

The Neutrino Flavour Puzzle in the Light of “Large” $\theta_{13}^{\text{PMNS}}$

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The last year has been an exciting time for models of neutrino masses and mixings ...

$$\theta_{13}^{PMNS} \approx 9^\circ \pm 1^\circ$$

Daya Bay Collaboration, arXiv:1203.1669

T2K, Minos, Double CHOOZ, Daya Bay, RENO

Finally, out of the many models ...

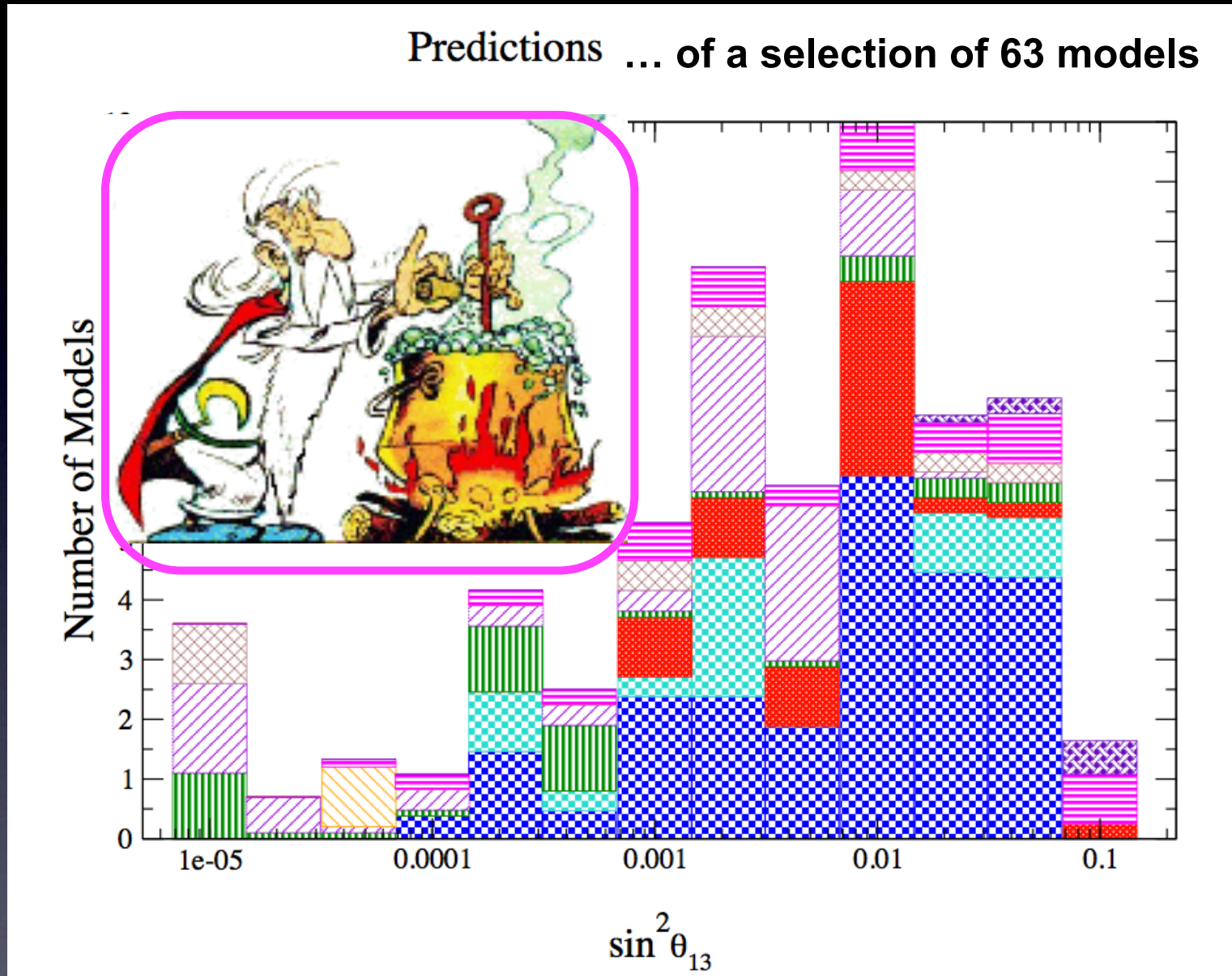


Figure shows only a small subset
of the existing models ... !

based on figure from [Albright, Mu-Chun Chen \('06\)](#)

Many new ideas and studies were inspired by the recent θ_{13} -measurements ...

Many works by various authors, e.g.:

A. Adulpravitchai, Y. H. Ahn, C. H. Albright, G. Altarelli, A. Aranda, T. Araki, F. Bazzocchi, W. Buchmuller, P. S. Bhupal Dev, G. C. Branco, Q.-H. Cao, H.-Y. Cheng, I. K. Cooper, S. Dev, G. Blankenburg, C. Bonilla, F. Gonzalez Canales, W. Chao, J.-M. Chen, M.-C. Chen, X. Chu, A. Datta, K. N. Deepthi, M. Dhen, D. A. Dicus, G.-J. Ding, P. V. Dong, V. Domcke, L. Dorame, B. Dutta, D. A. Eby, L. Everett, R. P. Feger, F. Feruglio, P. Ferreira, P. H. Frampton, M. Fukugita, R. R. Gautam, S. -F. Ge, D. K. Ghosh, R. Gonzalez Felipe, S. Gollu, S. Gupta, W. Grimus, C. Gross, N. Haba, C. Hagedorn, T. Hambye, J. E. Kim, Y. Koide, K. Hashimoto, K. Harigaya, H. -J. He, X. -G. He, J. Heek, D. Hernandez, M. Holthausen, R. S. Hundi, M. Ibe, H. Ishimori, F. R. Joaquim, A. S. Joshipura, S. K. Kang, T. W. Kephart, S. Khalil, S. F. King, T. Kobayashi, S. Kumar, L. Lavoura, X.-Q. Li, H. N. Long, P. O. Ludl, C. Luhn, B. Q. Ma, E. Ma, S. K. Majee, K.T. Mahanthappa, D. Marzocca, V. Maurer, D. Meloni, A. Merle, A. Meroni, R. Mohanta, R. N. Mohapatra, E. Molinaro, A. Mondragon, M. Mondragon, S. Morisi, C. H. Nam, H. Nishiura, S. Oh, H. Okada, K. M. Patel, K. M. Parattu, E. Peinado, S. T. Petcov, N. Qin, A. Rashed, W. W. Repko, A. D. Rojas, W. Rodejohann, A. Romanino, G. G. Ross, S. Rigolin, M. A. Schmidt, K. Schmitz, M. Severson, M.-S. Seo, H. Serodio, Y. Shimizu, J. I. Silva-Marcos, L. Singh, K. Siyeon, C. Sluka, A. Yu. Smirnov, M. Spinrath, E. Stamou, A. J. Stuart, R. Takahashi, M. Tanimoto, R. d. A. Toorop, J. W. F. Valle, I. d. M. Varzielas, V. V. Vien, B. Wang, Q. Wang, A. Watanabe, D. Wegman, A. Wingerter, Yue-Liang Wu, Z. -Z. Xing, T. T. Yanagida, W.-M. Yang, B. Zaldívar, F. -R. Yin, A. Zee, H. Zhang, Y. -j. Zheng, J.-J. Zhong, S. Zhou, R. Zwicky, ...

sorry for incompleteness!

Many unknowns remain ...

- What are the values of the Dirac CP phase δ^{PMNS}
- Is the mass scheme “normal” or “inverse”, i.e., what is $\text{sgn}(\Delta m_{31}^2)$?
- What is the deviation of $\theta_{23}^{\text{PMNS}}$ from maximal (i.e. from 45°)
- What is the absolute neutrino mass scale?
- Are neutrino masses of Dirac- or Majorana-type?
- If they are Majorana-type, what are the values of the Majorana phases?

Great for theorists! It means we can still make predictions ... !

Two main “neutrino puzzles”



Neutrino masses:
Why so small?
How to extend the SM?

Lepton mixing:
Why so large (compared to
the quark mixings)?

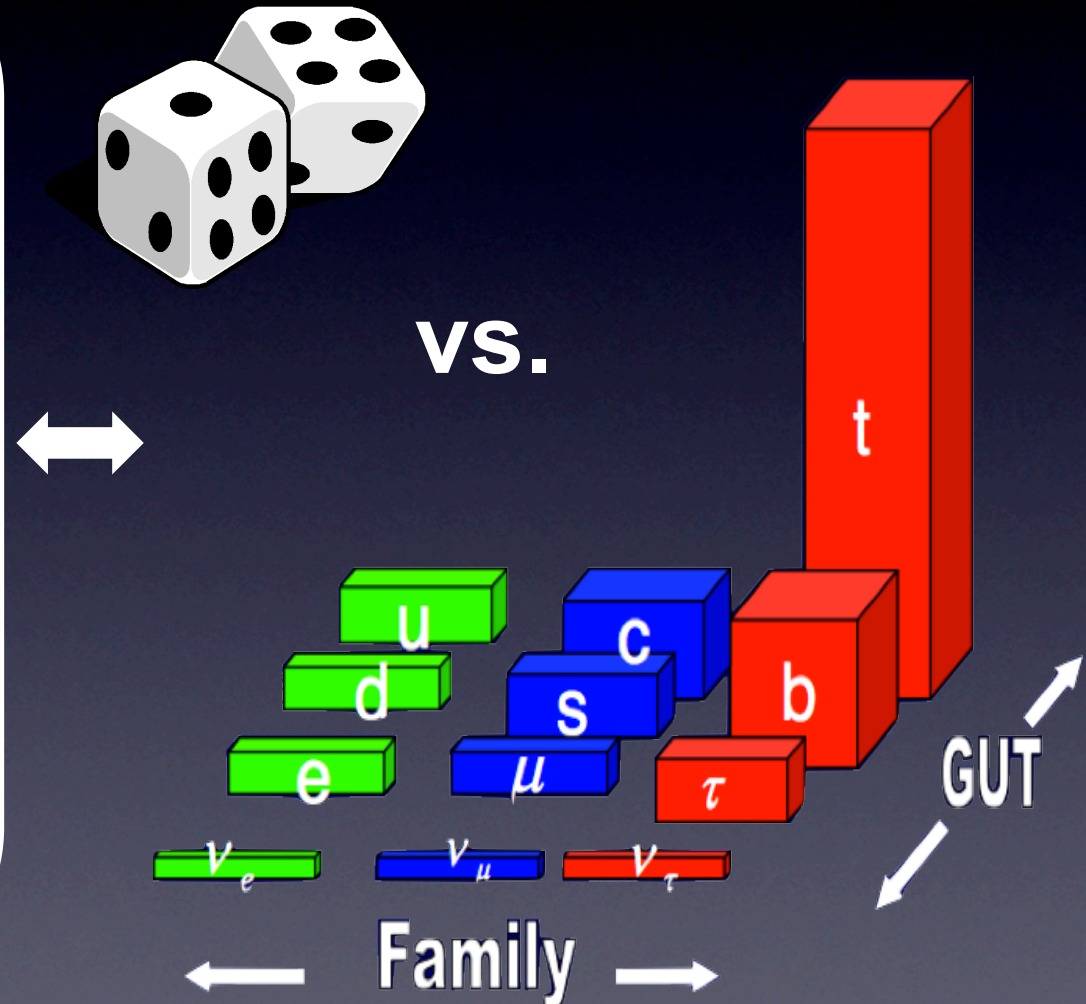
Main question for this talk:

What can we learn from the experimental results about the origin of the flavour structure?

Ideas and approaches ...

Various top-down possibilities:

- Random - Anarchical?
Hall, Murayama, Weiner ('99), ...
- Family symmetries?
Froggatt, Nielsen ('79); ... Ma, Rajasekaran, Babu, Valle, King, Ross, Altarelli, Feruglio, ...
- Grand Unified Theories (GUTs)?
Georgi, Glashow ('74), Fritsch, Minkowski ('75), ... Mohapatra, Senjanovic, Pati, Salam, Raby, Bajc, Aulakh, Vissani, ...
- Extra dimensions, string theory, ...
Arkani-Hamed, Dimopoulos, Dvali ('98), Dienes, Dudas, Ghergetta, March-Russel, Volkas, ...



Bottom-up suggestions ...

Tri-Bimaximal mixing:

$$\theta_{12}^{\text{PMNS}} = \arcsin\left(\frac{1}{\sqrt{3}}\right) \approx 35.3^\circ$$

$$\theta_{13}^{\text{PMNS}} = 0$$

$$\theta_{23}^{\text{PMNS}} = 45^\circ$$

Bimaximal mixing:

$$\theta_{12}^{\text{PMNS}} = 45^\circ$$

$$\theta_{13}^{\text{PMNS}} = 0$$

$$\theta_{23}^{\text{PMNS}} = 45^\circ$$

... but may be useful as leading order structures!?

Observations and suggestions:

- Tri-bimaximal (TB) mixing?
Harrison, Perkins, Scott ('02)
- Bimaximal mixing? Vissani ('97)
Barger, Pakvasa, Weiler, Whisnant ('98)
- Quark lepton complementarity (QLC):
 $\theta_{12}^{\text{PMNS}} + \theta_C = 45^\circ$?
Smirnov ('04); Raidal ('04), Minakata, Smirnov ('04)
- $\theta_{13}^{\text{PMNS}} = \theta_C / \sqrt{2}$?
- ...

Top-down vs. bottom-up ...

Various top-down possibilities:

- Random - Anarchical?
- Family symmetries?
- Grand Unified Theories (GUTs)?
- Extra dimensions
- String theory, ...
- ...



Observations and suggestions:

- Tri-bimaximal (TB) mixing?
- Bimaximal mixing?
- Quark lepton complementarity (QLC):
 $\theta_{12}^{\text{PMNS}} + \theta_C = 45^\circ$?
- $\theta_{13}^{\text{PMNS}} = \theta_C / \sqrt{2}$?
- ...

Various ideas and new developments have emerged ... and I can only discuss a small selection of them in my talk ...

One aspect which got a lot of attention:

Neutrino mixing patterns \leftrightarrow
Family symmetries ...

Most prominent example: Tri-bimaximal mixing $\leftrightarrow A_4, S_4, \dots$

Tri-bimaximal mixing

Harrison, Perkins, Scott ('02)

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

$\theta_{12} = 35.3^\circ$ $\theta_{23} = 45^\circ$ $\theta_{13} = 0^\circ$

➤ Remark: No conserved symmetry can enforce TB mixing!

Simple argument: Large mixing is no fixed point of the RG evolution

RGEs for the leptonic mixing angles (in LO in θ_{13} ; in the MSSM):

$$\dot{\theta}_{12} \approx -\frac{y_\tau^2}{32\pi^2} \sin 2\theta_{12} s_{23}^2 \frac{|m_1 e^{i\varphi_1} + m_2 e^{i\varphi_2}|^2}{\Delta m_{\text{sol}}^2} + \mathcal{O}(\theta_{13})$$

$$\dot{\theta}_{13} \approx \frac{y_\tau^2}{32\pi^2} \sin 2\theta_{12} \sin 2\theta_{23} \frac{m_3}{\Delta m_{\text{atm}}^2} [m_1 \cos(\varphi_1 - \delta) - m_2 \cos(\varphi_2 - \delta)] + \mathcal{O}(\theta_{13})$$

$$\dot{\theta}_{23} \approx -\frac{y_\tau^2}{32\pi^2} \sin 2\theta_{23} \frac{1}{\Delta m_{\text{atm}}^2} [c_{12}^2 |m_2 e^{i\varphi_2} + m_3|^2 + s_{12}^2 |m_1 e^{i\varphi_1} + m_3|^2] + \mathcal{O}(\theta_{13})$$

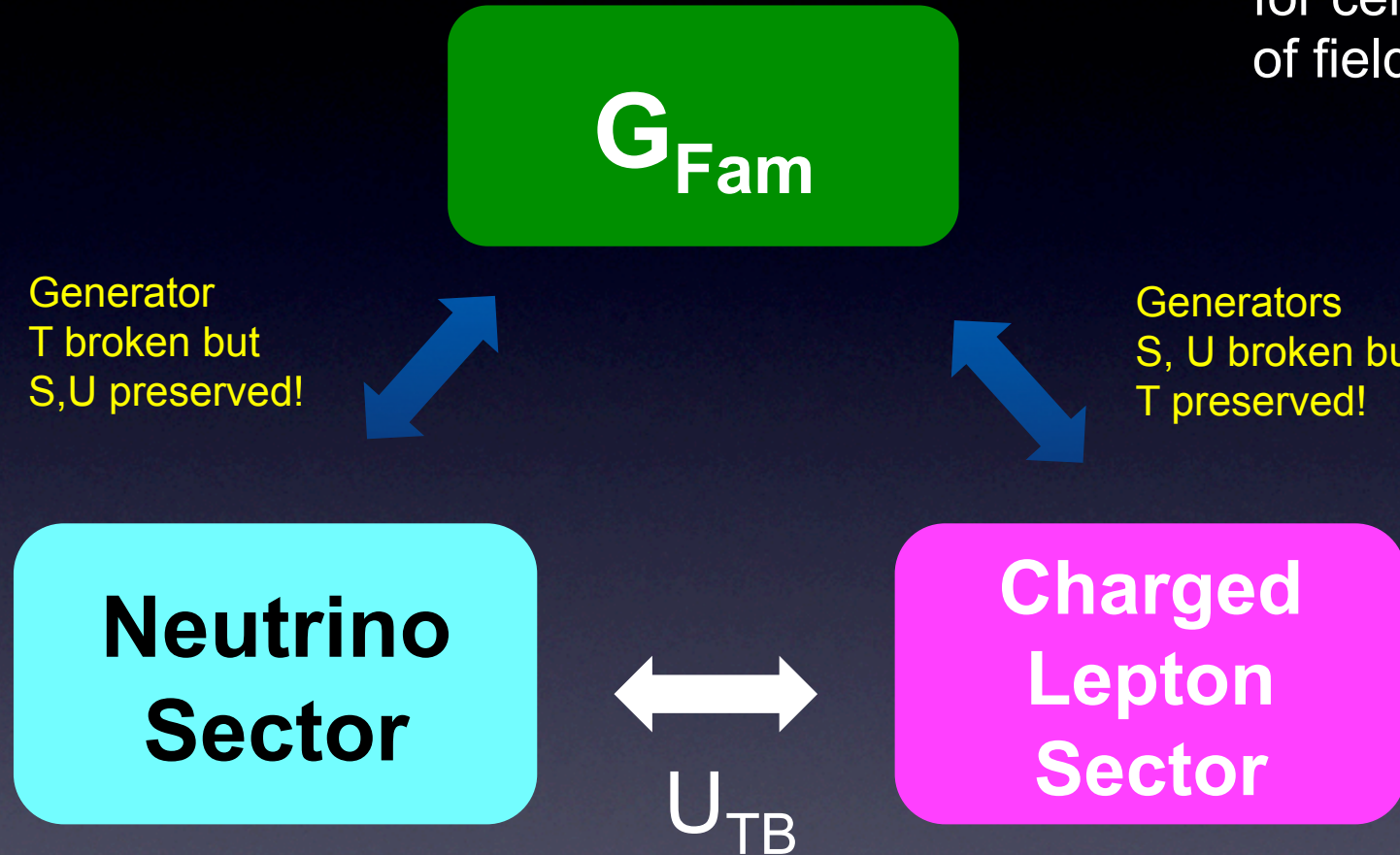
See e.g.: S.A., Kersten, Lindner, Ratz ('03)

(RGEs in the SM: above equations times -3/2)

I) "Direct" models:

Identify remnant symmetries!
 G_{Fam} generated by S, T and U
 $\rightarrow S_4$ family symmetry

A_4 possible as well
for certain selection
of field reps



Altarelli, Feruglio, Ma, Hagedorn, Merlo, Luhn ...

S_4 preferred: Lam

'Direct' vs. 'indirect' models:
King, Luhn ('09)

II) "Indirect" models:

Dynamical fields = Flavons
 $\sim A, B, C$ in 3 of G_{Fam} generate the
 flavour structure in the neutrino sector

$$C \sim \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$B \sim \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$A \sim \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

In seesaw:
Form Dominance
King, Chen ('09)

In string theory context:
S.A., Ibanez, Macri ('07)

G_{Fam} typically broken
 completely in m_ν
 \rightarrow flavour breaking
 directions are crucial!

King, Ross, de Medeiros Varzielas, Malinsky, Chen, Mahanthappa ...

***Both approaches point
to non-Abelian (discrete)
family symmetries G_{Fam} !***

Various models have been constructed
by many authors ...

TB mixing confronted with $\theta_{13}^{PMNS} = 9^\circ \pm 1^\circ$

- As already stated: The exact TB mixing pattern is ruled out by data. Two possible reactions are ...

- Look for alternatives ... such as for example
 - New flavour breaking directions (in “indirect” models), e.g. CSD2
S.A., King, Luhn, Spinrath ('11)
 - New discrete symmetries (leading to new “direct” models)
See e.g.: de Adelhart Toorop, Feruglio, Hagedorn ('11)

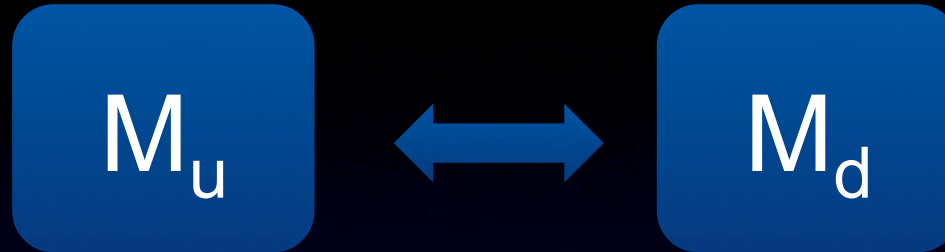
- View TB mixing as LO pattern only. Consider subleading effects, e.g. ...
 - Small correction to flavour breaking directions ($A \propto (0,1,1) \rightarrow (\varepsilon,1,1)$) King, Luhn ('11)
 - Include charged lepton mixing effects (with TB mixing in the neutrino sector)
See e.g.: King ('02), Frampton, Petcov, Rodejohann ('04), Altarelli, Feruglio, Masina ('04), S.A., King ('04), Ferrandis, Pakvasa ('04), Feruglio ('05), Datta, Everett, Ramond ('05), Mohapatra, Rodejohann ('05), S.A., King ('05), ...

The latter of these possibilities receives some motivation from the perspective of GUT models, because:

In GUT models, non-zero θ_{13}^{PMNS} is generically expected via charged lepton mixing contributions ...

Let us therefore discuss this connection in a bit more detail:

θ_{13}^{PMNS} from charged lepton mixing effects \leftrightarrow Grand Unified Theories



$$U_{\text{CKM}} = U^{u\dagger} U^d$$

Let us consider ...

Specific mixing pattern in m_ν , e.g.:



$$U_{\text{PMNS}} = U^{e\dagger} U^\nu$$

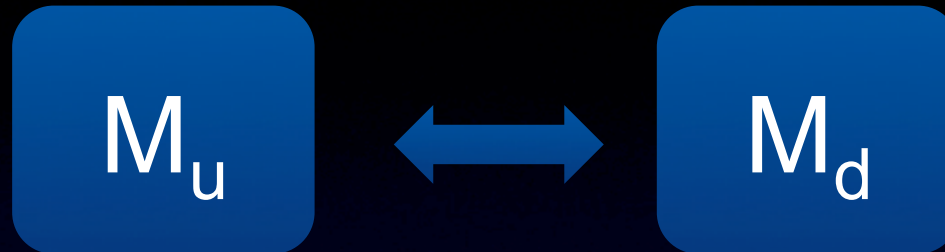
- Tri-bimaximal mixing: $U^\nu = U_{\text{TB}}$
- Bimaximal mixing: $U^\nu = U_{\text{Bimax}}$

with $\theta_{13}^\nu \approx 0$

Hierarchical masses in M_u and $M_d \Rightarrow \theta_{ij}^u \ll \theta_{ij}^d \Rightarrow$ typically

$$\theta_{12}^d \cong \theta_C$$

and $\theta_{13}^d, \theta_{23}^d \ll \theta_C$



$$U_{\text{CKM}} = U^{u\dagger} U^d$$

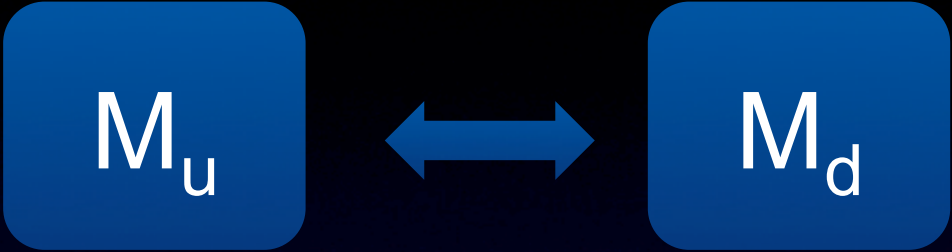
Let us consider U^{ν} with:

$$\theta_{13}^{\nu} \approx 0$$



$$U_{\text{PMNS}} = U^{e\dagger} U^{\nu}$$

$$\theta_{12}^d \cong \theta_C$$



$$U_{\text{CKM}} = U^{u\dagger} U^d$$

GUT relations

Let us consider U^{ν} with:

$$\theta_{13}^{\nu} \approx 0$$



$$U_{\text{PMNS}} = U^{e\dagger} U^{\nu}$$

$$\theta_{12}^d \cong \theta_C$$

$$M_u$$



$$M_d$$

Mass relations @ M_{GUT} ,
e.g.:

$$\frac{m_\tau}{m_b} = c_{33} = 1, \frac{3}{2}$$

$$U_{CKM} = U^{u\dagger} U^d$$



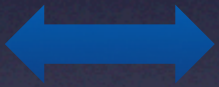
GUT
relations

$$\frac{m_\mu}{m_s} = c_{22} = 3, \frac{9}{2}, 6$$

Let us consider
 U^ν with:

$$\theta_{13}^\nu \approx 0$$

$$m_\nu$$



$$M_e$$

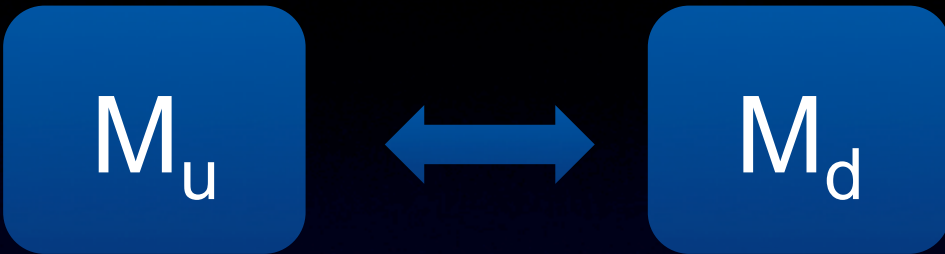
(c_{ij} : group theory Clebsch factors)

$$U_{PMNS} = U^{e\dagger} U^\nu$$

Clebsch factors $\in \{1,3\}$ in GUTs: Georgi, Jarlskog ('79)

New possible Clebsch factors $\in \{3/2,3,9/2,6,\dots\}$: S.A., Spinrath ('09)

$$\theta_{12}^d \cong \theta_C$$



But also mixing angle relations @ M_{GUT} , e.g.:

$$U_{CKM} = U^{u\dagger} U^d$$



Let us consider U^{ν} with:

$$\theta_{13}^{\nu} \approx 0$$

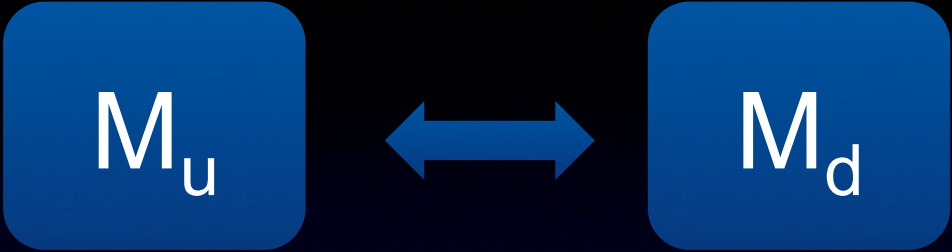


$$\theta_{12}^e = \frac{c_{12}}{c_{22}} \theta_{12}^d = \frac{c_{12}}{c_{22}} \theta_C$$

$$U_{PMNS} = U^{e\dagger} U^{\nu}$$

Possible new combinations of Clebsches leading to large θ_{13}^{PMNS} : S.A., Maurer ('11)
Mazocca, Petcov, Romanino, Spinrath ('11)

$$\theta_{12}^d \cong \theta_C$$



$$U_{\text{CKM}} = U^{u\dagger} U^d$$

GUT relations

Let us consider U^ν with:

$$\theta_{13}^\nu \approx 0$$



$$\theta_{12}^e = \frac{c_{12}}{c_{22}} \theta_C$$

and $\theta_{13}^e, \theta_{23}^e \ll \theta_C$

$$U_{\text{PMNS}} = U^{e\dagger} U^\nu$$

The PMNS mixing parameters result from U^ν and charged lepton mixing effects ($U^{e\dagger}$)

⇒ Two main effects

θ_C enters via
GUT relations

1) Prediction for $\theta_{13}^{\text{PMNS}}$:

$$\theta_{13}^{\text{PMNS}} = \frac{\theta_{12}^e}{\sqrt{2}} = \frac{\theta_C}{\sqrt{2}} \frac{c_{12}}{c_{22}}$$

⇒ Simple conditions
(including $c_{12} = c_{22}$)
in GUTs can lead to:

$$\theta_{13}^{\text{PMNS}} = \frac{\theta_C}{\sqrt{2}}$$

S.A., Maurer ('11)
Mazocca, Petcov, Romanino, Spinrath ('11)

S.A., Gross, Maurer, Sluka ('12); See also: King ('12);
Meroni, Petcov, Spinrath ('12);

With conventional Clebsch factors ($c_{12} = 1, c_{22} = 3$)
many models predicted non-zero but smaller value $\theta_{13}^{\text{PMNS}}$
 $= \theta_C / (3\sqrt{2}) \approx 3^\circ$ (... and are now ruled out by data):
works by many authors ...

Relation $\theta_{13}^{\text{PMNS}} = \theta_C / \sqrt{2}$ also appears in a special variant
of "Quark Lepton Complementarity" ... :
Minakata, Smirnov ('04); see also: Patel ('11)

1) Prediction for $\theta_{13}^{\text{PMNS}}$:

$$\theta_{13}^{\text{PMNS}} = \frac{\theta_{12}^e}{\sqrt{2}} = \frac{\theta_C}{\sqrt{2}} \frac{c_{12}}{c_{22}}$$

⇒ Simple conditions
(including $c_{12} = c_{22}$)
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$$\theta_{13}^{\text{PMNS}} = \frac{\theta_C}{\sqrt{2}}$$

S.A., Maurer ('11)
Mazocca, Petcov, Romanino, Spinrath ('11)

S.A., Gross, Maurer, Sluka ('12); See also: King ('12);
Meroni, Petcov, Spinrath ('12);

2) The “Lepton Mixing Sum Rule”:

$$\left. \begin{aligned} \theta_{13}^{\text{PMNS}} &= \frac{\theta_{12}^e}{\sqrt{2}} \\ \theta_{12}^{\text{PMNS}} &= \theta_{12}^\nu + \frac{\theta_{12}^e}{\sqrt{2}} \cos(\delta^{\text{PMNS}}) \end{aligned} \right\} \theta_{12}^{\text{PMNS}} = \theta_{12}^\nu + \theta_{13}^{\text{PMNS}} \cos(\delta^{\text{PMNS}})$$

King ('05); Masina ('05); S.A., King ('05)

characterizes a possible LO neutrino mixing
pattern (TB: $\theta_{12}^\nu \approx 35.3^\circ$; Bimax: $\theta_{12}^\nu = 45^\circ$)

Reconstructing θ_{12}^ν using the lepton mixing sum rule

$$\theta_{12}^{\text{PMNS}} = \theta_{12}^\nu + \theta_{13}^{\text{PMNS}} \cos(\delta^{\text{PMNS}})$$

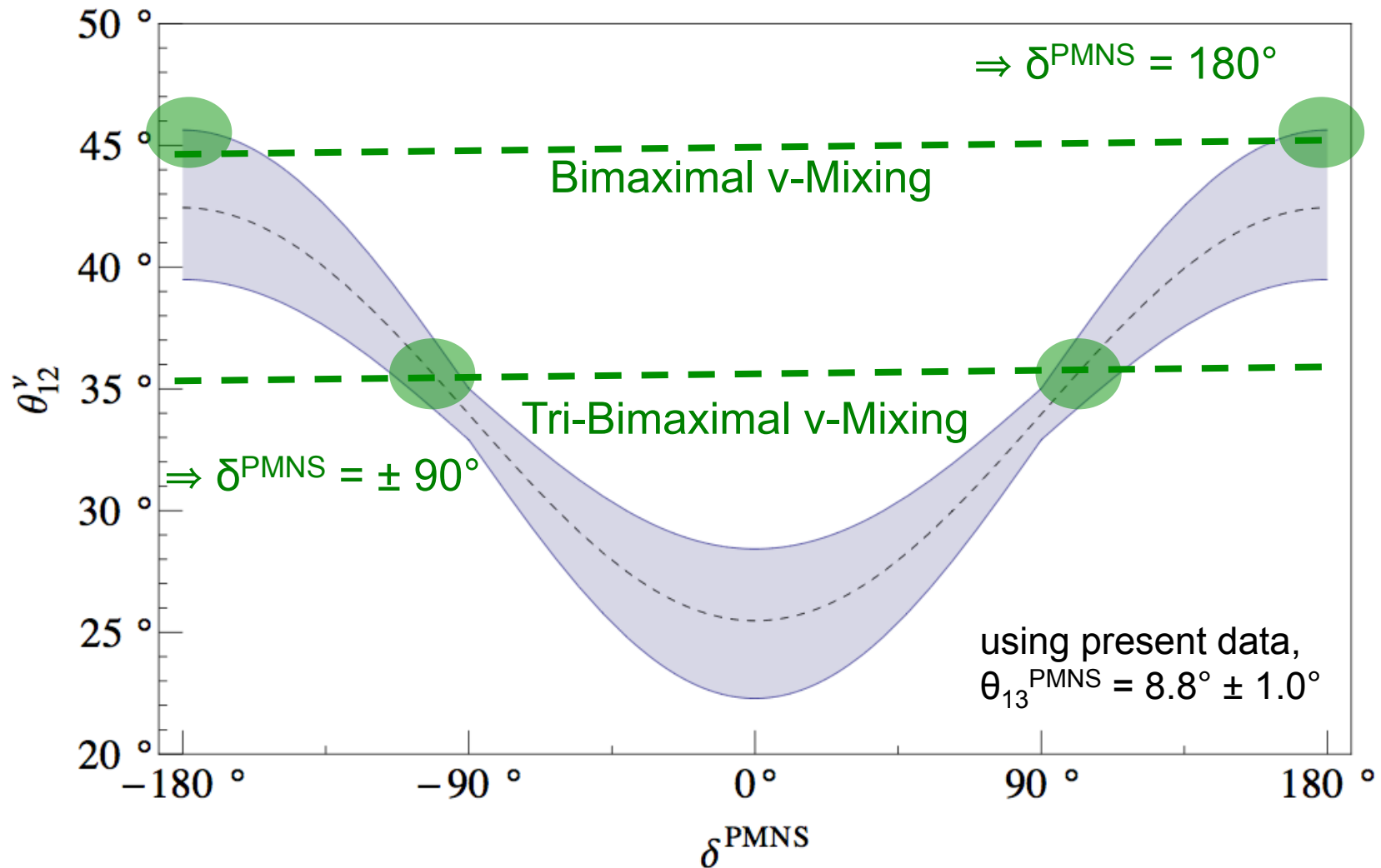


figure from S.A., Gross, Maurer, Sluka ('12)

Summary and conclusions

- Recent measurement of $\theta_{13}^{\text{PMNS}} \approx 9^\circ \pm 1^\circ \rightarrow$ Exciting time for neutrino mass and mixing models!
- While exact mixing patterns with $\theta_{13}^{\text{PMNS}} = 0$ (e.g. exact TB mixing of U_{PMNS}) are ruled out by data ... they may still provide viable LO structures, e.g.:
 - ✓ TB neutrino mixing + charged lepton contribution θ_{12}^e . Predicts $\delta^{\text{PMNS}} \approx \pm 90^\circ$
 - ✓ Bimaximal neutrino mixing + charged lepton contribution θ_{12}^e . Predicts $\delta^{\text{PMNS}} \approx 180^\circ$
- Independent of neutrino mixing pattern: $\theta_{13}^{\text{PMNS}} = \theta_C / \sqrt{2}$ can emerge in GUT flavour models under simple conditions ...
- Expected future (and already present) high experimental precision for the leptonic mixing parameters: **Accurate model analysis required ... !**