An Explicit SU(12) Family and Flavor Unification Model

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arXiv: 1204.5471

LexART Mathematica Package

ICHEP2012 Melbourne, 6 July 2012
MOTIVATION

• In standard GUT models such as SU(5), SO(10), and $E_6$, only appropriate chiral IRs available are $SU(5) : 10, \bar{5}$; $SO(10) : 16$; $E_6 : 27$
  so a flavor symmetry must be introduced to distinguish families in the direct product group $G_{\text{family}} \times G_{\text{flavor}}$

• Family and flavor unification requires a higher rank simple group. Some earlier studies made were based on SO(18), SU(11), SU(8), and SU(9), but these were not totally satisfactory.

• Here I describe an SU(12) model with interesting features that was constructed with the help of a Mathematica computer package called LieART written by Robert Feger and Tom Kephart. This allows one to compute tensor products, branching rules, etc., and perform detailed searches for satisfactory models: arXiv:1206.6362
SU(12) UNIFICATION MODEL

• SU(12) has 12 totally antisymmetric IRs:
  12, 66, 220, 495, 792, 924, 792, 495, 220, 66, 12, 1
  allowing 3 SU(5) families to be assigned to different IRs.

• Choose an anomaly-free set of SU(12) IRs which contains 3 chiral SU(5) families and pairs of fermions which become massive at SU(5) scale:

  6(495) + 4(792) + 4(220) + (66) + 4(12)
  → 3(10 + 5 + 1) + 238(5 + 5) + 211(10 + 10) + 484(1)

given the SU(12) → SU(5) branching rules:

  495 → 35(5) + 21(10) + 7(10) + 5 + 35(1)
  792 → 7(5) + 21(10) + 35(10) + 35(5) + 22(1)
  220 → 10 + 7(10) + 21(5) + 35(1)
  66 → 10 + 7(5) + 21(1)
  12 → 5 + 7(1)
PARTICLE ASSIGNMENTS

1st Family: $(10)_{495_1}$ $\supset u_L, u^c_L, d_L, e^c_L$
$(5)_{66_1}$ $\supset d^c_L, e_L, \nu_{1L}$
$(1)_{792_1}$ $\supset N^c_{1L}$

2nd Family: $(10)_{792_2}$ $\supset c_L, c^c_L, s_L, \mu^c_L$
$(5)_{792_2}$ $\supset s^c_L, \mu_L, \nu_{2L}$
$(1)_{220_2}$ $\supset N^c_{2L}$

3rd Family: $(10)_{220_3}$ $\supset t_L, t^c_L, b_L, \tau^c_L$
$(5)_{792_3}$ $\supset b^c_L, \tau_L, \nu_{3L}$
$(1)_{12_3}$ $\supset N^c_{3L}$

SU(12) Anomaly-free Set: $6(495) + 4(792) + 4(220) + (66) + 4(12)$

- Among SU(12) anomaly-free set, $5(495)\text{'s}$, $2(220)\text{'s}$, $3(12)\text{'s}$ are unassigned, become massive at SU(5) scale and decouple.
- Introduce massive fermion pairs $220 \times 220, 792 \times 292$ at SU(12) scale.
- $(1)_{66_H}, (1)_{66_H}, (1)_{220_H}, (1)_{220_H}$ conjugate Higgs pairs acquire SU(5) singlet VEVs at SU(5) scale, where $\epsilon \equiv M_{SU(5)}/M_{SU(12)} \sim 1/50$
- $(5)_{924_H}, (\bar{5})_{924_H}$ affect EW symmetry breaking at EW scale.
EFFECTIVE THEORY APPROACH

• Start with SU(12) SUSY model which can be broken down via a 143 adjoint Higgs

\[
\text{SU}(12) \rightarrow \text{SU}(5) \times \text{SU}(7) \times U(1)
\]

and finally to SU(5) via a set of antisymmetric chiral superfield IRs appropriately chosen to preserve SUSY.

• Unbroken SUSY at the SU(5) scale allows us to deal only with tree diagrams, for loop corrections are much suppressed.

• Examples: 33 contributions to Up and Down Quark Mass Matrices

\[
\begin{align*}
\text{U33:} & \quad (10)\overline{220}_3 \rightarrow (10)\overline{220}_3 & \quad (10)\overline{220}_3 \rightarrow (5)\overline{220} \rightarrow (5)220 \rightarrow (5)\overline{792}_3 \\
\text{D33:} & \quad \rightarrow (5)924_H & \quad \rightarrow (1)66_H
\end{align*}
\]

\[
\begin{align*}
& (10)\overline{220}_3.(5)924_H.(10)\overline{220}_3 \quad & (10)\overline{220}_3.(5)924_H.(5)\overline{220}(5)220.(1)66_H.(5)\overline{792}_3 \\
& \sim h^u_{33}v \quad u^T_{3L}u^c_{3L} \quad & \sim h^d_{33}v \quad d^T_{3L}d^c_{3L} \quad v = 174 \text{GeV}
\end{align*}
\]

• Every SU(5) Higgs singlet insertion introduces one power of epsilon.
One leading-order diagram for each matrix element
MASS MATRICES: LEADING ORDER TERMS

- Dropping the prefactors:

\[ M_U \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} v, \quad M_D \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon & \epsilon \end{pmatrix} v \]

\[ M_{DN} \sim \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} v, \quad M_{MN} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & \epsilon^2 & \epsilon^3 \\ \epsilon^2 & \epsilon^3 & 1 \end{pmatrix} \Lambda_R \]

\[ M_L \sim M_D^T, \quad M_\nu = -M_{DN}M^{-1}M_{DN}^T \sim \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} v^2/\Lambda_R \]

- \( M_U, M_{MN}, M_\nu \) are symmetric, \( M_D, M_L, M_{DN} \) doubly lopsided. Note that \( M_\nu \) has a mild hierarchy.
PHENOMENOLOGICAL FIT

• 25 leading independent prefactors + $\Lambda_R$ are used to fit 30 data parameters with fixed $\epsilon = (1/6.5)^2 = 0.0237$

• Best fit obtained with Normal Hierarchy and

$$\Lambda_R = M_{SU(5)} = 7.4 \times 10^{14} \text{ GeV}$$

$$\Rightarrow M_{SU(12)} = \Lambda_R / \epsilon = 3.1 \times 10^{16} \text{ GeV},$$

$$m_1 = 0, \quad M_1 = 1.67 \times 10^{12} \text{ GeV},$$

$$M_2 = 6.85 \times 10^{13} \text{ GeV},$$

$$M_3 = 5.30 \times 10^{14} \text{ GeV}$$
MATRICES GIVING BEST FIT

\[ M_U = \begin{pmatrix} -1.1 \varepsilon^4 & 7.1 \varepsilon^3 & 5.6 \varepsilon^2 \\ 7.1 \varepsilon^3 & -6.2 \varepsilon^2 & -0.10 \varepsilon \\ 5.6 \varepsilon^2 & -0.10 \varepsilon & -0.95 \end{pmatrix} v, \]
\[ M_D = \begin{pmatrix} -6.3 \varepsilon^4 & 8.0 \varepsilon^3 & -1.9 \varepsilon^3 \\ -4.5 \varepsilon^3 & 0.38 \varepsilon^2 & -1.3 \varepsilon^2 \\ 0.88 \varepsilon^2 & -0.23 \varepsilon & -0.51 \varepsilon \end{pmatrix} v, \]
\[ M_{DN} = \begin{pmatrix} h_{11}^{dn} \varepsilon^3 & 0.21 \varepsilon^2 & -2.7 \varepsilon \\ h_{21}^{dn} \varepsilon^2 & -0.28 \varepsilon & -0.15 \\ h_{31}^{dn} \varepsilon^2 & 2.1 \varepsilon & 0.086 \end{pmatrix} v, \]
\[ M_{MN} = \begin{pmatrix} -0.72 & -1.5 \varepsilon & h_{13}^{mn} \varepsilon^2 \\ -1.5 \varepsilon & 0.95 \varepsilon^2 & h_{23}^{mn} \varepsilon^3 \\ h_{13}^{mn} \varepsilon^2 & h_{23}^{mn} \varepsilon^3 & 0.93 \end{pmatrix} \Lambda_R \]
\[ M_\nu = \begin{pmatrix} -81. \varepsilon^2 & -4.3 \varepsilon & 2.4 \varepsilon \\ -4.3 \varepsilon & -0.25 & 0.28 \\ 2.4 \varepsilon & 0.28 & -1.1 \end{pmatrix} \frac{v^2}{\Lambda_R}, \]

• All prefactors except one are within \( O(0.1 - 10) \) of unity.
SUMMARY

• Unified SU(12) SUSY GUT model obtained by brute force computer scan over all SU(12) anomaly-free sets of IRs containing 3 SU(5) chiral families under the assumption that SU(12) → SU(5) → SM, looping over all SU(12) fermion and Higgs assignments that give good fits to the input data.

• For this purpose an effective theory approach was used to determine leading order tree-level diagrams for dim-(4+n) matrix elements in powers of $\epsilon^n$ where epsilon is the ratio of the SU(5) to SU(12) scale. Best fit obtained by requiring all prefactors be $O(1)$, but large number of them implies few predictions.

• This model is just one of many possibilities (including other smaller SU(N) groups), but its features were among most attractive found.

• Model serves as an existence proof for unification of family and flavor.