New Directions in Scattering Theory

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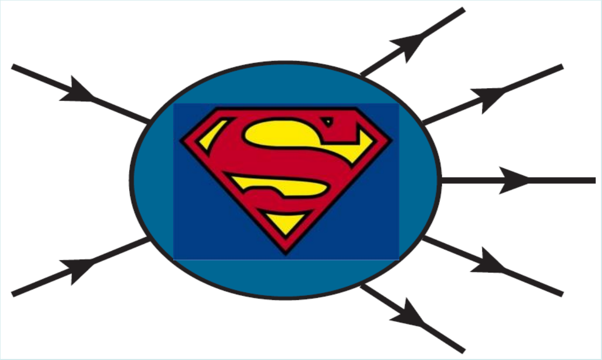
ICHEP 2012 Melbourne 11 July 2012

The *S* matrix

- Quantum field theory governs our description of high energy physics
- Contains much powerful, nonperturbative off-shell information
- But the "rubber meets the road" at the *S* matrix:



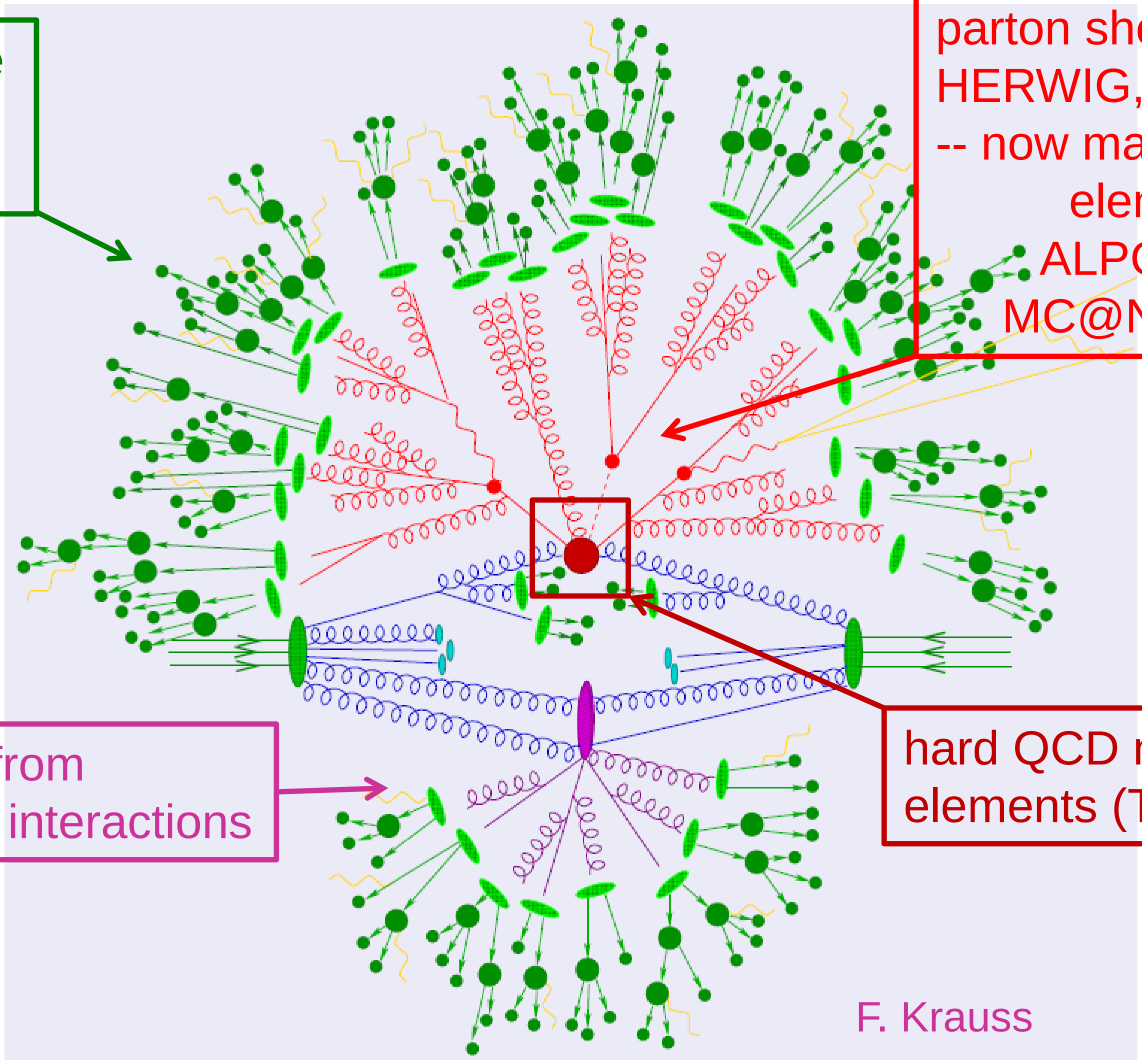
- LHC (and much other) experimental information based on **scattering** of on-shell states, evaluated in perturbation theory.

"Typical" hadron collider event

nonperturbative string/cluster fragmentation

See talk by J. Campbell for review of perturbative QCD status

underlying event from secondary parton interactions

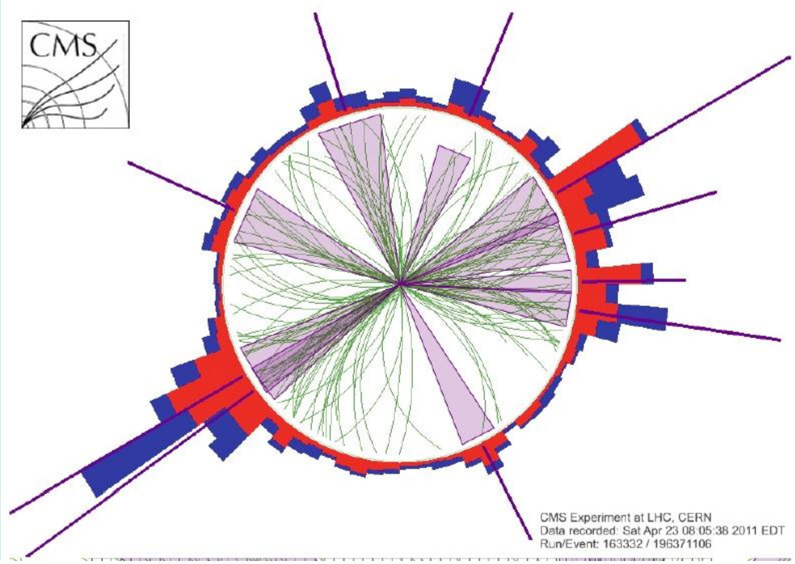


parton shower (PYTHIA, HERWIG, SHERPA)
-- now maintains matrix element accuracy: ALPGEN, SHERPA; MC@NLO, POWHEG

hard QCD matrix elements (THIS TALK)

A long road

- While accelerator physicists and experimenters were building the LHC and its detectors, theorists were improving their understanding of perturbative scattering amplitudes in order to:
- Enable more precise computations of complex, multi-jet LHC & Tevatron final states
- Help test Standard Model and search beyond it

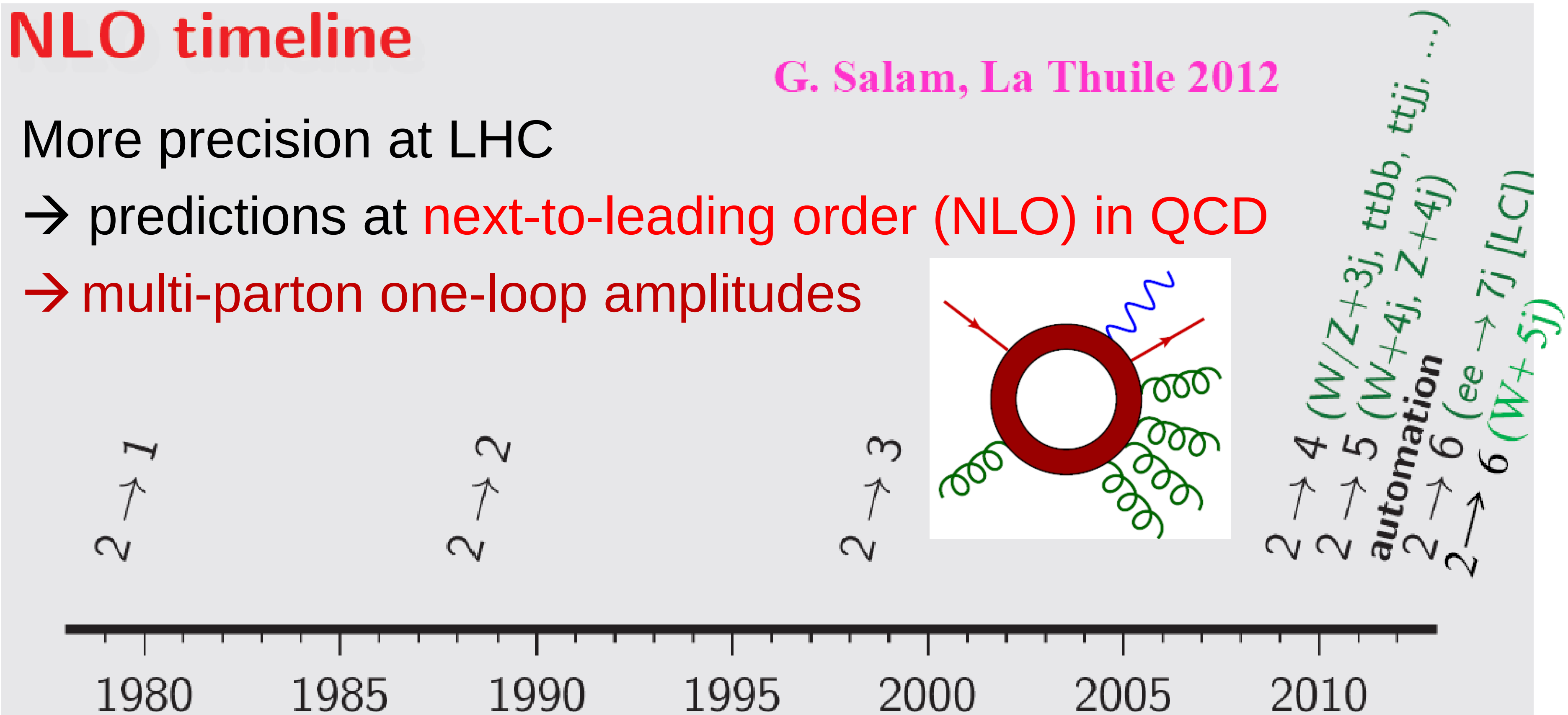


A long road (cont.)

More precision at LHC

→ predictions at next-to-leading order (NLO) in QCD

→ multi-parton one-loop amplitudes

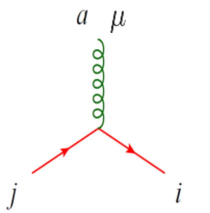


A winding road

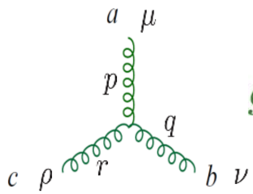
- New techniques were developed and tested out on an exotic cousin of QCD: maximally supersymmetric Yang-Mills theory (N=4 SYM)
- Along the way they also revealed:

1) remarkable properties of N=4 SYM
2) amazing duality between color and kinematics in gauge theory, leading to a picture of gravity as a "double copy" of gauge theory
3) remarkable UV behavior of N=8 supergravity

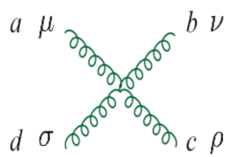
Tried and true perturbative technique: Feynman diagrams and rules



$$-ig_s(t^a)^i_j\gamma^\mu$$

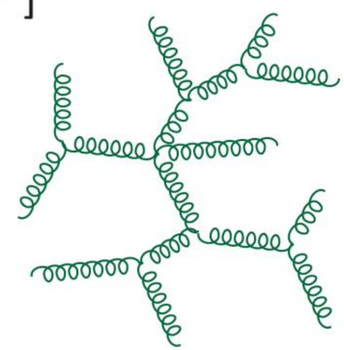


$$g_s f^{abc}[(p-q)^\rho\eta^{\mu\nu}+(q-r)^\mu\eta^{\nu\rho}+(r-p)^\nu\eta^{\rho\mu}]$$



$$-ig_s^2 f^{ace} f^{bde}[\eta^{\mu\nu}\eta^{\rho\sigma}-\eta^{\mu\sigma}\eta^{\rho\nu}]$$

+ 2 permutations

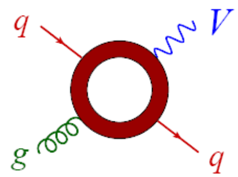
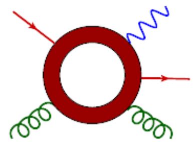
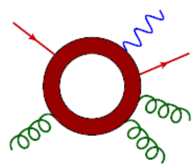
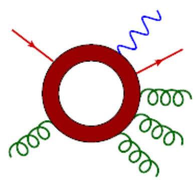
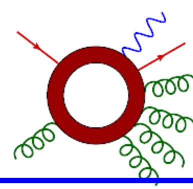


+ …

- Works very well at LHC for:
  - medium-multiplicity processes at tree level (LO)
  - low-multiplicity processes at one loop (NLO)

One loop QCD challenging at high multiplicity

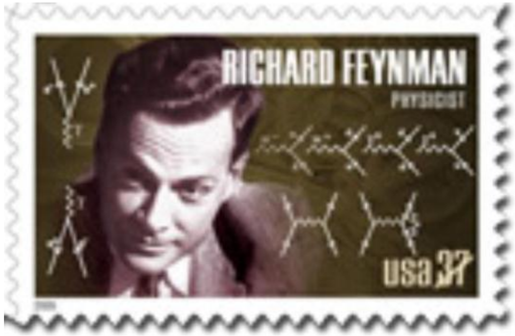
***pp* → *W* + *n* jets** (just amplitudes with most gluons)

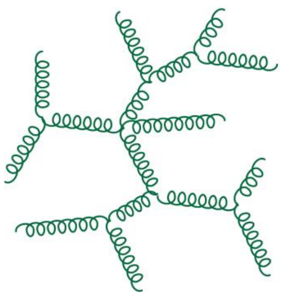
# of jets		# 1-loop Feynman diagrams
1		11
2		110
3		1,253
4		16,648
5		256,265

Current limit with Feynman diagrams

Current limit with on-shell methods

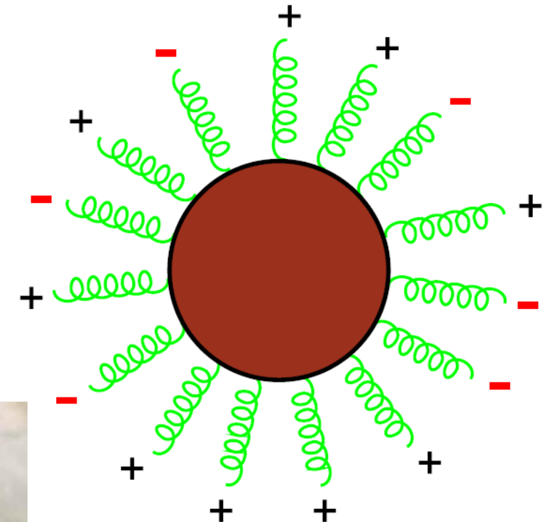
Granularity vs. Plasticity





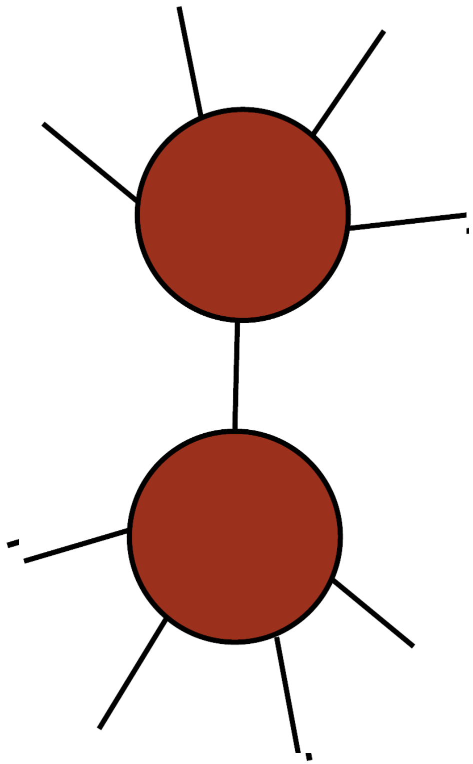
+ ...

on-shell methods



Recycling “Plastic” Amplitudes

Amplitudes fall apart into simpler ones in special limits – use information to reconstruct answer for any kinematics



Trees recycled into trees

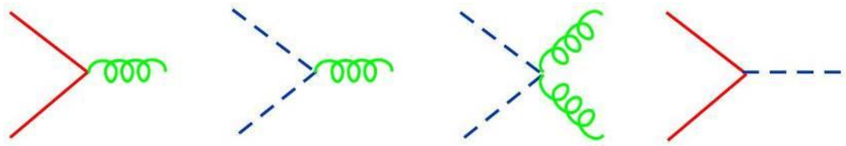
$$A_n(1,2,\ldots,n) \;=\; \sum_{h=\pm}\sum_{k=2}^{n-2} A_{k+1}(\hat{1},2,\ldots,k,-\hat{K}_{1,k}^{-h}) \times \frac{i}{K_{1,k}^2} A_{n-k+1}(\hat{K}_{1,k}^{h},k+1,\ldots,n-1,\hat{n})$$

on-shell recursion relations

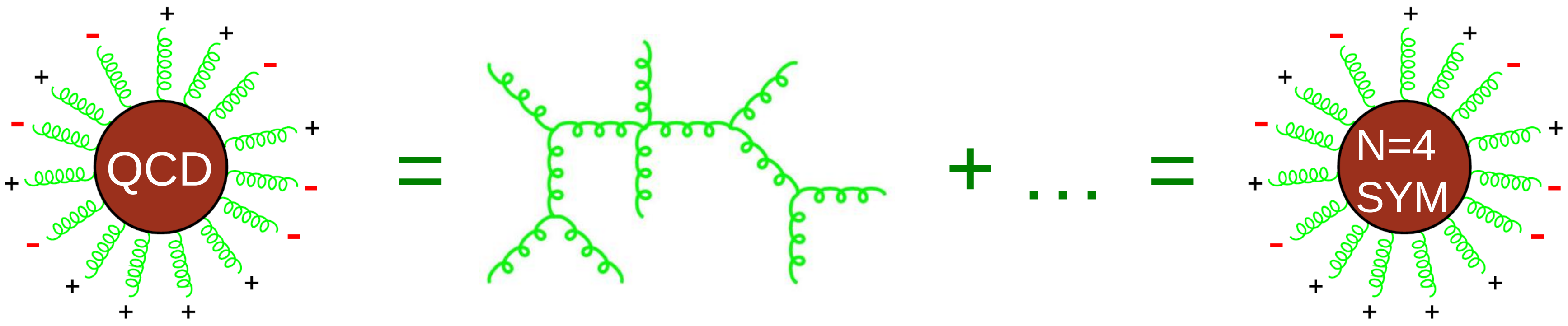
(Britto-Cachazo-Feng-Witten, 2004)

Universal Tree Amplitudes

- For pure-glue trees, matter particles (fermions, scalars) cannot enter, because they are always pair-produced

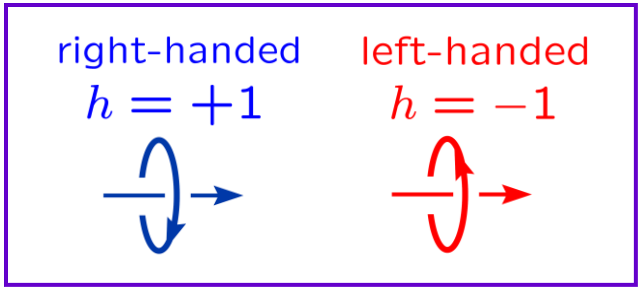


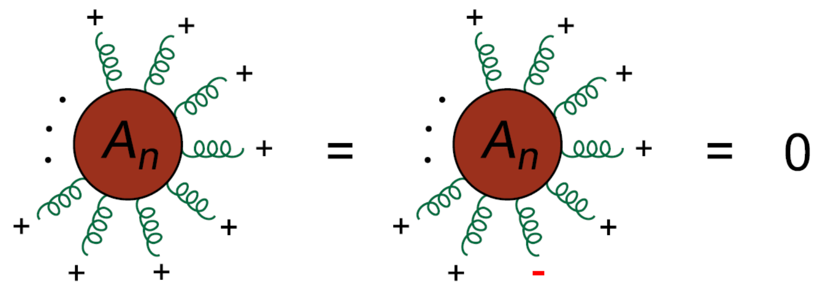
- Pure-glue trees are exactly the same in QCD as in N=4 SYM:

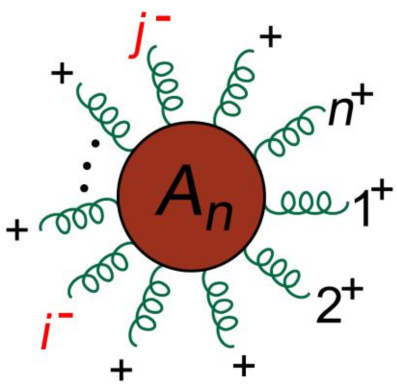


Tree-Level Simplicity

- When very simple QCD tree amplitudes were found, first in the 1980's







$$= \frac{\langle i\, j \rangle^4}{\langle 1\, 2\rangle \langle 2\, 3\rangle \cdots \langle n\, 1\rangle}$$

Parke-Taylor formula (1986)

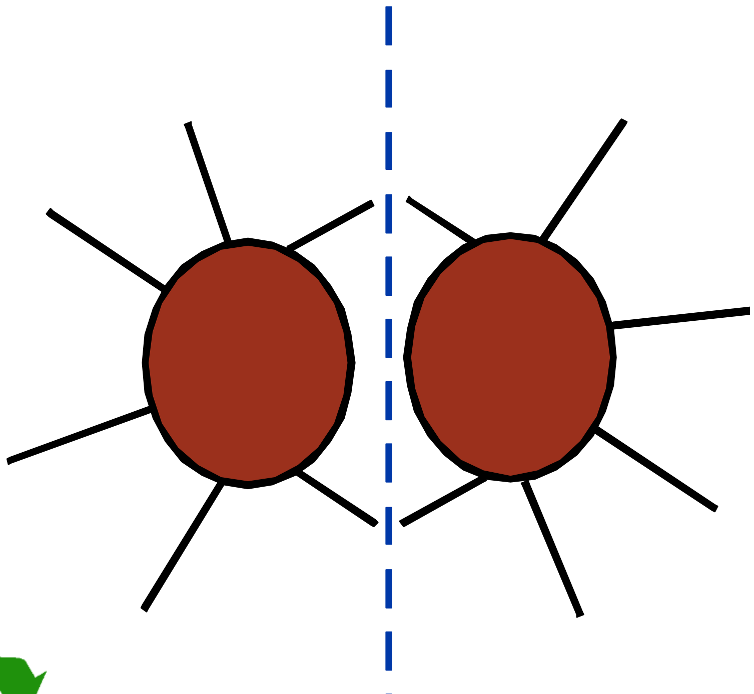
… the simplicity was secretly due to N=4 SYM

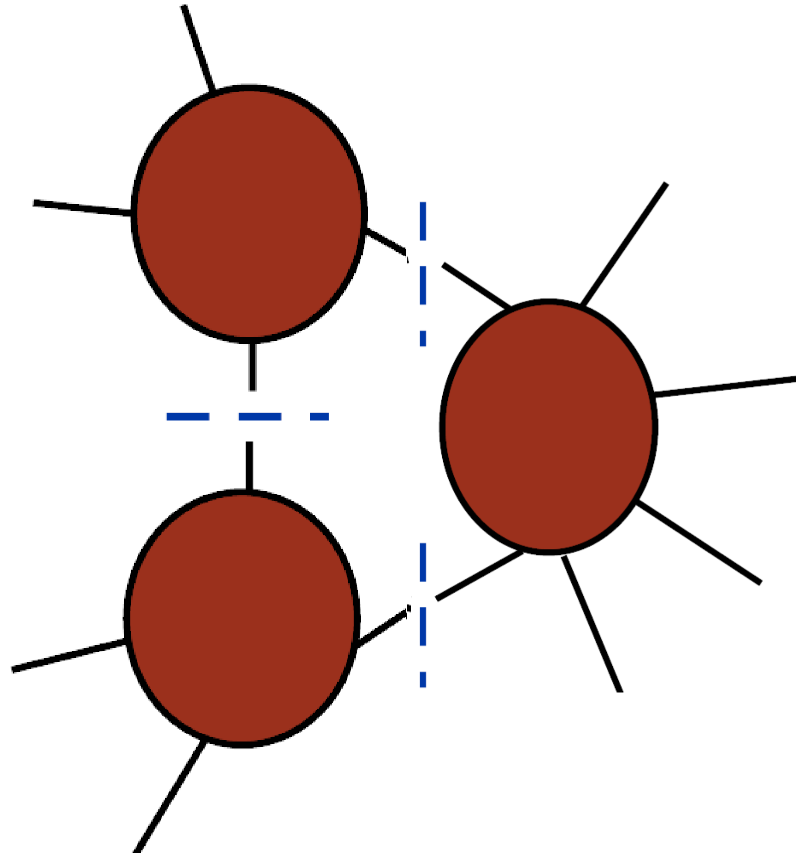
- Want to recycle this information at loop level in QCD

Generalized Unitarity (One-loop Plasticity)

Ordinary unitarity: $S^\dagger S = 1$
$S = 1 + iT \rightarrow \mathrm{Im}\, T = T^\dagger T$
At 1 loop puts 2 particles on shell

Generalized unitarity:
put 3 or 4 particles on shell



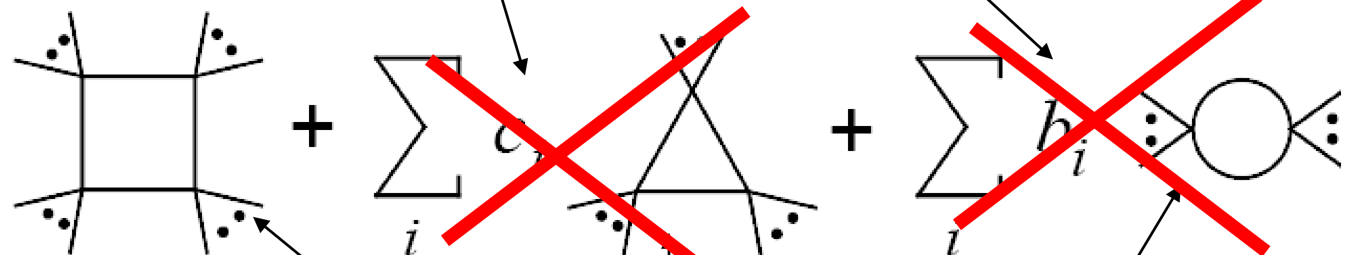


Trees recycled into loops!

Different theories differ at loop level

- ~~QCD~~ N=4 SYM at one loop:

coefficients are all rational functions – determine algebraically from products of trees using (generalized) unitarity

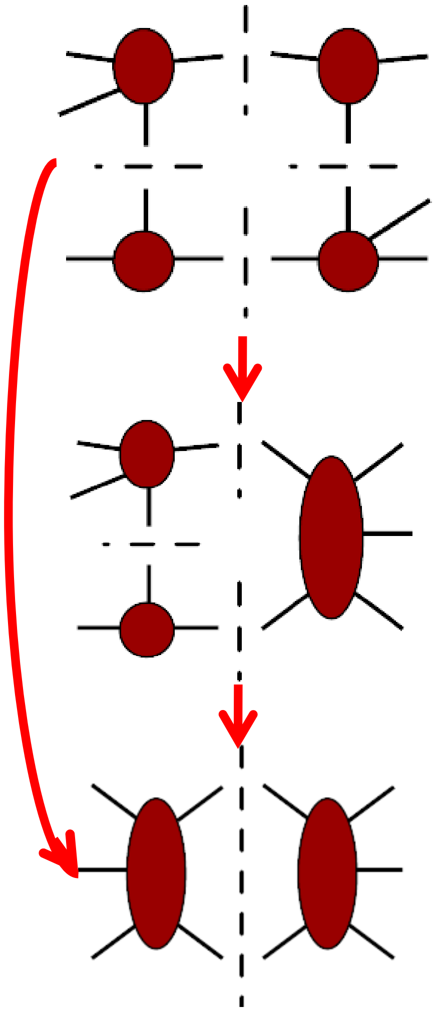


$$A^{1\text{-loop}} = \sum_i d_i \,[\text{box}] + \cancel{\sum_i c_i \,[\text{triangle}]} + \cancel{\sum_i b_i \,[\text{bubble}]} + \cancel{R} + \mathcal{O}(\epsilon)$$

rational part

well-known **scalar** one-loop integrals, master integrals, same for all amplitudes

QCD amplitude determined hierarchically



Each box coefficient comes uniquely from 1 "quadruple cut"

Britto, Cachazo, Feng, hep-th/0412103

Ossola, Papadopolous, Pittau, hep-ph/0609007;
Mastrolia, hep-th/0611091; Forde, 0704.1835;
Ellis, Giele, Kunszt, 0708.2398; Berger et al., 0803.4180;…

Each triangle coefficient from 1 triple cut, but "contaminated" by boxes

Each bubble coefficient from 1 double cut, removing contamination by boxes and triangles

Rational part depends on all of above

Bottom Line:

Trees recycled into loops!

Dramatic increase recently in rate of NLO QCD predictions for new processes:

The NLO revolution

Many Automated On-Shell One Loop Programs

Blackhat: Berger, Bern, LD, Diana, Febres Cordero, Forde, Gleisberg, Höche, Ita, Kosower, Maître, Ozeren, 0803.4180, 0808.0941, 0907.1984, 1004.1659, 1009.2338
\+ **Sherpa** → NLO *W,Z* + 3,4,5 jets   pure QCD 4 jets

CutTools: Ossola, Papadopolous, Pittau, 0711.3596
NLO *WWW, WWZ*, ... Binoth+OPP, 0804.0350
NLO $t\bar{t}b\bar{b}$, $t\bar{t}$ + 2 jets,…
Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, 0907.4723; 1002.4009
MadLoop → aMC@NLO: $t\bar{t}H$, *Wbb* Hirschi et al. 1103.0621, 1104.5613, …
HELAC-NLO: $t\bar{t}\,t\bar{t}$ Bevilacqua et al, 1110.1499, 1206.3064

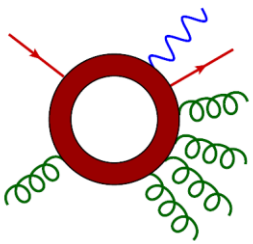
Rocket: Giele, Zanderighi, 0805.2152
Ellis, Giele, Kunszt, Melnikov, Zanderighi, 0810.2762
NLO *W* + 3 jets Ellis, Melnikov, Zanderighi, 0901.4101, 0906.1445
$W^+W^\pm$ + 2 jets Melia, Melnikov, Rontsch, Zanderighi, 1007.5313, 1104.2327

SAMURAI: Mastrolia, Ossola, Reiter, Tramontano, 1006.0710

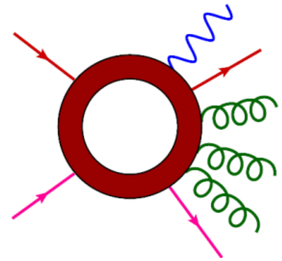
GoSam: W^+W^- + 2 jets Cullen et al. 1111.2034; Greiner et al. 1202.6004

NGluon: Badger, Biedermann, Uwer, 1011.2900

e.g. NLO $pp \rightarrow W + 5$ jets now feasible

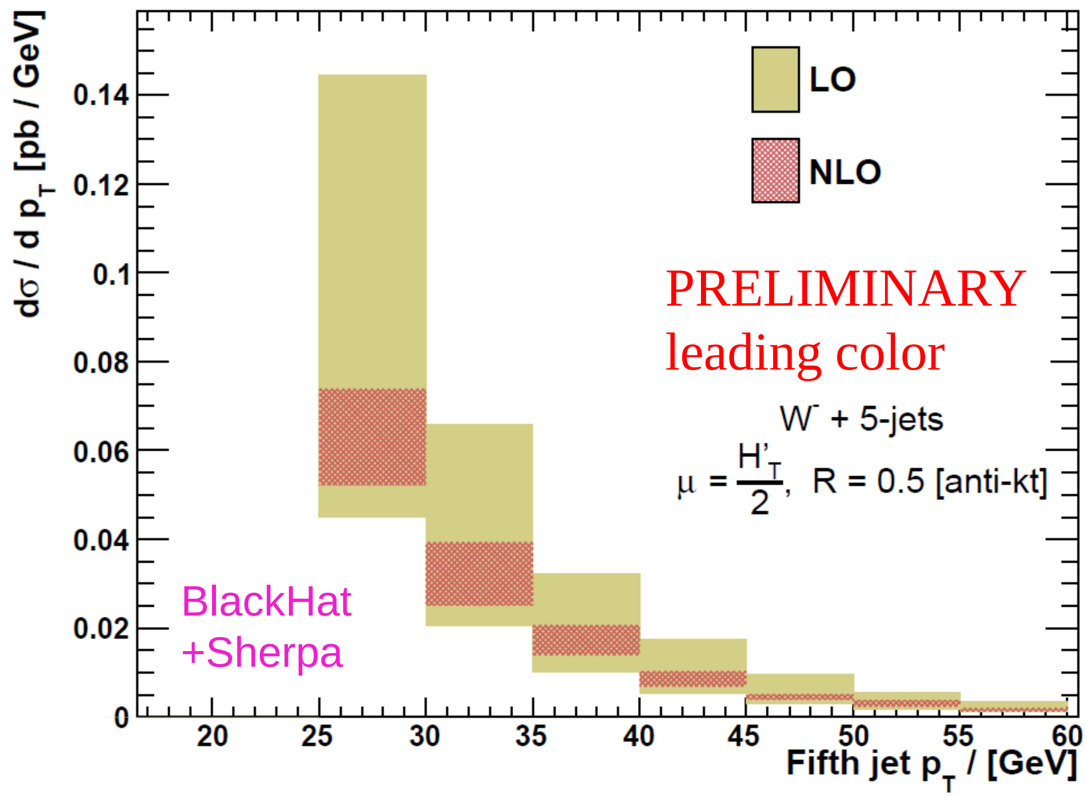


256,265 +



49,614 + …

number of one-loop Feynman diagrams not computed



See talks by D. Banturin & J. Campbell for more QCD applications

QCD's super-simple cousin:

N=4 supersymmetric Yang-Mills theory

"The lithium atom of the 21st century"

massless spin 1 gluon

4 massless spin 1/2 gluinos

6 massless spin 0 scalars

all states in adjoint representation, all linked by N=4 supersymmetry

- Interactions uniquely specified by gauge group, say $\mathrm{SU}(N_c)$ → only 1 coupling g
- Exactly scale-invariant (conformal) field theory: $\beta(g) = 0$ for all g
- What could be simpler?

Planar N=4 SYM and AdS/CFT

- In the 't Hooft limit, $N_c \to \infty$
  $\lambda = g^2 N_c$ fixed,

planar diagrams dominate

- AdS/CFT duality suggests planar N=4 SYM should be even simpler, because

large λ limit ←→ weakly-coupled gravity/string theory on $AdS_5 \times S^5$

Maldacena

Four Remarkable, Related Structures Found in Planar N=4 SYM Scattering

- Exact exponentiation of 4 & 5 gluon amplitudes
- Dual (super)conformal invariance
- Strong coupling → "soap bubbles"
- Equivalence between (MHV) amplitudes and Wilson loops

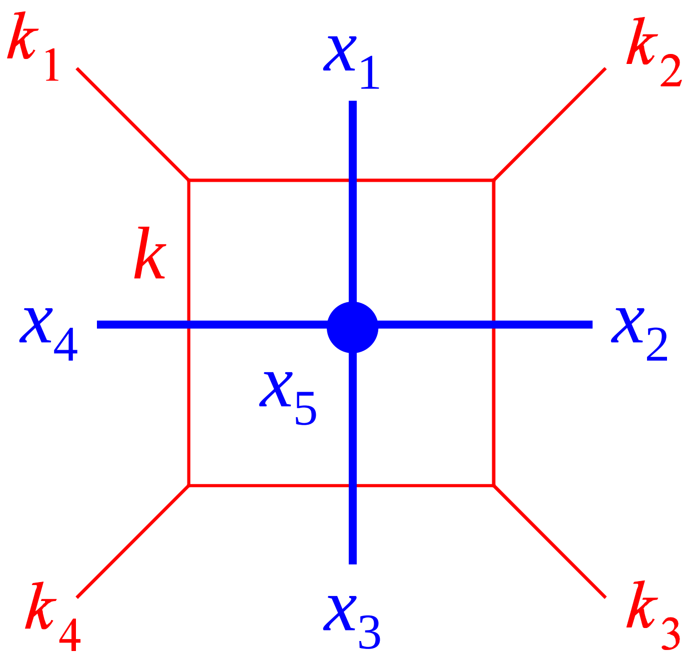
Can these structures be used to solve exactly for all planar N=4 SYM amplitudes?
What is the first nontrivial case to solve?

Dual Conformal Invariance

Broadhurst (1993); Lipatov (1999); Drummond, Henn, Smirnov, Sokatchev, hep-th/0607160

Conformal symmetry acting in momentum space, on dual or sector variables x_i

First seen in N=4 SYM planar amplitudes in the loop integrals



$$I = \int d^4k \frac{(k_1+k_2)^2(k_2+k_3)^2}{k^2(k-k_1)^2(k-k_1-k_2)^2(k+k_4)^2}$$

$$I = \int d^4x_5 \frac{x_{24}^2 x_{13}^2}{x_{15}^2 x_{25}^2 x_{35}^2 x_{45}^2}$$

$$k_1 = x_{41}$$
$$k_2 = x_{12}$$
$$k_3 = x_{23}$$
$$k_4 = x_{34}$$
$$k = x_{45}$$

invariant under inversion:

$$x_i^\mu \to \frac{x_i^\mu}{x_i^2}$$

$$x_{ij}^2 \to \frac{x_{ij}^2}{x_i^2 x_j^2}, \qquad \mathrm{d}^4 x_i \to \frac{\mathrm{d}^4 x_i}{x_i^8}$$

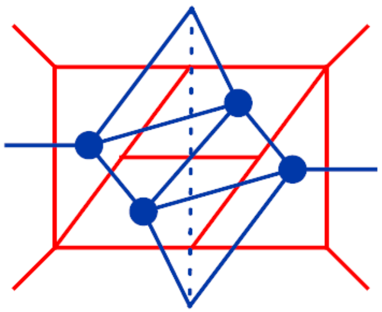
Dual conformal invariance at higher loops

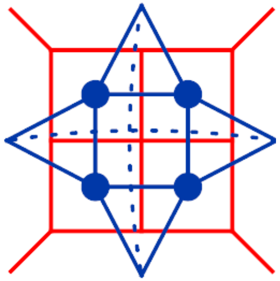
- Simple graphical rules:

4 (net) lines into inner x_i

1 (net) line into outer x_i

- Dotted lines are for numerator factors





All integrals entering planar 4-point amplitude at 2, 3, 4, 5, 6, 7 loops are of this form!

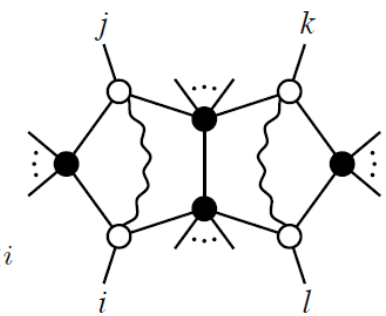
Bern, Czakon, LD, Kosower, Smirnov, hep-th/0610248
Bern, Carrasco, Johansson, Kosower, 0705.1864
Bourjaily et al., 1112.6432
Eden, Heslop, Korchemsky, Sokatchev, 1201.5329

All planar N=4 SYM integrands

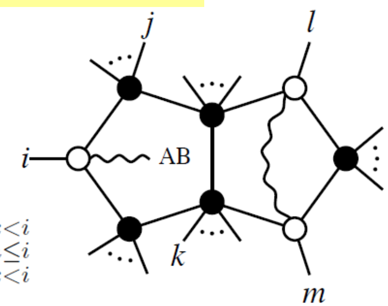
Arkani-Hamed, Bourjaily, Cachazo, Caron-Huot, Trnka, 1008.2958, 1012.6032

- All-loop BCFW recursion relation for integrand ☺
- Manifest Yangian invariance (huge group containing dual conformal symmetry).
- Multi-loop integrands written in terms of “momentum-twistors”.
- Still have to do integrals over the loop momentum ☹

$$\mathcal{A}^{2-\text{loop}}_{\text{MHV}} = \frac{1}{2}\sum_{i<j<k<l<i}$$

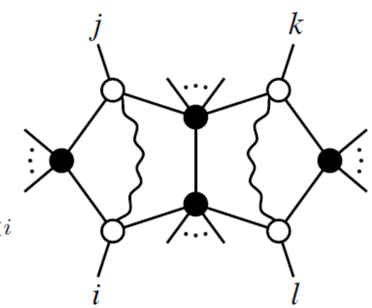


$$\mathcal{A}^{2-\text{loop}}_{\text{NMHV}} = \sum_{\substack{i<j<l<m\leq k<i\\ i<j<k<l<m\leq i\\ i\leq l<m\leq j<k<i}}$$



$$\times\,[i, j, j+1, k, k+1]$$

$$+\frac{1}{2}\sum_{i<j<k<l<i}$$



$$\times \left\{ \begin{array}{c} \mathcal{A}^{\text{tree}}_{\text{NMHV}}(j,\ldots,k;\ l,\ldots,i) \\ +\mathcal{A}^{\text{tree}}_{\text{NMHV}}(i,\ldots,j) \\ +\mathcal{A}^{\text{tree}}_{\text{NMHV}}(k,\ldots,l) \end{array} \right\}$$

Do we even need integrands?

In many cases, symmetries and other constraints on the multi-loop planar N=4 SYM amplitude are so powerful that we don't even need to know the integrand at all! ☺

Dual conformal constraints

- Symmetry fixes form of amplitude, up to functions of dual conformally invariant cross ratios:

$$u_{ijkl} \equiv \frac{x_{ij}^2 x_{kl}^2}{x_{ik}^2 x_{jl}^2}$$

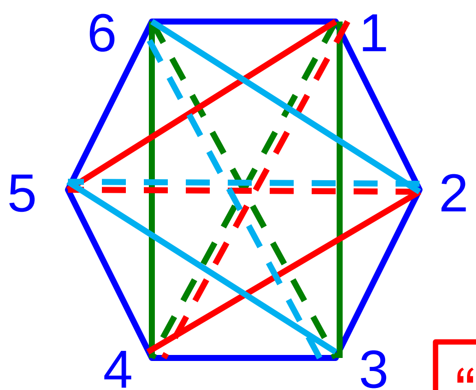
- Because $x_{i-1,i}^2 = k_i^2 = 0$ there are no such variables for n = 4,5
- Amplitude known to all orders (fixed to BDS ansatz):

$$\boxed{\mathcal{A}_{4,5}(\epsilon; s_{ij}) \; = \; \mathcal{A}_{4,5}^{\mathrm{BDS}}(\epsilon; s_{ij})}$$

For $n = 6$, precisely 3 ratios:

$$u_1 = \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2} = \frac{s_{12} s_{45}}{s_{123} s_{345}}$$

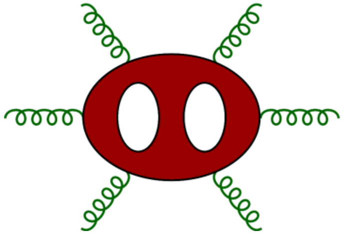
+ 2 cyclic perm's



$$\boxed{\mathcal{A}_6(\epsilon; s_{ij}) \; = \; \mathcal{A}_6^{\mathrm{BDS}}(\epsilon; s_{ij}) \exp[R_6(u_1, u_2, u_3)]}$$

"first obstruction" to solving planar N=4 SYM

2 loops 6 gluons: $R_6^{(2)}(u_1,u_2,u_3)$



• First worked out analytically from Wilson loop integrals
Del Duca, Duhr, Smirnov, 0911.5332, 1003.1702
17 pages of Goncharov polylogarithms

• Simplified to just a few classical polylogarithms $\mathrm{Li}_n(x)$:
Goncharov, Spradlin, Vergu, Volovich, 1006.5703

$$\mathrm{Li}_n(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^n}$$

$$\boxed{\begin{aligned} R_6^{(2)}(u_1,u_2,u_3) &= \sum_{i=1}^{3}\left(L_4(x_i^+,x_i^-) - \frac{1}{2}\mathrm{Li}_4(1-1/u_i)\right) \\ &- \frac{1}{8}\left(\sum_{i=1}^{3}\mathrm{Li}_2(1-1/u_i)\right)^2 + \frac{1}{24}J^4 + \frac{\pi^2}{12}J^2 + \frac{\pi^4}{72} \end{aligned}}$$

$$L_4(x^+,x^-) = \frac{1}{8!!}\log(x^+x^-)^4 + \sum_{m=0}^{3}\frac{(-1)^m}{(2m)!!}\log(x^+x^-)^m(\ell_{4-m}(x^+)+\ell_{4-m}(x^-))$$

$$\ell_n(x) = \frac{1}{2}\left(\mathrm{Li}_n(x) - (-1)^n\,\mathrm{Li}_n(1/x)\right)$$

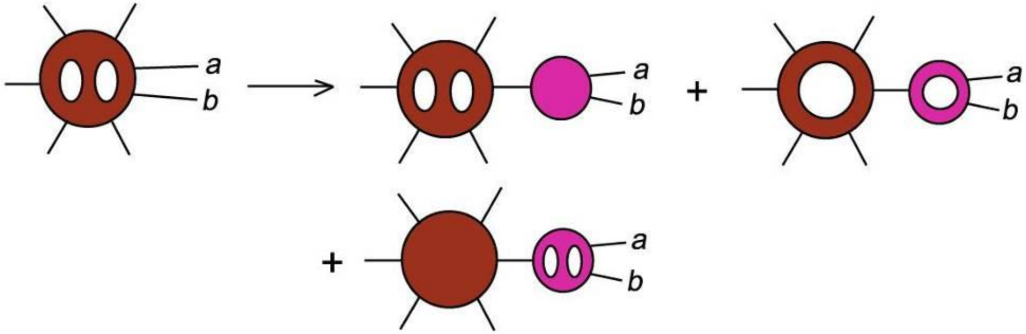
$$J = \sum_{i=1}^{3}(\ell_1(x_i^+) - \ell_1(x_i^-))$$

$$\boxed{x_i^\pm = u_i x^\pm, \qquad x^\pm = \frac{u_1+u_2+u_3-1\pm\sqrt{\Delta}}{2u_1u_2u_3} \qquad \Delta = (u_1+u_2+u_3-1)^2 - 4u_1u_2u_3}$$

A collinear constraint

Alday, Gaiotto, Maldacena, Sever, Vieira, 1006.2788; GMSV, 1010.5009; 1102.0062

- $R_6^{(2)}(u_1, u_2, u_3)$ recovered directly from analytic properties, using "near collinear limits"



- Limit controlled by an operator product expansion (OPE) for (equivalent) polygonal Wilson loops
- Allows bootstrapping to yet more complicated amplitudes:

n=7,8,… , L=2 Sever, Vieira, 1105.5748

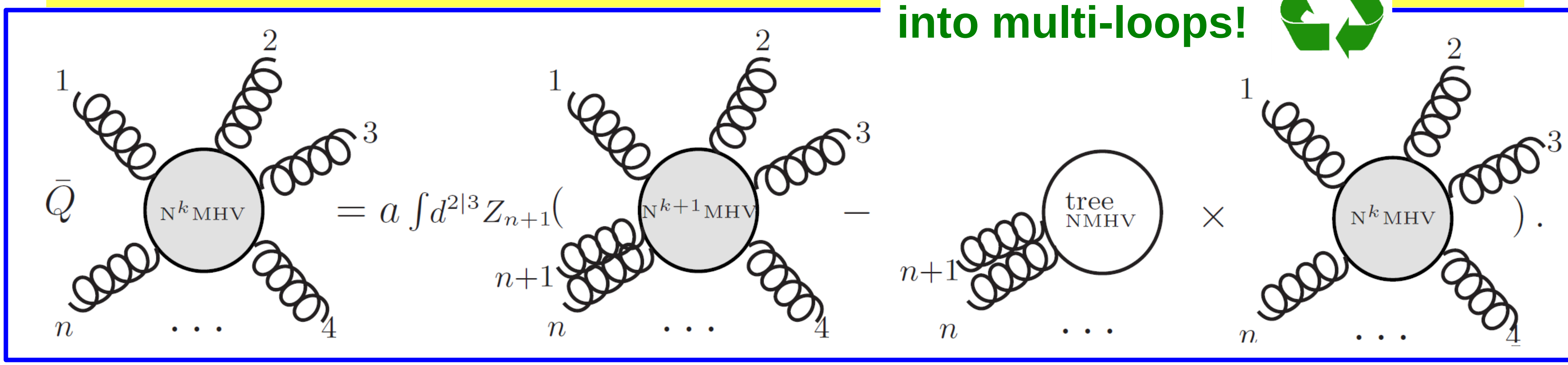
n=6, L=3,4 LD, Drummond, Henn, 1108.4461; LD, Duhr, Pennington, 1207.0186

An integro-differential equation

Bullimore, Skinner, 1112.1056; Caron-Huot, He, 1112.1060

- Based on a first order differential operator (dual superconformal generator) $\bar{Q}$, which has anomaly due to virtual collinear configurations:

Loops recycled into multi-loops!

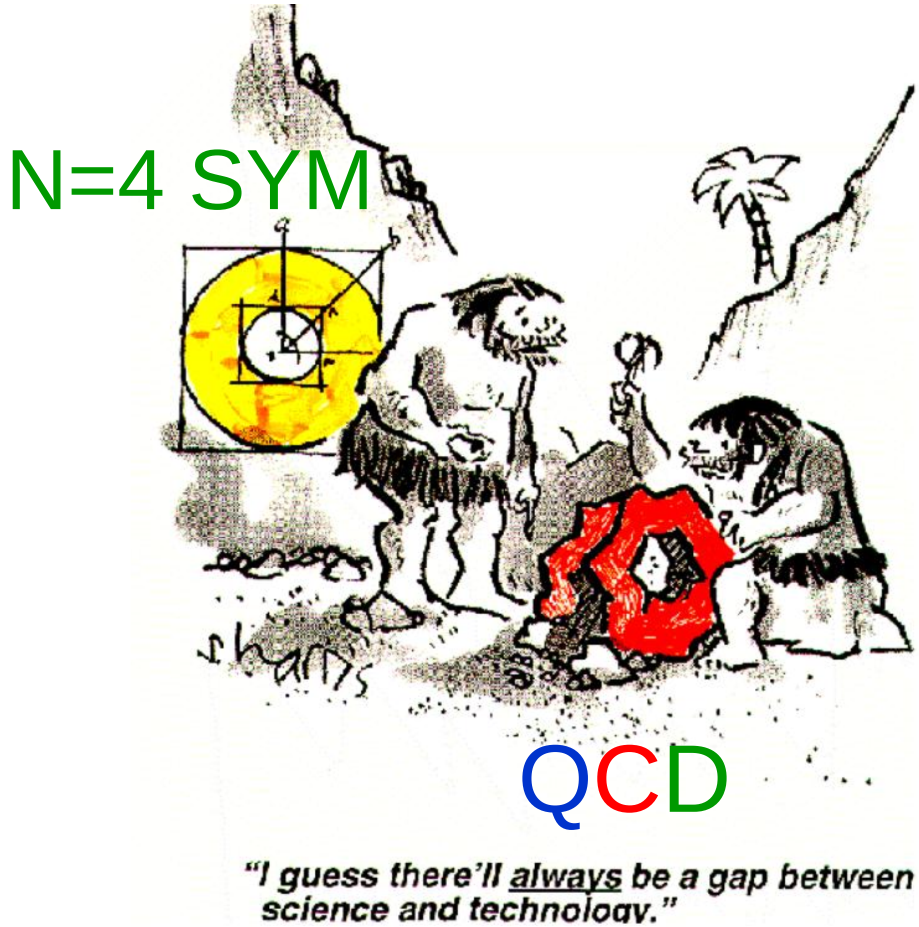


- Already used to recover several other state-of-the-art results

Caron-Huot, He, 1112.1060

- An iterative solution to the all-loop all-multiplicity *S* matrix!?!

Can multi-loop N=4 SYM advances help for QCD?



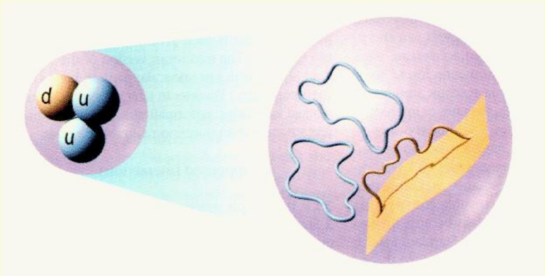
Yes and no.

"Most complicated" analytic functions are common to both theories.

But simpler parts (e.g. rational part at one loop) are distinct.

What's next after the Higgs?

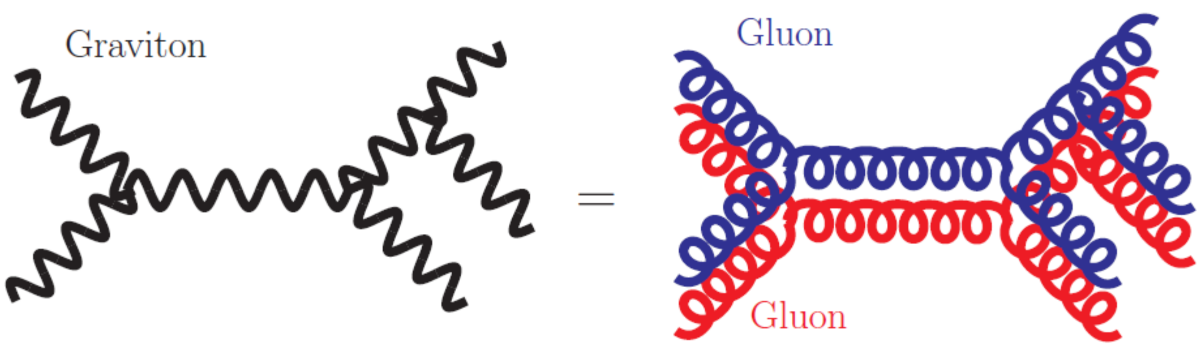
- Unifying gravity with quantum mechanics?
- Quantum gravity diverges badly in ultraviolet, because Newton's constant, $G_N = 1/M_{\mathrm{Pl}}^2$ is dimensionful
- String theory cures divergences, but particles are no longer pointlike.



- Is this really necessary? Or could enough symmetry, e.g. **N=8 supersymmetry**, allow a point particle theory of quantum gravity to be perturbatively ultraviolet finite?

UV Finiteness of N=8 Supergravity?

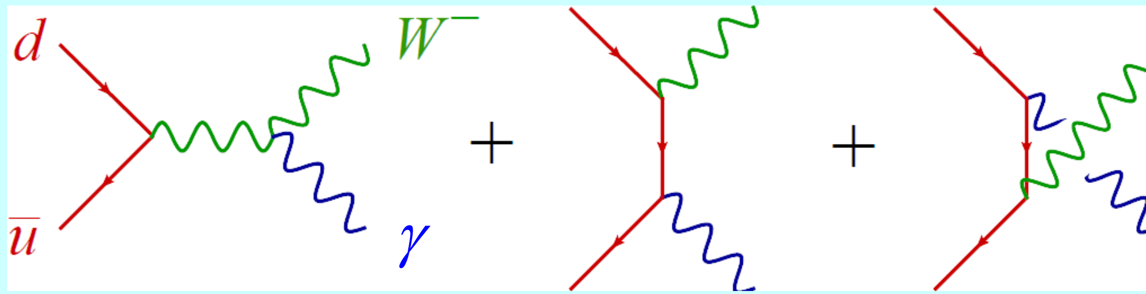
- Investigate by computing multi-loop amplitudes in N=8 supergravity DeWit, Freedman (1977); Cremmer, Julia, Scherk (1978); Cremmer, Julia (1978,1979) and examining their ultraviolet behavior. Bern, Carrasco, LD, Johansson, Kosower, Roiban, hep-th/0702112; BCDJR, 0808.4112, 0905.2326, 1008.3327, 1201.5366
- But gravity is very complicated, so try to write spin-2 graviton as two spin-1 gluons stitched together:



Radiation Zeroes

- In 1979, Mikaelian, Samuel and Sahdev computed

$$\frac{d\sigma(d\bar{u}\to W^-\gamma)}{d\cos\theta}$$



- They found a "radiation zero" at

$$\cos\theta = -(1+2Q_d) = -1/3$$

- Zero independent of (W,γ) helicities
- Implied a connection between "color" (here ~ electric charge Q_d) and kinematics ($\cos\theta$)

Radiation Zeroes → Color-Kinematic Duality

- MSS result was soon generalized to other 4-point non-Abelian gauge theory amplitudes

  Zhu (1980), Goebel, Halzen, Leveille (1981)

- Massless adjoint gauge theory result:

$$\mathcal{A}_4^{\text{tree}} = \frac{n_s C_s}{s} + \frac{n_t C_t}{t} + \frac{n_u C_u}{u}$$

- 3 color factors are not independent:

$C_i \sim f^{abe} f^{cde}$ → Jacobi relation: $C_t - C_u = C_s$

- In a suitable "gauge", one finds that

  kinematic factors obey same relation: $n_t - n_u = n_s$

  Same structure extends to **arbitrary amplitudes!**

  Bern, Carrasco, Johansson, 0805.3993

From Color-Kinematic Duality to Double Copy Formula

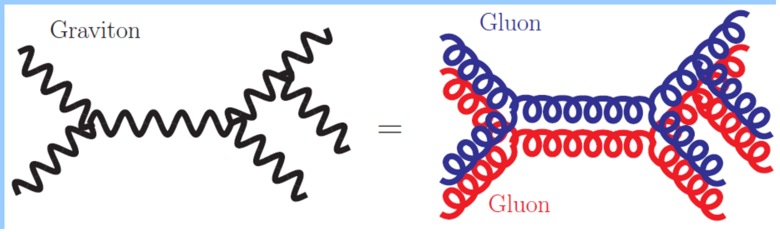
- When kinematic numerators are arranged to obey same Jacobi-type relations as the color factors, then gravity amplitudes are given simply by squaring gauge-theory numerators $n_i(s,t)$:

$$\mathcal{A}_4^{\text{tree}} = \frac{n_s C_s}{s} + \frac{n_t C_t}{t} + \frac{n_u C_u}{u}$$

$$\rightarrow \quad M_4^{\text{tree}} = \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

Bern, Carrasco, Johansson, 0805.3993

- Same procedure works for complicated loop amplitudes!

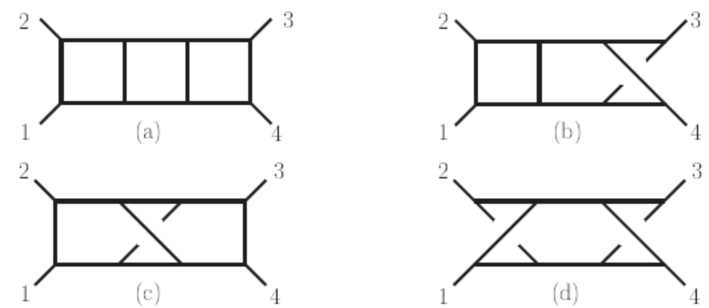


Double-copy formula at 3 loops

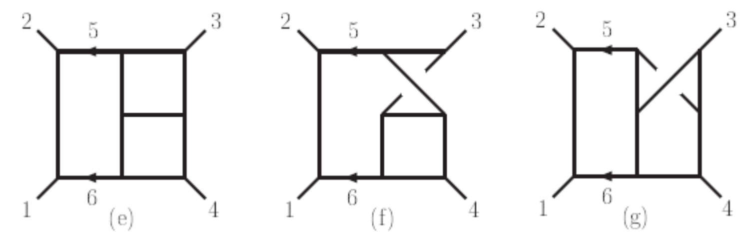
Bern, Carrasco, Johansson, 1004.0476

N=8 SUGRA

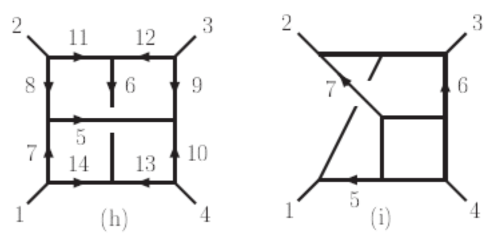
$$[\, s^2 \,]^2$$



$$\Big[\ \tfrac{1}{3}\ \ [s(t-\tau_{36}-\tau_{46}) \\ -\ t(\tau_{26}+\tau_{46}) \\ +\ u(\tau_{26}+\tau_{36}) - s^2]\ \Big]^2$$



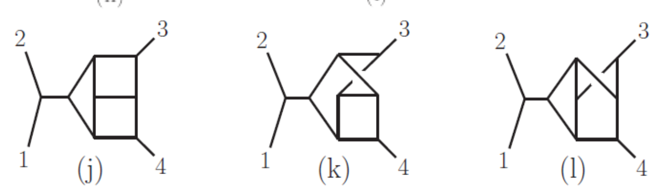
$$\left[N^{(h)}(\tau_{ij})\right]^2$$



$$\left[N^{(i)}(\tau_{ij})\right]^2$$

4 loop amplitude has similar representation!
BCDJR, 1201.5366

$$\left[\frac{1}{3}s(t-u)\right]^2$$



N=8 SUGRA as good as N=4 SYM in ultraviolet

Using double-copy formula, N=8 SUGRA amplitudes found to have the same UV behavior as N=4 SYM through 4 loops.

- But N=4 SYM is known to be finite for all loops.

- Therefore, either this pattern breaks at some point (5 loops??) or else **N=8 supergravity** would be a **perturbatively finite point-like theory of quantum gravity**, against all conventional wisdom!

Conclusions

- Scattering amplitudes have a very rich structure, in QCD, but especially in highly supersymmetric gauge theories and supergravity.
- Much structure impossible to see using Feynman diagrams, unveiled with help of on-shell methods
- Among other applications, they have
  - had practical payoffs in the "NLO QCD revolution"
  - provided clues that planar N=4 SYM amplitudes might be solvable in closed form
  - shown that N=8 supergravity has amazingly good ultraviolet properties
- More surprises are surely in store in the future!

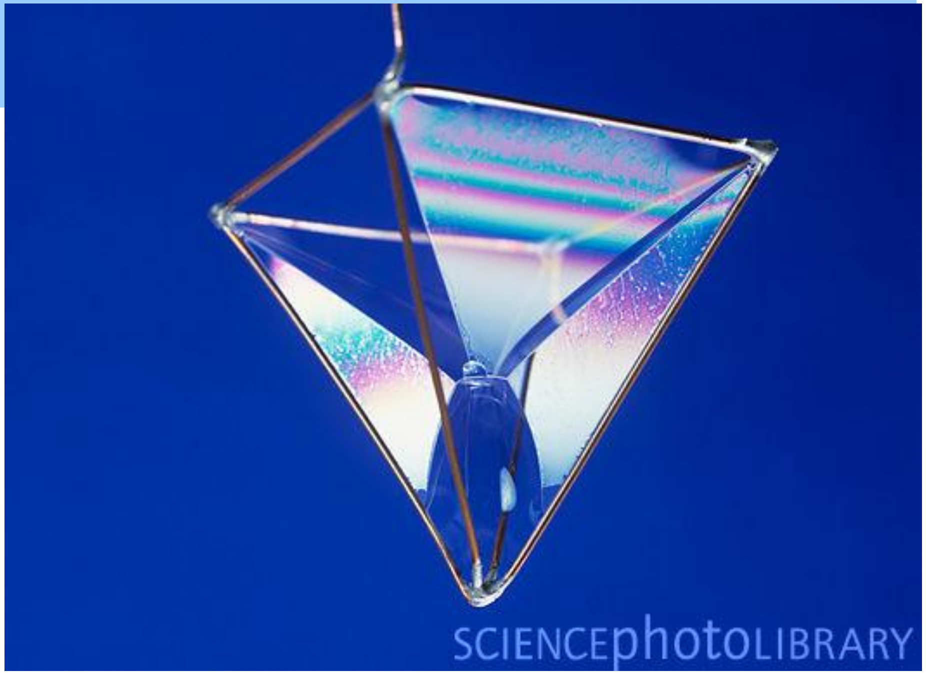
Extra Slides

Strong coupling and soap bubbles

Alday, Maldacena, 0705.0303

- Use AdS/CFT to compute scattering amplitude
- High energy scattering in string theory semi-classical: two-dimensional string world-sheet stretches a long distance, classical solution minimizes area

Gross, Mende (1987,1988)



Classical action imaginary
→ exponentially suppressed tunnelling configuration

$$A_n \sim \exp[-\sqrt{\lambda} S_{\rm cl}^{\rm E}]$$

$\mathcal{N}=8$ vs. $\mathcal{N}=4$ SYM

$2^8 = 256$ massless states, ~ expansion of $(x+y)^8$

$$\mathcal{N}=8: \quad 1 \leftrightarrow 8 \leftrightarrow 28 \leftrightarrow 56 \leftrightarrow 70 \leftrightarrow 56 \leftrightarrow 28 \leftrightarrow 8 \leftrightarrow 1$$

$$\text{helicity}: \quad -2 \quad -\tfrac{3}{2} \quad -1 \quad -\tfrac{1}{2} \quad 0 \quad \tfrac{1}{2} \quad 1 \quad \tfrac{3}{2} \quad 2$$

SUSY $\longleftrightarrow$

$$h^- \quad \psi_i^- \quad v_{ij}^- \quad \chi_{ijk}^- \quad s_{ijkl} \quad \chi_{ijk}^+ \quad v_{ij}^+ \quad \psi_i^+ \quad h^+$$

$$\mathcal{N}=4 \text{ SYM}: \quad 1 \quad 4 \quad 6 \quad 4 \quad 1$$

$2^4 = 16$ states ~ expansion of $(x+y)^4$

$$g^- \quad \lambda_A^- \quad \phi_{AB} \quad \lambda_A^+ \quad g^+$$

all in adjoint representation

$$\Rightarrow \quad [\mathcal{N}=8] = [\mathcal{N}=4] \otimes [\mathcal{N}=4]$$