# Bulk viscosity, chemical equilibration and flow at RHIC

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**Abstract**

We study the effects of bulk viscosity on $p_T$ spectra and elliptic flow in heavy ion collisions at RHIC. We argue that direct effect of the bulk viscosity on the evolution of the velocity field is small, but corrections to the freezeout distributions can be significant. These effects are dominated by chemical non-equilibration in the hadronic phase. We show that a non-zero bulk viscosity in the range $\zeta/s \lesssim 0.05$ improves the description of spectra and flow at RHIC.

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## 1. Introduction

The observation of nearly perfect hydrodynamic flow is one of the central discoveries of the heavy program at RHIC, and a significant amount of effort is being devoted to a precise determination of the shear viscosity to entropy density ratio $\eta/s$. In this contribution we will try to estimate the bulk viscosity $\zeta$ of the excited matter created at RHIC.

Bulk viscosity enters the equations of fluid dynamics as an additional contribution to the stress tensor, $\delta T^{\mu\nu} = -\Delta^{\mu\nu}\zeta\partial_k u^k$. Here, $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$ is a projector on the fluid rest frame and $u^\mu$ is the velocity of the fluid. Comparing with the stress tensor of an ideal fluid, $T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu}$ where $\epsilon$ is the energy density and $P$ is the pressure, we observe that bulk viscosity reduces the pressure of an expanding fluid relative to its equilibrium value. In a heavy ion collision this implies that bulk viscosity reduces the amount of radial flow.

Bulk viscosity also effects the spectra of produced particles. Particle spectra are computed by matching the stress tensor across the freeze-out surface. This yields the standard Cooper-Frye formula

$$E_p \frac{dN}{d^3p} = \frac{1}{(2\pi)^3}\int_\sigma f(E_p)p^\mu d\sigma_\mu \, , \tag{1}$$

where $dN/d^3p$ is the spectrum of produced particles, $E_p$ is the single particle energy, and $\sigma$ is the freeze-out surface. The distribution function $f(E_p) = f_0(E_p) + \delta f(E_p)$ contains an equilibrium part $f_0$ and a viscous correction $\delta f$. In the case of bulk viscosity $\delta f$ is proportional to the expansion rate $\partial_k u^k$, but the overall magnitude and dependence on energy is sensitive to the underlying non-equilibrium reactions.

## 2. Theories and models of the single particle spectra

The bulk viscosity only constrains a moment of $\delta f$, and determining the full functional form of the non-equilibrium distribution function requires a microscopic model or theory. The simplest model is based on the Boltzmann equation in the relaxation time approximation. In this approximation the complicated collision term in the Boltzmann equation is parameterized in terms of a

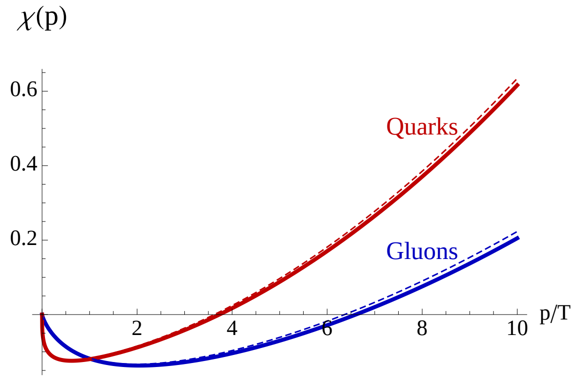


Figure 1: Non-equilibrium distribution $\chi$ of quarks and gluons in leading order perturbative QCD. The quantity $\chi$ is defined by $\delta f = -f_0(1 \pm f_0)\chi(p)(\partial \cdot u)$. The dashed curves show an approximate solution that does not exactly conserve energy.

single collision time $\tau(E_p)$. Energy and momentum conservation restrict the functional form of $\tau(E_p)$ [1]. The bulk viscosity of an ultra-relativistic gas in the relaxation time approximation is [2]

$$\zeta = 15\left(\frac{1}{3} - c_s^2\right)^2 \eta\,, \tag{2}$$

where $c_s$ is the speed of sound. The non-equilibrium distribution function is of the form

$$\delta f \sim f_p^0 \frac{\eta}{sT}\frac{p^2}{T^2}\left(\frac{1}{3} - c_s^2\right)(\partial \cdot u)\,. \tag{3}$$

We observe that the bulk viscosity scales as the second power of the conformal breaking parameter $(c_s^2 - 1/3)$, whereas $\delta f$ scales as the first power. This implies that for a nearly conformal fluid corrections to the freeze-out distribution are typically more important than corrections to the velocity fields. This conclusion does not depend on taking the relativistic limit. In general, the conformal breaking parameter is

$$\mathcal{F} = \int \frac{d^3p}{E_p(2\pi)^3}\left(\frac{p^2}{3} - c_s^2 E_p^2\right) f_0(p)\,(1 \pm f_0(p)) \tag{4}$$

where $\pm$ corresponds to bosons/fermions. The factor $\mathcal{F}$ vanishes in both the relativistic limit $E_p \sim p$, and in the non-relativistic limit $E_p \sim m + p^2/(2m)$. We find that $\zeta \sim \mathcal{F}^2$ and $\delta f \sim \mathcal{F}$.

The off-equilibrium distribution can be studied more rigorously in perturbative QCD [3]. In QCD the process of emitting an extra soft gluon is efficient, and bulk viscosity is determined by the time scale for equilibrating the momenta of produced gluons via elastic $2 \leftrightarrow 2$ scattering. The off-equilibrium distribution of gluons with momenta $p \gg T$ is

$$\delta f \sim f_p^0 \frac{p^2}{2\mu_A T}\left(\frac{1}{3} - c_s^2\right)(\partial \cdot u)\,, \tag{5}$$

where $\mu_A \sim g^2 m_D^2 \log(T/m_D)$ is the drag coefficient and $m_D$ is the Debye screening mass. In perturbative QCD $(c_s^2 - 1/3) = O(\alpha_s^2)$. In pure gauge theory $\zeta = 0.44\alpha_s^2 T^3/\log(\alpha_s^{-1})$ which

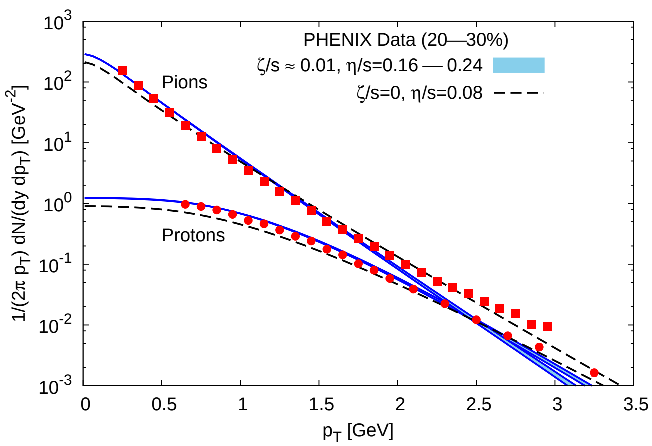


Figure 2: Transverse momentum spectrum of pions and protons at RHIC (data from [6]). The lines show hydrodynamic calculations with and without bulk viscosity.

implies $\zeta \simeq 48(c_s^2 - 1/3)^2\eta$. In full QCD we find interesting differences between the distribution functions of quarks and gluons, see Fig. 1. Both off-equilibrium distribution functions change sign, as is required by energy conservation, but the zero crossing occurs for different momenta.

The difference between quarks and gluon distribution functions may manifest itself in viscous corrections to penetrating probes, but more direct observables are related to the differences between hadronic distribution functions. A problem that can be studied rigorously is the bulk viscosity of a pion gas [4]. In this case bulk viscosity is determined by particle number changing processes, in particular the rate for the process $\pi+\pi \leftrightarrow 4\pi$. In an expanding gas of massive pions the equilibrium value of the total pion number decreases with time, but pion number changing processes are slow and a pion excess is created. The distribution function can be parametrized by an off-equilibrium chemical potential for the total number of pions

$$\delta f = f_0\left(\frac{\delta\mu}{T} + \frac{E_p\delta T}{T^2}\right) = -f_0\left(\chi_0 - \chi_1 E_p\right)(\partial\cdot u)\,, \tag{6}$$

where $\delta\mu$ is related to the bulk viscosity $\zeta$ and $\delta T$ is fixed by energy conservation. The bulk viscosity is controlled by the inelasticity $\Delta E = 2m_\pi$, $\zeta \sim (f_\pi^8/m_\pi^5)\exp(-2m_\pi/T)$. We have extended equ. (6) to a hadronic resonance gas, see [5] for earlier studies of chemical equilibration. We assume that the relative sizes of $\delta\mu$ for different species are determined by fast reactions like $\rho \leftrightarrow \pi\pi$ or $p+\bar{p} \leftrightarrow n\pi$ ($n \simeq 5$ in the regime of interest). This implies, for example, $\mu_\rho = 2\mu_\pi$ and $\mu_p = 2.5\mu_\pi$. The overall scale of $\delta\mu$ is related to the bulk viscosity $\zeta$ and can be extracted from experiment. As before $\delta T$ is determined by energy conservation.

## 3. Spectra and flow at RHIC

We have applied this model to $p_T$ spectra and flow at RHIC, see Figs. 2 and 3. The hydrodynamic model incorporates a single freezeout temperature $T_{fr} \simeq 140$ MeV and no hadronic afterburner, but the spectra include feed-down from hadronic resonances. The model for the

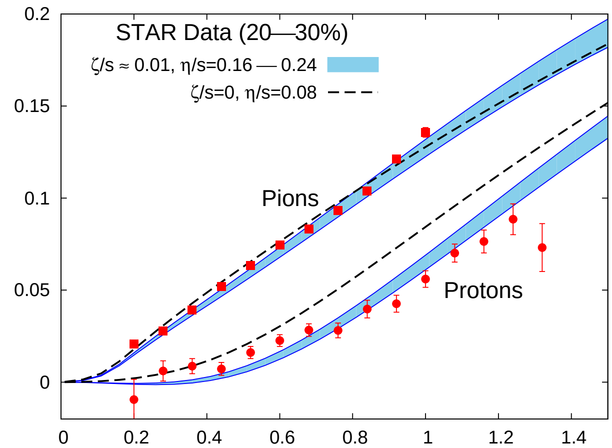


Figure 3: Differential elliptic flow of pions and protons at RHIC (data from [7]) compared to hydrodynamic calculations with and without bulk viscosity.

temperature dependence of the bulk viscosity is described in [1]. The value of $\zeta/s$ quoted in the Figure refers to the freezeout surface. We have solved second order hydrodynamic equations and we have verified that the gradient expansion is convergent. Viscous corrections to the spectra become large for $p_T \gtrsim 2$ GeV, and results in this regime cannot be trusted. We observe that a non-zero bulk viscosity improves the description of the spectra and flow. In particular, bulk viscosity raises the single particle spectra at low $p_T$, and increases the splitting between the pion and proton $v_2(p_T)$. These effects cannot be described in purely hydrodynamic models without bulk viscosity, but they have been explained in terms of hadronic non-equilibrium effects in kinetic afterburners. Our results show that these effects can be described efficiently in terms of bulk viscosity. We also note that many important hadronic reactions, such as $p\bar{p}$ annihilation into several pions, are difficult to include in kinetic models. Finally, we observe that the value of $\zeta$ extracted from the data is surprisingly small, $\zeta/s \simeq 0.01$. The corresponding pion chemical potential is of the order $\mu_\pi \simeq (10-20)$ MeV, depending on the local expansion rate.

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