The QCD equation of state with 2+1 flavors of Highly Improved Staggered Quarks (HISQ)

Alexei Bazavov (for the HotQCD Collaboration)[1]

Brookhaven National Laboratory, Upton, NY 11793

Abstract

One of the fundamental properties of the quark-gluon plasma (QGP), the equation of state, is a subject of extensive studies in lattice QCD and an essential requirement for the correct hydrodynamic modeling of heavy-ion collisions. Lattice QCD provides first-principle calculations for the physics in the non-perturbative regime. In this contribution, we report on recent progress by the HotQCD collaboration in studying the 2+1 flavor equation of state on lattices with the temporal extent $N_\tau = 6$, 8, 10 and 12 in Highly Improved Staggered Quarks (HISQ) discretization scheme. Comparisons with equation of state calculations with different fermion actions are also discussed.

At fixed cutoff (in our case, lattice spacing, a) the trace of the energy-momentum tensor, or interaction measure, can be related to the partition function of the system:

$$\varepsilon - 3p = -\frac{T}{V}\frac{d\ln Z}{d\ln a}, \quad Z = \int DUD\bar{\psi}D\psi \exp\left(-S_{gauge} - S_{fermion}\right), \tag{1}$$

where ε is the energy density and p is pressure.

The equation of state, *i.e.*, the dependence of pressure on temperature can be expressed then as an integral:

$$\frac{p}{T^4} - \frac{p_0}{T_0^4} = \int_{T_0}^{T} dT' \frac{\varepsilon - 3p}{T'^5}. \tag{2}$$

To calculate the interaction measure in 2+1 flavor QCD we work on a Euclidean space-time lattice and evaluate path integrals with the importance sampling technique. We use a tree-level improved action for gauge fields and the Highly Improved Staggered Quarks (HISQ) action for fermions [1]. To get finite values, subtraction of UV divergences is required:

$$\frac{\varepsilon - 3p}{T^4} = R_\beta[\langle S_{gauge}\rangle_0 - \langle S_{gauge}\rangle_T] - R_\beta R_m[2m_l(\langle\bar{l}l\rangle_0 - \langle\bar{l}l\rangle_T) + m_s(\langle\bar{s}s\rangle_0 - \langle\bar{s}s\rangle_T)], \tag{3}$$

where the subscript "0" refers to $T = 0$ and "T" to finite temperature for observables evaluated at the same value of the cutoff. On a hyper-cubic lattice $N_s^3 \times N_\tau$ the physical temperature is set by

[1] The HotQCD collaboration members are: A. Bazavov, T. Bhattacharya, M. Buchoff, M. Cheng, N. Christ, C. DeTar, H.-T. Ding, S. Gottlieb, R. Gupta, P. Hegde, U. Heller, C. Jung, F. Karsch, E. Laermann, L. Levkova, Z. Lin, R. Mawhinney, S. Mukherjee, P. Petreczky, D. Renfrew, C. Schmidt, C. Schroeder, W. Soeldner, R. Soltz, R. Sugar, D. Toussaint, P. Vranas.

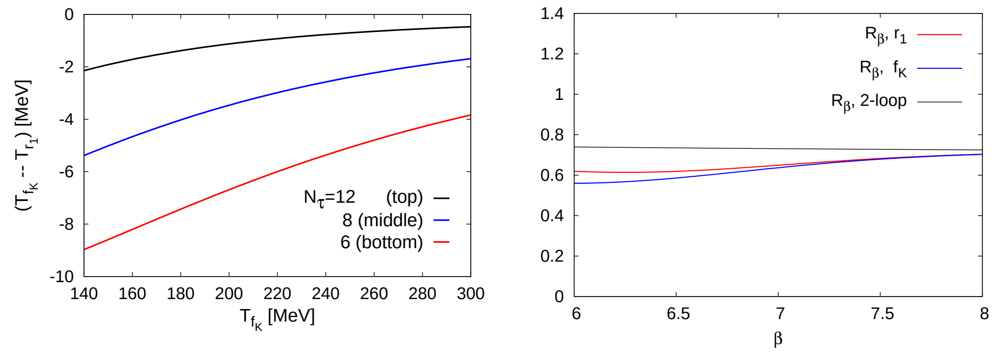

Figure 1: The difference in temperature (left) set by r_1 and f_K scale, described in the text. The running of the gauge coupling (right).

the size of the temporal dimension and the lattice spacing as $T = 1/(N_\tau a)$. For $T = 0$ calculations we use $N_\tau \geqslant N_s$, and for $T > 0$ we keep $N_s/N_\tau = 4$ and at fixed N_τ vary the lattice spacing a by varying the gauge coupling $\beta = 10/g^2$. The continuum limit in this setup is controlled by $N_\tau \to \infty$, therefore, we carried out this study on several sets of ensembles with $N_\tau = 6$, 8, 10 and 12. The strange quark mass m_s is tuned to the physical value, while the two degenerate light quarks have masses $m_l = m_s/20$, slightly heavier than physical ($m_l \simeq m_s/27$). To set the lattice spacing in physical units (fm) we use the Sommer-type scale [2] $r_1 = 0.3106$ fm [3], or, alternatively, the kaon decay constant, $f_K = 156.1$ MeV. From Fig. 1 (left) one can see what effect using different reference observables has on converting the temperature from lattice units to MeV. Over the temperature range of interest on $N_\tau = 6$ lattices the difference is within 9 MeV, and on $N_\tau = 12$ within 2 MeV. Another effect of using different scales is related to running of the corresponding β-functions:

$$R_\beta(\beta) = -a\frac{d\beta}{da}, \quad R_m(\beta) = \frac{1}{m_s(\beta)}\frac{dm_s(\beta)}{d\beta}, \tag{4}$$

where $m_s(\beta)$ defines a line of constant physics (LCP), *i.e.*, such combination of the gauge coupling and the strange quark mass so that pion and kaon masses (in MeV) stay approximately constant in the whole β range used in the simulation. R_β for r_1 and f_K scale is shown in Fig. 1

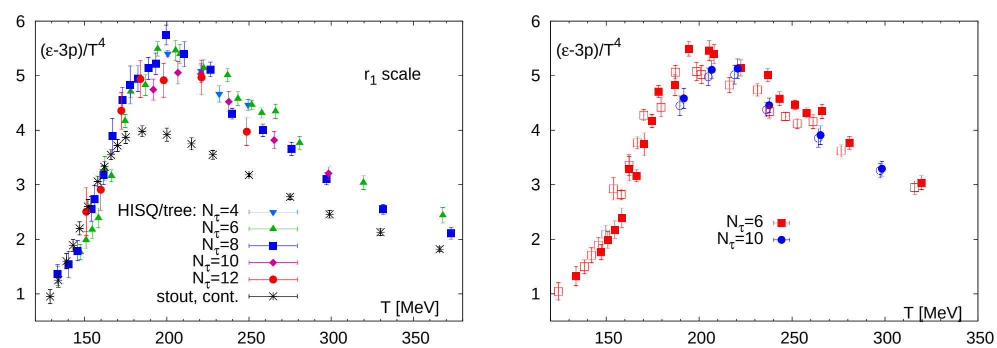

Figure 2: Comparison of the HISQ/tree interaction measure on $N_\tau = 6$, 8, 10 and 12 lattices with the stout continuum estimate [4] (left), $N_\tau = 6$ and 10 HISQ/tree data (right). Filled (open) symbols in the right panel correspond to the r_1 (f_K) scale, see text.

(right), together with the 2-loop perturbative result. As is clear from Eq. (3), the β-functions enter into the interaction measure multiplicatively.

Present status of the interaction measure with HISQ/tree is shown in Fig. 2 together with the stout continuum estimate of Ref [4]. Although we have not yet performed the continuum limit, the HISQ/tree results seem to disagree with stout in $T = 170 - 350$ MeV range.

The interaction measure for $N_\tau = 6$ and 10 is shown in Fig. 2 (right), $N_\tau = 8$ and $N_\tau = 12$ in Fig. 3. We also include our data with the asqtad action, where available. For comparison we plot the HISQ/tree and asqtad results with the temperature scale set with both r_1 (filled symbols) and f_K (open symbols).

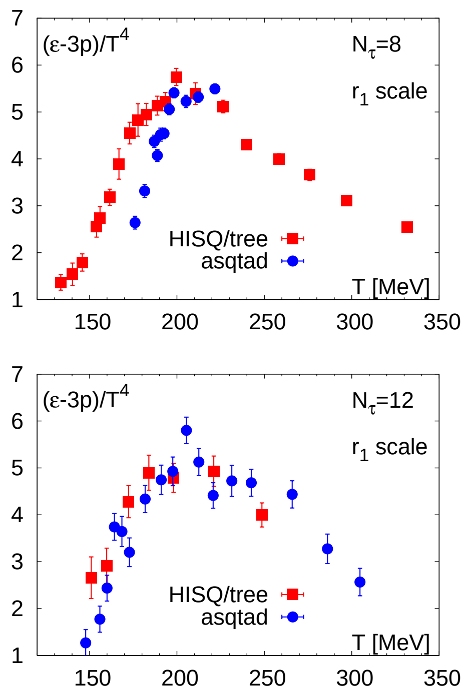

Figure 3: The interaction measure for the HISQ/tree action on $N_\tau = 8$ (top) and $N_\tau = 12$ (bottom) ensembles with r_1 (left) and f_K (right) scale.

On the coarsest lattice, $N_\tau = 6$, Fig. 2 (right) the cutoff effects for the HISQ/tree action are easily observed: the data plotted with f_K scale (red open symbols) lies lower and is shifted to the left, compared to the data with the r_1 scale (red filled symbols). On $N_\tau = 8$ the cutoff effects for HISQ/tree are milder, but still are very substantial for the asqtad action, compare the filled blue symbols in the left panel to the open blue symbols in the right panel of the upper part of Fig. 3. On $N_\tau = 10$ and 12 for both HISQ/tree and asqtad, Fig. 2 (right) and the lower part of Fig. 3, the cutoff effects are comparable with the statistical errors and agreement between the two actions is good for r_1 and f_K scale.

The low-temperature behavior of the interaction measure is shown in Fig. 4 (left). The lines represent the hadron resonance gas (HRG) calculation with the physical pion and also with the full unphysical heavier pion multiplet, as encountered on the lattice. (The effects of heavier pion multiplet at fixed lattice spacing for the HISQ/tree action are quantitatively addressed in Ref. [5].) There are two interesting observations: 1. Although the root-mean-squared pion mass

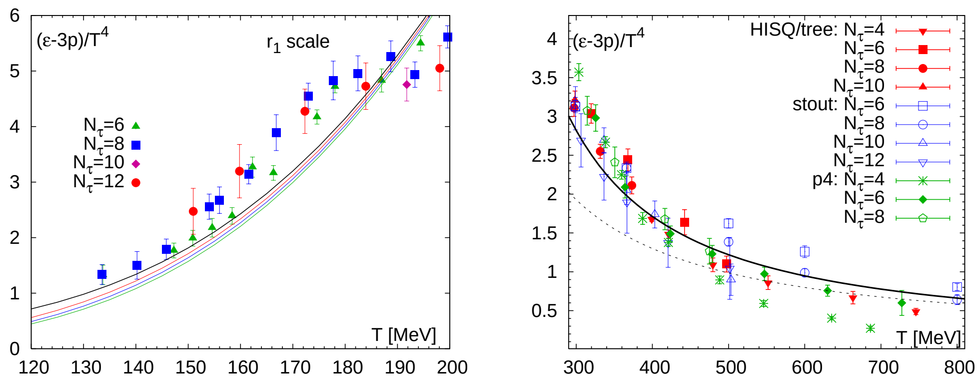

Figure 4: The interaction measure at low (left) and high (right) temperature. The curves on the left panel represent the HRG calculation, see text. On the right the dashed curve shows 1-loop and the solid curve 2-loop perturbative results.

varies from 400 to 200 MeV for $N_\tau = 6 - 12$ ensembles, the interaction measure in HRG is not very sensitive to this variation. Thus, better agreement of the HISQ/tree lattice data for various N_τ in the low-temperature region is due to both, improved properties of the HISQ/tree action and little sensitivity of this observable to the cutoff effects in the pion sector. 2. The lattice result starts to disagree with the HRG model at $T \sim 150 - 160$ MeV.

The interaction measure at high temperature is presented in Fig. 4 (right) for the HISQ/tree, p4 and stout action. The lines represent perturbative calculations. For T up to 800 MeV cutoff effects expected in the $T \to \infty$ limit are observed only for the p4 action.

1. Conclusion

We continued the calculation of the 2+1 flavor QCD equation of state with the HISQ/tree action on $N_\tau = 6$, 8, 10 and 12 ensembles. Using different observables to set the scale, r_1 and f_K, allows for a crude estimate of the magnitude of cutoff effects, which seem to be small at the finest $N_\tau = 12$ lattices. Better control over the statistics on $N_\tau = 10$ and 12 ensembles is required before an extrapolation to the continuum may be attempted.

Acknowledgments

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