One-Loop Calculation of the Oblique S Parameter in Higgsless Electroweak Models

ICHEP2012, 4 - 11 July 2012

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arXiv:1206.3454 [hep-ph]
Related works:
JHEP **07** (2008) 014 [arXiv:0803.1567]

OUTLINE

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1. Motivation

i) The Standard Model (SM) provides an extremely succesful description of the electroweak and strong interactions.

ii) A key feature is the particular mechanism adopted to break the electroweak gauge symmetry to the electroweak subgroup, $SU(2)_L \times U(1)_Y \rightarrow U(1)_{QED}$, so that the W and Z bosons become massive. ➔ The Higgs Hunting

iii) The LHC has just discovered a new particle around 125 GeV*.

iv) What if this new particle is not a Higgs boson? Or a not standard one? Or a scalar resonance? We should look for alternative mechanisms of mass generation. ➔ Higgsless Electroweak Models

vi) Strongly-coupled models: usually they do contain resonances. Many possibilities in the market: Technicolour, Walking Technicolour, Conformal Technicolour, Extra Dimensions…

v) They should fulfilled the existing phenomenological tests. ➔ Oblique Electroweak Observables**

* Preliminiminary CMS and ATLAS Collaborations.

** Peskin and Takeuchi '92.

Similarities to Chiral Simmetry Breaking in QCD

i) In the limit where the $U(1)_Y$ coupling g' is neglected, the Lagrangian is invariant under global $SU(2)_L \times SU(2)_R$ transformations. The Electroweak Symmetry Breaking (EWSB) turns out to be $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$ (custodial symmetry).

ii) Absolutely similar to the Chiral Symmetry Breaking (ChSB) occuring in QCD. So the same pion Lagrangian describes the Goldstone boson dynamics associated with the EWSB, being replaced f_π by $v=1/\sqrt{(2G_F)}=246$ GeV. Same procedure as in Chiral Perturbation Theory (ChPT)*.

$$\Delta\mathcal{L}^{(2)}_{\rm ChPT} = \frac{f_\pi^2}{4}\langle u_\mu u^\mu\rangle \quad \rightarrow \quad \Delta\mathcal{L}^{(2)}_{\rm EW} = \frac{v^2}{4}\langle u_\mu u^\mu\rangle$$

iii) We can introduce the resonance fields needed in strongly-coupled Higgsless modes in a similar way as in ChPT: Resonance Chiral Theory (RChT)**.

✓ Note the implications of a naïve rescaling from QCD to EW:

$$\left\{ \begin{array}{rcl} f_\pi = 0.090\,\text{GeV} & \longrightarrow & v = 0.246\,\text{TeV} \\ M_\rho = 0.770\,\text{GeV} & \longrightarrow & M_V = 2.1\,\text{TeV} \\ M_{a1} = 1.260\,\text{GeV} & \longrightarrow & M_A = 3.4\,\text{TeV} \end{array} \right.$$

iv) Actually, the estimation of the S parameter in strongly-coupled EW models is equivalent to the determination of L_{10} in ChPT***.

* Weinberg '79
* Gasser & Leutwyler '84 '85
* Bijnens et al. '99 '00

**Ecker et al. '89
** Cirigliano et al. '06

*** Pich, IR, Sanz-Cillero '08.

2. Oblique Electroweak Observables

✓ Universal oblique corrections via the EW boson self-energies (transverse in the Landau gauge)

$$\mathcal{L}_{\mathrm{v.p.}} \doteq -\frac{1}{2} W_\mu^3\, \Pi_{33}^{\mu\nu}(q^2) W_\nu^3 - \frac{1}{2} B_\mu\, \Pi_{00}^{\mu\nu}(q^2) B_\nu - W_\mu^3\, \Pi_{30}^{\mu\nu}(q^2) B_\nu - W_\mu^+\, \Pi_{WW}^{\mu\nu}(q^2) W_\nu^-$$

✓ S parameter: new physics in the difference between the Z self-energies at $Q^2=M_Z^2$ and $Q^2=0$.

$$e_3 = \frac{g}{g'}\, \widetilde{\Pi}_{30}(0)\,, \qquad \Pi_{30}(q^2) = q^2\, \widetilde{\Pi}_{30}(q^2) + \frac{g^2 \tan\theta_W}{4} v^2\,, \qquad S = \frac{16\pi}{g^2}\left(e_3 - e_3^{\mathrm{SM}}\right).$$

✓ We follow the useful dispersive representation introduced by Peskin and Takeuchi*.

$$\begin{aligned} S &= \frac{16}{g^2 \tan\theta_W} \int_0^\infty \frac{\mathrm{d}s}{s} \left(\mathrm{Im}\widetilde{\Pi}_{30}(s) - \mathrm{Im}\widetilde{\Pi}_{30}^{\mathrm{SM}}(s) \right) = \\ &= \int_0^\infty \frac{\mathrm{d}s}{s} \left(\frac{16}{g^2 \tan\theta_W}\, \mathrm{Im}\widetilde{\Pi}_{30}(s) - \frac{1}{12\pi} \left[1 - \left(1 - \frac{M_H^2}{s} \right)^3 \theta\left(s - M_H^2\right) \right] \right) \end{aligned}$$

- ✓ The convergence of the integral needs a vanishing $\mathrm{Im}\widetilde{\Pi}_{30}(s)$ at short distances.
- ✓ S parameter is defined for a reference value for the SM Higgs mass.

* Peskin and Takeuchi '92.

3. The Effective Lagrangian

Let us consider a low-energy effective theory containing the SM gauge bosons coupled to the electroweak Goldstones and the lightest vector and axial-vector resonances:

$$\mathcal{L} = \mathcal{L}_{\rm EW}^{(2)} + \mathcal{L}_{\rm GF} + \mathcal{L}_V + \mathcal{L}_A + \mathcal{L}_{VV}^{\rm kin} + \mathcal{L}_{AA}^{\rm kin} + \mathcal{L}_{VA}$$

$$\mathcal{L}_{\rm EW}^{(2)} = -\frac{1}{2g^2}\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle - \frac{1}{2g'^2}\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle + \frac{v^2}{4}\langle u_\mu u^\mu\rangle\,, \qquad \mathcal{L}_{\rm GF} = -\frac{1}{2\xi}(\partial^\mu \vec{W}_\mu)^2\,,$$

$$\mathcal{L}_V + \mathcal{L}_A = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{i\,G_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle + \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle\,,$$

$$\mathcal{L}_{RR}^{\rm kin} = -\frac{1}{2}\langle \nabla^\lambda R_{\lambda\mu}\nabla_\nu R^{\nu\mu} - \frac{M_R^2}{2} R_{\mu\nu}R^{\mu\nu}\rangle\,, \qquad (R = V, A)$$

$$\begin{aligned}\mathcal{L}_{VA} = {}& i\,\lambda_2^{VA}\langle [V^{\mu\nu}, A_{\nu\alpha}] h_\mu^\alpha\rangle + i\,\lambda_3^{VA}\langle [\nabla^\mu V_{\mu\nu}, A^{\nu\alpha}] u_\alpha\rangle \\ &+ i\,\lambda_4^{VA}\langle [\nabla_\alpha V_{\mu\nu}, A^{\alpha\nu}] u^\mu\rangle + i\,\lambda_5^{VA}\langle [\nabla_\alpha V_{\mu\nu}, A^{\mu\nu}] u^\alpha\rangle\,,\end{aligned}$$

$$\kappa = -2\lambda_2^{\rm VA} + \lambda_3^{\rm VA}\,, \qquad \sigma = 2\lambda_2^{\rm VA} - 2\lambda_3^{\rm VA} + \lambda_4^{\rm VA} + 2\lambda_5^{\rm VA}\,.$$

We have seven resonance parameters: F_V, G_V, F_A, κ, σ, M_V and M_A. ⇒ The high-energy constraints are fundamental.

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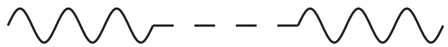
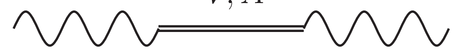
4. The Calculation of S

i) At leading-order (LO)*



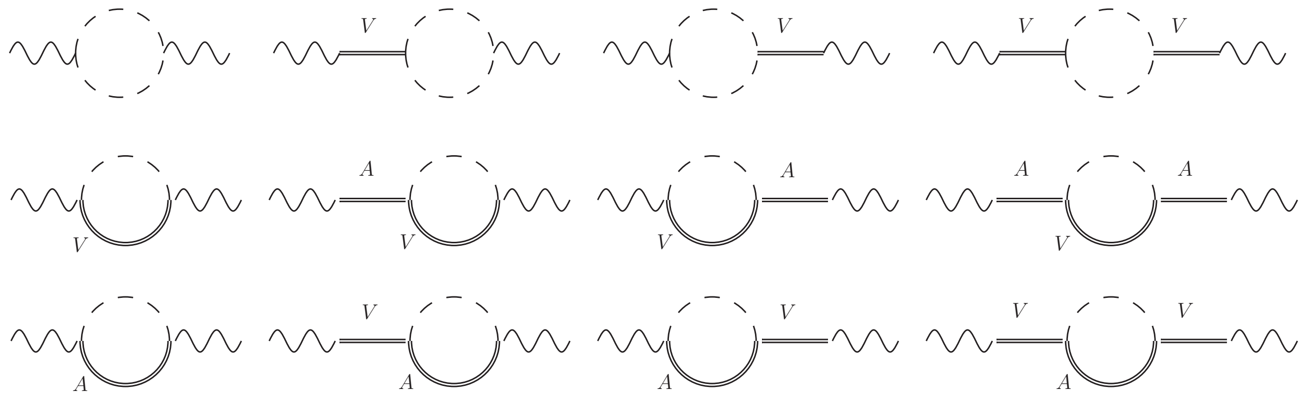


$$\Pi_{30}(s)|_{\mathrm{LO}} = \frac{g^2 \tan\theta_W}{4} s \left(\frac{v^2}{s} + \frac{F_V^2}{M_V^2 - s} - \frac{F_A^2}{M_A^2 - s} \right)$$

$$S_{\mathrm{LO}} = 4\pi \left(\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2} \right)$$

* Peskin and Takeuchi '92.

ii) At next-to-leading order (NLO)**



- ✓ Once-subtracted dispersive relation
- ✓ Contributions from ππ, Vπ and Aπ cuts, since higher cuts are supposed to be suppressed.
- ✓ F_R^r and M_R^r are *renormalized* couplings which define the resonance poles at the one-loop level.

$$\Pi_{30}(s) = \Pi_{30}(0) + \frac{s}{\pi}\int_0^\infty \frac{\mathrm{d}t}{t\,(t-s)}\mathrm{Im}\Pi_{30}(t)$$

$$\Pi_{30}(s)|_{\mathrm{NLO}} = \frac{g^2\tan\theta_W}{4}\, s\left(\frac{v^2}{s} + \frac{F_V^{r\,2}}{M_V^{r\,2}-s} - \frac{F_A^{r\,2}}{M_A^{r\,2}-s} + \overline{\Pi}(s)\right)$$

$$S_{\mathrm{NLO}} = 4\pi\left(\frac{F_V^{r\,2}}{M_V^{r\,2}} - \frac{F_A^{r\,2}}{M_A^{r\,2}}\right) + \overline{S}$$

* Barbieri et al.'08
* Cata and Kamenik '08
* Orgogozo and Rynchov '08

5. High-energy Constraints

- ✓ We have seven resonance parameters: F_V, G_V, F_A, κ, σ, M_V and M_A.
- ✓ The number of unknown couplings can be reduced by using short-distance information.
- ✓ In contrast with the QCD case, we ignore the underlying dynamical theory.

i) Weinberg Sum Rules (WSR)*

$$\Pi_{30}(s) = \frac{g^2 \tan\theta_W}{4}\, s\, [\Pi_{VV}(s) - \Pi_{AA}(s)]$$

$$\left\{ \begin{array}{rcl} \frac{1}{\pi}\int_0^\infty dt\, [\mathrm{Im}\Pi_{VV}(t) - \mathrm{Im}\Pi_{AA}(t)] & = & v^2 \\ \frac{1}{\pi}\int_0^\infty dt\, t\, [\mathrm{Im}\Pi_{VV}(t) - \mathrm{Im}\Pi_{AA}(t)] & = & 0 \end{array} \right.$$

i.i) LO

$$F_V^2 - F_A^2 = v^2$$
$$F_V^2 M_V^2 - F_A^2 M_A^2 = 0$$

⬇

1 or 2 constraints

i.ii) Imaginary NLO

$$\mathrm{Im}\Pi_{30}(s) \sim \mathcal{O}\left(\frac{1}{s}\right)$$

⬇

3 or 4 constraints

i.iii) Real NLO: fixing of $F_{V,A}^r$ or lower bounds**

$$F_V^{r\,2} - F_A^{r\,2} = v^2\,(1 + \delta_{\rm NLO}^{(1)})$$
$$F_V^{r\,2} M_V^{r\,2} - F_A^{r\,2} M_A^{r\,2} = v^2 M_V^{r\,2} \delta_{\rm NLO}^{(2)}$$

⬇

Constraints on F_V^r and F_A^r

* Weinberg'67
* Bernard et al.'75.
** Pich et al.'08

ii) Additional short-distance constraints

ii.i) $W_L W_L \to W_L W_L$ scattering* $\quad G_V = \frac{v}{\sqrt{3}}$

ii.ii) Vector Form Factor** $\quad F_V G_V = v^2$

ii.iii) Axial Form Factor*** $\quad F_V - 2G_V = F_A\,(2\kappa + \sigma)$

3 additional constraints!

- ✓ We have up to 9 (7) constraints with 2 (1) WSR and 7 resonance parameters: we cannot consider all the constraints at the same time, some approximately.
- ✓ As a check of consistency we consider different combination of constraints.

* Bagger et al.'94
* Barbieri et al.'08
** Ecker et al.'89
*** Pich et al.'08

6. Phenomenology

S = 0.04 ± 0.10 * (M_H=0.120 TeV)

i) LO results

i.i) 1st and 2nd WSRs

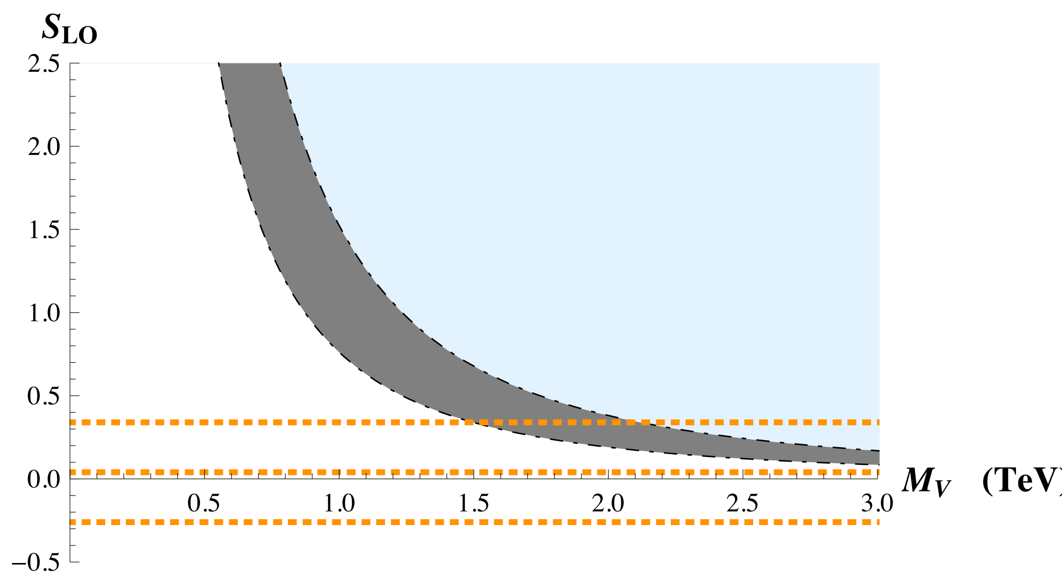
$$S_{\rm LO} \;=\; \frac{4\pi v^2}{M_V^2}\left(1+\frac{M_V^2}{M_A^2}\right)$$

$$\frac{4\pi v^2}{M_V^2} < S_{\rm LO} < \frac{8\pi v^2}{M_V^2}$$

i.ii) Only 1st WSR

$$S_{\rm LO} \;=\; 4\pi\left\{\frac{v^2}{M_V^2}+F_A^2\left(\frac{1}{M_V^2}-\frac{1}{M_A^2}\right)\right\}$$

$$S_{\rm LO} > \frac{4\pi v^2}{M_V^2}$$

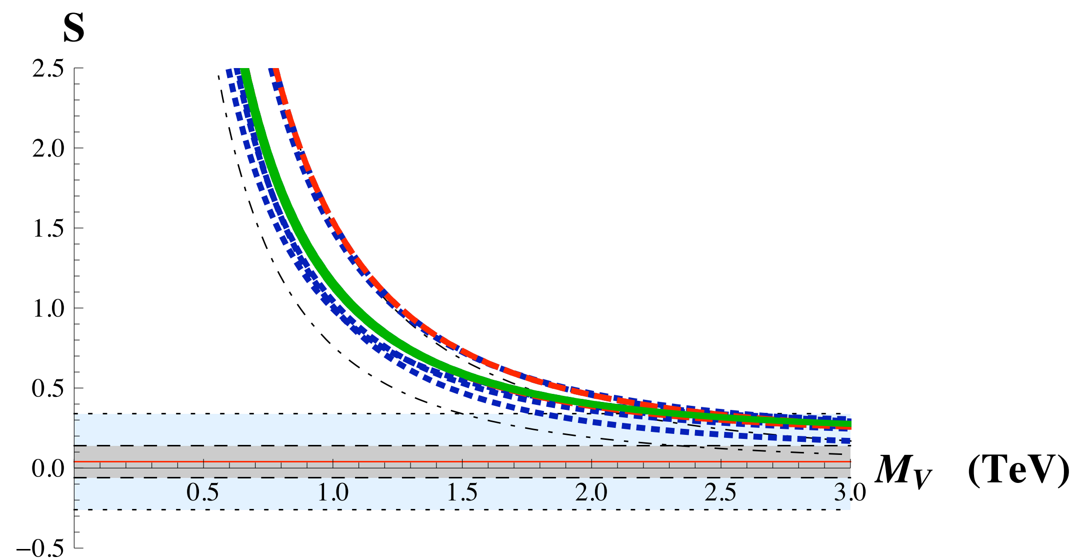


At LO M_V > 1.5 TeV at 3σ

* Gfitter
* LEP EWWG
* Zfitter

ii) NLO results: 1st and 2nd WSRs

- ✓ 1st and 2nd WSRs at LO and at NLO:
  - ✓ 6 constraints
  - ✓ M_V the only free parameter
- ✓ 8 solutions.
- ✓ Only 2 approximately compatible with VFF, AFF and scattering constraints (green).

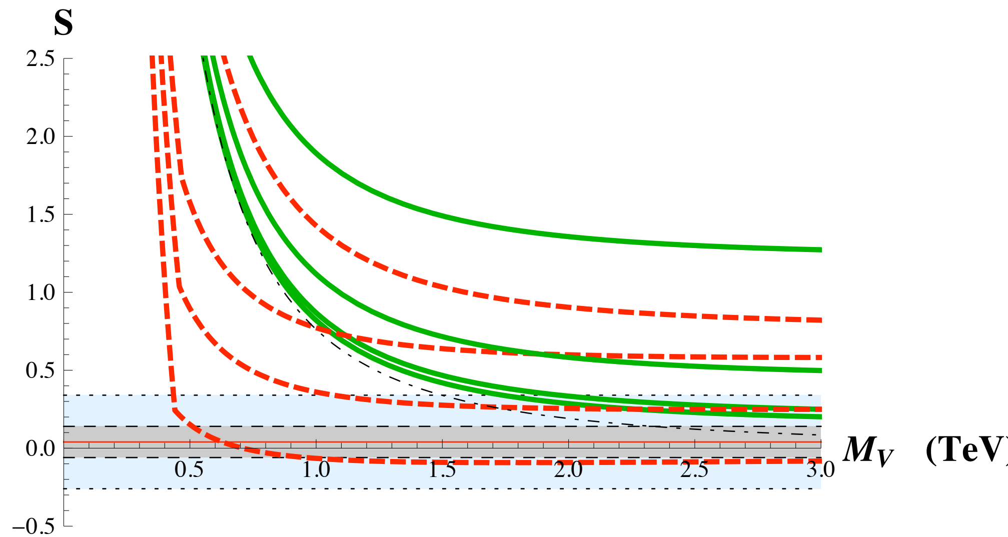


- ✓ If, alternatively, we consider the 1st and the 2nd WSR only at NLO with the VFF and AFF constraints (6 constraints), a heavier result is found: $M_V > 2.4$ TeV at 3σ.

At NLO with the 1st and 2nd WSRs
$M_V > 1.8$ TeV at 3σ

iii) NLO results: only 1st WSR

- ✓ 1st WSR at NLO + VFF and AFF constraints:
  - ✓ 5 constraints
  - ✓ M_V and M_A the only free parameters are.
- ✓ Without the 2nd WSR we can only derive lower bounds on S.
- ✓ Imposing that $F_V^2 - F_A^2 > 0$ we have found only 2 solutions.
- ✓ One of them (red) is clearly disfavoured:
  - ✓ Sharply violation of the 2nd WSR at LO and at NLO
  - ✓ Large NLO correction
  - ✓ Big splitting between M_V and M_A.



- ✓ Without the 2nd WSR it is possible the analysis with only the ππ cut. The same result is found: $M_V > 1.8$ TeV at 3σ.

At NLO with only the 1st WSR
$M_V > 1.8$ TeV at 3σ

7. Summary

1. What? One-loop calculation of the oblique S parameter within Higgsless models of EWSB

2. Why? What if this new particle around 125 GeV is not a Higgs bosson?

- ✓ We should look for alternative ways of mass generation: strongly-coupled higgsless models.
- ✓ They should fulfilled the existing phenomenological tests.

3. Where? Effective approach

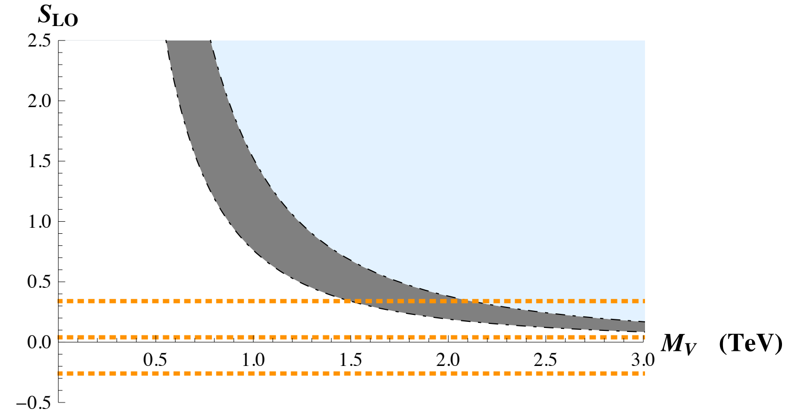
a) EWSB: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$: similar to ChSB in QCD: ChPT.

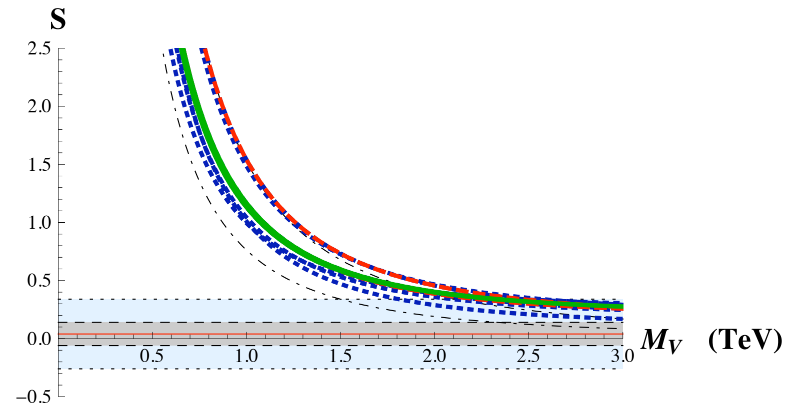
b) Strongly-coupled Higgsless models: similar to resonances in QCD: RChT.

c) General Lagrangian with at most two derivatives and short-distance information.

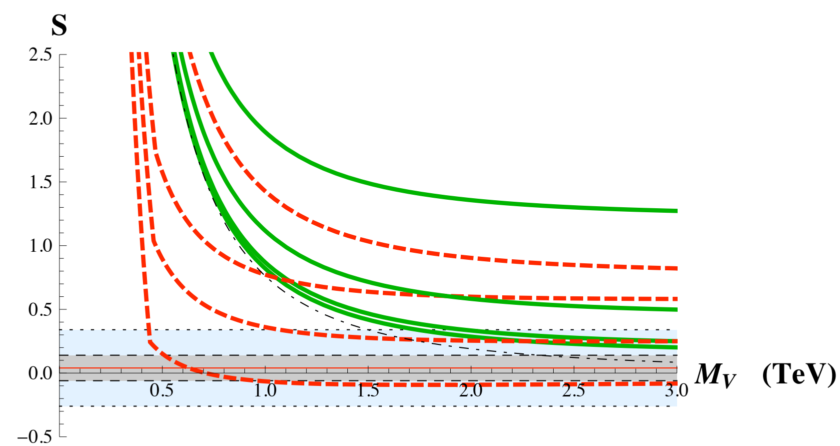
4. How? Dispersive representation of Peskin and Takeuchi'92.

- ✓ Improvements over previous NLO calculation:
  - ✓ Dispersive calculation: no unphysical cut-offs.
  - ✓ A more general Lagrangian.
  - ✓ Short-distance information as a crucial ingredient.
- ✓ We have considered different possibilites:
  - ✓ LO
  - ✓ NLO with the 1st and 2nd WSR
  - ✓ NLO with only the 1st WSR





- ✓ Similar results:
  - ✓ At LO $M_V > 1.5$ TeV at 3σ.
  - ✓ At NLO $M_V > 1.8$ TeV at 3σ.
- ✓ In these reasonable strongly coupled models the S parameter requires a high resonance mass scale, beyond the 1 TeV.



8*. Future work

- ✓ Consideration of this new scalar with a mass around 125 GeV in our calculation:
  - ✓ Higgs boson? Which one?
  - ✓ A scalar resonance of strongly-coupled models?

→ A new $S\pi$ or $H\pi$ cut!, but only at NLO.

- ✓ Oblique T parameter

→ Absence of a known dispersive representation.