Universal behavior in the scattering of heavy, weakly interacting dark matter on nuclear targets

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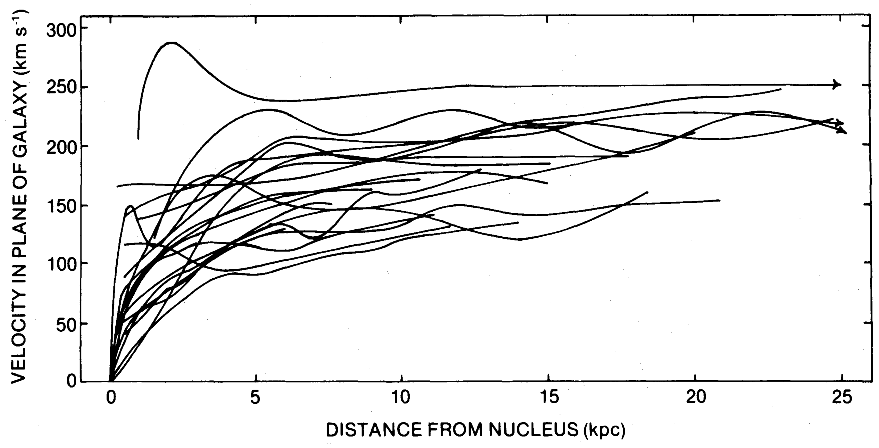
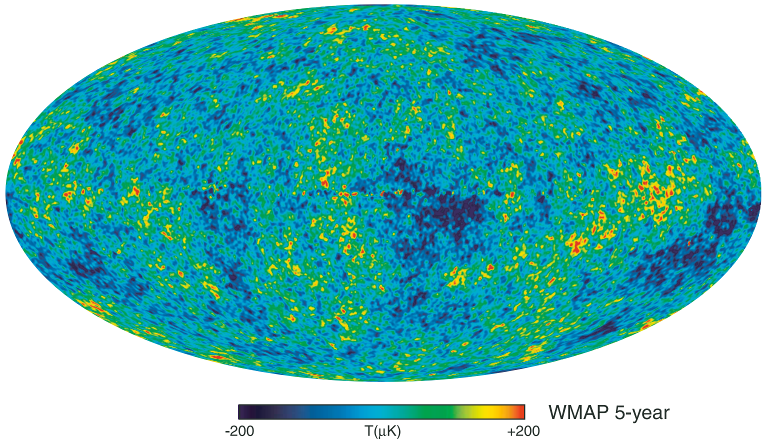
Outline

- simple model of SU(2) charged DM
- heavy particle effective theory
  - electroweak symmetric theory
  - low energy theory
  - matching
  - running
  - hadronic inputs
  - universal cross section for SU(2) charged WIMP
- summary and outlook

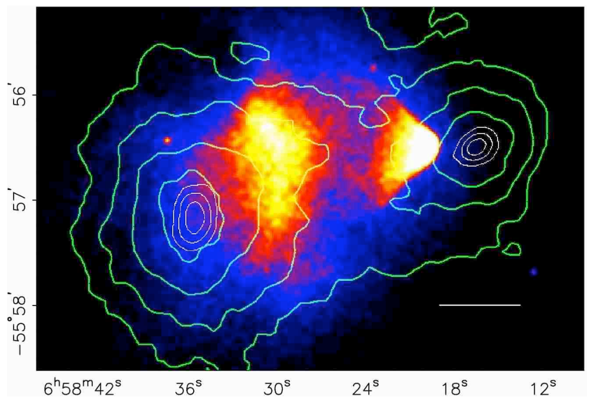
Multiple astrophysical indications of cold dark matter



modification of galactic rotation curves



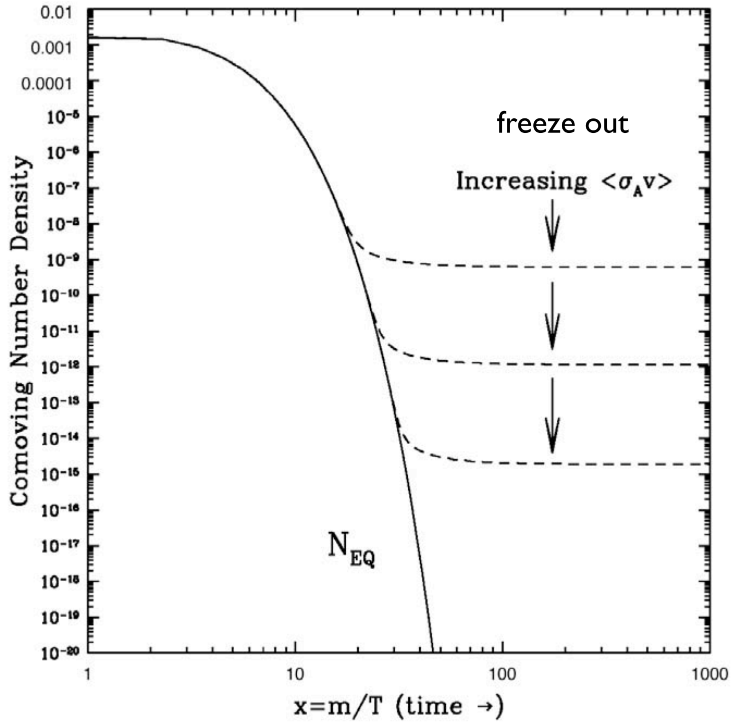
imprints on microwave background

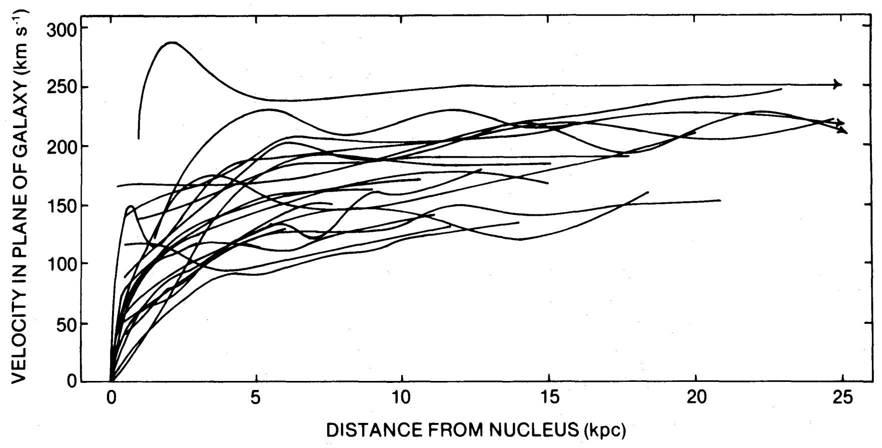


apparent extra collisionless matter from lensing measurements

Multiple astrophysical indications of cold dark matter

Indication of thermal relic, weakly interacting, particles beyond the standard model ?





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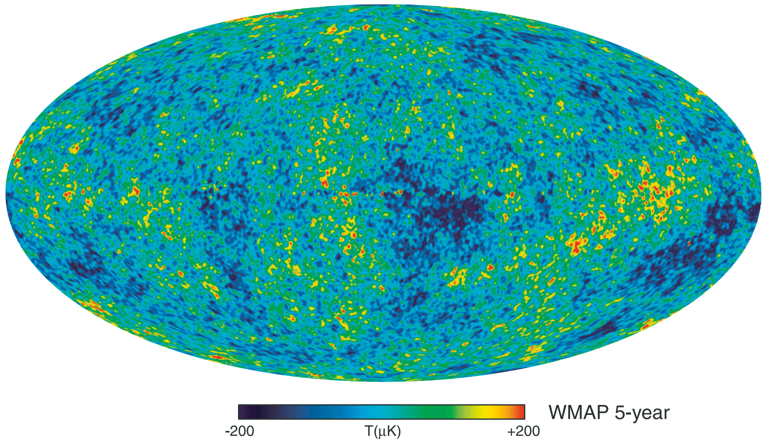

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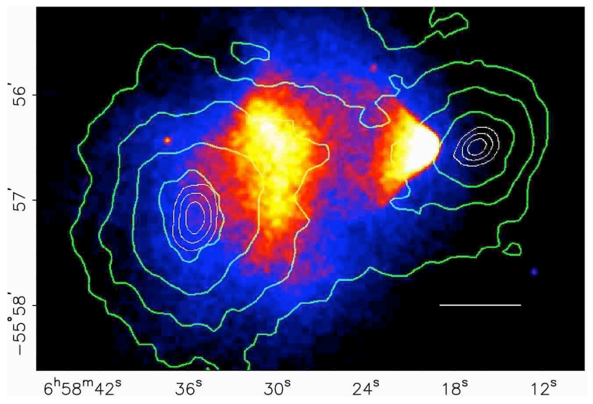


apparent extra collisionless matter from lensing measurements

on-ramps to the talk

- a motivation for electroweak-SU(2) charged dark matter
- QCD anatomy of dark matter direct detection
- (formalism in heavy particle effective theories)

Will find a general mechanism that can lead to stable particles,

- M ~ TeV for particle to be significant component of thermal relic dark matter

- predictive scattering cross section on nucleon in limit M >> m_W

technical part of talk: compute this universal cross section in terms of Standard Model parameters

A nontrivial problem involving multiple scales

Parameter	Value
$\lvert V_{td}\rvert$	~ 0
$\lvert V_{ts}\rvert$	~ 0
$\lvert V_{tb}\rvert$	~ 1
m_u/m_d	0.49(13)
m_s/m_d	19.5(2.5)
$\Sigma_{\pi N}^{\rm lat}$	0.047(9) GeV
$\Sigma_s^{\rm lat}$	0.050(8) GeV
$\Sigma_{\pi N}$	0.064(7) GeV
Σ_0	0.036(7) GeV
m_W	80.4 GeV
m_t	172 GeV
m_b	4.75 GeV
m_c	1.4 GeV
m_N	0.94 GeV
$\alpha_s(m_Z)$	0.118
$\alpha_2(m_Z)$	0.0338

m_h ?

A prototype for systematic computation of QCD effects in DM - nucleus scattering

Recall axion: UV completions realizing Peccei Quinn mechanism generically involve fermions coupled to color SU(3)

old gluon

$$\mathcal{L} = |\partial_\mu \sigma|^2 + \bar{q}(i\partial\!\!\!/ + g A\!\!\!/)q - V(\sigma) - \lambda \sigma \bar{q}_L q_R + h.c.$$

new scalar

new quark

Low energy: $a(x) \to a(x) + c$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{a(x)}{f} \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma} + \dots$$

Are there (beyond SM) Dirac fermions coupled to SM gauge fields (e.g. axion models: SU(3)) ?

Weakly interacting stable pions

- Consider confined Dirac fermions, coupled to weak SU(2)

old SU(2) gauge bosons

$$\Delta\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + g_2 W\!\!\!\!/^{\,a} t^a + \hat{g}\hat{A\!\!\!/})\psi$$

new "quarks" and "gluons"

SU(2) is a special group: all representations are self-conjugate:

$$-t^{a*} = S^\dagger t^a S$$

e.g.

$$-\frac{\sigma^{a*}}{2} = (i\sigma^2)^\dagger \frac{\sigma^a}{2}(i\sigma^2), \quad a = 1, 2, 3$$

Implies an invariance of the action: $\Longrightarrow \quad \mathcal{L} \to \mathcal{L}$

$$\psi \to S\psi^{\mathcal{C}} = Si\gamma^2\psi^*$$

$$W^a \to W^a$$

Call this discrete symmetry "G parity" after (ungauged) QCD operation.
Consider lightest G-odd particle: Lorentz scalar, weak SU(2) triplet

☐ Mass spectrum

$$j \times j - 0 = 1 + 2 + \cdots + 2j$$

fermion isospin

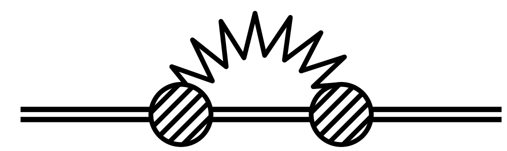
pion isospin

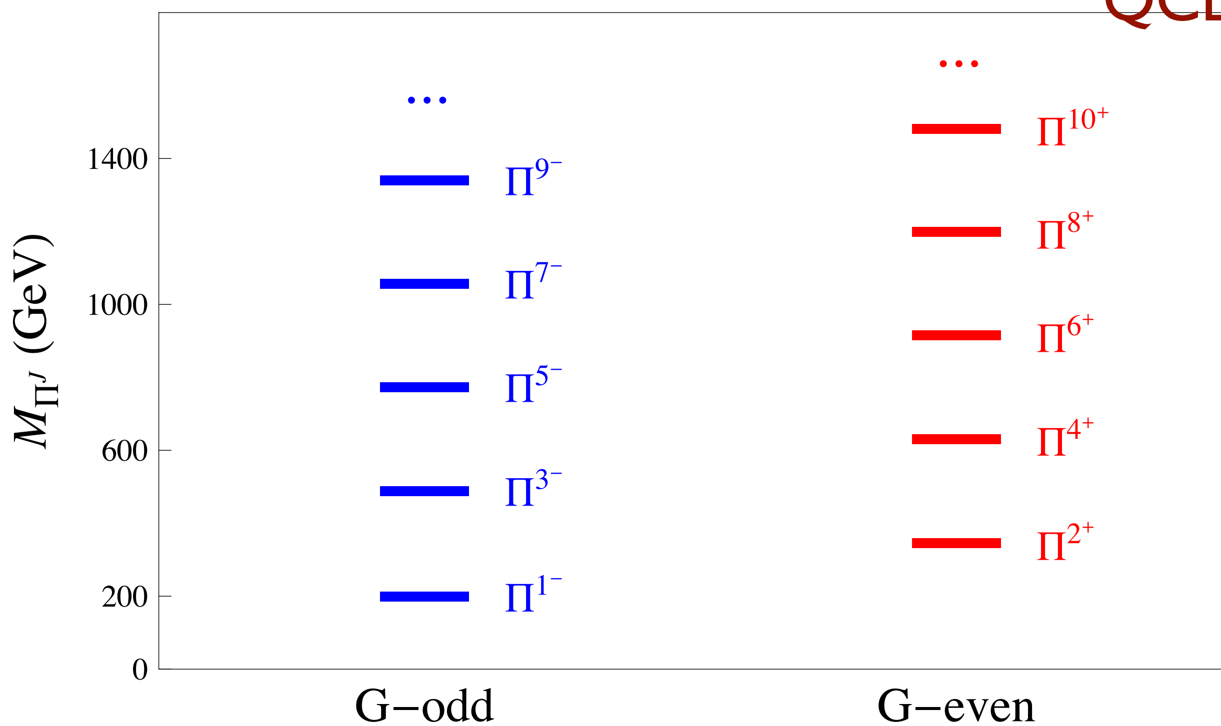
Masses induced by radiative corrections, proportional to total isospin

$$m^2_{\Pi(JQ)} \sim J(J+1)\alpha_2 \Lambda_h^2$$

isospin

QCD-like confinement scale (~ TeV)





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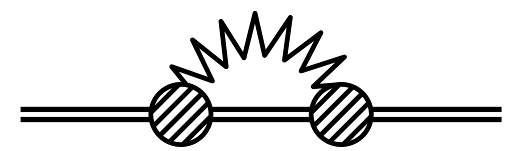
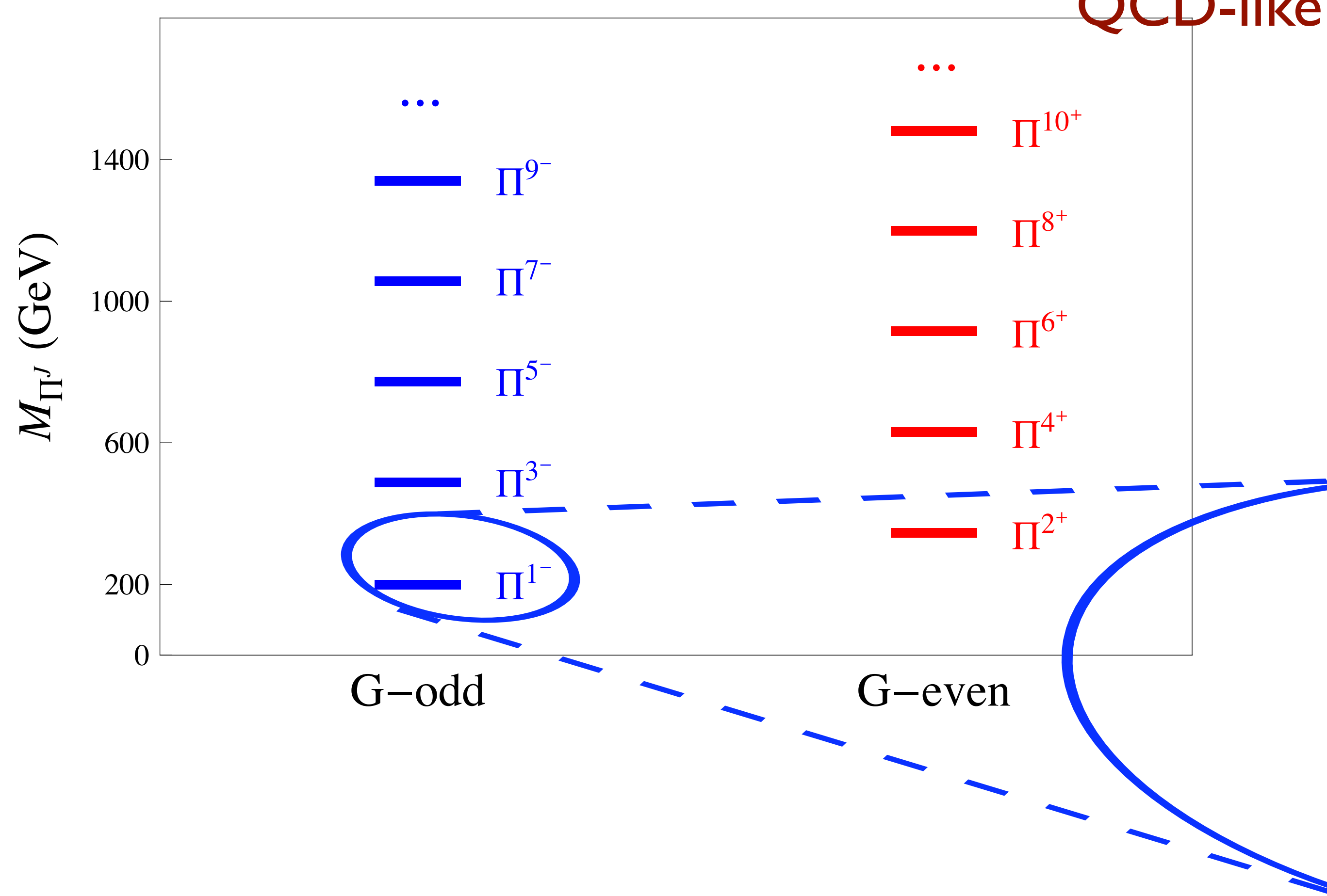
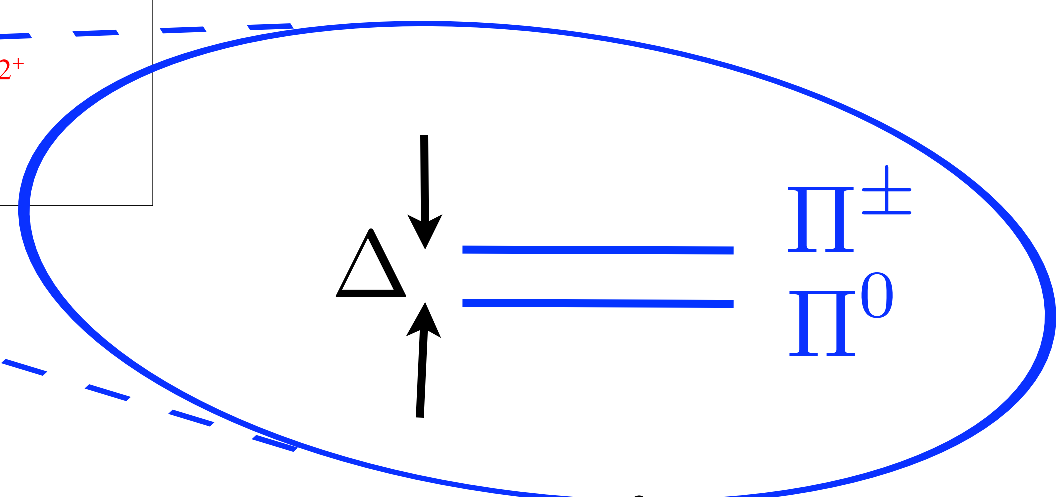




charged-neutral "hyperfine" mass splitting

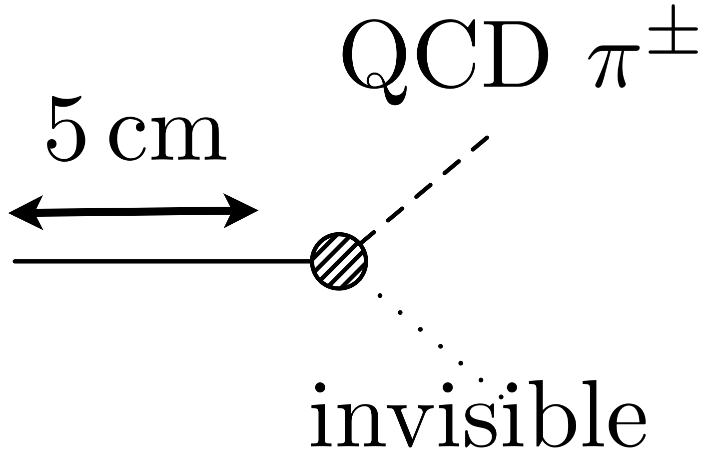


$$\Delta = m_{\Pi(1,\pm1)} - m_{\Pi(10)} \approx \alpha_2 m_W \sin^2 \frac{\theta_W}{2} \approx 170\,\mathrm{MeV}$$

Decay of excited states

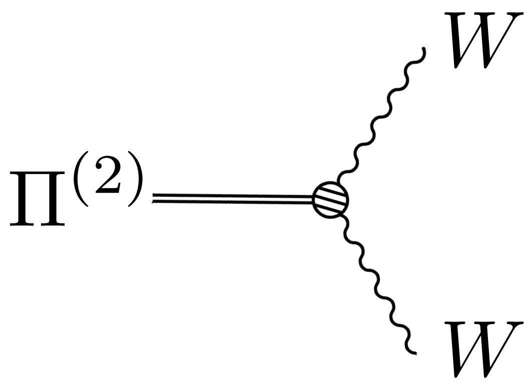
charged decay rate

$$\Gamma(\Pi^{(1,\pm 1)} \to \Pi^{(10)} + \pi^\pm) = \frac{4G_F^2}{\pi}\Delta^2\sqrt{\Delta^2 - m_\pi^2} f_\pi^2 \sim 1/(5\,\mathrm{cm})$$

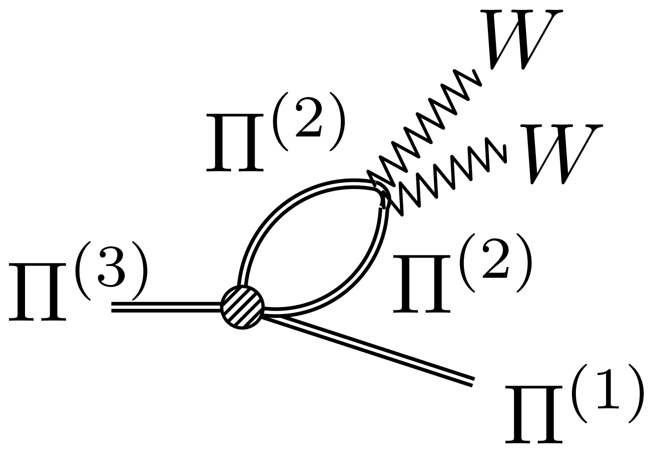


⇒ interesting collider signature

excited G-even states decay to SM



excited G-odd states decay to LGP + SM



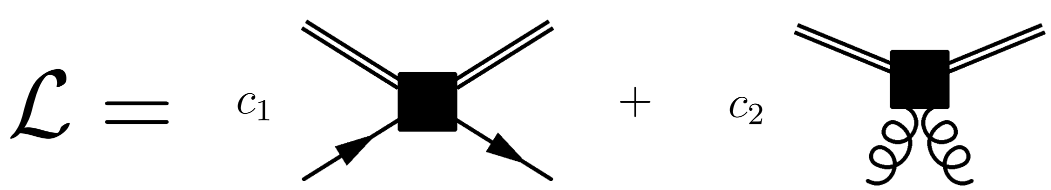
Enter heavy particle effective theory

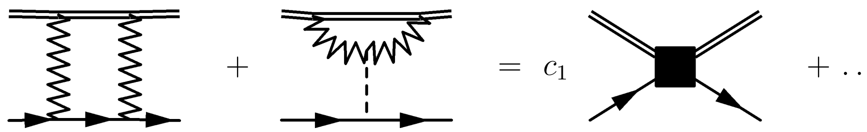
Found a mechanism that generates an isotriplet of real scalars

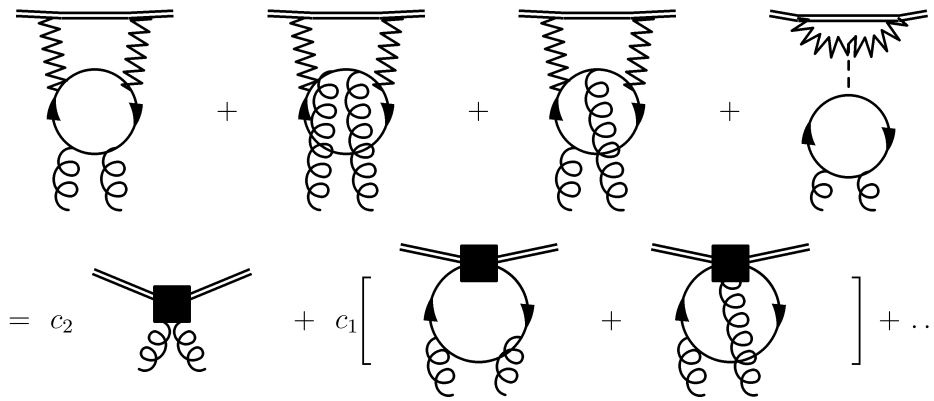
If the neutral component is a significant component of thermal relic dark matter, can estimate it's mass in the ~TeV range

Consider any such SU(2) electroweak multiplet

Universal properties emerge in the limit M >> m_W, described by the relevant heavy particle effective theory







Scattering on nucleon is completely determined, up to controlled corrections

$$m_W/M, \quad \Lambda^2_{\rm QCD}/m_c^2, \quad m_b/m_W \ldots$$

Multiple scales:

Renormalization analysis required to sum large logarithms

$$\alpha_s(\mu)\log\frac{m_t}{\mu} \sim \alpha_s(1\,\mathrm{GeV})\log\frac{170\,\mathrm{GeV}}{1\,\mathrm{GeV}}$$

Consider effective theory at each scale:

mass scale	Energy ↑	effective theory	
		$?\,,\mathrm{SM}$	
M		$\phi_v\,,\mathrm{SM}$	} Universal to any SU(2)-multiplet scenario
$m_W \sim m_h \sim m_t$			
		$\phi_v^{(Q=0)}\,, u, d, s, c, b, g$	} Universal to theories without low-mass non-SM fields
$m_b\,,\quad m_c$			
		$\phi_v^{(Q=0)}\,, u, d, s, g$	
Λ_{QCD}			

Electroweak symmetric theory

Operator basis

Building blocks:

$$\phi_v(x)\,, \quad v^\mu\,, \quad D_{\perp\mu} = D_\mu - v^\mu v\cdot D$$

Everything not forbidden is allowed:

$$\mathcal{L}_\phi = \phi_v^* \Big\{ iv\cdot D - c_1 \frac{D_\perp^2}{2M} + c_2 \frac{D_\perp^4}{8M^3} + g_2 c_D \frac{v^\alpha [D_\perp^\beta, W_{\alpha\beta}]}{8M^2} + i g_2 c_M \frac{\{D_\perp^\alpha, [D_\perp^\beta, W_{\alpha\beta}]\}}{16M^3}$$
$$+ g_2^2 c_{A1} \frac{W^{\alpha\beta} W_{\alpha\beta}}{16M^3} + g_2^2 c_{A2} \frac{v_\alpha v^\beta W^{\mu\alpha} W_{\mu\beta}}{16M^3} + g_2^2 c_{A3} \frac{\mathrm{Tr}(W^{\alpha\beta} W_{\alpha\beta})}{16M^3} + g_2^2 c_{A4} \frac{\mathrm{Tr}(v_\alpha v^\beta W^{\mu\alpha} W_{\mu\beta})}{16M^3}$$
$$+ g_2^2 c'_{A1} \frac{\epsilon^{\mu\nu\rho\sigma} W_{\mu\nu} W_{\rho\sigma}}{16M^3} + g_2^2 c'_{A2} \frac{\epsilon^{\mu\nu\rho\sigma} v^\alpha v_\mu W_{\nu\alpha} W_{\rho\sigma}}{16M^3} + g_2^2 c'_{A3} \frac{\epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}(W_{\mu\nu} W_{\rho\sigma})}{16M^3}$$
$$+ g_2^2 c'_{A4} \frac{\epsilon^{\mu\nu\rho\sigma} v^\alpha v_\mu \mathrm{Tr}(W_{\nu\alpha} W_{\rho\sigma})}{16M^3} + \dots \Big\} \phi_v\,,$$

Electroweak symmetric theory

Operator basis

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$$\phi_v(x)\,, \quad v^\mu\,, \quad D_{\perp\mu} = D_\mu - v^\mu v\cdot D$$

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$$+ g_2^2 c_{A1}\frac{W^{\alpha\beta}W_{\alpha\beta}}{16M^3} + g_2^2 c_{A2}\frac{v_\alpha v^\beta W^{\mu\alpha}W_{\mu\beta}}{16M^3} + g_2^2 c_{A3}\frac{\mathrm{Tr}(W^{\alpha\beta}W_{\alpha\beta})}{16M^3} + g_2^2 c_{A4}\frac{\mathrm{Tr}(v_\alpha v^\beta W^{\mu\alpha}W_{\mu\beta})}{16M^3}$$

$$+ g_2^2 c'_{A1}\frac{\epsilon^{\mu\nu\rho\sigma}W_{\mu\nu}W_{\rho\sigma}}{16M^3} + g_2^2 c'_{A2}\frac{\epsilon^{\mu\nu\rho\sigma}v^\alpha v_\mu W_{\nu\alpha}W_{\rho\sigma}}{16M^3} + g_2^2 c'_{A3}\frac{\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}(W_{\mu\nu}W_{\rho\sigma})}{16M^3}$$

$$+ g_2^2 c'_{A4}\frac{\epsilon^{\mu\nu\rho\sigma}v^\alpha v_\mu \mathrm{Tr}(W_{\nu\alpha}W_{\rho\sigma})}{16M^3} + \dots \Big\}\phi_v\,,$$

Generalized polarizability operators

Standard model interactions

$$\mathcal{L}_{\phi,\mathrm{SM}} = \phi_v^* \Big\{ c_H \frac{H^\dagger H}{M} + \dots + c_Q \frac{t_J^a \bar{Q}_L \tau^a \slashed{v} Q_L}{M^2} + c_X \frac{i\bar{Q}_L \tau^a \gamma^\mu Q_L \{t_J^a, D_\mu\}}{2M^3} + c_{DQ} \frac{\bar{Q}_L \slashed{v}\, iv\cdot D Q_L}{M^3}$$

$$+ c_{Du} \frac{\bar{u}_R \slashed{v}\, iv\cdot D u_R}{M^3} + c_{Dd} \frac{\bar{d}_R \slashed{v}\, iv\cdot D d_R}{M^3} + c_{Hd} \frac{\bar{Q}_L H d_R + h.c.}{M^3} + c_{Hu} \frac{\bar{Q}_L \tilde{H} u_R + h.c.}{M^3}$$

$$+ g_3^2 c_{A1}^{(G)} \frac{G^{A\,\alpha\beta} G^A_{\alpha\beta}}{16M^3} + g_3^2 c_{A2}^{(G)} \frac{v_\alpha v^\beta G^{A\,\mu\alpha} G^A_{\mu\beta}}{16M^3} + g_3^2 c_{A1}^{(G)\prime} \frac{\epsilon^{\mu\nu\rho\sigma} G^A_{\mu\nu} G^A_{\rho\sigma}}{16M^3} + g_3^2 c_{A2}^{(G)\prime} \frac{\epsilon^{\mu\nu\rho\sigma} v^\alpha v_\mu G^A_{\nu\alpha} G^A_{\rho\sigma}}{16M^3}$$

$$+ \dots \Big\} \phi_v \,.$$

Reparameterization invariance:

$$c_Q = c_X$$

All of these are suppressed by 1/M

(Ignore for now, but give universal subleading corrections)

Low energy theory

Operator basis

$$\mathcal{L} = \mathcal{L}_{\phi_0} + \mathcal{L}_{\rm SM} + \mathcal{L}_{\phi_0,{\rm SM}} + \dots\,,$$

Heavy neutral scalar:

$$\mathcal{L}_{\phi_0} = \phi^*_{v,Q=0}\left\{ iv\cdot\partial - \frac{\partial_\perp^2}{2M_{(Q=0)}} + \mathcal{O}(1/m_W^3) \right\}\phi_{v,Q=0}$$

c_D=0 (reality constraint)

SM interactions:

$$\mathcal{L}_{\phi_0,{\rm SM}} = \frac{1}{m_W^3}\phi_v^*\phi_v\left\{ \sum_q \left[c_{1q}^{(0)} O_{1q}^{(0)} + c_{1q}^{(2)} v_\mu v_\nu O_{1q}^{(2)\mu\nu} \right] + c_2^{(0)} O_2^{(0)} + c_2^{(2)} v_\mu v_\nu O_2^{(2)\mu\nu} \right\} + \dots$$

Convenient to choose basis of definite spin

$$O_{1q}^{(0)} = m_q \bar{q} q\,, \qquad O_2^{(0)} = (G^A_{\mu\nu})^2\,,$$

$$O_{1q}^{(2)\mu\nu} = \bar{q}\left(\gamma^{\{\mu} iD^{\nu\}} - \frac{1}{d} g^{\mu\nu} i\not{D} \right) q\,, \qquad O_2^{(2)\mu\nu} = -G^{A\mu\lambda} G^{A\nu}{}_\lambda + \frac{1}{d} g^{\mu\nu} (G^A_{\alpha\beta})^2\,.$$

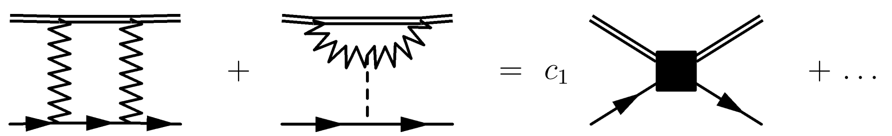
Matching (μ=M)

Heavy particle Feynman rules simplify matching calculations

quark operators

$$c_{1U}^{(0)}(\mu_t) = \mathcal{C}\left[-\frac{1}{x_h^2}\right]\,, \qquad c_{1D}^{(0)}(\mu_t) = \mathcal{C}\left[-\frac{1}{x_h^2} - |V_{tD}|^2\frac{x_t}{4(1+x_t)^3}\right]\,,$$

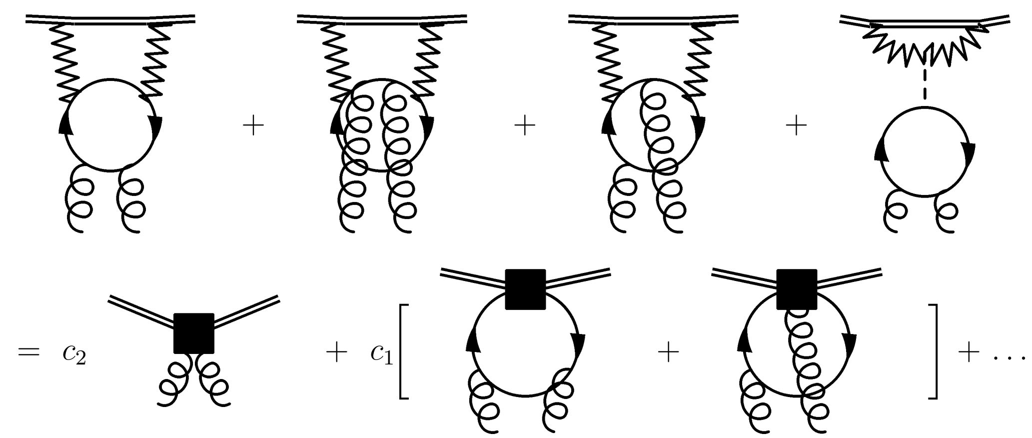
$$c_{1U}^{(2)}(\mu_t) = \mathcal{C}\left[\frac{2}{3}\right]\,, \qquad c_{1D}^{(2)}(\mu_t) = \mathcal{C}\left[\frac{2}{3} - |V_{tD}|^2\frac{x_t(3+6x_t+2x_t^2)}{3(1+x_t)^3}\right]\,,$$



gluon operators

$$c_2^{(0)}(\mu_t) = \mathcal{C}\frac{\alpha_s(\mu_t)}{4\pi}\left[\frac{1}{3x_h^2} + \frac{3+4x_t+2x_t^2}{6(1+x_t)^2}\right]\,,$$

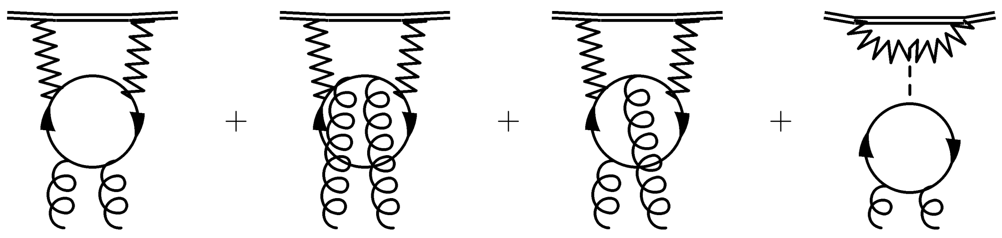
$$c_2^{(2)}(\mu_t) = \mathcal{C}\frac{\alpha_s(\mu_t)}{4\pi}\Bigg[-\frac{32}{9}\log\frac{\mu_t}{m_W} - 4 - \frac{4(2+3x_t)}{9(1+x_t)^3}\log\frac{\mu_t}{m_W(1+x_t)}$$
$$- \frac{4(12x_t^5 - 36x_t^4 + 36x_t^3 - 12x_t^2 + 3x_t - 2)}{9(x_t-1)^3}\log\frac{x_t}{1+x_t} - \frac{8x_t(-3+7x_t^2)}{9(x_t^2-1)^3}\log 2$$
$$- \frac{48x_t^6 + 24x_t^5 - 104x_t^4 - 35x_t^3 + 20x_t^2 + 13x_t + 18}{9(x_t^2-1)^2(1+x_t)}\Bigg]\,.$$



high scale matching for
quark + spin-0 gluon agree with
[Hisano, Ishiwata, Nagata, Takesako (2011)]

spin-2 gluon new

Full theory side:



$$i\mathcal{M} = -g_2^2 \int (dL) \left[\frac{1}{-v\cdot L + i0} + \frac{1}{v\cdot L + i0} \right] \frac{1}{(L^2 - m_W^2 + i0)^2} v_\mu v_\nu \Pi^{\mu\nu}(L)$$

quark propagator in background gluon field

Electroweak gauge invariance is immediate:

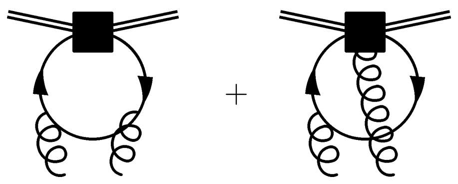
$$v^\mu \left[g_{\mu\mu'} - (1-\xi) \frac{L_\mu L_{\mu'}}{L^2 - \xi m_W^2} \right] = v_{\mu'} + \mathcal{O}(v\cdot L)$$

crossed and uncrossed diagrams cancel

Fock-Schwinger gauge (x.A=0) :

$$iS(p) = \frac{i}{\not p - m} + g \int (dq) \frac{i}{\not p - m} i \not A(q) \frac{i}{\not p - \not q - m}$$
$$+ g^2 \int (dq_1)(dq_2) \frac{i}{\not p - m} i\not A(q_1) \frac{i}{\not p - \not q_1 - m} i \not A(q_2) \frac{i}{\not p - \not q_1 - \not q_2 - m} + \dots$$

Effective theory side:



Ignoring quark masses, effective theory onshell loop diagrams vanish in dim.reg.

$$\int \frac{d^d L}{(2\pi)^d} f(L^2) = 0 = \frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}$$

After equating full theory = effective theory, all remaining divergences are UV

(can also work with finite quark masses)

Solution to RG equations

$$O_{1q}^{(0)} = m_q \bar{q} q \,, \qquad O_2^{(0)} = (G^A_{\mu\nu})^2 \,,$$

$$O_{1q}^{(2)\mu\nu} = \bar{q}\left(\gamma^{\{\mu} iD^{\nu\}} - \frac{1}{d} g^{\mu\nu} i\not{D}\right) q \,, \qquad O_2^{(2)\mu\nu} = -G^{A\mu\lambda} G^{A\nu}{}_{\lambda} + \frac{1}{d} g^{\mu\nu} (G^A_{\alpha\beta})^2$$

$$\frac{d}{d\log\mu} O_i^{(S)} = -\sum_j \gamma_{ij}^{(S)} O_j \qquad \frac{d}{d\log\mu} c_i^{(S)} = \sum_j \gamma_{ji}^{(S)} c_j^{(S)}$$

Spin 0:

$$c_2^{(0)}(\mu) = c_2^{(0)}(\mu_t) \frac{\frac{\beta}{g}[\alpha_s(\mu)]}{\frac{\beta}{g}[\alpha_s(\mu_t)]}$$

$$\hat{\gamma}^{(0)} = \left(\begin{array}{ccc|c} 0 & & & 0 \\ & \ddots & & \vdots \\ & & 0 & 0 \\ \hline -2\gamma'_m & \cdots & -2\gamma'_m & (\beta/g)' \end{array}\right)$$

$$c_1^{(0)}(\mu) = c_1^{(0)}(\mu_t) - 2[\gamma_m(\mu) - \gamma_m(\mu_t)] \frac{c_2^{(0)}(\mu_t)}{\frac{\beta}{g}[\alpha_s(\mu_t)]}$$

Spin 2:

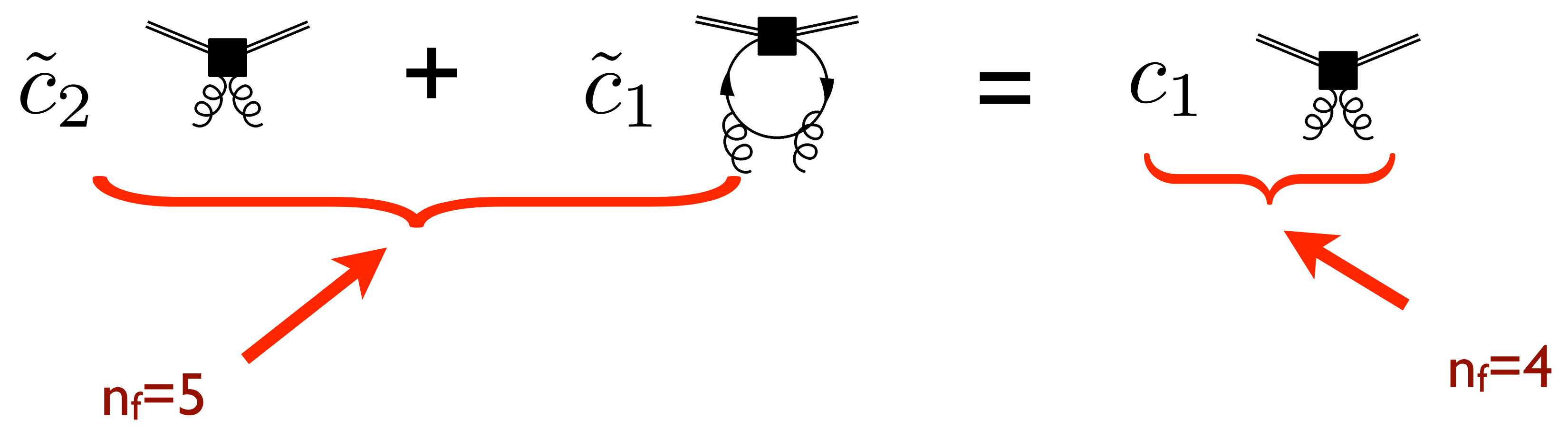
Diagonalize anomalous dimension matrix
(familiar from PDF analysis)

As check, can evaluate spin-2 matrix elements at high scale (spin-0 and spin-2 decoupled)

$$\hat{\gamma}^{(2)} = \frac{\alpha_s}{4\pi} \left(\begin{array}{ccc|c} \frac{64}{9} & & & -\frac{4}{3} \\ & \ddots & & \vdots \\ & & \frac{64}{9} & -\frac{4}{3} \\ \hline -\frac{64}{9} & \cdots & -\frac{64}{9} & \frac{4n_f}{3} \end{array}\right) + \ldots$$

Matching ($\mu = m_b$)

Integrate out heavy quarks



$$c_2^{(0)}(\mu_b) = \tilde{c}_2^{(0)}(\mu_b)\left(1 + \frac{4\tilde{a}}{3}\log\frac{m_b}{\mu_b}\right) - \frac{\tilde{a}}{3}\tilde{c}_{1b}^{(0)}(\mu_b)\left[1 + \tilde{a}\left(11 + \frac{4}{3}\log\frac{m_b}{\mu_b}\right)\right] + \mathcal{O}(\tilde{a}^3)$$

$$c_{1q}^{(0)}(\mu_b) = \tilde{c}_{1q}^{(0)}(\mu_b) + \mathcal{O}(\tilde{a}^2),$$

$$c_2^{(2)}(\mu_b) = \tilde{c}_2^{(2)}(\mu_b) - \frac{4\tilde{a}}{3}\log\frac{m_b}{\mu_b}\tilde{c}_{1b}^{(2)}(\mu_b) + \mathcal{O}(\tilde{a}^2),$$

$$c_{1q}^{(2)}(\mu_b) = \tilde{c}_{1q}^{(2)}(\mu_b) + \mathcal{O}(\tilde{a}),$$

[Ovrut, Schnitzer, 1982]
[Inami, Kubota, Okada, 1983]

Contribution to gluon operators familiar from h→gg

Heavy quark mass scheme enters at higher order

Charm quark treated similarly (after running to mc)

Hadronic matrix elements: Spin - 0

$$\langle N(k)|T^{\mu\nu}|N(k)\rangle = \frac{k^\mu k^\nu}{m_N} = \frac{1}{m_N}\left(k^\mu k^\nu - \frac{1}{4}g^{\mu\nu}m_N^2\right) + m_N \frac{1}{4}g^{\mu\nu}$$

Spin-0 operators determine contributions to nucleon mass

$$m_N = (1-\gamma_m)\sum_q \langle N|m_q\bar{q}q|N\rangle + \frac{\beta}{2g}\langle N|(G^a_{\mu\nu})^2|N\rangle$$

$$\langle N|O^{(0)}_{1q}|N\rangle \equiv m_N f^{(0)}_{q,N}\,, \qquad \frac{-9\alpha_s(\mu)}{8\pi}\langle N|O^{(0)}_2(\mu)|N\rangle \equiv m_N f^{(0)}_{G,N}(\mu)$$

significant uncertainty in this quantity

$$m_N(f^{(0)}_{u,N} + f^{(0)}_{d,N}) \approx \Sigma_{\pi N}\,, \quad m_N f^{(0)}_{s,N} = \frac{m_s}{m_u+m_d}(\Sigma_{\pi N} - \Sigma_0) = \Sigma_s$$

$$f^{(0)}_{G,N}(\mu) \approx 1 - \sum_{q=u,d,s} f^{(0)}_{q,N}$$

but NLO, NNLO corrections significant and are included

Hadronic matrix elements: Spin - 2

$$\langle N(k)|T^{\mu\nu}|N(k)\rangle = \frac{k^\mu k^\nu}{m_N} = \frac{1}{m_N}\left(k^\mu k^\nu - \frac{1}{4}g^{\mu\nu}m_N^2\right) + m_N\frac{1}{4}g^{\mu\nu}$$

Spin-2 operators determine momentum fraction carried by partons

$$\langle N|O_{1q}^{(2)\mu\nu}(\mu)|N\rangle \equiv \frac{1}{m_N}\left(k^\mu k^\nu - \frac{g^{\mu\nu}}{4}m_N^2\right) f_{q,N}^{(2)}(\mu)$$

$$\langle N|O_{2}^{(2)\mu\nu}(\mu)|N\rangle \equiv \frac{1}{m_N}\left(k^\mu k^\nu - \frac{g^{\mu\nu}}{4}m_N^2\right) f_{G,N}^{(2)}(\mu)$$

μ(GeV)	$f_{u,p}^{(2)}(\mu)$	$f_{d,p}^{(2)}(\mu)$	$f_{s,p}^{(2)}(\mu)$	$f_{G,p}^{(2)}(\mu)$
1.0	0.404(6)	0.217(4)	0.024(3)	0.36(1)
1.2	0.383(6)	0.208(4)	0.027(2)	0.38(1)
1.4	0.370(5)	0.202(4)	0.030(2)	0.40(1)

[MSTW 0901.0002]

$$f_{q,p}^{(2)}(\mu) = \int_0^1 dx\, x[q(x,\mu) + \bar{q}(x,\mu)]$$

Approximate isospin symmetry:

$$f_{u,n}^{(2)} = f_{d,p}^{(2)}\,, \quad f_{d,n}^{(2)} = f_{u,p}^{(2)}\,, \quad f_{s,n}^{(2)} = f_{s,p}^{(2)}$$

Cross section

Parameter	Value
$\lvert V_{td}\rvert$	~ 0
$\lvert V_{ts}\rvert$	~ 0
$\lvert V_{tb}\rvert$	~ 1
m_u/m_d	$0.49(13)$
m_s/m_d	$19.5(2.5)$
$\Sigma_{\pi N}^{\rm lat}$	$0.047(9)$ GeV
$\Sigma_s^{\rm lat}$	$0.050(8)$ GeV
$\Sigma_{\pi N}$	$0.064(7)$ GeV
Σ_0	$0.036(7)$ GeV
m_W	80.4 GeV
m_t	172 GeV
m_b	4.75 GeV
m_c	1.4 GeV
m_N	0.94 GeV
$\alpha_s(m_Z)$	0.118
$\alpha_2(m_Z)$	0.0338
m_h	?

Consider result as a function of higgs boson mass

Cross section is completely determined, given standard model inputs

$$\sigma_{A,Z} = \frac{m_r^2}{\pi}\,|Z\mathcal{M}_p + (A-Z)\mathcal{M}_n|^2 \approx \frac{m_r^2 A^2}{\pi}|\mathcal{M}_p|^2$$

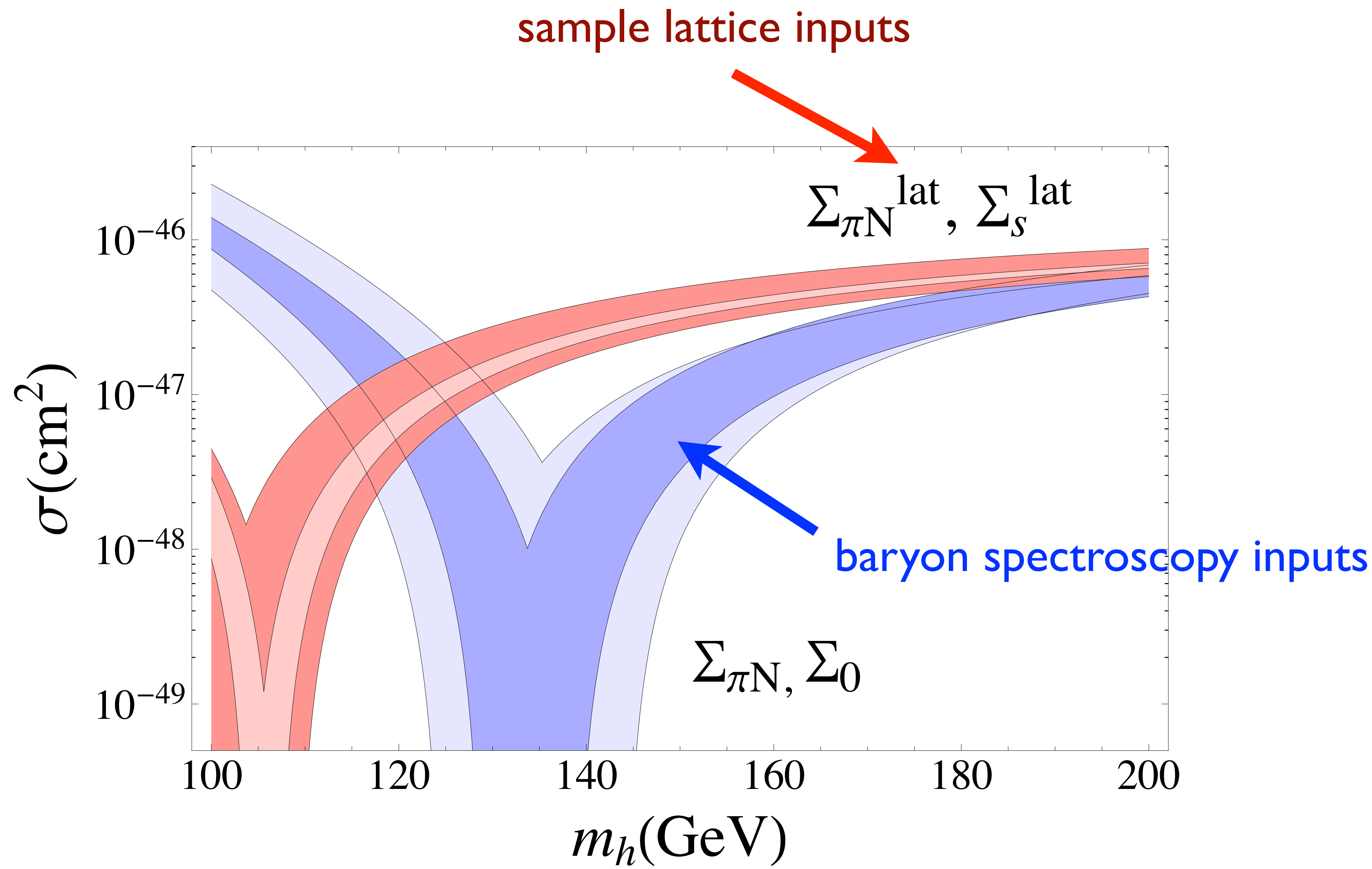
Previous estimates range over several orders of magnitude, errors not specified

[Cirelli, Strumia (2006-2009)]

[Essig (2008)]

[Hisano, Ishiwata, Nagata, Takesako (2011)]

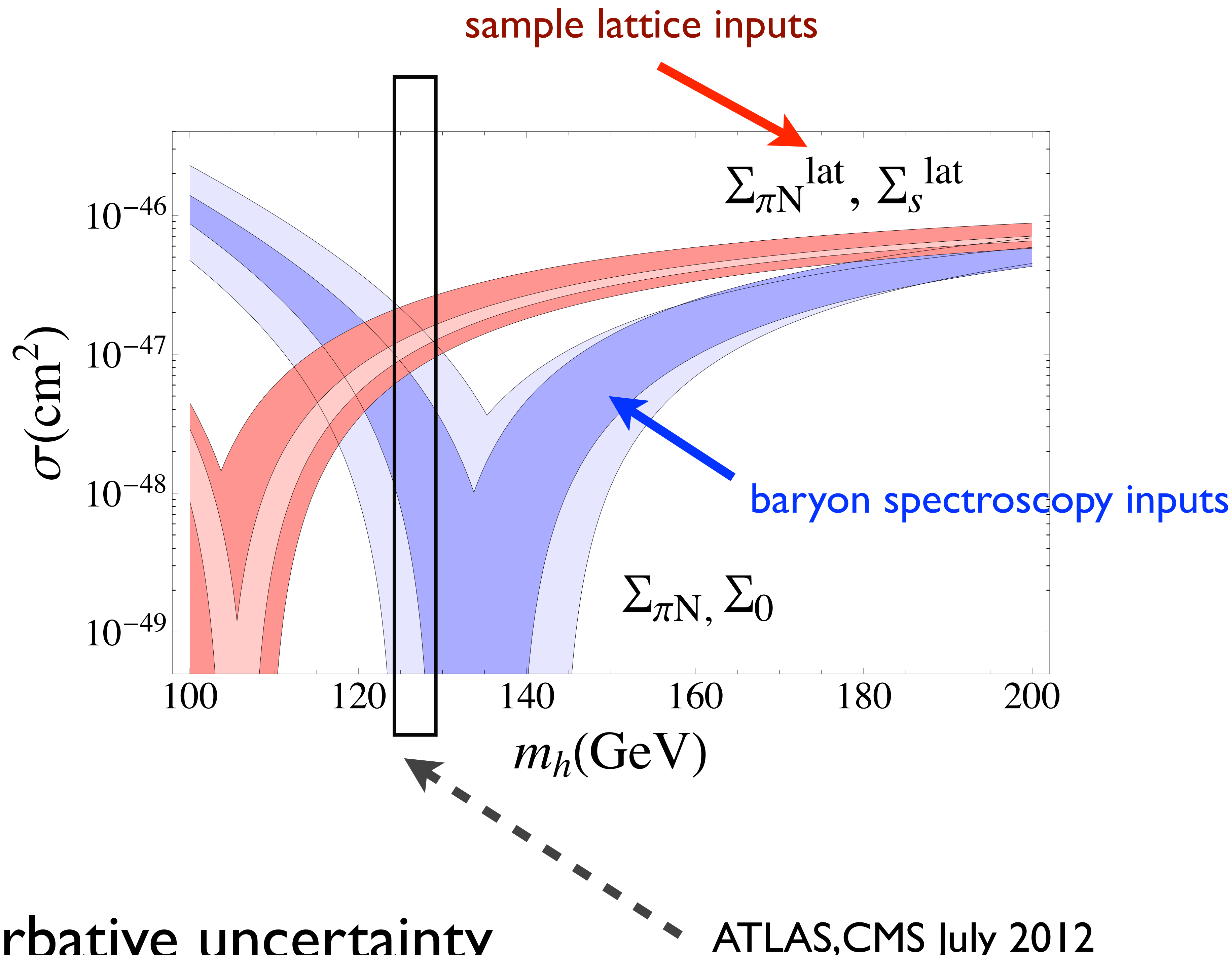
Numerical benchmark: low velocity, spin independent cross section on nucleon



Dark band: perturbative uncertainty

Light band: hadronic input uncertainty

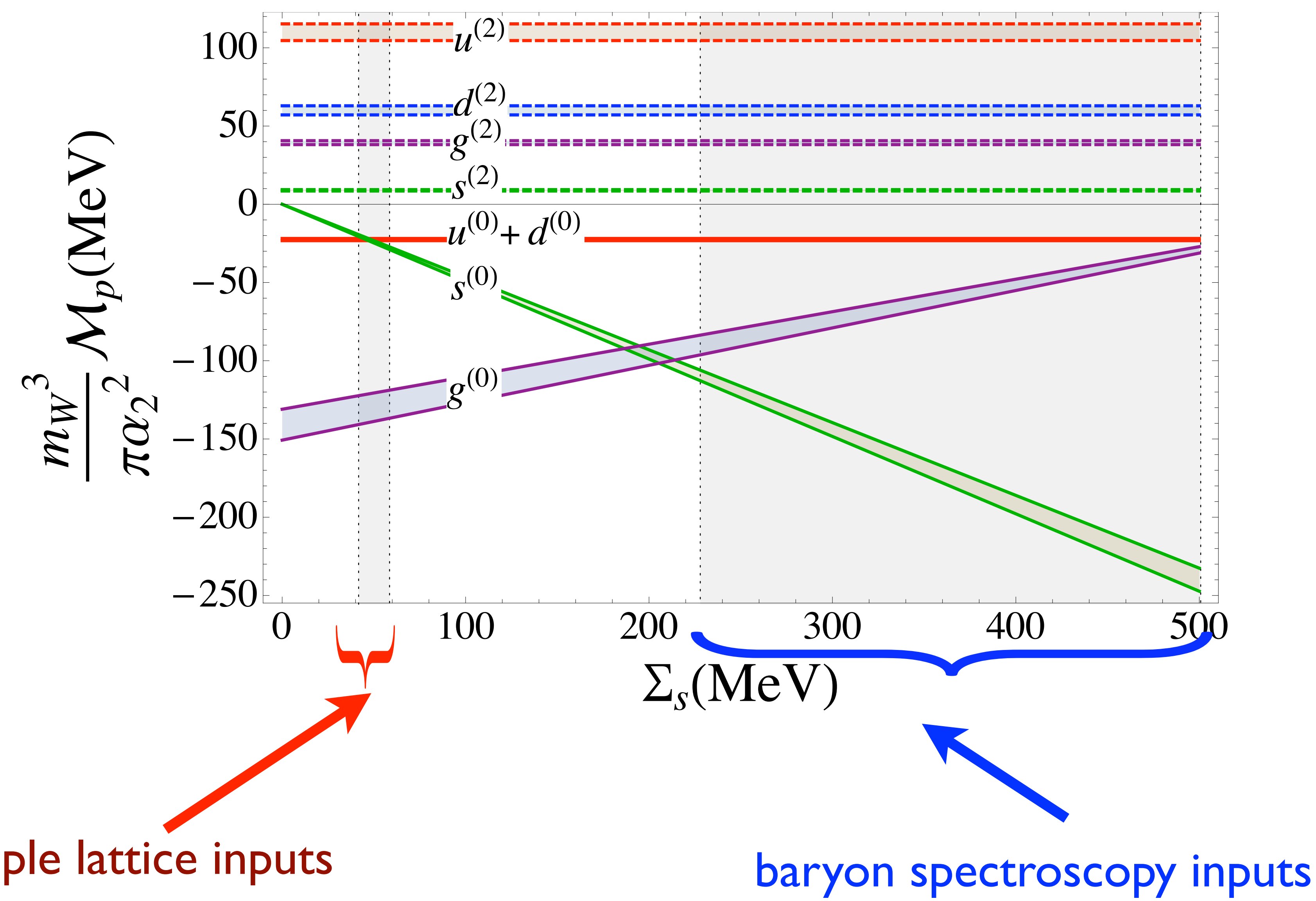
Numerical benchmark: low velocity, spin independent cross section on nucleon



Dark band: perturbative uncertainty
Light band: hadronic input uncertainty

Strange quark scalar matrix element dependence

strange matrix element (and correlated gluon matrix element) a prominent uncertainty

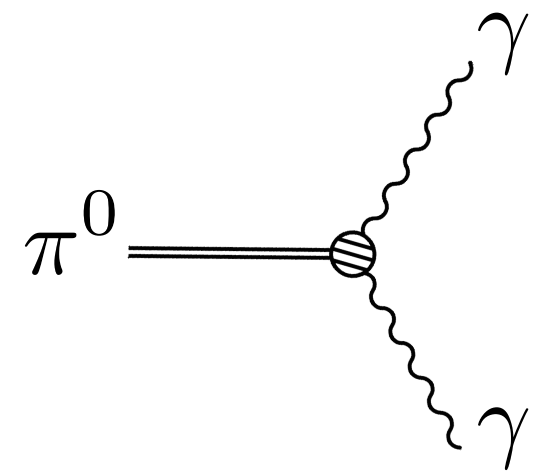


Summary and Outlook

- universal cross section: (small) target for future experiments
- heavy particle formalism applies to DM candidates heavy compared to m_W (recall $m_h/m_t << 1$ often useful)
- RG analysis universal to DM computations (not just $M>>m_W$): error analysis!
- Simplified computations, e.g. 2-loop matching for gluon operators

Recall in QCD:

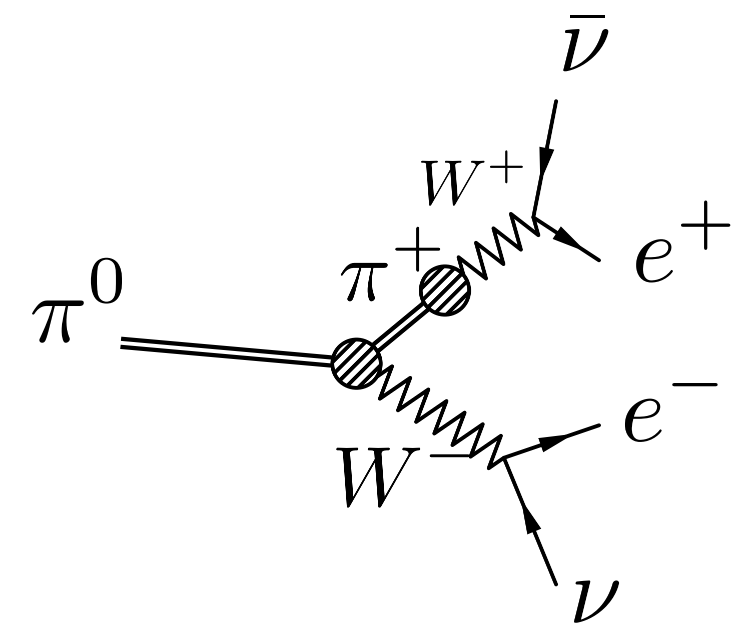
2 flavors: π^+ π^- π^0 odd under G parity



Neutral pion is not stable

These decay modes would be absent if:

- no coupling to $U(1)_Y$
- both L and R are 2 of SU(2)



3 flavors

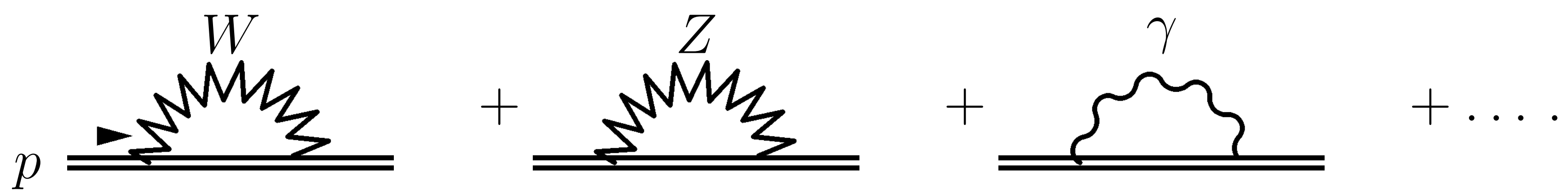
Similarly, for SU(3) multiplet of QCD,
consider (u,d,s) to transform as spin-1 of SU(2)

π^+ - π^-, K^+ - K^-, K^0 - $\bar{K}^0$ *odd* under "G", remaining π, K, η *even*

In contrast to "NGB parity" (all pions odd), this parity not broken by "anomalous" five-pion Wess Zumino Witten interactions

Universal mass splitting induced by EWSB

$$-i\Sigma(p) =$$



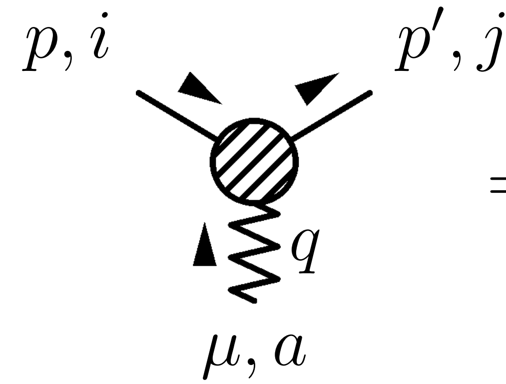
$$-i\Sigma_2(v\cdot p) = -g_2^2 \int \frac{d^dL}{(2\pi)^L}\frac{1}{v\cdot(L+p)}\left[J^2\frac{1}{L^2-m_W^2} + J_3^2\left(\frac{c_W^2}{L^2-m_Z^2} - \frac{1}{L^2-m_W^2} + \frac{s_W^2}{L^2}\right)\right] + \mathcal{O}(1/M)$$

heavy particle Feynman rules

$$\delta M = \Sigma(v\cdot p = 0) = \alpha_2 m_W\left[-\frac{1}{2}J^2 + \sin^2\frac{\theta_W}{2}J_3^2\right]$$

$$M_{(Q)} - M_{(Q=0)} = \alpha_2 Q^2 m_W \sin^2\frac{\theta_W}{2} + \mathcal{O}(1/M) \approx (170\,\mathrm{MeV})Q^2$$

Sample matching calculation



$$= i g_2 (p+p')^\mu F(q^2) (t_J^a)_{ji}$$

full theory:

$$F(q^2) = 1 + \frac{g_2^2}{(4\pi)^2}\frac{q^2}{M^2}\left\{ C_2(r)\left[-\frac{2}{3\epsilon_{\rm IR}} - 1 + \frac{4}{3}\log\frac{M}{\mu}\right] + C_2(G)\left[-\frac{1}{24\epsilon_{\rm IR}} + \frac{3}{4} + \frac{1}{12}\log\frac{M}{\mu}\right]\right\} + \dots .$$

full theory = effective theory:

$$F(0) - F'(0)\boldsymbol{q}^2 + \dots = 1 - c_D \frac{\boldsymbol{q}^2}{8M^2} + \dots ,$$

$$(p+p')^i \left[-F(0)\left(1 - \frac{\boldsymbol{p}^2 + \boldsymbol{p}'^2}{4M^2}\right) + F'(0)\boldsymbol{q}^2 + \dots \right]$$

$$= (p+p')^i \left[-1 + \frac{\boldsymbol{p}^2 + \boldsymbol{p}'^2}{4M^2} + c_M \frac{\boldsymbol{q}^2}{8M^2}\right] + q^i \frac{\boldsymbol{p}'^2 - \boldsymbol{p}^2}{8M^2}(c_D - c_M) + \dots .$$

$\Rightarrow$

$$c_D(\mu) = c_M(\mu) = \frac{\alpha_2(\mu)}{(4\pi)}\left[-8J(J+1) + 12 + \left(\frac{32J(J+1)}{3} + \frac{4}{3}\right)\log\frac{M}{\mu}\right]$$