An Explicit SU(12) Family and Flavor Unification Model

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LieART Mathematica Package
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MOTIVATION

- In standard GUT models such as SU(5), SO(10), and $\mathbf{E}_6$, only appropriate chiral IRs available are

$$\mathbf{SU(5)}:\mathbf{10},\overline{\mathbf{5}};\quad \mathbf{SO(10)}:\mathbf{16};\quad \mathbf{E_6}:\mathbf{27}$$

  so a flavor symmetry must be introduced to distinguish families in the direct product group $\mathbf{G_{family}} \times \mathbf{G_{flavor}}$

- Family and flavor unification requires a higher rank simple group. Some earlier studies made were based on SO(18), SU(11), SU(8), and SU(9), but these were not totally satisfactory.

- Here I describe an SU(12) model with interesting features that was constructed with the help of a Mathematica computer package called LieART written by Robert Feger and Tom Kephart. This allows one to compute tensor products, branching rules, etc., and perform detailed searches for satisfactory models: arXiv: 1206.6362

SU(12) UNIFICATION MODEL

- SU(12) has 12 totally antisymmtric IRs:
  $\mathbf{12}, \mathbf{66}, \mathbf{220}, \mathbf{495}, \mathbf{792}, \mathbf{924}, \overline{\mathbf{792}}, \overline{\mathbf{495}}, \overline{\mathbf{220}}, \overline{\mathbf{66}}, \overline{\mathbf{12}}, \mathbf{1}$
  allowing 3 SU(5) families to be assigned to different IRs.

- Choose an anomaly-free set of SU(12) IRs which contains 3 chiral SU(5) families and pairs of fermions which become massive at SU(5) scale:

$$\mathbf{6(495) + 4(\overline{792}) + 4(\overline{220}) + (\overline{66}) + 4(\overline{12})}$$
$$\mathbf{\rightarrow 3(10 + \overline{5} + 1) + 238(5 + \overline{5}) + 211(10 + \overline{10}) + 484(1)}$$

given the SU(12) $\rightarrow$ SU(5) branching rules:

$$\begin{array}{rcl}
\mathbf{495} & \rightarrow & \mathbf{35(5) + 21(10) + 7(\overline{10}) + \overline{5} + 35(1)} \\
\overline{\mathbf{792}} & \rightarrow & \mathbf{7(5) + 21(10) + 35(\overline{10}) + 35(\overline{5}) + 22(1)} \\
\overline{\mathbf{220}} & \rightarrow & \mathbf{10 + 7(\overline{10}) + 21(\overline{5}) + 35(1)} \\
\overline{\mathbf{66}} & \rightarrow & \mathbf{\overline{10} + 7(\overline{5}) + 21(1)} \\
\overline{\mathbf{12}} & \rightarrow & \mathbf{\overline{5} + 7(1)}
\end{array}$$

PARTICLE ASSIGNMENTS

1st Family: $(\mathbf{10})\mathbf{495_1} \supset \mathbf{u_L}, \mathbf{u^c_L}, \mathbf{d_L}, \mathbf{e^c_L}$

$(\overline{\mathbf{5}})\overline{\mathbf{66}}_\mathbf{1} \supset \mathbf{d^c_L}, \mathbf{e_L}, \nu_{\mathbf{1L}}$

$(\mathbf{1})\overline{\mathbf{792}}_\mathbf{1} \supset \mathbf{N^c_{1L}}$

2nd Family: $(\mathbf{10})\overline{\mathbf{792}}_\mathbf{2} \supset \mathbf{c_L}, \mathbf{c^c_L}, \mathbf{s_L}, \mu^\mathbf{c}_\mathbf{L}$

$(\overline{\mathbf{5}})\overline{\mathbf{792}}_\mathbf{2} \supset \mathbf{s^c_L}, \mu_\mathbf{L}, \nu_{\mathbf{2L}}$

$(\mathbf{1})\overline{\mathbf{220}}_\mathbf{2} \supset \mathbf{N^c_{2L}}$

3rd Family: $(\mathbf{10})\overline{\mathbf{220}}_\mathbf{3} \supset \mathbf{t_L}, \mathbf{t^c_L}, \mathbf{b_L}, \tau^\mathbf{c}_\mathbf{L}$

$(\overline{\mathbf{5}})\overline{\mathbf{792}}_\mathbf{3} \supset \mathbf{b^c_L}, \tau_\mathbf{L}, \nu_{\mathbf{3L}}$

$(\mathbf{1})\overline{\mathbf{12}}_\mathbf{3} \supset \mathbf{N^c_{3L}}$

SU(12) Anomaly-free Set:

$$\mathbf{6(495) + 4(\overline{792}) + 4(\overline{220}) + (\overline{66}) + 4(\overline{12})}$$

- Among SU(12) anomaly-free set, $\mathbf{5(495)'s}, \mathbf{2(\overline{220})'s}, \mathbf{3(\overline{12})'s}$ are unassigned, become massive at SU(5) scale and decouple.
- Introduce massive fermion pairs $\mathbf{220} \times \overline{\mathbf{220}}, \mathbf{792} \times \overline{\mathbf{292}}$ at SU(12) scale.
- $(\mathbf{1})\mathbf{66_H}, (\mathbf{1})\overline{\mathbf{66}}_\mathbf{H}, (\mathbf{1})\mathbf{220_H}, (\mathbf{1})\overline{\mathbf{220}}_\mathbf{H}$ conjugate Higgs pairs acquire SU(5) singlet VEVs at SU(5) scale, where $\epsilon \equiv \mathbf{M_{SU(5)}/M_{SU(12)}} \sim \mathbf{1/50}$
- $(\mathbf{5})\mathbf{924_H}, (\overline{\mathbf{5}})\mathbf{924_H}$ affect EW symmetry breaking at EW scale.

EFFECTIVE THEORY APPROACH

- Start with SU(12) SUSY model which can be broken down via a 143 adjoint Higgs

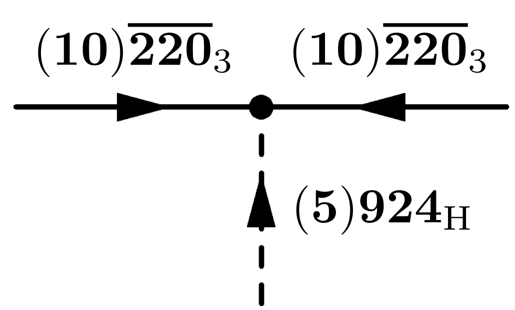
$$\mathbf{SU(12) \rightarrow SU(5) \times SU(7) \times U(1)}$$

  and finally to SU(5) via a set of antisymmetric chiral superfield IRs appropriately chosen to preserve SUSY.

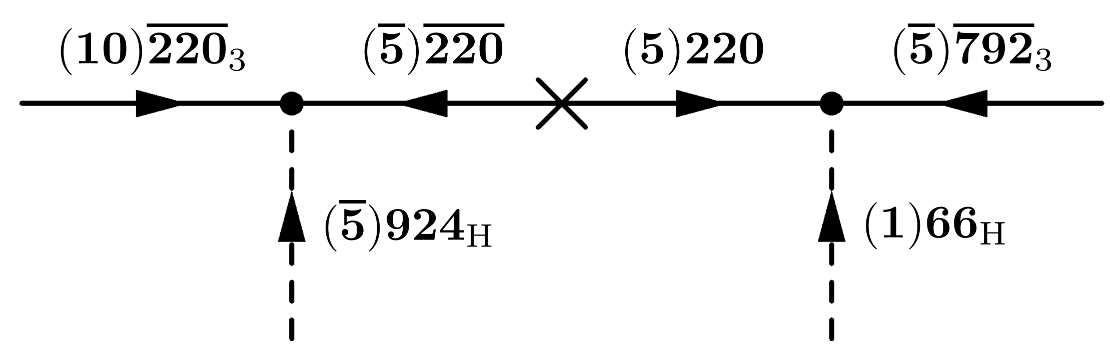
- Unbroken SUSY at the SU(5) scale allows us to deal only with tree diagrams, for loop corrections are much suppressed.

- Examples: 33 contributions to Up and Down Quark Mass Matrices

U33:



D33:



$$\mathbf{(10)\overline{220}_3.(5)924_H.(10)\overline{220}_3}$$

$$\mathbf{\sim h^u_{33} v\ u^T_{3L} u^c_{3L}}$$

$$\mathbf{(10)\overline{220}_3.(\bar{5})924_H.(\bar{5})\overline{220} \times (5)220.(1)66_H.(\bar{5})\overline{792}_3}$$

$$\mathbf{\sim h^d_{33} \epsilon v\ d^T_{3L} d^c_{3L}} \qquad \mathbf{v = 174 GeV}$$

- Every SU(5) Higgs singlet insertion introduces one power of epsilon.

Up-Type Quark Mass-Term Diagrams

Dim 4: **U33**: $(10)\overline{220}_3.(5)924_{\mathrm{H}}.(10)\overline{220}_3$
Dim 5: **U23**: $(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}_3$
U32: $(10)\overline{220}_3.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}_2$
Dim 6: **U13**: $(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}_3$
U31: $(10)\overline{220}_3.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}\times(\overline{10})792.(1)220_{\mathrm{H}}.(10)495_1$
U22: $(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}_2$
Dim 7: **U12**: $(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}_2$
U21: $(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}\times(\overline{10})792.(1)220_{\mathrm{H}}.(10)495_1$
Dim 8: **U11**: $(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220$
$.(1)66_{\mathrm{H}}.(10)\overline{792}\times(\overline{10})792.(1)220_{\mathrm{H}}.(10)495_1$

Down-Type Quark Mass-Term Diagrams

Dim 5: **D32**: $(10)\overline{220}_3.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}_2$
D33: $(10)\overline{220}_3.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}_3$
Dim 6: **D31**: $(10)\overline{220}_3.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}\times(5)792.(1)\overline{220}_{\mathrm{H}}.(\overline{5})\overline{66}_1$
D22: $(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}_2$
D23: $(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}_3$
Dim 7: **D12**: $(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}_2$
D21: $(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}\times(5)792.(1)\overline{220}_{\mathrm{H}}.(\overline{5})\overline{66}_1$
D13: $(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}_3$
Dim 8: **D11**: $(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220$
$.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}\times(5)792.(1)\overline{220}_{\mathrm{H}}.(\overline{5})\overline{66}_1$

Dirac-Neutrino Mass-Term Diagrams

Dim 4: **DN23**: $(\overline{5})\overline{792}_2.(5)924_{\mathrm{H}}.(1)\overline{12}_3$
DN33: $(\overline{5})\overline{792}_3.(5)924_{\mathrm{H}}.(1)\overline{12}_3$
Dim 5: **DN13**: $(\overline{5})\overline{66}_1.(1)\overline{220}_{\mathrm{H}}.(5)792\times(\overline{5})\overline{792}.(5)924_{\mathrm{H}}.(1)\overline{12}_3$
DN22: $(\overline{5})\overline{792}_2.(1)66_{\mathrm{H}}.(5)220\times(\overline{5})\overline{220}.(5)924_{\mathrm{H}}.(1)\overline{220}_2$
DN32: $(\overline{5})\overline{792}_3.(1)66_{\mathrm{H}}.(5)220\times(\overline{5})\overline{220}.(5)924_{\mathrm{H}}.(1)\overline{220}_2$
Dim 6: **DN12**: $(\overline{5})\overline{66}_1.(1)\overline{220}_{\mathrm{H}}.(5)792\times(\overline{5})\overline{792}.(1)66_{\mathrm{H}}.(5)220\times(\overline{5})\overline{220}.(5)924_{\mathrm{H}}.(1)\overline{220}_2$
DN21: $(\overline{5})\overline{792}_2.(1)66_{\mathrm{H}}.(5)220\times(\overline{5})\overline{220}.(5)924_{\mathrm{H}}.(1)\overline{220}\times(1)220.(1)66_{\mathrm{H}}.(1)\overline{792}_1$
DN31: $(\overline{5})\overline{792}_3.(1)66_{\mathrm{H}}.(5)220\times(\overline{5})\overline{220}.(5)924_{\mathrm{H}}.(1)\overline{220}\times(1)220.(1)66_{\mathrm{H}}.(1)\overline{792}_1$
Dim 7: **DN11**: $(\overline{5})\overline{66}_1.(1)\overline{220}_{\mathrm{H}}.(5)792\times(\overline{5})\overline{792}.(1)66_{\mathrm{H}}.(5)220\times(\overline{5})\overline{220}.(5)924_{\mathrm{H}}.(1)\overline{220}\times(1)220.(1)66_{\mathrm{H}}.(1)\overline{792}_1$

One leading-order diagram for each matrix element

Majorana-Neutrino Mass-Term Diagrams

Dim 4: **MN11**: $(1)\overline{792}_1.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}_1$
MN33: $(1)\overline{12}_3.(1)66_{\mathrm{H}}.(1)\overline{12}_3$
Dim 5: **MN12**: $(1)\overline{792}_1.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\mathrm{H}}.(1)\overline{220}_2$
MN21: $(1)\overline{220}_2.(1)\overline{66}_{\mathrm{H}}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}_1$
Dim 6: **MN13**: $(1)\overline{792}_1.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\mathrm{H}}.(1)\overline{220}\times(1)220.(1)\overline{66}_{\mathrm{H}}.(1)\overline{12}_3$
MN31: $(1)\overline{12}_3.(1)\overline{66}_{\mathrm{H}}.(1)220\times(1)\overline{220}.(1)\overline{66}_{\mathrm{H}}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}_1$
MN22: $(1)\overline{220}_2.(1)\overline{66}_{\mathrm{H}}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\mathrm{H}}.(1)\overline{220}_2$
Dim 7: **MN23**: $(1)\overline{220}_2.(1)\overline{66}_{\mathrm{H}}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\mathrm{H}}.(1)\overline{220}\times(1)220.(1)\overline{66}_{\mathrm{H}}.(1)\overline{12}_3$
MN32: $(1)\overline{12}_3.(1)\overline{66}_{\mathrm{H}}.(1)220\times(1)\overline{220}.(1)\overline{66}_{\mathrm{H}}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\mathrm{H}}.(1)\overline{220}_2$

MASS MATRICES: LEADING ORDER TERMS

- Dropping the prefactors:

$$\mathbf{M_U} \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \mathbf{v}, \qquad \mathbf{M_D} \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon & \epsilon \end{pmatrix} \mathbf{v}$$

$$\mathbf{M_{DN}} \sim \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \mathbf{v}, \qquad \mathbf{M_{MN}} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & \epsilon^2 & \epsilon^3 \\ \epsilon^2 & \epsilon^3 & 1 \end{pmatrix} \mathbf{\Lambda_R}$$

$$\mathbf{M_L} \sim \mathbf{M_D^T}, \qquad \mathbf{M_\nu} = -\mathbf{M_{DN}} \mathbf{M^{-1}} \mathbf{M_{DN}^T} \sim \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \mathbf{v^2}/\mathbf{\Lambda_R}$$

- $\mathbf{M_U}, \mathbf{M_{MN}}, \mathbf{M_\nu}$ are symmetric, $\mathbf{M_D}, \mathbf{M_L}, \mathbf{M_{DN}}$ doubly lopsided. Note that $\mathbf{M_\nu}$ has a mild hierarchy.

PHENOMENOLOGICAL FIT

- 25 leading independent prefactors + $\mathbf{\Lambda_R}$ are used to fit 30 data parameters with fixed $\epsilon = \mathbf{(1/6.5)^2 = 0.0237}$

- Best fit obtained with Normal Hierarchy and

$$\begin{aligned} \mathbf{\Lambda_R} &= \mathbf{M_{SU(5)}} = \mathbf{7.4 \times 10^{14}}\ \mathrm{GeV} \\ \Rightarrow \mathbf{M_{SU(12)}} &= \mathbf{\Lambda_R}/\epsilon = \mathbf{3.1 \times 10^{16}}\ \mathrm{GeV}, \end{aligned}$$

$$\begin{aligned} \mathbf{m_1} = \mathbf{0}, \qquad \mathbf{M_1} &= \mathbf{1.67 \times 10^{12}}\ \mathrm{GeV}, \\ \mathbf{M_2} &= \mathbf{6.85 \times 10^{13}}\ \mathrm{GeV}, \\ \mathbf{M_3} &= \mathbf{5.30 \times 10^{14}}\ \mathrm{GeV} \end{aligned}$$

MATRICES GIVING BEST FIT

$$M_{\mathrm{U}} = \begin{pmatrix} -1.1\varepsilon^4 & 7.1\varepsilon^3 & 5.6\varepsilon^2 \\ 7.1\varepsilon^3 & -6.2\varepsilon^2 & -0.10\varepsilon \\ 5.6\varepsilon^2 & -0.10\varepsilon & -0.95 \end{pmatrix} v, \qquad M_{\mathrm{D}} = \begin{pmatrix} -6.3\varepsilon^4 & 8.0\varepsilon^3 & -1.9\varepsilon^3 \\ -4.5\varepsilon^3 & 0.38\varepsilon^2 & -1.3\varepsilon^2 \\ 0.88\varepsilon^2 & -0.23\varepsilon & -0.51\varepsilon \end{pmatrix} v,$$

$$M_{\mathrm{DN}} = \begin{pmatrix} h_{11}^{\mathrm{dn}}\varepsilon^3 & 0.21\varepsilon^2 & -2.7\varepsilon \\ h_{21}^{\mathrm{dn}}\varepsilon^2 & -0.28\varepsilon & -0.15 \\ h_{31}^{\mathrm{dn}}\varepsilon^2 & 2.1\varepsilon & 0.086 \end{pmatrix} v, \quad M_{\mathrm{MN}} = \begin{pmatrix} -0.72 & -1.5\varepsilon & h_{13}^{\mathrm{mn}}\varepsilon^2 \\ -1.5\varepsilon & 0.95\varepsilon^2 & h_{23}^{\mathrm{mn}}\varepsilon^3 \\ h_{13}^{\mathrm{mn}}\varepsilon^2 & h_{23}^{\mathrm{mn}}\varepsilon^3 & 0.093 \end{pmatrix} \Lambda_{\mathrm{R}}$$

$$M_\nu = \begin{pmatrix} -81.\varepsilon^2 & -4.3\varepsilon & 2.4\varepsilon \\ -4.3\varepsilon & -0.25 & 0.28 \\ 2.4\varepsilon & 0.28 & -1.1 \end{pmatrix} \frac{v^2}{\Lambda_{\mathrm{R}}},$$

- All prefactors except one are within $\mathcal{O}(\mathbf{0.1-10})$ of unity.

SUMMARY

- Unified SU(12) SUSY GUT model obtained by brute force computer scan over all SU(12) anomaly-free sets of IRs containing 3 SU(5) chiral families under the assumption that SU(12) $\rightarrow$ SU(5) $\rightarrow$ SM, looping over all SU(12) fermion and Higgs assignments that give good fits to the input data.

- For this purpose an effective theory approach was used to determine leading order tree-level diagrams for dim-(4+n) matrix elements in powers of $\epsilon^{\mathbf{n}}$ where epsilon is the ratio of the SU(5) to SU(12) scale. Best fit obtained by requiring all prefactors be $\mathcal{O}(1)$, but large number of them implies few predictions.

- This model is just one of many possibilities (including other smaller SU(N) groups), but its features were among most attractive found.

- Model serves as an existence proof for unification of family and flavor.