Meson Spectrum of a Walking Gauge Theory

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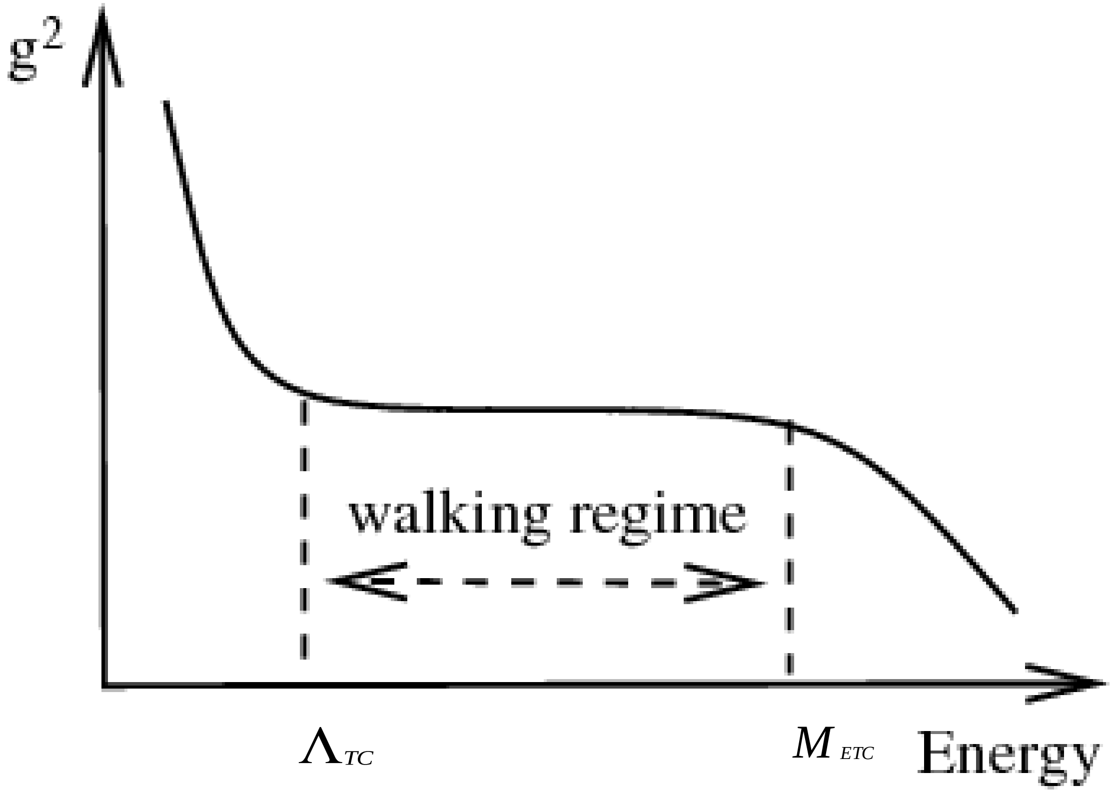
arXiv:1105.4185, arXiv:1203.1968

Plan

- Review of walking.
- Gravity duals of CSB a la Sakai-Sugimoto.
- Walking backgrounds.
- Flavor embeddings.
- Computations of meson masses.
- Summary and a list of open questions.

Walking Behavior



Introduction to walking

- Walking Gauge Field Theories have been studied since the mid 1980's.
- Serious Lattice studies in the past few years.
- Phenomenological applications?

My first encounter with walking

- QED 2+1 shows walking behaviour.
- Interesting model field theory.
- Could have applications in CM Physics.

Walking in 3+1 In the 1980's

- Holdom, Yamawaki, Bando Matumoto, Appelquist, Karabali, Wijewardhana. Akiba Yanagida.
- Based on UV or IR fixed point behavior.
- Analyzed by solving Dyson Schwinger Gap Equations.

Walking in Non Abelian Theories

- Walking is supposed to happen when the gluon contribution to the beta function is cancelled by a large number of fermions.
- Above a critical Number of flavors

  the theory becomes deconfining and chirally symmetric.

QCD with large number of massless quark flavors

- Loss of asymptotic freedom at 16.5 Dirac flavors.
- Chiral symmetry restored and confinement lost at a number of flavors below 16. Most probably around 12.
- Running of the coupling slows down with increasing flavor number.
- Conformal window?

Critical N_f using gap equation Method

- The Zero temperature chiral phase transition in SU(N) gauge theories.
  Appelquist Terning, Wijewardhana
  Published in **Phys.Rev.Lett. 77 (1996) 1214-1217** e-Print: **hep-ph/9602385.**
- **No narrow light scalar degree of freedom at this transition.**

Questions

- What is the spectrum of a walking theory?
- What is the starting point of the conformal window?
- Parity doubling in walking theories?
- Parity inversion? Are the vector mesons heavier than the axial vector mesons?
- How large is the anomalous dimensions of $\overline{Q}Q$?

Are there Nambu-Goldstone degrees of freedom due to spontaneous scale breaking? i.e. Dilatons?

Gauge Gravity Duality

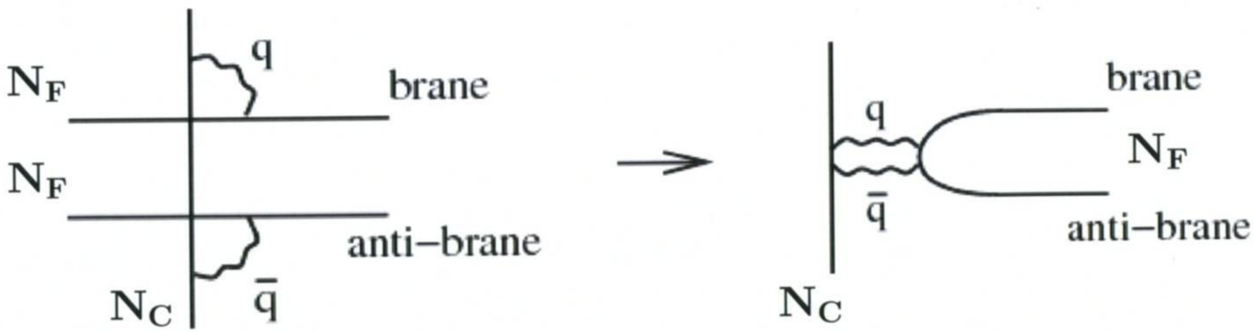
- Two different perspectives on D-branes in string theory:
- gravity background
- [SUGRA solution] open strings BCs
- A stack of large number of D-branes:
- Two sides of duality encode same degrees of freedom
- [The two sides have equal partition functions!]

Gravity Duals of Chiral SB

Need chiral fermions $\rightarrow$ consider intersecting D-branes

Shorthand notation:

Chiral symmetry breaking: [Sakai and Sugimoto (2004)]



both chiralities

chiral sym. breaking: $\langle \bar{q} q \rangle \neq 0$

Gravity dual of Dynamical Chiral Symmetry Breaking

- Create a Background by stacking N_{TC} D Branes.

  Embed N_{TF} Probe branes and anti branes in this background. A U shaped embedding signals spontaneous chiral symmetry breaking with accompanying Nambu-Goldstone(NG) bosons.

  Introduced by Sakai Sugimoto for QCD.

Sakai-Sugimoto Model

- Stack of N D4 color branes.
- N_f   D8   and   D 8 bar flavor Branes.
- U shaped Embedding.
- Chiral symmetry breaking with Nambu Goldtone phenomena.
- Probe Approximation where N_f<<N .
- Gauge coupling does not walk.

Walking Backgrounds

- Nunez, Papadimitriou and Piai found a gravity dual of a walking theory(0812.3655-th).
- Based on one of the IIB solutions found by Casero,Nunez,Paredes(0602027-th).
- Deformations of the Maldacena, Nunez background(hep-th/0008001).Wrapping of D5 branes around a two sphere.
- Sourced by D5 Branes.

Running Coupling

- Computation of running by Nunez, Papadimitriou , Piai.
- Base on the method proposed by

  Di Vecchia,Lerda,Merlatti, th/0205204.

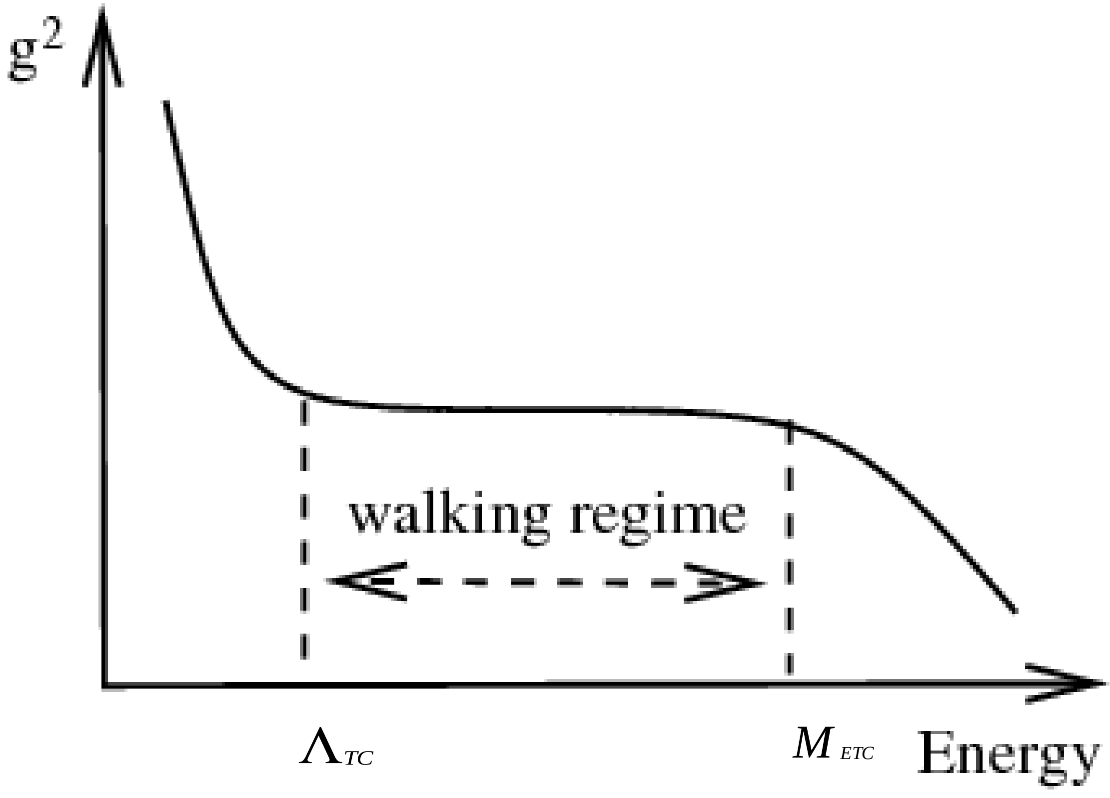
Running Coupling

- Running Coupling read off from the DBI action of a supersymmetric D5 brane probe in the above background. Walking region is the interval $(1, \rho_*)$.Walking requires $\beta = \sin^3(\alpha)$ <<1

$$\rho_* = \frac{\log(2) + 3\log\cot(\alpha)}{4}$$

$$\frac{g^2}{8\pi^2} = \frac{\exp(\mathrm{arcsin}ch(\frac{1}{\sin ch(2\rho)}))}{c P_1}$$

Walking Behavior



Chiral Symmetry Breaking

- Anguelova found a U shaped embedding of D7 branes and anti branes in this D5 brane background.
- Published in **JHEP 1010:065,2010**.
  e-Print: **arXiv:1007.4793** [hep-th]).
- **Model for WTC.**

Meson Masses and Decay Constants

- Using the D7 profile and integrating over the compact directions we get

$$S_{DBI} = -K \int d^4x \, d\rho \, (a(\rho) F_{\mu\nu} F^{\mu\nu} + 2b(\rho) F_{\mu\rho} F^{\mu}_{\rho})$$

- Where μ, ν are space-time coordinates

$$K = T(2\pi\alpha')^2 V_3 / g_s$$

- Use the gauge $A_\rho = 0$

Mode Expansion

Fourier transform the Minkowsky CDT , x and express

$$A_\mu(q,\rho) = V_\mu(q)\Psi_V^0(q^2,\rho) + A_\mu(q)\Psi_A^0(q^2,\rho)$$
$$+\sum_n (V_\mu^n \Psi_{V_n}(\rho) + A_\mu^n \Psi_{A_n}(\rho))$$

$\Psi_v^0 \, and \, \Psi_A^0$ are non-normalizable modes that correspond to sources for the V and A boundary currents respectively.

Boundary Conditions

- Summed over n are normalizable bulk gauge fields.
- Parity in the field theory is related to reflection on the flavor brane embedding about the meeting point of the D7 and anti D7 branes.
- Vectors are symmetric and axial vectors are anti symmetric w.r.t. reflection.

Boundary Conditions

- Normalizable modes go to zero as ρ tends to infinity along both branches of D7 and anti D7. Ψ_V^0 goes to 1 on both branches while Ψ_A^0 goes to 1 on D7 and goes to -1 on anti D7 as ρ goes to infinity.

$$\frac{d}{d\rho}\Psi_{V_n}(\rho)\Big|_{\rho_0}=0 \qquad\qquad \Psi_{A_n}(\rho_0)=0$$

EQN of Motion

$$\frac{\frac{d}{d\rho}(b(\rho)\frac{d}{d\rho}\Psi_n(\rho))}{a(\rho)} = -m_n{}^2\Psi_n$$

$$\frac{\frac{d}{d\rho}(b(\rho)\frac{d}{d\rho}\Psi^0(\rho))}{a(\rho)} = -q^2\Psi^0$$

Correlation Functions of the Field Theory

- In gauge gravity duality: Dual Gravitational action is the generating functional of the correlation functions of the field theory.
- One can compute explicitly the vector meson decay constants in terms of $\Psi^{0}_{V,A}$.

Meson decay constants

$$g_{V_n} = -K \int_{D7+\overline{D7}} \Psi_V^0(\rho)\,\partial_\rho [b(\rho)\,\partial_\rho \Psi_{V_n}]\,d\rho$$

$$g_{A_n} = -K \int_{D7+\overline{D7}} \Psi_A^0(\rho)\,\partial_\rho [b(\rho)\,\partial_\rho \Psi_{A_n}]\,d\rho$$

$$g_n = -2K \int_{\rho_0}^{\rho_\Lambda} \partial_\rho [\Psi^0 b(\rho)\,\partial_\rho \Psi_n] + 2K \int_{\rho_0}^{\rho_\Lambda} \partial_\rho \Psi^0 b(\rho)\,\partial_\rho \Psi_n$$

$$F_\pi^2 = 2K[b(\rho)\,\Psi_A^0(0,\rho)\,\partial_\rho \Psi_A^0(0,\rho)]\Big|_{\rho=\infty}$$

$$\frac{g_v}{m_v} = F_v$$

$$\frac{g_A}{m_A} = F_A$$

How about S? Is it small?

$$S = -4\pi \frac{d}{dq^2}(\Pi_V - \Pi_A)\Big|_{q^2=0}$$

Vector Meson Dominance

$$S = 4\pi \sum \left(\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2} \right)$$

Computation of S

$$\Pi_V(q^2) = < J^V_\mu(q^2) J^V_\nu(0) > = -\frac{\delta}{\delta V_\mu}\frac{\delta}{\delta V_\nu} S_{DBI}\Big|_{V=0} =$$

$$2K[b(\rho)\Psi^0_V(q^2,\rho)\frac{d}{d\rho}\Psi^0_V(q^2,\rho)]\Big|_{\rho=\infty}$$

$$\Pi_A(q^2) = < J^A_\mu(q^2) J^A_\nu(0) > = -\frac{\delta}{\delta A_\mu}\frac{\delta}{\delta A_\nu} S_{DBI}\Big|_{A=0} =$$

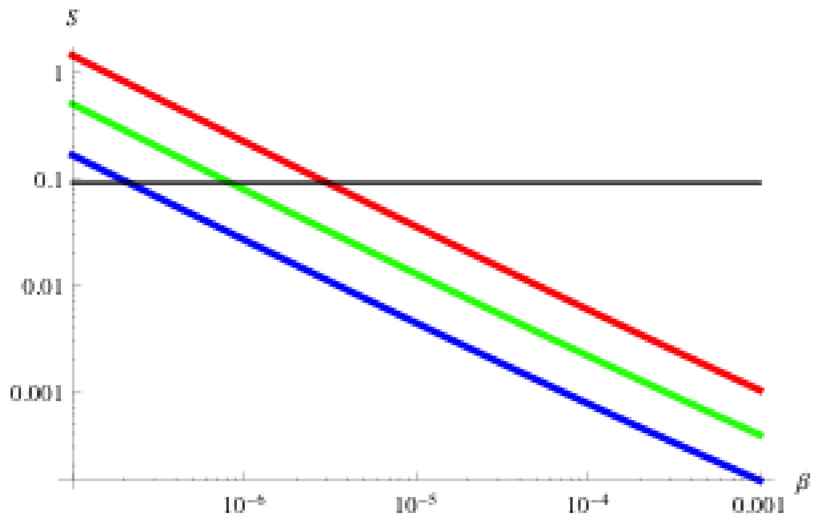
$$2K[b(\rho)\Psi^0_A(q^2,\rho)\frac{d}{d\rho}\Psi^0_A(q^2,\rho)]\Big|_{\rho=\infty}$$

S Parameter

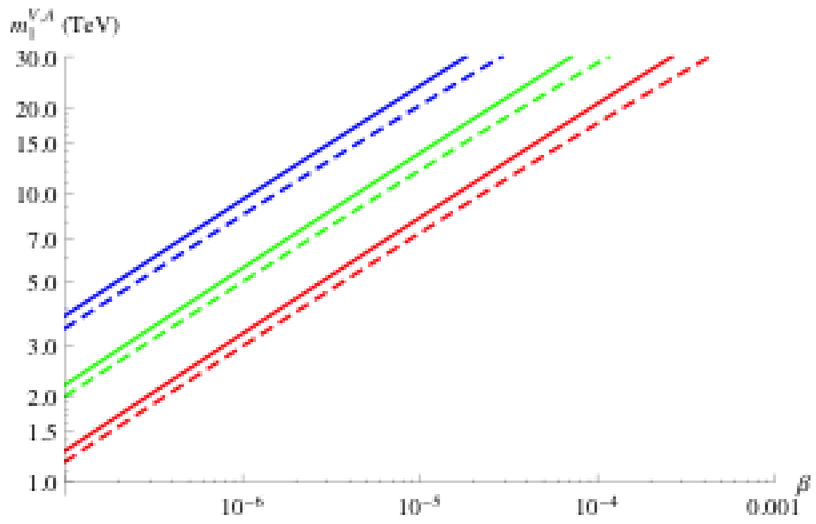
$$S = -8\pi K[b(\rho)\frac{\partial}{\partial q^2}(\Psi_V^0 \partial_\rho \Psi_V^0 - \Psi_A^0 \partial_\rho \Psi_A^0)]\Big|_{\rho=\infty, q=0}$$

Direct Computation of S

S parameter plotted as a function of Beta=1/L



Vector and Axial vector masses as a function of Beta.



Stability

- Studied by studying linear fluctuations around the 7, 7 bar embedding.
- Embedding is stable.
- 2 embedding functions. Each fluctuation has positive mass squared.

Scalar Masses

$(m^{\varphi,\theta}{}_n)^2 > 0$

Stable under linear fluctuations.

$m^{\varphi,\theta}{}_n$ almost degenerate with $m^V{}_n$.

$m^{\varphi}{}_n > m^V{}_n$ and $m^{\theta}{}_n < m^V{}_n$.

Spectrum

- No parity doubling.
- Scalar and vector mass differences are small.
- S is small but not due to cancellation between the V and A sectors.

Conclusions

- S is small because the decay constant to mass ratios are small. No cancellation between V and A sectors.
- Positive S. It is saturated by the first few resonances.
- No parity doubling. Vector bosons are always lighter than Axial Vectors.
- S increases with the length of the walking region. We expected the opposite to happen.

Conclusions

- Scalar and vector mass differences are small.
- We do not understand the reason for it.

Open Questions

- Yet to compute the anomalous dimension of the condensate.
- Are there light scalars? Possibly light Dilatons?
- Are there ways to compute the back reaction due to flavor embedding.
- Separating Chiral Breaking and Confinement scales?