# The NLO inclusive forward hadron production in pA collisions

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**Abstract**

Recently, the one-loop factorization for inclusive hadron productions in pA collisions in the saturation formalism has been established via the complete next-to-leading order calculation. The differential cross section is written into a factorization form in the coordinate space at the next-to-leading order, while the naive form of the convolution in the transverse momentum space does not hold. The rapidity divergence with small-x dipole gluon distribution of the nucleus is factorized into the energy evolution of the dipole gluon distribution function, which is known as the Balitsky-Kovchegov equation. Furthermore, the collinear divergences associated with the incoming parton distribution of the nucleon and the outgoing fragmentation function of the final state hadron are factorized into the splittings of the associated parton distribution and fragmentation functions, which allows us to reproduce the well-known DGLAP equation. The hard coefficient function, which is finite and free of divergence of any kind, is evaluated at one-loop order. This result is important, not only for the phenomenological applications to the inclusive hadron production in p-A collisions at RHIC and future LHC experiment, but also for theoretically promoting the rigorous developments towards factorizations in small-x physics.

## 1. Introduction

Small-$x$ physics provides us the dense parton densities at high energy limit and predicts the onset of the saturation phenomenon as a result of nonlinear dynamics. Inclusive hadron production in $pA$ collisions, which is considered as one of the most important processes to reveal and test the small-$x$ physics, has attracted much of theoretical interests in recent years [1, 2, 3, 4, 6, 5, 7, 8, 9, 10, 11, 12, 13]. In particular, the suppression of hadron production in the forward $dAu$ scattering at RHIC observed in the experiments [14, 15] has been regarded as one of the evidences for the gluon saturation at small-$x$ in a large nucleus [7, 8]. Saturation phenomenon at small-$x$ in nucleon and nucleus plays an important role in high energy hadronic scattering [16, 17, 18]. In this proceeding for Quark Matter 2012, we summarize our results which calculate the one-loop perturbative corrections. Our results, which is an important step toward a complete description of hadron production in $pA$ collisions in the small-$x$ limit, have been published earlier in Ref. [19].

## 2. The one-loop calculation of inclusive hadron productions in $pA$ collisions

Inclusive hadron production in $pA$ collisions,

$$p + A \to h + X\,, \tag{1}$$

can be viewed as a process where a parton from the nucleon (with momentum $p$) scatters on the nucleus target (with momentum $P_A$), and fragments into final state hadron with momentum $P_h$. In the dense medium of the large nucleus and at small-$x$, the multiple interactions become important, and we need to perform the relevant resummation to make the reliable predictions. This is particularly important because the final state parton is a colored object. Its interactions with the nucleus target before it fragments into the hadron is crucial to understand the nuclear effects in this process. In our calculations, we follow the high energy factorization, also called color-dipole or color-glass-condensate (CGC), formalism [20, 21, 22, 23] to evaluate the above process up to one-loop order.

According to our calculations, the QCD factorization formalism for the above process reads as,

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \sum_a \int \frac{dz}{z^2}\frac{dx}{x}\xi x f_a(x,\mu)D_{h/c}(z,\mu)\int [dx_\perp]S^Y_{a,c}([x_\perp])\mathcal{H}_{a\to c}(\alpha_s,\xi,[x_\perp]\mu)\ ,\quad(2)$$

where $\xi = \tau/xz$ with $\tau = p_\perp e^y/\sqrt{s}$, $y$ and $p_\perp$ the rapidity and transverse momentum for the final state hadron and $s$ the total center of mass energy square $s = (p + P_A)^2$, respectively. Schematically, the incoming parton described by the parton distribution $f_a(x)$ scatters off the nuclear target represented by multiple-point correlation function $S^Y([x_\perp])$, and fragments into the final state hadron defined by the fragmentation function $D_{h/c}(z)$. All these quantities have clear operator definitions in QCD. In particular, $f_a(x)$ and $D_{h/c}(z)$ are collinear parton distribution and fragmentation functions which only depend on the longitudinal momentum fraction $x$ of the nucleon carried by the parton $a$, and the momentum fraction $z$ of parton $c$ carried by the final state hadron $h$, respectively. From the nucleus side, it is the multi-point correlation functions denoted as $S^Y_{a,c}(x_\perp)$ (see the definitions below) that enters in the factorization formula, depending on the flavor of the incoming and outgoing partons and the gluon rapidity $Y$ associated with the nucleus: $Y \approx \ln(1/x_g)$ with $x_g$ being longitudinal momentum fraction.

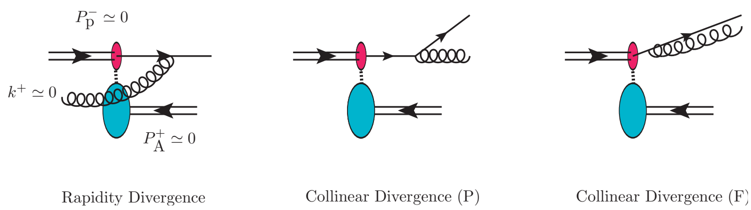


Figure 1: Illustrations of the kinematic region of various divergences appearing in NLO calculation.

To evaluate NLO corrections, we need to calculate the gluon radiation contributions. At one-loop order, the gluon radiation will introduce various divergences. The factorization formula in Eq. (2) is to factorize these divergences into the relevant factors.

For example, there is the rapidity divergence associated with $S^Y([x_\perp])$ when the radiated gluon is collinear to the target nucleus. The physical interpretation of the rapidity divergence subtraction is quite interesting. The soft gluon is emitted from the projectile proton with momentum $(1-\xi)p^+$, and it is easy to see that the rapidity of this soft gluon goes to $-\infty$ when $\xi \to 1$ since the radiated gluon is now in the region $k_g^- \gg k_g^+$. As a matter of fact, this soft gluon can be regarded as collinear to the target nucleus which is moving on the backward light cone with the rapidity close to $-\infty$ and $P_A^- \gg P_A^+$. Therefore, it is quite natural to renormalize this soft gluon into the gluon distribution function of the target nucleus through the BK evolution equation[21, 22]. In addition, there will be collinear divergences associated with the incoming parton distribution or the final state fragmentation functions when the radiated gluon is either collinear to the incoming parton or to the outgoing parton as illustrated in last two figures of Fig. 1. As the standard procedure, we use the dimensional regularization ($D = 4 - 2\epsilon$) and follow the $\overline{\text{MS}}$ subtraction scheme, in order to compute and move the collinear divergence into the DGLAP equation.

The leading order results have been calculated before, from which we have

$$\mathcal{H}_{2qq}^{(0)} = e^{-ik_\perp \cdot r_\perp}\delta(1-\xi)\,, \tag{3}$$

where $k_\perp = p_\perp/z$ and $r_\perp = x_\perp - y_\perp$. The NLO calculations are obtained by using the dijet results calculated in Ref. [24, 25] and then integrating over the phase space of the unobserved parton. After removing all the divergences as illustrated in Fig. 1, we can reach finite hard coefficients for the production cross section. For the quark channel contribution: $qA \to q + X$, the above factorization formula can be explicitly written as

$$\begin{aligned}
\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} &= \int \frac{dz}{z^2}\frac{dx}{x}\xi xq(x,\mu)D_{h/q}(z,\mu)\int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} \\
&\quad \left\{S^{(2)}(x_\perp,y_\perp)\left[\mathcal{H}_{2qq}^{(0)} + \frac{\alpha_s}{2\pi}\mathcal{H}_{2qq}^{(1)}\right]\right. \\
&\quad \left. + \int \frac{d^2b_\perp}{(2\pi)^2}S^{(4)}(x_\perp,b_\perp,y_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}_{4qq}^{(1)}\right\}\,,
\end{aligned} \tag{4}$$

up to one-loop order. The two-point and four-point functions are defined as

$$S^{(2)}(x_\perp,y_\perp) = \frac{1}{N_c}\langle U(x_\perp)U^\dagger(y_\perp)\rangle_Y \tag{5}$$

$$S^{(4)}(x_\perp,b_\perp,y_\perp) = \frac{1}{N_c^2}\langle \mathrm{Tr}[U(x_\perp)U^\dagger(b_\perp)]\mathrm{Tr}[U(b_\perp)U^\dagger(y_\perp)]\rangle_Y\,, \tag{6}$$

with $N_c$ the number of color in QCD, and $U(x_\perp)$ is the Wilson line in the small-$x$ formalism.

After subtracting the above divergences, we obtain the hard coefficients at large $N_c$ limit

$$\begin{aligned}
\mathcal{H}_{2qq}^{(1)} &= C_F\mathcal{P}_{qq}(\xi)\ln\frac{c_0^2}{r_\perp^2\mu^2}\left(e^{-ik_\perp\cdot r_\perp} + \frac{1}{\xi^2}e^{-i\frac{k_\perp}{\xi}\cdot r_\perp}\right) \\
&\quad -3C_F\delta(1-\xi)e^{-ik_\perp\cdot r_\perp}\ln\frac{c_0^2}{r_\perp^2k_\perp^2},
\end{aligned} \tag{7}$$

$$\begin{aligned}
\mathcal{H}_{4qq}^{(1)} &= -4\pi N_c e^{-ik_\perp\cdot r_\perp}\left\{e^{-i\frac{1-\xi}{\xi}k_\perp\cdot(x_\perp-b_\perp)}\frac{1+\xi^2}{(1-\xi)_+}\frac{1}{\xi}\frac{x_\perp-b_\perp}{(x_\perp-b_\perp)^2}\cdot\frac{y_\perp-b_\perp}{(y_\perp-b_\perp)^2}\right. \\
&\quad -\delta(1-\xi)\int_0^1 d\xi'\frac{1+\xi'^2}{(1-\xi')_+} \\
&\quad \left.\times\left[\frac{e^{-i(1-\xi')k_\perp\cdot(y_\perp-b_\perp)}}{(b_\perp-y_\perp)^2} - \delta^{(2)}(b_\perp-y_\perp)\int d^2r'_\perp\frac{e^{ik_\perp\cdot r'_\perp}}{r'^2_\perp}\right]\right\},
\end{aligned} \tag{8}$$

where $c_0 = 2e^{-\gamma_E}$ with $\gamma_E$ the Euler constant. These hard coefficients do not contain any divergence. The next-to-leading order differential cross section is written into a factorization form in the coordinate space, while It is straightforward to see that the naive form of the convolution in the transverse momentum space does not hold. Furthermore, as part of the consistency check, we find that the above result is in agreement with the collinear factorization result in the dilute limit in the forward pA collisions. The calculations for all other partonic channels $q \to qg$, $g \to gg$ and $g \to q\bar{q}$ follow the same procedure, and the corresponding hard coefficients are calculated up to one-loop order in Ref. [19].

## 3. Conclusion

Our calculations, together with NLO DGLAP and NLO BK[26, 27] evolution equations, as well as the one-loop running coupling correction, provide the complete formula for inclusive hadron production at NLO. The corrections to this NLO order cross section are either of order $\alpha_s^2$ or suppressed by $\frac{1}{N_c^2}$. The numerical implementation of the above NLO correction will help us understand the forward inclusive hadron production in pA collisions phenomenologically and make the important precision test of saturation physics possible. In terms of resummation, we will be able to resum $\alpha_s(\alpha_s \ln k_\perp^2)^n$ and $\alpha_s(\alpha_s \ln 1/x)^n$ terms. The extra factor of $\alpha_s$ can either come from the hard factor, which is calculated in this manuscript, or arise from the NLO DGLAP and BK evolution equations. The complete study of various processes[28] will provide us important steps to understand factorizations and phenomenology in the small-$x$ physics.

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