Introduction to the Color Glass Condensate

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Aim

• Understand, at the conceptual level, the importance of Color Glass Condensate (CGC) in the high-energy hadron scattering

• Learn how to apply it to heavy-ion collisions: “Glasma”

• Know how it works in comparison with the most recent experimental data from RHIC and LHC
Cross sections **GROW** with increasing energies and amount to **> 100 mb** at cosmic ray or LHC energies.
How to “read” these data?

- Proton’s “geometric” cross section $\pi r_c^2 \sim 30$ mb (charge radius $r_c \sim 1$ fm)
  - maximum absorption “shadowing” $\sim 2\pi r_c^2 \sim 60$ mb $< 100$ mb
  - Proton is “expanding” !?

- Particle Data Group (COMPETE Collab. Phys. Rev.D65 (2002))

\[
\sigma_{total}^{ab}(s) = Z^{ab} + B \ln^2 s + ...
\]

- $Z^{ab}$: constant
- $B$: independent of hadron species $a, b$
- $\ln^2 s$ : consistent with the Froissart bound (unitarity bound)
This example implies ...

- At high energies, *something “unusual and interesting”* must be happening in hadrons.
  
  - “expansion” of a target
  
  - unitarity
  
  - universal picture!

→ consistent with the Color Glass Condensate.

(but only qualitatively at present for the total cross section)
We want to understand ...

- universal picture of hadrons/nuclei in the high-energy limit (if any)
- if we can describe it in QCD, in particular, in weak-coupling technique
- at which energy scale it starts to appear
- to what extent we can understand the experimental results at current energies with this picture
What is the CGC?

- **Dense gluonic states** in hadrons which **universally** appear in the **high-energy limit** of scattering

  **Color** ... gluons have “colors”

  **Glass** ... gluons with small longitudinal mom. fractions ($x << 1$) are created by long-lived partons that are distributed randomly on the transverse disk

  **Condensate** ... gluon density is very high, and saturated

- **Most advanced** (and still developing) **theoretical picture of high energy scattering in QCD**
  
  Based on QCD (weak coupling due to $Q_s >> \Lambda_{QCD}$, but non-perturbative)

  Unitarity effects (multiple scattering, nonlinear effects)

  LO description completed around 2000
Proton composition changes with energy

Deep inelastic scattering (DIS: ep \( \rightarrow \) eX) can probe quarks and gluons in a proton

\[ Q^2 : \text{transverse resolution} \]
\[ x : \text{longitudinal mom. fraction of partons} \]

Gluons are the dominant component at high energy (small x)

Gluons (must be multiplied by 20)
Life and death of fluctuation

fluctuation of a fast moving parton

\[ \begin{align*} p & \rightarrow p - k \\ k & \end{align*} \]

\[ p^\mu = (p, 0_\perp, p) \]

\[ k^\mu = (E_k, k^i, k^z = xp) \]

Lifetime of fluctuation

\[ (xp \gg k_t) \]

\[ \Delta t \sim \frac{1}{\Delta E} = \frac{1}{E_k + E_{p-k} - p} \sim \frac{2x(1-x)p}{k_\perp^2} \]

- If parent parton has large energy \( xp \gg k_t \),
  \( \rightarrow \) fluctuation becomes long lived
- With increasing \( p \), long-lived fluct. w/ smaller \( x \) becomes possible
- If daughter parton is long-lived, it can further fluctuate:
  \( \rightarrow \) multiple parton (gluon) production
- One gluon emission is enhanced: \( \alpha_s \ln 1/x \gg 1 \) at small \( x \ll 1 \)
  Need to sum up many-gluon emissions \( (\alpha_s \ln 1/x)^n \)
- When the density of gluons becomes high, they start to interact with each other \( \rightarrow \) CGC
- Fluctuations become real particles in reactions
Phase diagram of a proton as seen in DIS

Q_s^{-1} is typical transverse size. Q_s^2(x) \sim 1/x^\lambda increases (x \to 0)
\alpha_s(Q_s^2) \ll 1 weak coupling

Recombination of gluons

N_g \leq 1 unitarity

1/x in log scale

Multiple gluon emissions

N_g \sim e^{\omega \ln 1/x}

Parton number increases, but density decreases

Q^2

Transverse resolution

Color glass condensate

Gluon density

Parton gas

Higher energies

No region

Saturation scale Q_s(x)

DGLAP

BK

BFKL
Saturation scale : $Q_s(x)$

• Gluon distribution function: $xG(x, Q^2)$
  number of gluons having longitudinal fraction in the interval “$x - x + dx$”
  looked at transverse resolution scale $1/Q$.

• Typical pQCD cross section : $\sigma \sim \alpha_s/Q^2$

Gluons fill the transverse area of hadron ($\pi R^2$) when
$$\frac{\alpha_s}{Q^2} \cdot xG(x, Q^2) = \pi R^2$$

$Q$ satisfying this is called “saturation momentum” $Q_s$

Intuitive picture:

1/$Q_s$ is the “transverse size” of gluons when they fill the transverse area of a hadron.

Typical transverse momentum carried by gluons in a hadron.
Saturation scale: $Q_s(x)$

$$\frac{\alpha_s}{Q_s^2} \cdot x G(x, Q_s^2) = \pi R^2$$

- **Small-$x$ limit of DGLAP equation** (Double Log App.)

  $$x G(x, Q^2) \sim e^{\sqrt{4\bar{\alpha}_s y \rho}}$$
  $$y = \ln \frac{1}{x}, \quad \rho = \ln \frac{Q^2}{Q_0^2}, \quad \bar{\alpha}_s = \frac{N_c \alpha_s}{\pi}$$

  $$Q_s^2 \propto e^{4\bar{\alpha}_s y} = \left(\frac{1}{x}\right)^{4\bar{\alpha}_s}$$

- **BFKL equation** (resummation: LO $(\alpha_s \ln 1/x)^n$, NLO: $\alpha_s (\alpha_s \ln 1/x)^n$)

  Gluon number (LO) $\sim e^{\omega y}$

  $$\omega = 4 \ln 2 = 2.77 \bar{\alpha}_s$$

  $$Q_s^2 \propto \left(\frac{1}{x}\right)^{\lambda}$$

  LO $\lambda = 4.88 \bar{\alpha}_s$ \[\text{Iancu, Itakura, McLerran’02}\]

  NLO $\lambda \sim 0.3$ \[\text{Triantafyllopoulos, ’03}\]

$Q_s$ grows with increasing energy (decreasing $x$) → weak-coupling at high energies
Going up higher energies: evolution eqs.

**Evolution wrt** $x$ (or rapidity $y = \ln \frac{1}{x}$)

- **BFKL** (LO: $(\alpha_s \ln \frac{1}{x})^n$, NLO: $\alpha_s (\alpha_s \ln \frac{1}{x})^n$)
  \[
  \frac{\partial \phi(x, k_t)}{\partial \ln(x_0/x)} \approx \mathcal{K} \otimes \phi(x, k_t)
  \]
  $K$: gluon splitting $g \rightarrow gg$
  $\phi$: unintegrated gluon distr.

- **BK** (includes the nonlinear effects)
  \[
  \frac{\partial \phi(x, k_t)}{\partial \ln(x_0/x)} \approx \mathcal{K} \otimes \phi(x, k_t) - \phi(x, k_t)^2
  \]

Known up to full NLO accuracy. [Balitsky, Chirilli 2008]
But for practical purposes, we use **BK with running coupling** $\rightarrow$ “rcBK”

\[
K^{\text{run}}(r, r_1, r_2) = \frac{N_c \alpha_s(r^2)}{2\pi^2} \left[ \frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2} \left( \frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{1}{r_2^2} \left( \frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right]
\]
Evidence: Geometric Scaling

**DIS (ep, eA) cross sections scale with $Q^2/Q_s^2$**

- Stasto, Golec-Biernat, Kwiecinski
  - PRL 86 (2001) 596
- Freund, Rummukainen, Weigert, Schafer
  - PRL 90 (2003) 222002
- Marquet, Schoeffel

**Ep**

- $\gamma*p$ total

**eA**

- $F_2^A/A$ scaled

**Diffraactive ep**

- $\beta = 0.2$ for $Q^2$ in (5-90) GeV
- $\beta = 0.4$ for $Q^2$ in (5-90) GeV

- $Q_s^2(x,p)$

- $k_t < Q_s^2/\Lambda_{QCD}$ (lancu, itakura, McLerran)

- **Existence of saturation scale $Q_s$**
- Can determine $x$ and $A$ dependences of $Q_s$
- Extends outside of the saturation regime $k_t < Q_s^2/\Lambda_{QCD}$ (lancu, itakura, McLerran)

*Figure 1.* Experimental data on $\sigma_{\gamma*p}$ from the region $x < 0.01$ plotted versus the scaling variable $\tau = Q^2 R_0^2(x)$.

*Figure 2.* The diffractive cross-section $\beta d\sigma_{\gamma*p \to Xp}/d\beta$ from H1 and ZEUS measurements, as a function of $t_\perp$ in bins of $\beta$ for $Q^2$ values in the range [5, 90] GeV$^2$ and for $x_g < 0.01$. Only statistical uncertainties are shown.

*Figure 3.* Scaling behavior of NMC and E665 $F_2^A$ data vs $\tau = (Q^2/R_0^2(x))$. The vertical axis corresponds to the left-hand side of Eq. (5). The dashed line corresponds to the geometric scaling curve obtained from HERA data. These are shown offset by a factor of 5.
CGC turns into Glasma after heavy-ion collision

**Glasma**: non-equilibrium matter between Color Glass Condensate (CGC) and Quark Gluon Plasma (QGP). Created in heavy-ion collisions.

Solve Yang Mills eq. \([D_\mu, F^{\mu\nu}] = 0\) in expanding geometry with the CGC initial condition.

\[ \alpha_1^i = \text{pure gauge} \]
\[ \alpha_2^i = \text{pure gauge} \]
\[ \rho_1, \rho_2 \]
\[ \alpha_{1,2}^i = 0 \]

Randomly distributed
CGC as the initial condition for H.I.C.

HIC = Collision of two sheets of CGCs

Each source creates the gluon field for each nucleus. $\leftarrow$ Initial condition

$$J^\mu = \delta^{\mu+} \delta(x^-) \rho_1(x_T) + \delta^{\mu-} \delta(x^+) \rho_2(x_T)$$

$$- D_i \alpha^i_{(m)} = \rho_{(m)}(x_{\perp}) \quad \alpha_1, \alpha_2: \text{gluon fields of nuclei}$$

In Region (3), and at $\tau=0^+$, the gauge field is determined by $\alpha_1$ and $\alpha_2$

$$A^\pm = \pm x^\pm \alpha(\tau, x_T)$$

$$A^i = \alpha^i_3(\tau, x_T).$$

$$\alpha^i_3 \big|_{\tau=0} = \alpha^i_1 + \alpha^i_2$$

$$\alpha \big|_{\tau=0} = \frac{ig}{2} \left[ \alpha^i_1, \alpha^i_2 \right]$$

$$\partial_\tau \alpha \big|_{\tau=0} = \partial_\tau \alpha^i_3 \big|_{\tau=0} = 0.$$
Glasma flux tube structure

Just after the collision: only $E^z$ and $B^z$ are nonzero
(Initial CGC is transversely random)
→ Glasma = electric and magnetic flux tubes extending in the longitudinal direction

Unstable dynamics

Color-electric flux tube
→ gluon pair, qqbar pair production via Schwinger mechanism

Color-magnetic flux tube
→ Unstable against rapidity dependent fluctuation via Nielsen-Olesen instability [Fujii, Itakura 2008]

When both are present
→ Schwinger production of gluons enhanced by the N-O instability [TANJI, ITAKURA 2012]

Typical configuration of a single event just after the collision
Unsolved issues on glasma

• How does the glasma thermalize into QGP?
  unstable dynamics? → turbulent distribution leading to isotropization
  Bose-Einstein Condensation? → see talks by J.P.Blaizot and F.Gelis
  Other mechanisms, such as induced cascade by high pt partons?

• What is the observable consequence?
  Ridge as remnants of longitudinal flux tube structure?

• What kind of fluctuations are there?
  Color fluctuation inherent to CGC generates higher harmonics of flow?
Examples of new progress (1)

- Schenke, Tribedy and Venugopalan, 2012 computed eccentricity and triangularity with IP-Glasma model

- IP-Glasma
- MC-KLN
- MC-Glauber

*Fluctuations from nucleon position and color dynamics*

*Only fluctuations from nucleon positions included*

*Also computed many other observables*
Examples of new progress (2)

- Global analysis of DIS data with rcBK solution (AAMQS) and its application to forward particle production in dAu at RHIC
To reduce ambiguity
- construct a nucleus by randomly placing nucleons
- use AAMQS parameters for proton IC optimized for DIS at small-x
- quantum evolution is performed “locally” in b space

(to avoid IR div. in b-dep BK)

Large x partons
\( f_i(x, k_t) : \text{CTEQ6M NLO} \)

local rcBK evolution
MC-DHJ/rcBK : results

modified MV model ($\gamma = 1.118$)

“running coupling” version of MV model [Iancu-Itakura-Triantafylopoulos] : to be consistent with rcBK evolution

- Set $h$ works well even in $pp$, but not as good as Albacete-Marquet
- rcMV is not “tuned” (similar param as MV)
- However, both work quite well in dAu (IC dependence reduces at high rapidity)

Best results from theoretical point of view, but still needs better (global) description including $pp$ data (tuning of rcMV is necessary)

reproduce the data nicely
- AAMQS set $h$ and rcMV for $N(r,y)$
- $Q_{s0A}^2$ fixed by MC; no additional parameter
New ALICE data on pPb @ 5.02 TeV

arXiv:1210.3615
Summary

• CGC is the universal picture of hadrons at high energies, which appears as a result of gluon 3-point vertex. Its theoretical framework is established at the LO level, but is developing beyond the LO.

• CGC provides the initial conditions for the heavy ion collisions, and turns into Glasma. The Glasma is responsible for thermalization, but is not solved yet.

• CGC picture is getting precise and is now seriously compared with experimental data at RHIC (forward rapidity) and LHC. MC-DHJ/rcBK model works well in describing the forward dAu data.
backup
High-energy scattering

High-energy limit = “Regge limit”

total scatt. energy >> typical energy/momentum scale in reaction

**Hadron-hadron scattering**

Momentum transfer squared

\[ t = (p_a - p_c)^2 \]

Total scattering energy squared

\[ s = (p_a + p_b)^2 \]

**Deep inelastic ep scattering**

Total γ*p scattering energy squared

\[ W^2 = (p + q)^2 \]

Virtuality of photon

\[ Q^2 = -q^2 \]

\[ W^2 \gg Q^2 \]

\[ x \sim Q^2 / (W^2 + Q^2) \rightarrow 0 \]