition become the 1st order at large theta.

Sign problem vs. theta vacuum

We propose the new reweighting technique in which the theory with no P-odd mass is used as a reference theory.