Evolution of singularities in unequal time correlator in thermalization of quark gluon plasma

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Abstract

We studied thermalization of strongly coupled gauge theory using a gravitational collapse model. In particular, we studied unequal time correlator for the glue ball operator. We found singularities of the correlator for zero momentum sector is consistent with a geometric optics picture in the gravitational theory. The singularities of the correlator indicates strong temporal correlation, thus the time after which the singularities disappear provides a measure of the temporal decoherence in the thermalization process.

1. Introduction

Hydrodynamics has shown successful description of collective flow at low transverse momenta observed in heavy ion collisions at the relativistic heavy ion collider (RHIC). While hydrodynamic simulations assume a short thermalization time, $\tau \sim 0.5 fm$, which is believed to be attributed to strong coupling of QCD at early stage of collisions, the mechanism of thermalization remain less understood. A theoretical understanding of the thermalization mechanism requires knowledge of Quantum Chromodynamics (QCD) dynamics at strong coupling and far from equilibrium, which is inaccessible to lattice simulations and to perturbative field theory techniques. Gauge/gravity duality offers a useful tool to study the dynamics of strongly coupled gauge theory. While the dual microscopic theory $\mathcal{N} = 4$ Super Yang-Mills theory is different from QCD, understanding its thermalization at strong coupling is interesting from theoretical perspective and might give us insight to thermalization in heavy ion collisions.

In [1], we used a gravitational collapse model to describe thermalization of homogeneous and isotropic gauge field. We found one-point function of stress tensor thermalizes instantaneously, while two-point correlator shows deviation from thermal counterpart. It suggested two-point correlators take longer time to thermalize than one-point function. This is consistent with recent studies on equal-time correlator, Wilson loop, entanglement entropy etc [2, 3, 4]. In [1] we traced a sequence of quasi-static states, which corresponds to an adiabatic thermalization process, which allowed us to obtain the evolution of spectral functions as thermal equilibrium is approached. In experiments of heavy ion collisions, thermalization always occurs in finite amount of time, therefore going beyond quasi-static state (or adiabaticity) is a must for realistic description of thermalization process.

[1] In Collaboration with Johanna Erdmenger and Carlos Hoyos.

2. Gravitational Collapse Model

Let us give a brief review of the gravitational collapse model. It is composed of a homogeneous shell collapsing under its own gravity. The shell separates the spacetime into the parts above and below. By above and below, we refer to the region between AdS boundary and the shell and the region between the shell and the AdS interior, respectively. The corresponding metrics are given by AdS_5-Schwarzschild and by pure AdS,

$$\begin{aligned} &\text{above} : ds^2 = \frac{-f dt_f^2 + d\vec{x}^2 + dz^2/f}{z^2} \\ &\text{below} : ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}, \\ &\text{with} \quad f = 1 - \frac{z^4}{z_h^4}. \end{aligned} \tag{1}$$

The z_h is the position of horizon, which defines a temperature $z_h = \frac{1}{\pi T}$. As we will see soon, in Schwarzschild coordinates, the horizon is always "protected" by the shell, but in Kruskal coordinates, the shell will be able to cross the horizon. Note that we have used t_f for the time coordinate above the shell to distinguish it from the coordinate t below. However we choose the radial coordinate z to be continuous across the shell.

The falling trajectory of the shell is determined by the Israel junction conditions:

$$[K_{ij} - \gamma_{ij} K] = \kappa S_{ij}, \qquad \{K_{ij}\} S^{ij} = 0. \tag{2}$$

We use Greek letters for spacetime coordinates and Latin letters for coordinates on the hypersurface traced out by the shell. γ_{ij} is the induced metric and K_{ij} is the extrinsic curvature. The square (curly) bracket denotes the difference (sum) of the quantities above and below the shell. S_{ij} is the stress tensor of the shell, for which we use the ideal fluid type stress tensor: $S_{ij} = (\epsilon(z) + p(z)) u_i u_j + p(z)\gamma_{ij}$ and a conformal equation of state $\epsilon = 3p$. The resulting trajectory of the shell takes the following form:

$$\dot{z} = \sqrt{\frac{1}{4}\left(bz^4 + \frac{1}{bz_h^4}\right)^2 - 1}, \quad \dot{t}_f = \frac{\sqrt{f + \dot{z}^2}}{f} = \frac{\frac{1}{bz_h^4} - bz^4}{2f}, \tag{3}$$

where the dot refers to derivative with respect to proper time. Three parameters are needed to specify the trajectory of the shell: z_h, b and z_s. b characterizes the energy density of the shell and z_s is the initial radial coordinate of the shell, which is related to the intrinsic scale of the collisions. They are not independent, but are related by

$$bz_s^4 + \frac{1}{bz_h^4} = 2. \tag{4}$$

Note that due to the warping factor, the shell freezes near the horizon asymptotically. As we will see, this will have strong influence on the singularities of the unequal time correlator.

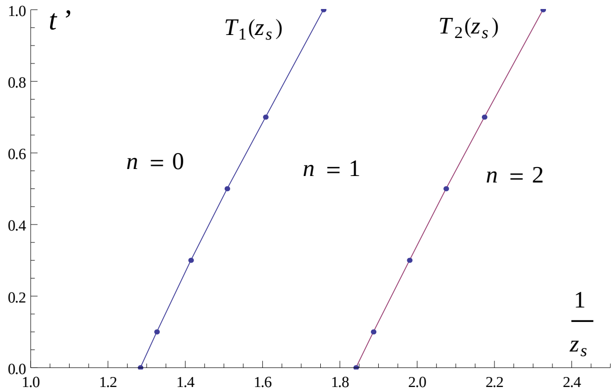

Figure 1: The regions of parameter space marked by the number bouncings of the light ray on the shell. A smaller t' and z_s always lead to more bouncings. We observe a good linear relation between $\frac{1}{z_s}$ and t' along the boundaries of regions T_n.

3. Geometric optics in the gravitational background

The field theory quantity we are interested in is the spatially integrated (or zero momentum) unequal time correlator for glue ball operator, defined as

$$G_R(t,t') = \int d^3x\theta(t-t')\langle[O(t,x),O(t',0)]\rangle. \tag{5}$$

The gauge/gravity duality relates this quantity to the massless scalar wave in the gravitational collapse model. Generically, (5) contains contributions from various frequency modes. The key observation here is the wave equation for massless scalar reduces to a classical picture of bouncing light ray, provided that we focus on contributions from high frequency modes. This can be justified in a WKB approximation. Furthermore, a divergence matching method based on this picture allows us to determine the precise form of the divergences [5].

To apply the divergence matching method to our model, we need to know the trajectory of bouncing light ray in the gravitational collapse background. We consider light ray leaving the boundary at time t'. It follows null geodesics in the bulk and bounces between the shell and the AdS boundary. Simple numerics shows that only finite number of bouncing is possible before both the light ray and the shell asymptote to the horizon, which is a consequence of red-shift by the warping factor. Fig.1 shows a chart of the number of bouncings in the parameter space of intrinsic scale z_s and the time parameter t'. For a given z_s, we may define $T_n(z_s)$ as the critical value of t', beyond which an n-th bouncing does not occur. The T_n are curves separating regions with different number of bouncings.

For a given t', we have obtained the time $\bar{t}_n$ that the light ray returns to the boundary after n-th bouncing on the shell. With this information, we are ready to apply the divergence matching method. The key ingredient of the method is to focus on contributions from high frequency modes, that is $|\omega| \gg T$, which leads to a clear separation between positive/negative frequency modes. Applying the divergence matching method, $G_R(t,t')$ has a singularities as $t \to \bar{t}_n$. The precise form of the singularities are given by [6]

$$G_n^> = \frac{A_n(-i)^{n-1}}{(-t+\bar{t}_n+i\epsilon)^{5-n}}, \qquad G_n^< = -\frac{A_n i^{n-1}}{(-t+\bar{t}_n+i\epsilon)^{5-n}}. \tag{6}$$

The $i\epsilon$ prescription clearly separate the contribution from positive/negative frequency modes. For $n = 0$, the singularities is simply the spatially integrated light-cone singularity. The singularities corresponding to $n > 1$ encodes the spectrum of the glue ball operator. As the shell asymptotes to the horizon, the resonances in the high frequency part of the glue ball spectrum becomes more and more dense, but the contribution from each resonance becomes smaller and smaller. This is consistent with our previous results on the spectral density [1].

Furthermore, the singularities in coordinate space is of particular interest. It measures the strongest temporal correlation between two time points. We have shown before that $\overline{t_n} \to +\infty$ as $t' \to T_n(z_s)$ from below, the retarded correlator $G^R(t', t)$ will be free of non-trivial singularities when $t' > T_1(z_s)$. Therefore we can interpret $T_1(z_s)$ as time scale for temporal decoherence, which should be distinguished the full thermalization time. Restoring the unit, we find the decoherence time is given by

$$t_{\rm de} = \frac{T_1(\pi T z_s)}{\pi T}, \tag{7}$$

4. Summary

We have studied thermalization of strongly couple gauge field using a gravitational collapse model. We focused on spatially integrated (or zero momentum) unequal time correlator for glue ball operator, defined as $G_R(t, t') = \int d^3x \theta(t-t')\langle [O(t, x), O(t', 0)]\rangle$. For varying t', we analyze the singularities of the correlator $G_R(t, t')$ and find that singularities are consistent with a geometric optics picture, i.e. singularities occur at $t = \overline{t_n}$, where $\overline{t_n}$ ($\overline{t_1} < \overline{t_2} < \cdots$) is the time for a light ray originally leaving the boundary at t' to return to the boundary after the n-th bouncing off the shell. Furthermore, the n-th singularity of $\overline{t_n}$ moves monotonously to $+\infty$ as t' approaches a critical value T_n ($T_1 > T_2 > \cdots$) from below. For $t' > T_1$, the last non-trivial singularity $\overline{t_1}$ escapes from detection, thus we use T_1 as the time scale for temporal decoherence, after which strong temporal correlation is lost.

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