

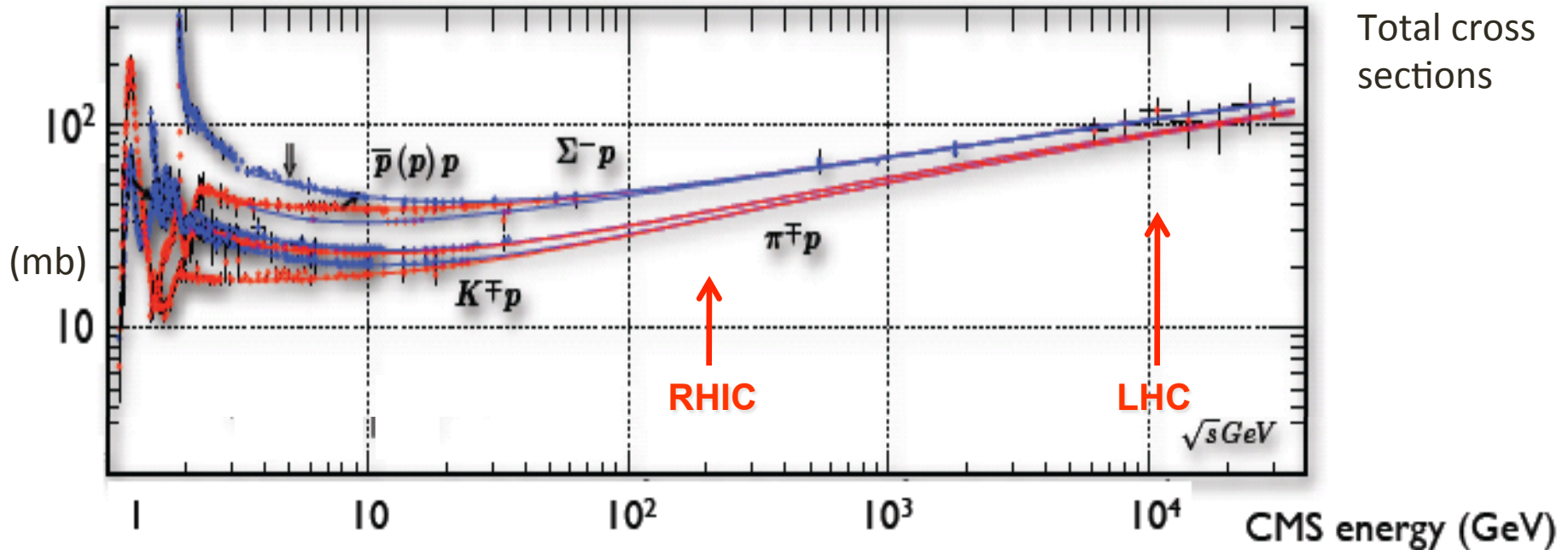
Introduction to the Color Glass Condensate

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Aim

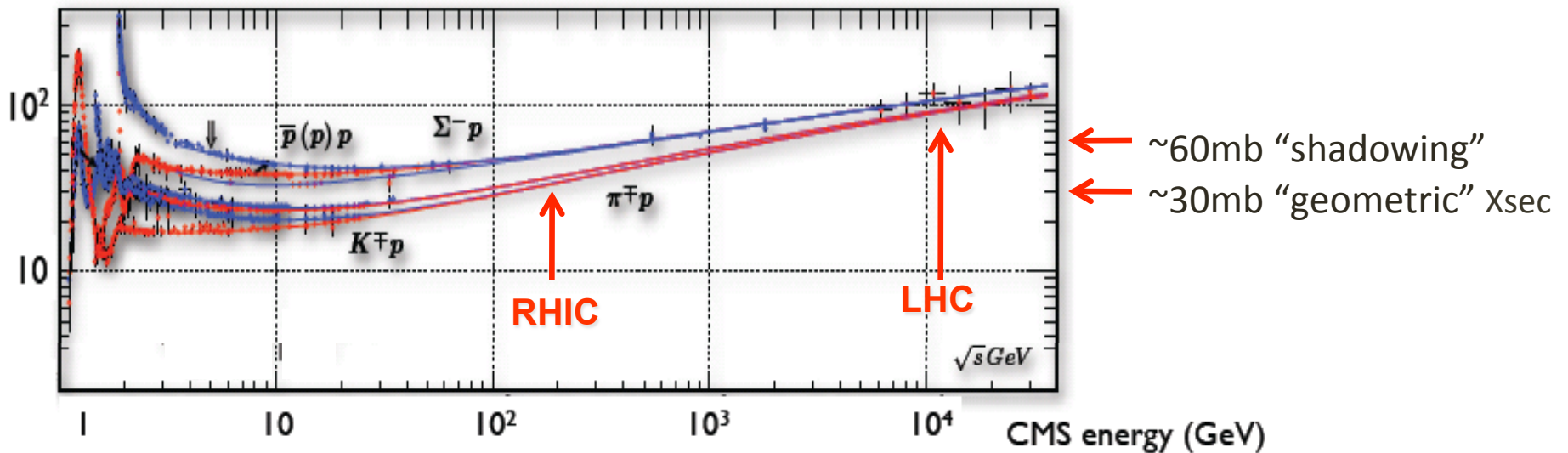
- Understand, at the conceptual level, the importance of Color Glass Condensate (CGC) in the high-energy hadron scattering
- Learn how to apply it to heavy-ion collisions: “Glasma”
- Know how it works in comparison with the most recent experimental data from RHIC and LHC

Hadron cross sections



Cross sections **GROW** with increasing energies and amount to **> 100 mb** at cosmic ray or LHC energies

How to “read” these data?



- Proton’s “geometric” cross section $\pi r_c^2 \sim 30 \text{ mb}$ (charge radius $r_c \sim 1 \text{ fm}$)
 maximum absorption “shadowing” $\sim 2\pi r_c^2 \sim 60 \text{ mb} < 100 \text{ mb}$

→ Proton is “expanding” !?

- Particle Data Group (COMPETE Collab. Phys. Rev.D65 (2002))

$$\sigma_{total}^{ab}(s) = Z^{ab} + B \ln^2 s + \dots$$

Z^{ab} : constant

B : independent of hadron species a, b

$\ln^2 s$: consistent with the Froissart bound (unitarity bound)

This example implies ...

- At high energies, *something “unusual and interesting”* must be happening in hadrons.
 - “expansion” of a target
 - unitarity
 - universal picture!
- consistent with the Color Glass Condensate.
- (but only qualitatively at present for the total cross section)

We want to understand ...

- **universal picture** of hadrons/nuclei in the high-energy limit (if any)
- if we can describe it in **QCD**, in particular, in **weak-coupling technique**
- at which energy scale it starts to appear
- to what extent we can understand the experimental results at current energies with this picture

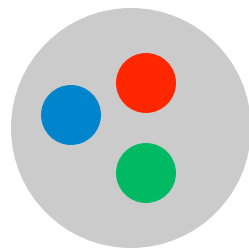
What is the CGC?

- ***Dense gluonic states*** in hadrons which *universally appear in the high-energy limit of scattering*

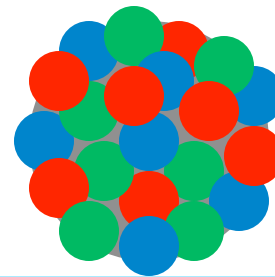
Color ... gluons have “colors”

Glass ... gluons with small longitudinal mom. fractions ($x \ll 1$) are created by long-lived partons that are distributed randomly on the transverse disk

Condensate ... gluon density is very high, and saturated



High energy



Color Glass
Condensate (CGC)

- **Most advanced** (and still developing) **theoretical picture of high energy scattering in QCD**

Based on QCD (weak coupling due to $Q_s \gg \Lambda_{\text{QCD}}$, but non-perturbative)

Unitarity effects (multiple scattering, nonlinear effects)

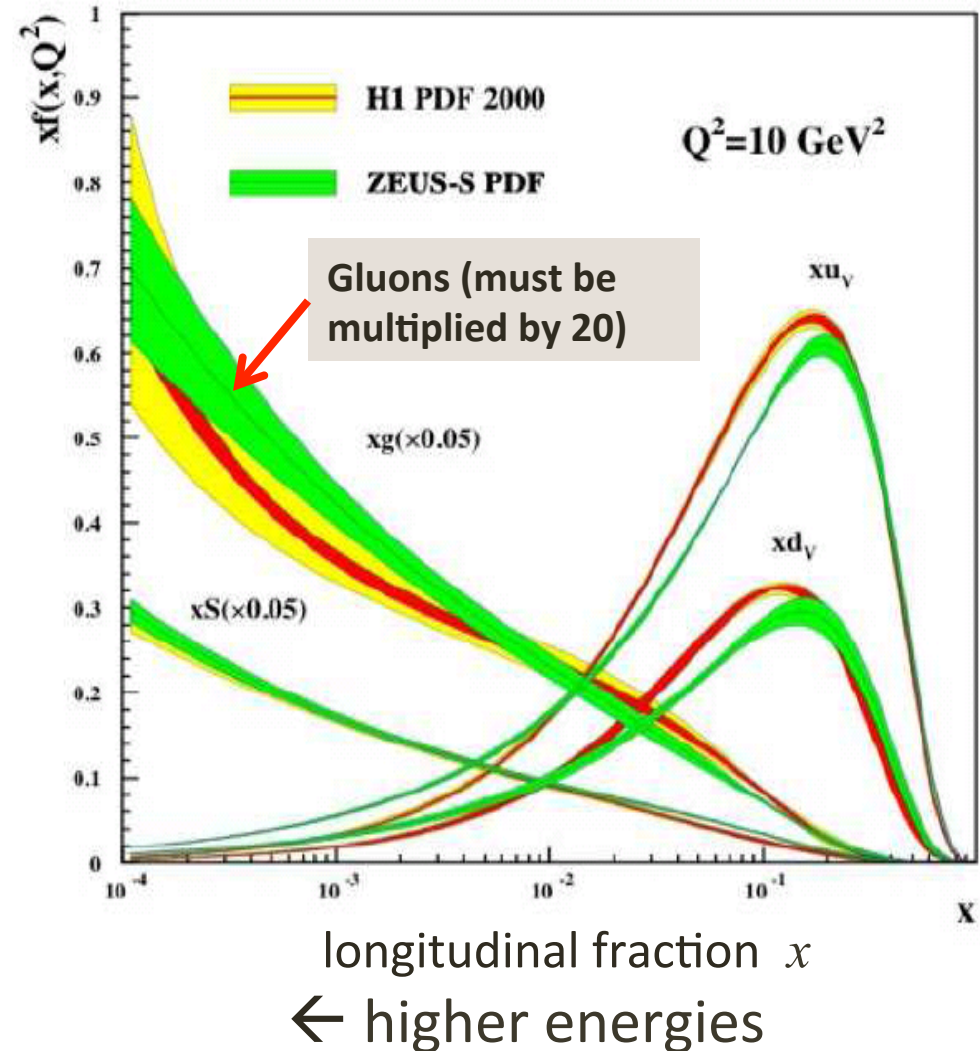
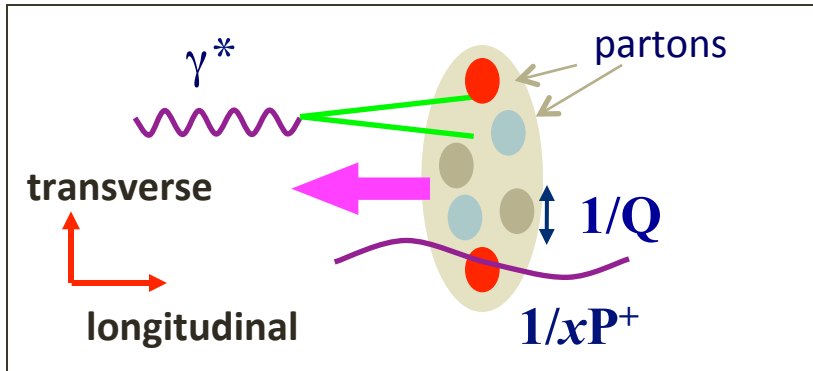
LO description completed around 2000

Proton composition changes with energy

Deep inelastic scattering (DIS: $ep \rightarrow eX$)

can probe quarks and gluons in a proton

HERA @ DESY



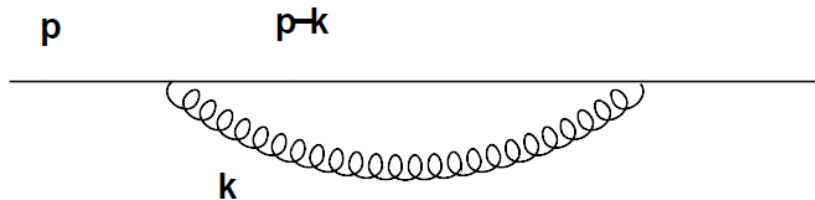
Two kinematical variables

- Q^2 : transverse resolution
- x : longitudinal mom. fraction of partons

Gluons are the dominant component at high energy (small x)

Life and death of fluctuation

fluctuation of a fast moving parton



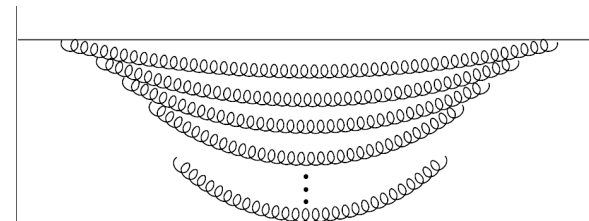
$$p^\mu = (p, 0_\perp, p)$$

$$k^\mu = (E_k, k_\perp^i, k^z = xp)$$

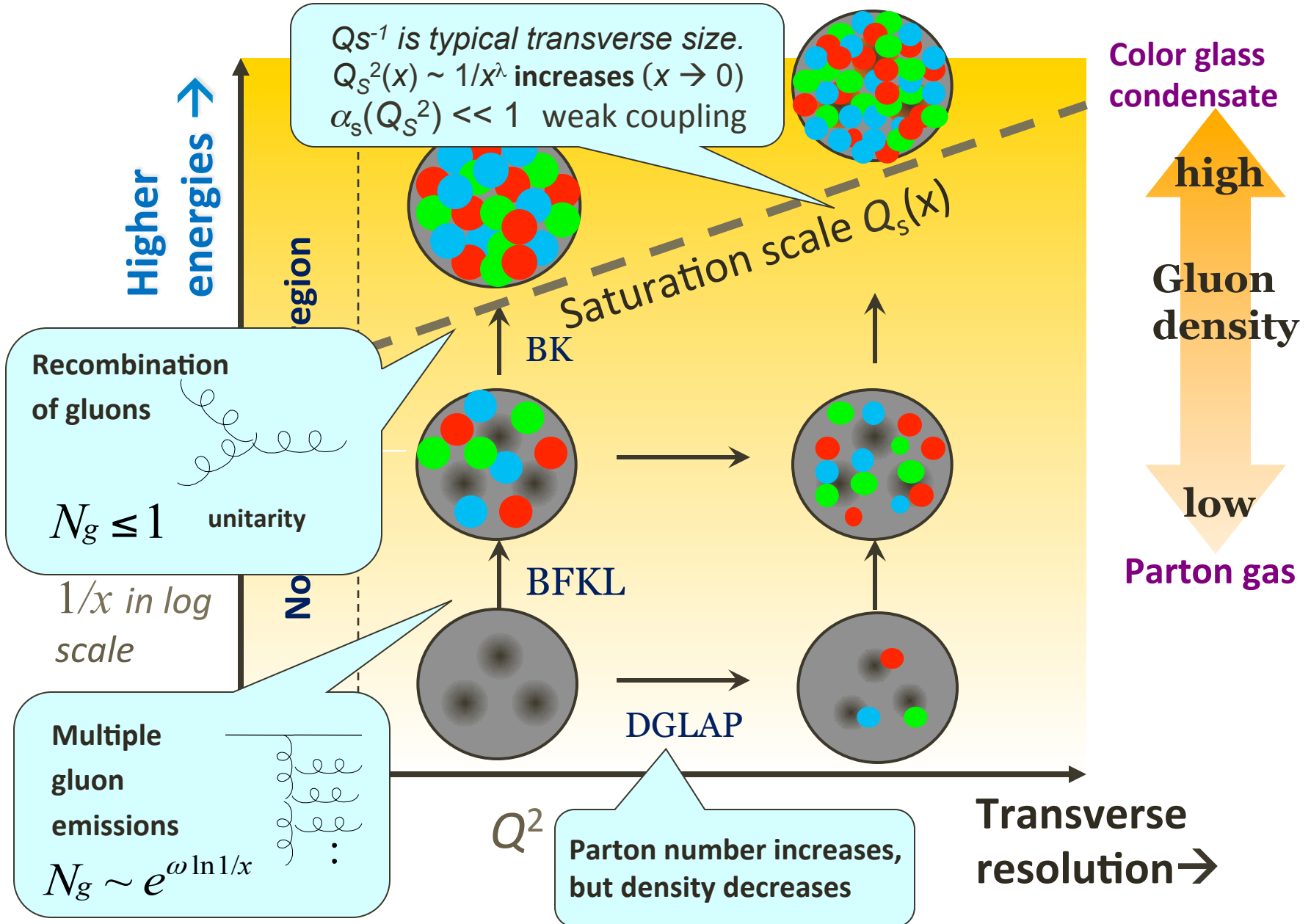
Lifetime of fluctuation
($xp \gg k_t$)

$$\Delta t \sim \frac{1}{\Delta E} = \frac{1}{E_k + E_{p-k} - p} \sim \frac{2x(1-x)p}{k_\perp^2}$$

- If parent parton has large energy $xp \gg k_t$,
 → fluctuation becomes **long lived**
- With increasing p , long-lived fluct. w/ smaller x becomes possible
- If daughter parton is long-lived, it can further fluctuate:
 → **multiple parton (gluon) production**
- One gluon emission is enhanced: **$\alpha_s \ln 1/x \gg 1$ at small $x \ll 1$**
 Need to sum up many-gluon emissions $(\alpha_s \ln 1/x)^n$
- When the density of gluons becomes high, they start to interact with each other → **CGC**
- Fluctuations become real particles in reactions



Phase diagram of a proton as seen in DIS

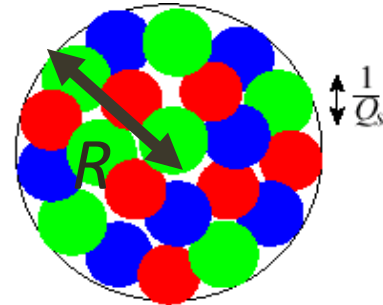


Saturation scale : $Q_s(x)$

- Gluon distribution function: $xG(x, Q^2)$
number of gluons having longitudinal fraction in the interval “ $x - x+dx$ ”
looked at transverse resolution scale $1/Q$.
- Typical pQCD cross section : $\sigma \sim \alpha_s/Q^2$

Gluons fill the transverse area of hadron (πR^2)

when
$$\frac{\alpha_s}{Q^2} \cdot xG(x, Q^2) = \pi R^2$$



Q satisfying this is called “saturation momentum” Q_s

Intuitive picture :

$1/Q_s$ is the “**transverse size**” of gluons when they fill the transverse area of a hadron.

Typical transverse momentum carried by gluons in a hadron

Saturation scale : $Q_s(x)$

$$\frac{\alpha_s}{Q_s^2} \cdot xG(x, Q_s^2) = \pi R^2$$

- **Small- x limit of DGLAP equation** (Double Log App.)

$$xG(x, Q^2) \sim e^{\sqrt{4\bar{\alpha}_s y \rho}} \quad y = \ln \frac{1}{x}, \quad \rho = \ln \frac{Q^2}{Q_0^2}, \quad \bar{\alpha}_s = \frac{N_c \alpha_s}{\pi}$$

$$Q_s^2 \propto e^{4\bar{\alpha}_s y} = (1/x)^{4\bar{\alpha}_s}$$

- **BFKL equation** (resummation : LO $(\alpha_s \ln 1/x)^n$, NLO: $\alpha_s (\alpha_s \ln 1/x)^n$)

$$\text{gluon number (LO)} \sim e^{\omega y} \quad \omega = \bar{\alpha}_s 4 \ln 2 = 2.77 \bar{\alpha}_s$$

$$Q_s^2 \propto (1/x)^\lambda \quad \text{LO } \lambda = 4.88 \bar{\alpha}_s \quad [\text{lancu, Itakura, McLerran '02}]$$

$$\text{NLO } \lambda \sim 0.3 \quad x = 10^{-2} - 10^{-4} \\ [\text{Triantafyllopoulos, '03}]$$

Q_s grows with increasing energy (decreasing x)

→ weak-coupling at high energies

Going up higher energies: evolution eqs.

Evolution wrt x (or rapidity $y = \ln 1/x$)

- **BFKL** (LO : $(\alpha_s \ln 1/x)^n$, NLO: $\alpha_s (\alpha_s \ln 1/x)^n$)

$$\frac{\partial \phi(\mathbf{x}, \mathbf{k}_t)}{\partial \ln(\mathbf{x}_0/\mathbf{x})} \approx \mathcal{K} \otimes \phi(\mathbf{x}, \mathbf{k}_t)$$

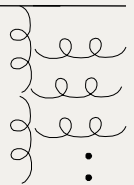
\mathcal{K} : gluon splitting $g \rightarrow gg$
 ϕ : unintegrated gluon
 distr.

- **BK** (includes the nonlinear effects)

$$\frac{\partial \phi(\mathbf{x}, \mathbf{k}_t)}{\partial \ln(\mathbf{x}_0/\mathbf{x})} \approx \mathcal{K} \otimes \phi(\mathbf{x}, \mathbf{k}_t) - \phi(\mathbf{x}, \mathbf{k}_t)^2$$

Multiple gluon
emissions

$$N_g \sim e^{\omega \ln 1/x}$$



Recombination of gluons

$$N_g \leq 1$$

Unitarity



Known up to full NLO accuracy. [Balitsky, Chirilli 2008]

[Balitsky, Gardi et al.,
Kovchegov-Weigert]

But for practical purposes, we use **BK with running coupling** \rightarrow "rcBK"

$$K^{\text{run}}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) = \underbrace{\frac{N_c \alpha_s(r^2)}{2\pi^2} \left[\frac{r^2}{r_1^2 r_2^2} \right]}_{\text{LO}} + \frac{1}{r_1^2} \left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{1}{r_2^2} \left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right)$$

Evidence : Geometric Scaling

DIS (ep, eA) cross sections scale with Q^2/Q_s^2

Stasto, Golec-Biernat, Kwiecinski
PRL 86 (2001) 596

Freund, Rummukainen, Weigert, Schafer
PRL 90 (2003) 222002

Marquet, Schoeffel
Phys. Lett. B639 (2006) 471

ep

eA

Diffractive ep

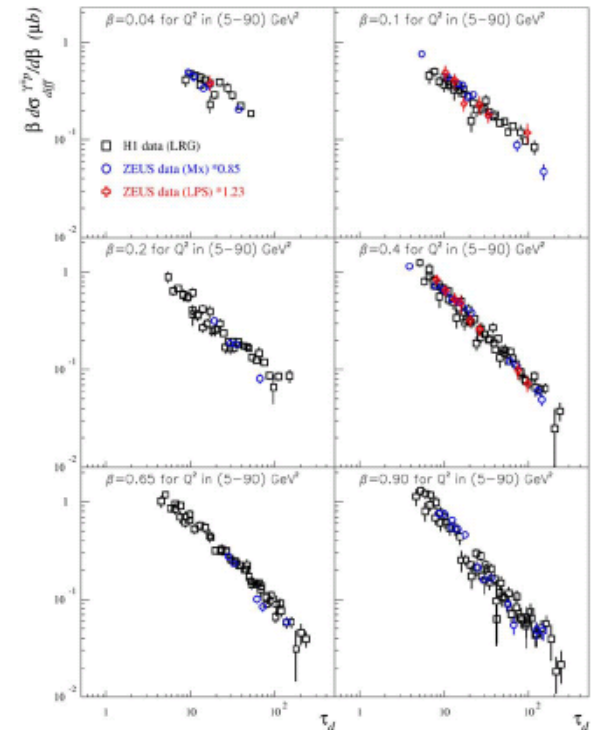
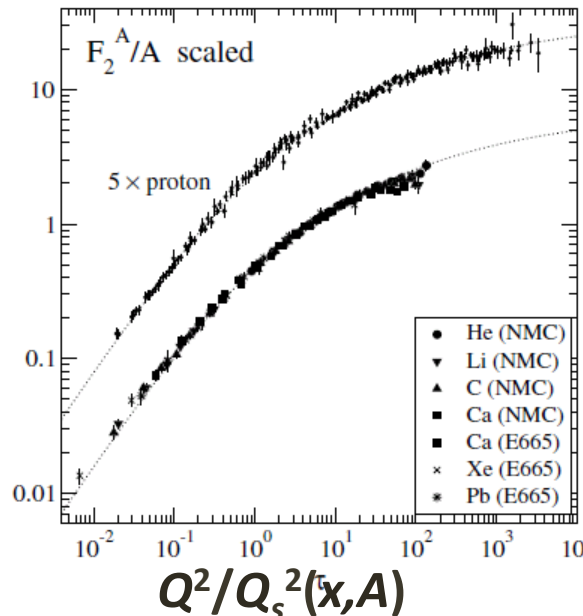
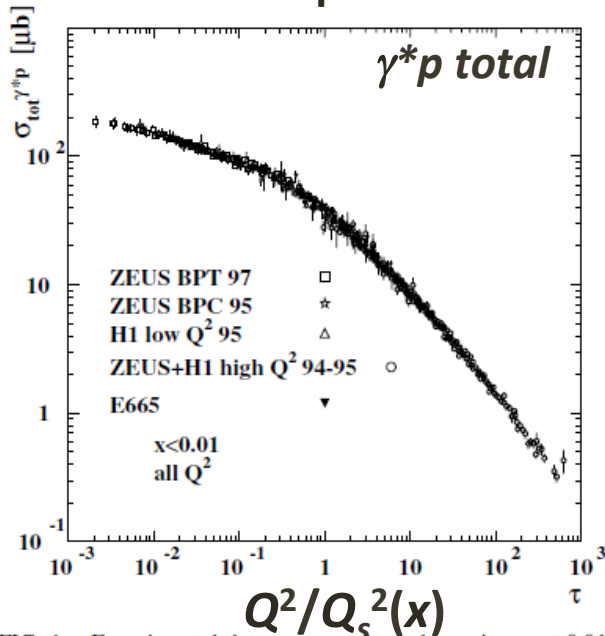


FIG. 1. Experimental data on σ_{γ^*p} from the region $x < 0.01$ plotted versus the scaling variable $\tau = Q^2 R_0^2(x)$.

FIG. 3. Scaling behavior of NMC and E665 F_2^A data vs $\tau = \left(\frac{x_0}{x_0}\right)^{2A} \frac{Q^2}{A^{1/\beta}}$. The vertical axis corresponds to the left-hand side of Eq. (5). The dashed line corresponds to the geometric scaling curve obtained from HERA data. These are shown offset by a factor of 5.

Fig. 2. The diffractive cross-section $\beta d\sigma_{\text{diff}}^{\gamma^*p \rightarrow Xp} / d\beta$ from HI and ZEUS measurements, as a function of τ_d in bins of β for Q^2 values in the range [5; 90] GeV² and for $x_p < 0.01$. Only statistical uncertainties are shown.

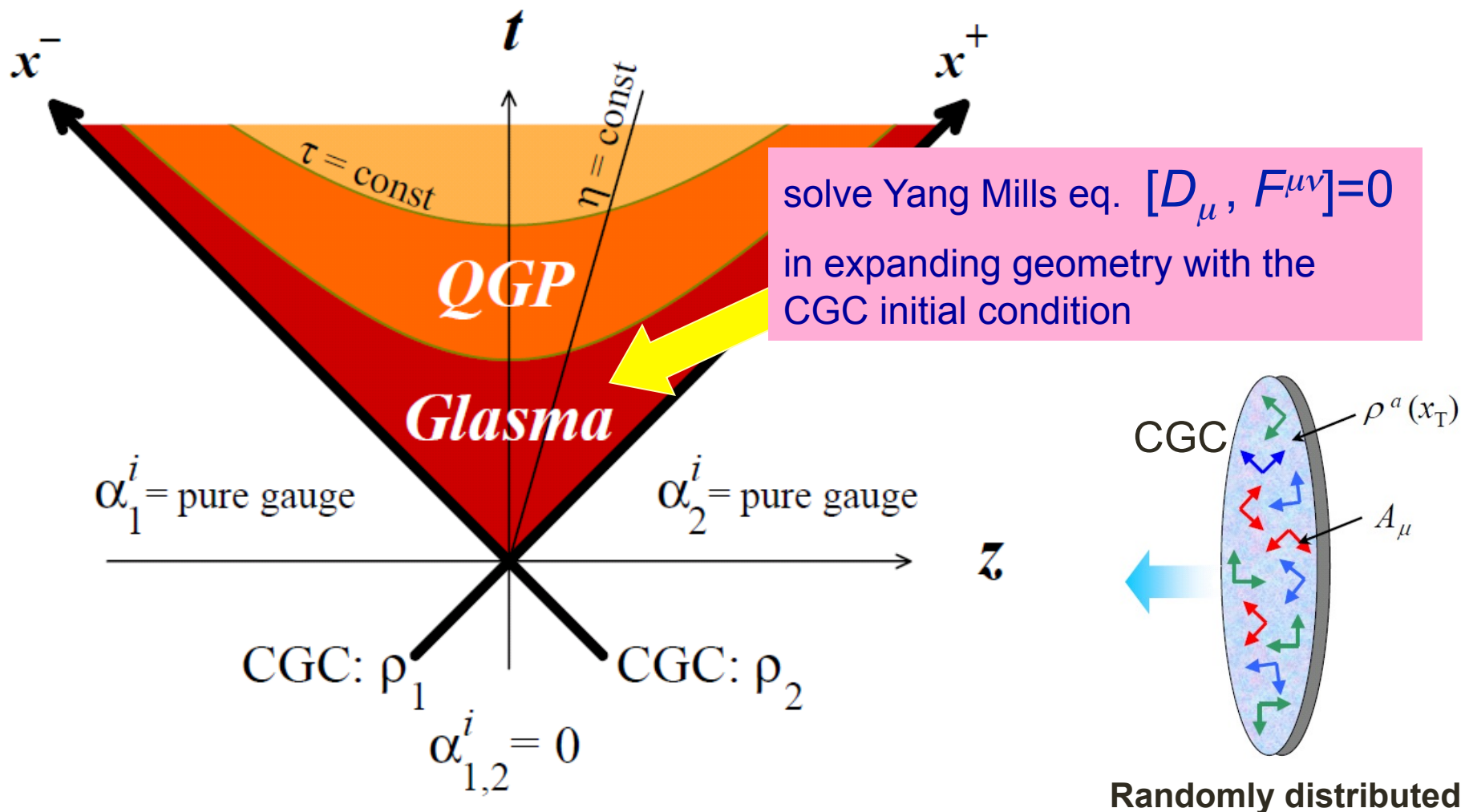
- **Existence of saturation scale Q_s**
- Can determine x and A dependences of Q_s
- Extends outside of the saturation regime $k_t < Q_s^2 / \Lambda_{\text{QCD}}$ (Iancu, Itakura, McLerran)

$$Q^2/Q_s^2(x_p)$$

CGC turns into Glasma

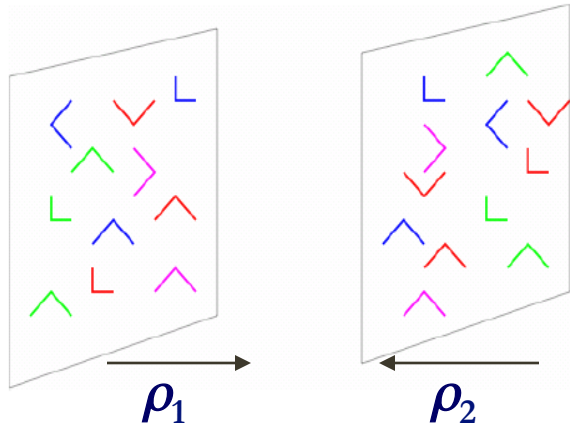
after heavy-ion collision

Glasma : non-equilibrium matter between Color **G**lass Condensate (CGC) and Quark Gluon Plasma (QGP). Created in heavy-ion collisions.

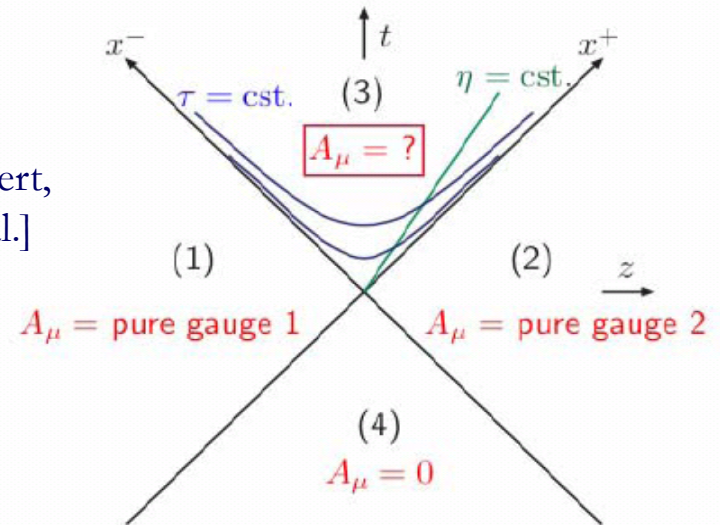


CGC as the initial condition for H.I.C.

HIC = Collision of two sheets of CGCs



[Kovner, Weigert, McLerran, et al.]



Each source creates the gluon field for each nucleus. ← Initial condition

$$J^\mu = \delta^{\mu+} \delta(x^-) \rho_1(\mathbf{x}_T) + \delta^{\mu-} \delta(x^+) \rho_2(\mathbf{x}_T)$$

$$-D_i \alpha_{(m)}^i = \rho_{(m)}(\mathbf{x}_\perp). \quad \alpha_1, \alpha_2 : \text{gluon fields of nuclei}$$

In Region (3), and at $\tau=0+$, the gauge field is determined by α_1 and α_2

$$A^\pm = \pm x^\pm \alpha(\tau, x_T) \quad \alpha_3^i |_{\tau=0} = \underline{\alpha_1^i} + \underline{\alpha_2^i}$$

$$A^i = \alpha_3^i(\tau, x_T). \quad \alpha |_{\tau=0} = \frac{ig}{2} [\underline{\alpha_1^i}, \underline{\alpha_2^i}]$$

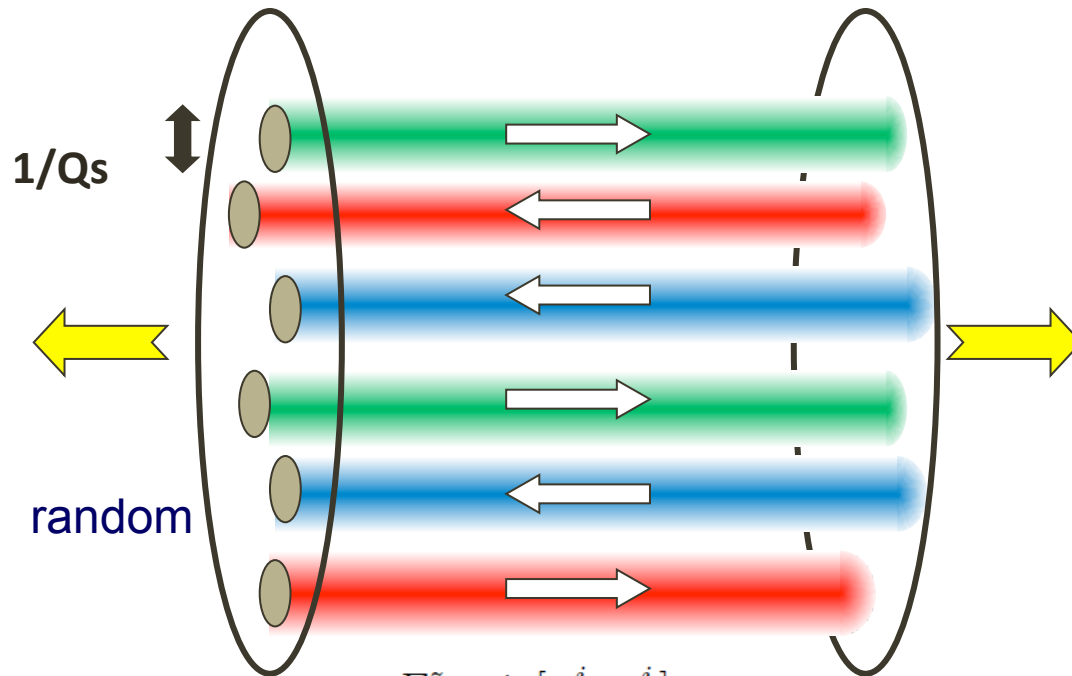
$$\partial_\tau \alpha |_{\tau=0} = \partial_\tau \alpha_3^i |_{\tau=0} = 0.$$

Glasma flux tube structure

Just after the collision: only E^z and B^z are nonzero

(Initial CGC is transversely random)

→ Glasma = **electric and magnetic flux tubes** extending in the longitudinal direction



$$E^z = ig[\alpha_1^i, \alpha_2^i]$$
$$B^z = ig\epsilon^{ij}[\alpha_1^i, \alpha_2^j].$$

Typical configuration of a single event
just after the collision

Unstable dynamics

Color-electric flux tube

→ gluon pair, qqbar pair
production via **Schwinger
mechanism**

Color-magnetic flux tube

→ Unstable against rapidity
dependent fluctuation via
Nielsen-Olesen instability

[Fujii, Itakura 2008]

When both are present

→ **Schwinger production of
gluons enhanced by the N-O
instability** [Tanji, Itakura 2012]

Unsolved issues on glasma

- **How does the glasma thermalize into QGP?**

unstable dynamics? → turbulent distribution leading to isotropization

Bose-Einstein Condensation? → see talks by J.P.Blaizot and F.Gelis

Other mechanisms, such as induced cascade by high p_t partons?

- **What is the observable consequence?**

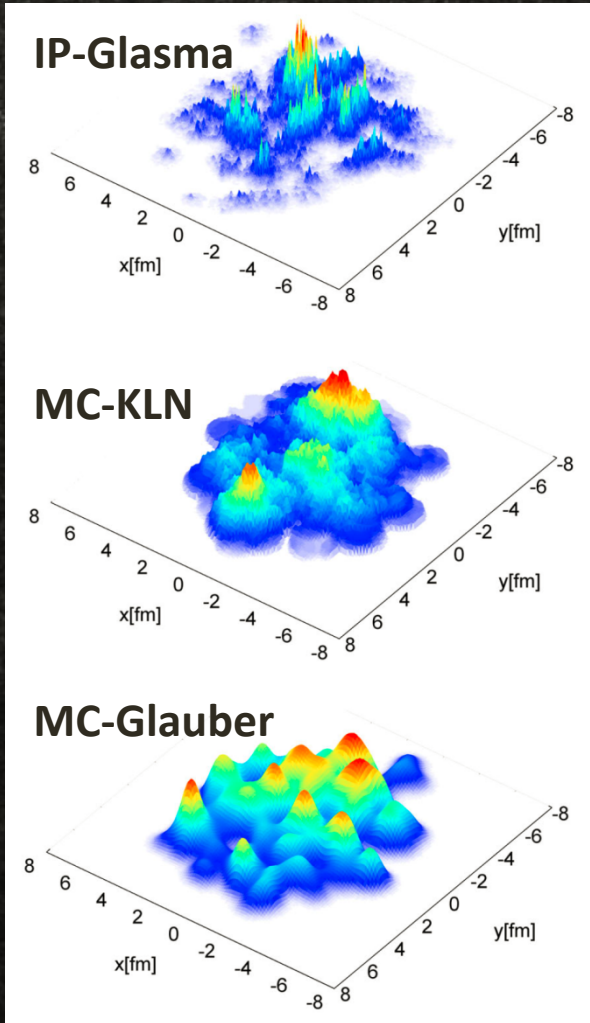
Ridge as remnants of longitudinal flux tube structure?

- **What kind of fluctuations are there?**

Color fluctuation inherent to CGC generates higher harmonics of flow?

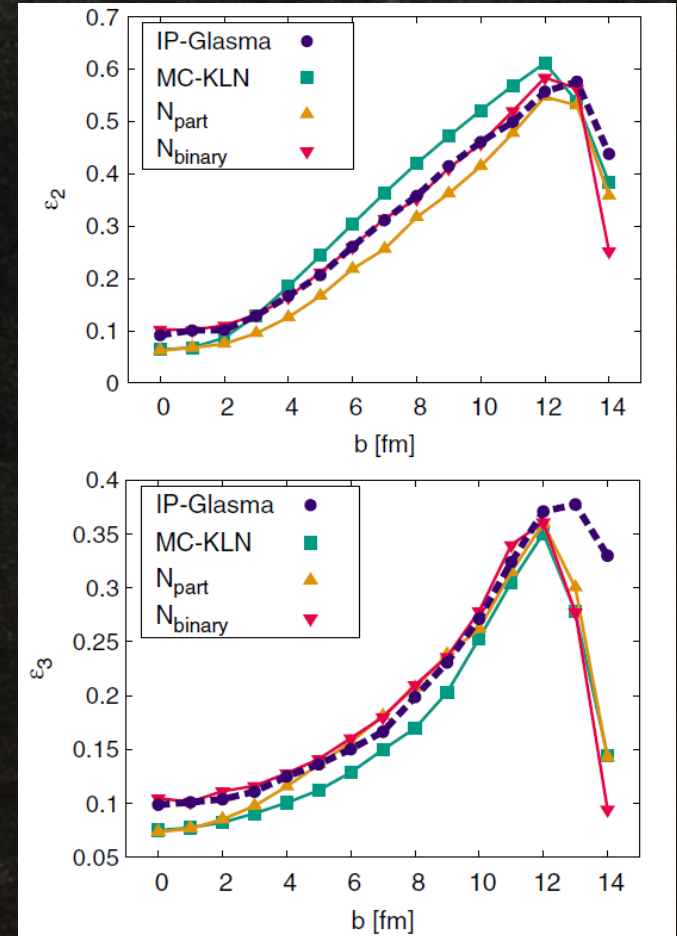
Examples of new progress (1)

- Schenke, Tribedy and Venugopalan, 2012 computed eccentricity and triangularity with IP-Glasma model



Fluctuations from nucleon position and color dynamics

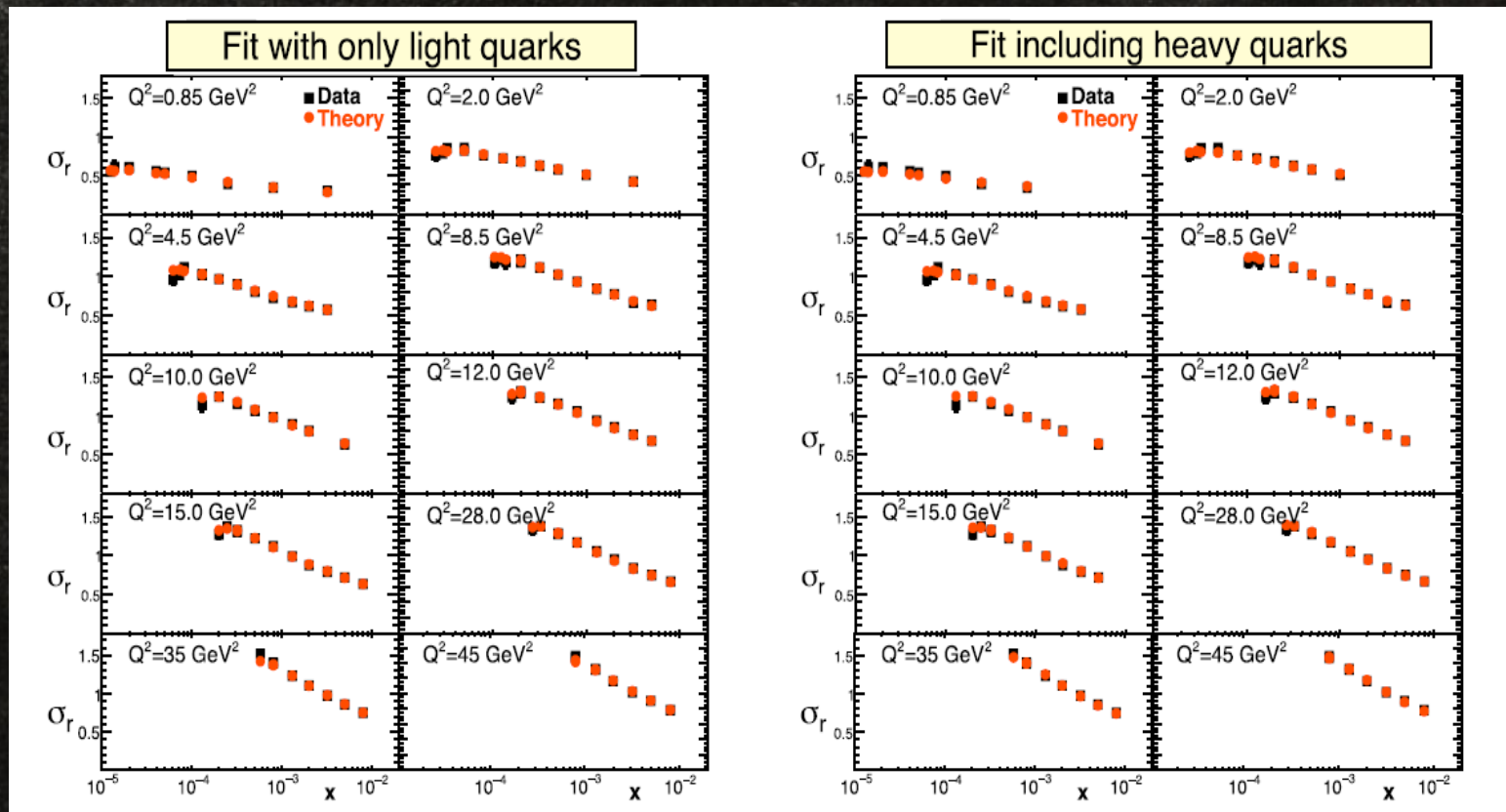
Only fluctuations from nucleon positions included



Also computed many other observables

Examples of new progress (2)

- Global analysis of DIS data with rcBK solution (AAMQS) and its application to forward particle production in dAu at RHIC



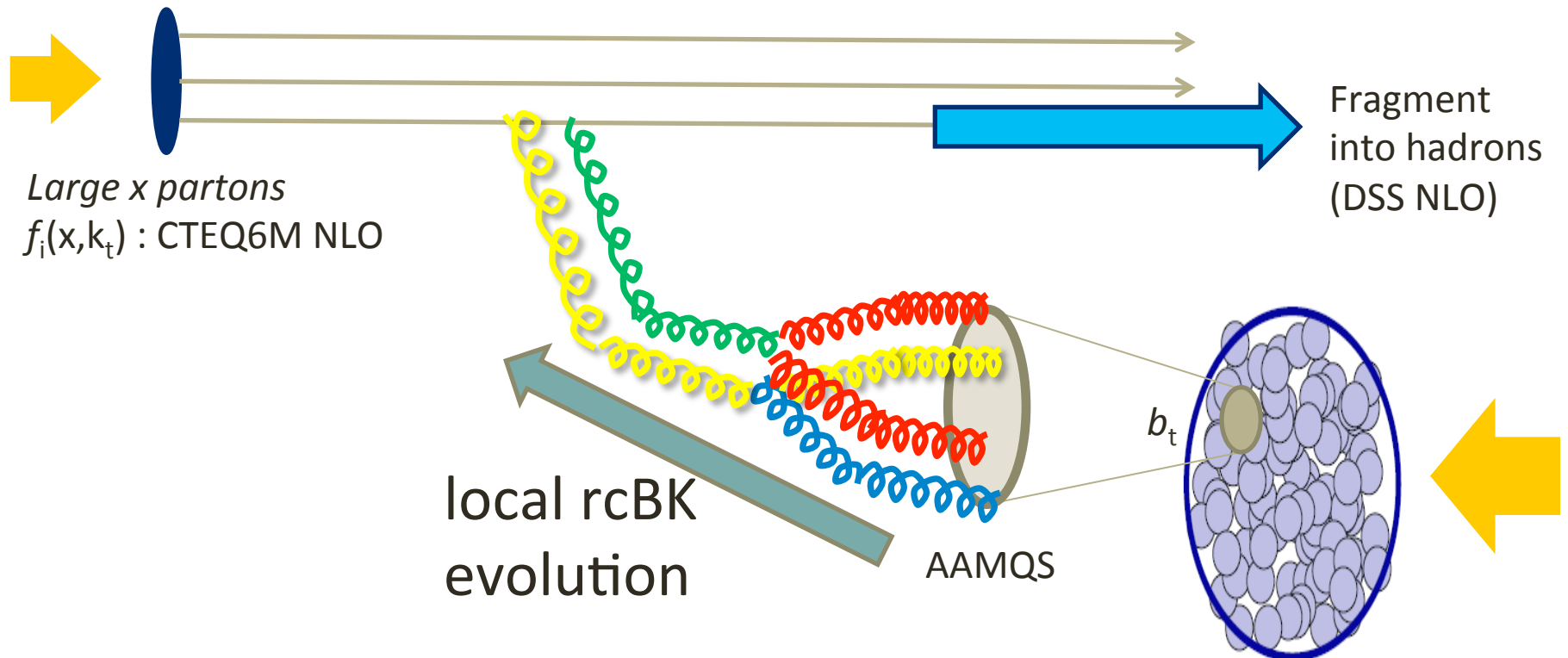
MC-DHJ/rcBK

[Fujii, Itakura, Nara,
2011, 2012]

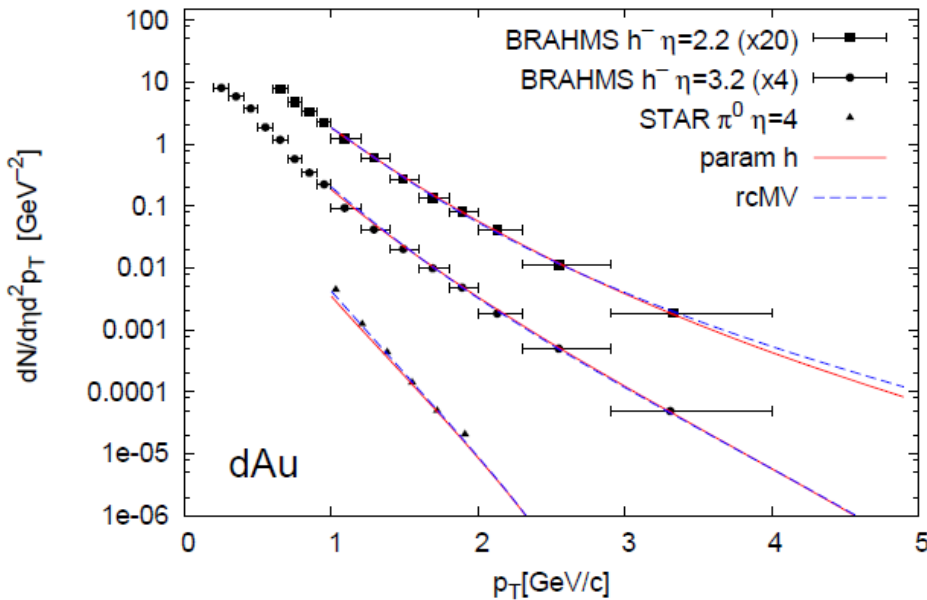
To reduce ambiguity

- construct a nucleus by randomly placing nucleons
- use AAMQS parameters for proton IC optimized for DIS at small- x
- quantum evolution is performed “locally” in b space

(to avoid IR div. in b -dep BK)



MC-DHJ/rcBK : results

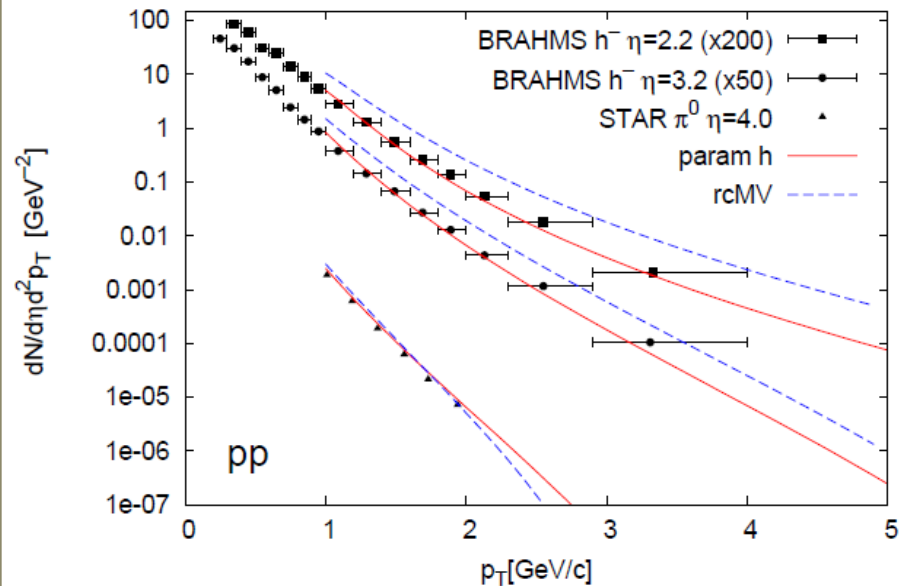


modified MV model ($\gamma = 1.118$)
 “running coupling” version of MV model [Iancu-Itakura-Triantafylopoulos] :
 to be consistent with rcBK evolution

- reproduce the data nicely
- AAMQS set h and rcMV for $\mathcal{N}(r, y)$
- Q_{s0A}^2 fixed by MC; **no additional parameter**

Best results from theoretical point of view, but still needs better (global) description including pp data (tuning of rcMV is necessary)

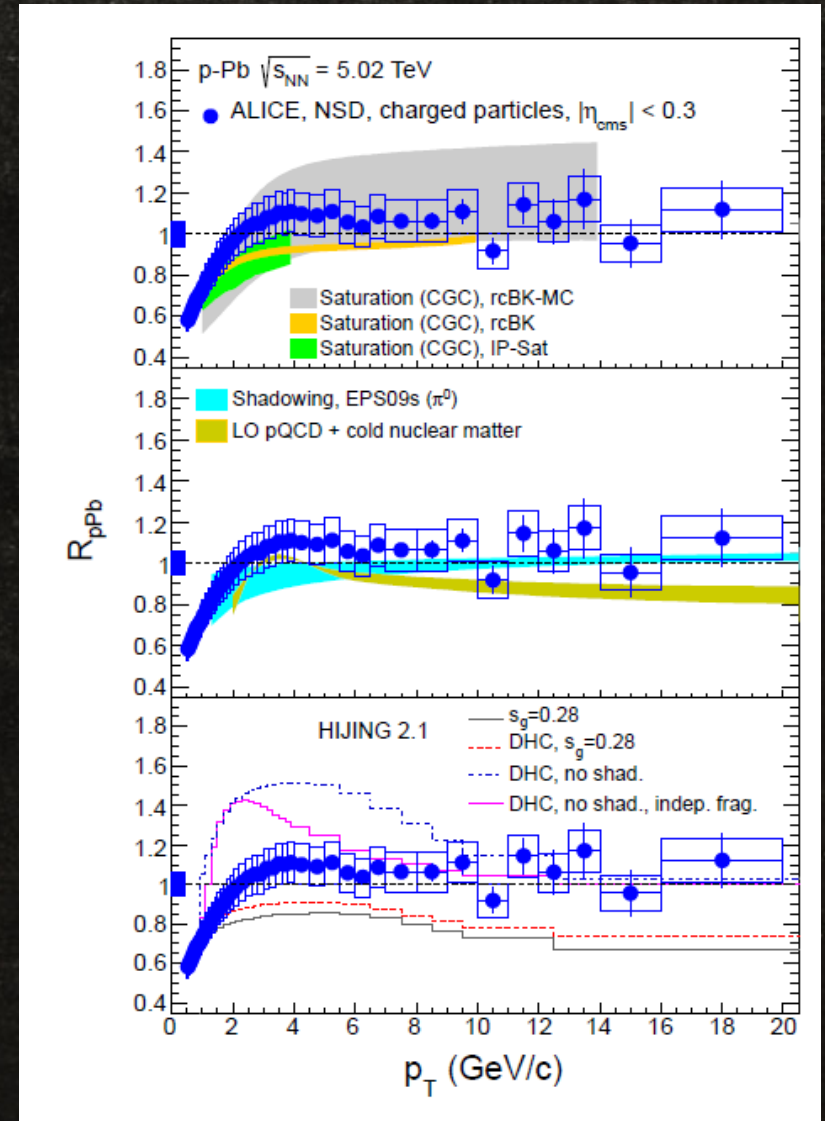
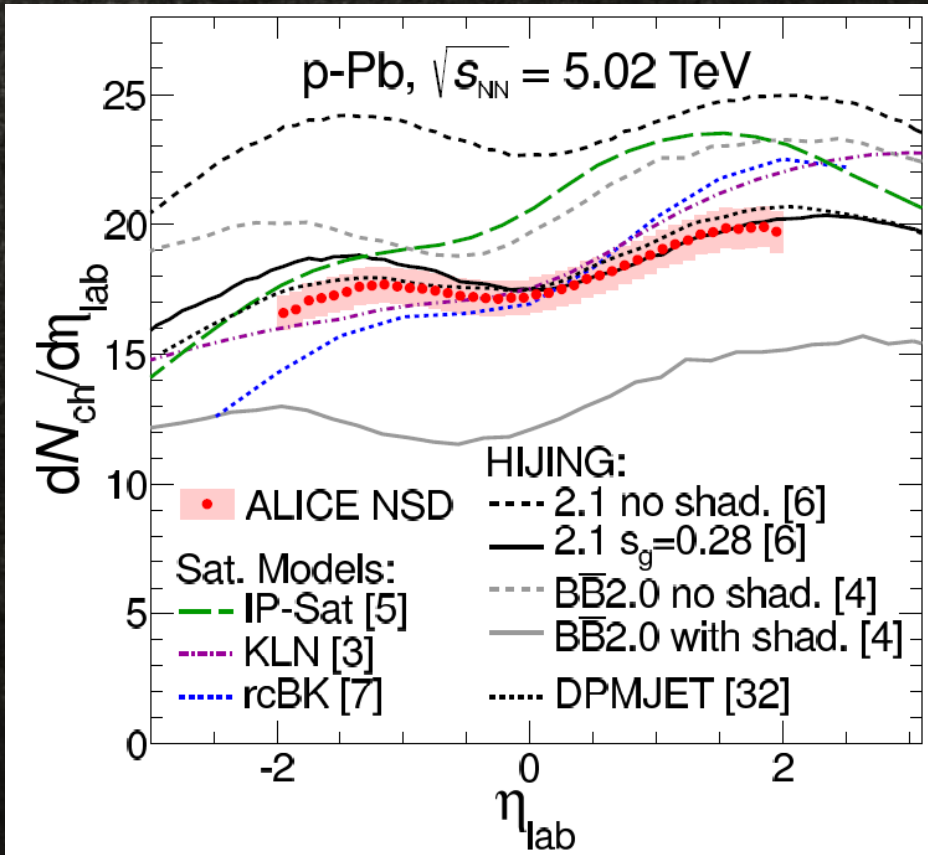
- Set h works well even in pp , but not as good as Albacete-Marquet
- rcMV is not “tuned” (similar param as MV)
- However, both work quite well in dAu (IC dependence reduces at high rapidity)



New ALICE data on pPb @ 5.02 TeV

arXiv: 1210.4520

arXiv:1210.3615



Summary

- CGC is the universal picture of hadrons at high energies, which appears as a result of gluon 3-point vertex. Its theoretical framework is established at the LO level, but is developing beyond the LO.
- CGC provides the initial conditions for the heavy ion collisions, and turns into Glasma. The Glasma is responsible for thermalization, but is not solved yet.
- CGC picture is getting precise and is now seriously compared with experimental data at RHIC (forward rapidity) and LHC. MC-DHJ/rcBK model works well in describing the forward dAu data.

backup

High-energy scattering

High-energy limit = "Regge limit"

total scatt. energy \gg typical energy/momentum scale in reaction

Hadron-hadron scattering

Momentum transfer squared

$$t = (p_a - p_c)^2$$

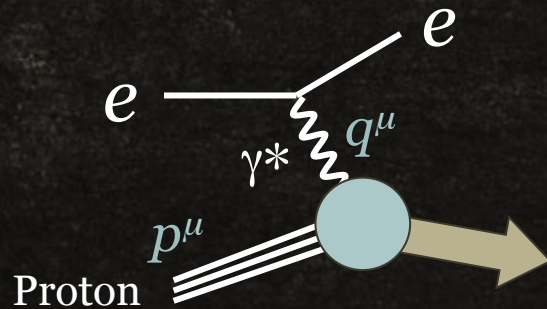
Total scattering energy squared

$$s = (p_a + p_b)^2$$



$$s \gg |t|$$

Deep inelastic ep scattering



$$W^2 = (p + q)^2$$

$$Q^2 = -q^2$$

Total γ^*p scattering energy squared

Virtuality of photon

$$W^2 \gg Q^2$$

$$x \sim Q^2 / (W^2 + Q^2) \rightarrow 0$$