A model independent determination of the $B \to X_s \gamma$ decay rate

SIMBA Collaboration:

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Talk overview

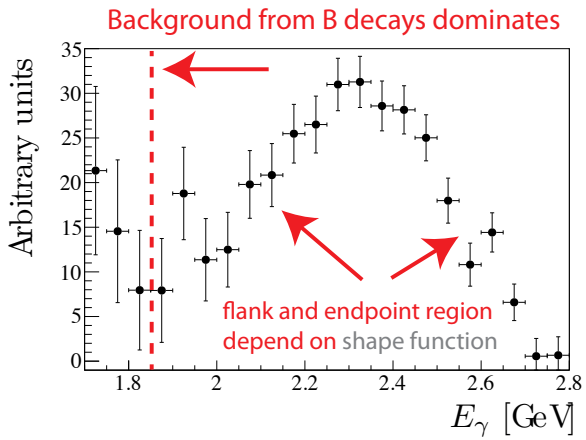
I. Introduction
II. Analyzed $B \to X_s \gamma$ measurements and Fit
III. Determination of the shape function and C_7^{incl}
IV. Theory uncertainties
V. Summary and Conclusion

I.a Introduction

- $B \to X_s \gamma$ very promising to probe Flavor sector for new physics
- Most precise measurements at high E_γ

$\Updownarrow$

Theory most precise with low E_γ cut

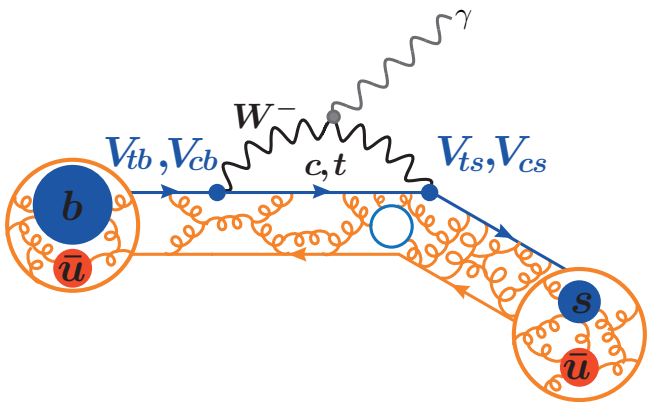


[PRL:103241801]

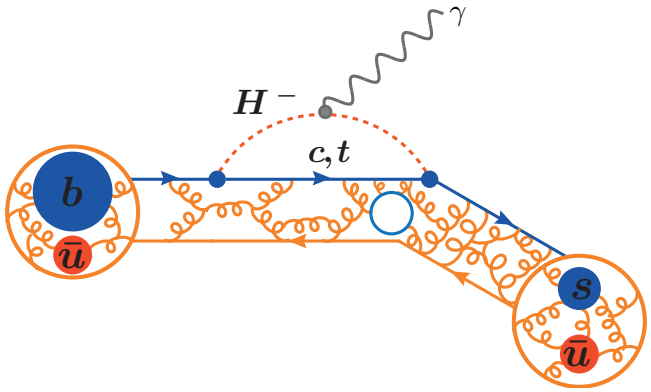
- Rising E_γ cut $\leftrightarrow$ dependence on parton distribution function of b-quark ($\widehat{=}$ Shape function)
- **HFAG** extrapolates $\Delta\mathcal{B}$ to a lower cut $E_\gamma > 1.6$

Reference	$\Delta\mathcal{B}(E_\gamma > 1.6\,\text{GeV})$
HFAG [arXiv:1010.1589]	$(3.55 \pm 0.24 \pm 0.09) \times 10^{-4}$
Misiak et al. [PRL:98:022002]	$(3.15 \pm 0.23) \times 10^{-4}$

Standard Model $B \to X_s \gamma$:



2HDM contribution:



$\Rightarrow$ **SIMBA** tests Standard Model without need of extrapolations

I.b Shape function and Master fomulae

- Treat unknown shape function $\widehat{F}(k)$ as expansion of set of basis functions:

$$\widehat{F}(k) = \frac{1}{\lambda}\left[\sum_n c_n\, f_n(k)\right]^2 \qquad \text{with} \qquad \int_0^\infty \mathrm{d}k\, \widehat{F}(k) = \sum_n c_n^2 = 1$$

λ is a parameter of the basis, f_n are the basis functs. with coeff. c_n.

- Non-perturbative physics in coefficients $c_n \rightarrow$ determine from measured differential E_γ spectra
- finite exp. input $\leftrightarrow$ series must be truncated

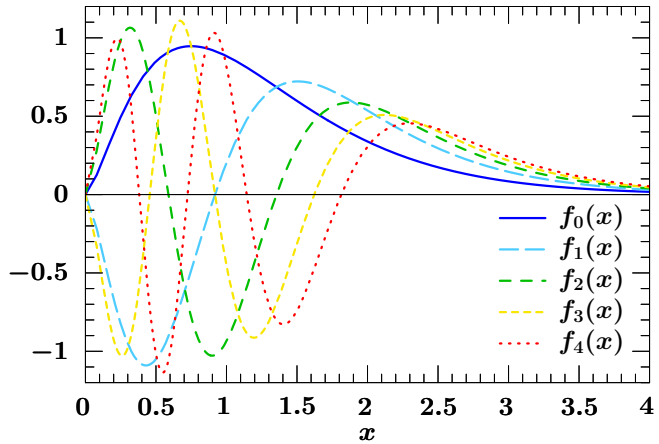
Aim negligible model dependence w.r.t. exp. uncert.

- Master formula for differential decay rate:

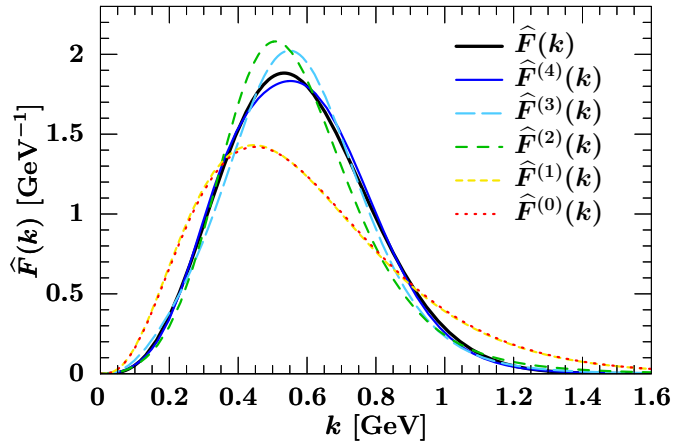
$$\mathrm{d}\Gamma_s \propto |V_{tb}\, V_{ts}^*|^2\, m_b^2 \Big\{ \left|C_7^{\text{incl}}\right|^2 \left[\left(\widehat{W}_{77}^{\text{sing}} + \widehat{W}_{77}^{\text{nons}}\right) \otimes \widehat{F} + \sum_n W_{77,n}\, F_n^{subl}\right] + \sum_{i,j\neq 7}\left[\Re(C_7^{incl})\, 2C_i\, \widehat{W}_{7i}^{nons} + C_i C_j \widehat{W}_{ij}^{nons}\right] \otimes \widehat{F} + \ldots \Big\}$$

C_7^{incl} sums all contributions creating same effective $b \rightarrow s\gamma$ vertex prop. to C_7. Included at full **NNLL+NNLO**. $C_{i\neq 7}$ fixed at SM values.

Used basis functions:



Expansion of model function:



[PRD:78:114014]

- Absorb sub-leading $1/m_b$ corrections: $\widehat{\mathcal{F}}(k) = \widehat{F}(k) + \frac{1}{m_b}\sum_n F_n^{subl}$

I.c Truncation uncertainty and basis

- finite exp. input $\leftrightarrow$ series must be truncated

$$\boxed{\widehat{F}(k) = \frac{1}{\lambda}\left[\sum_n c_n\, f_n(k)\right]^2}$$

  $\Rightarrow$ Induces residual basis (= model) dependence

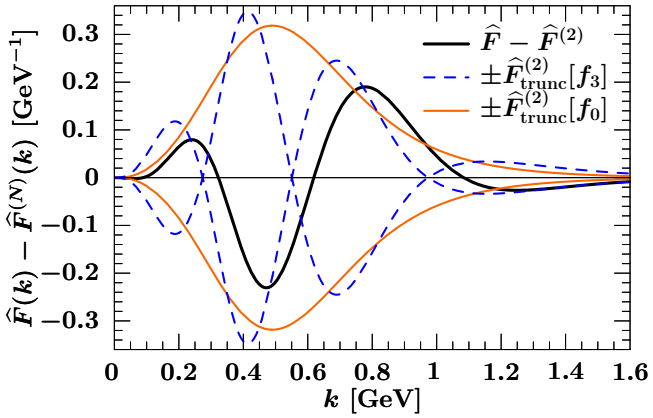
- Truncation error scales with truncation order N

$$1 - \sum_{n=0}^{N} c_n$$

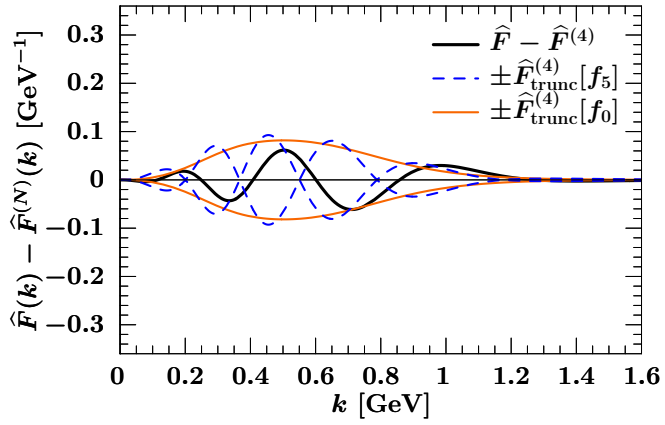
- Optimal N and λ (= basis) determined from data
  - $\Rightarrow$ Choose λ so series converges quickly
  - $\Rightarrow$ Choose N so truncation error is small w.r.t. exp. uncert.
  - $\Rightarrow$ Add more terms with more precise data

$\Rightarrow$ Must be careful not to 'overtune' things

Truncation error with $N = 2$:



Truncation error with $N = 4$:



[PRD:78:114014]

II. Experimental input and Fit

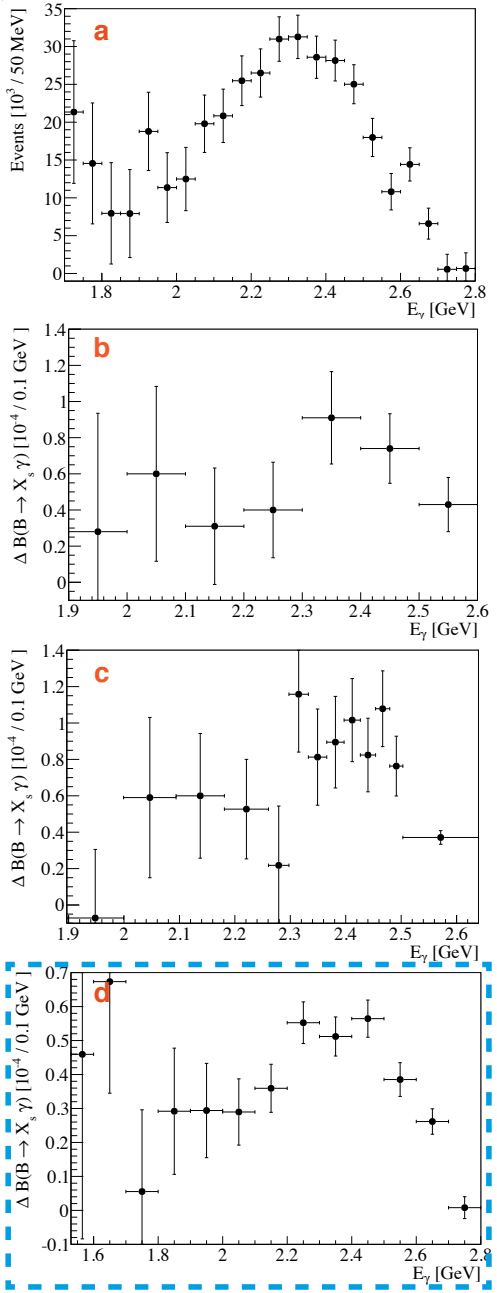
- Analyze four E_γ spectra from *BaBar* and *Belle*

  a *Belle* inclusive (in $\Upsilon(4S)$ frame): Inclusive E_γ spectrum using **605 fb^{-1}** and a leptonic tag.
  $\Rightarrow$ Use eff. corrected spectrum and smear theory

  b *BaBar* with hadronic tags (in B frame): E_γ spectrum using **210 fb^{-1}** and a hadronic tag.

  c *BaBar* sum-over-exclusive modes (in B frame): E_γ spectrum is recon. using the had. mass m_X using **82 fb^{-1}**.

  d *BaBar* inclusive (in $\Upsilon(4S)$ frame): Inclusive E_γ spectrum using **347 fb^{-1}** $\rightarrow$ see previous talk
  $\Rightarrow$ Use eff. corrected and resol. unfolded spectrum

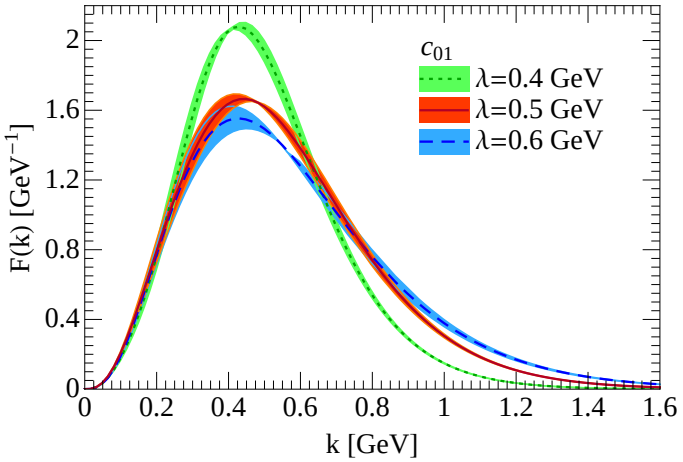


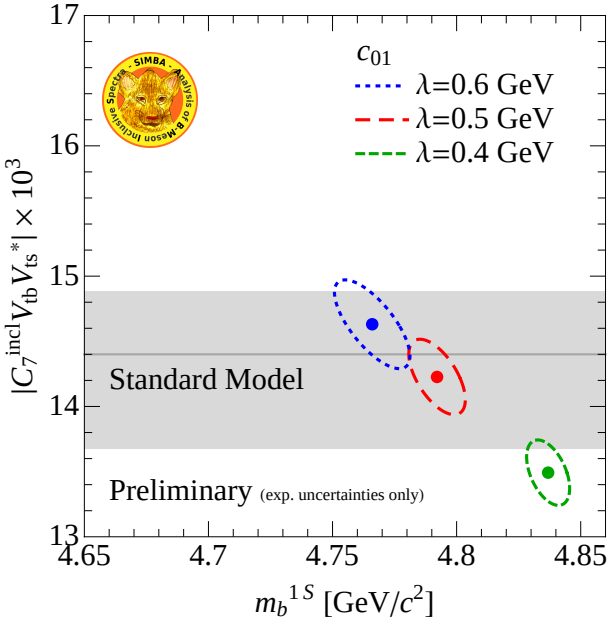
Fit Procedure: Use a χ^2 fit

- Float C_7^{incl} and number of c_n coefficients
  ($C_{i\neq 7}$ fixed at SM values)
- Evaluate model dependence for several bases:
  Different Bases $\leftrightarrow$ $\lambda = 0.4 - 0.6$ GeV
- Pick an expansion with negligible model dependence w.r.t. experimental uncertainty

III.a Basis independence

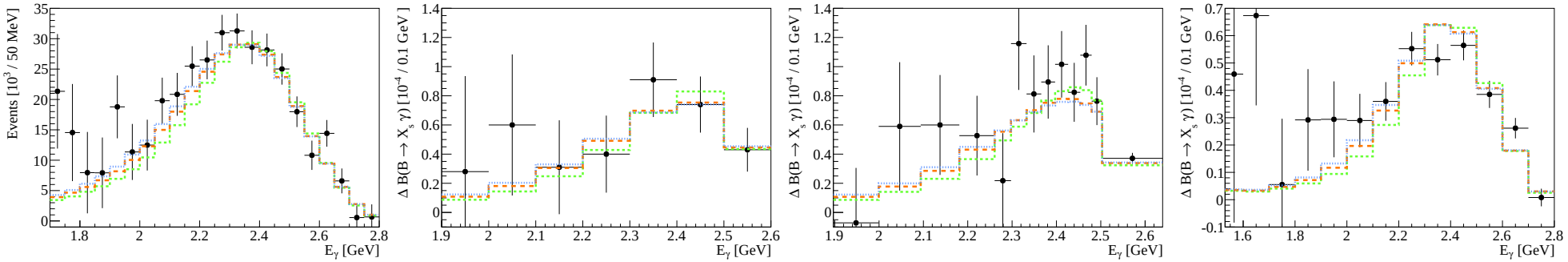
- Fit with two basis functions (c_{01}):
  - → Equivalent to fixed model with fitted 1st moment
  - → All fits with good χ^2/ndf: 53.8/50; 44.0/50; 42.3/50





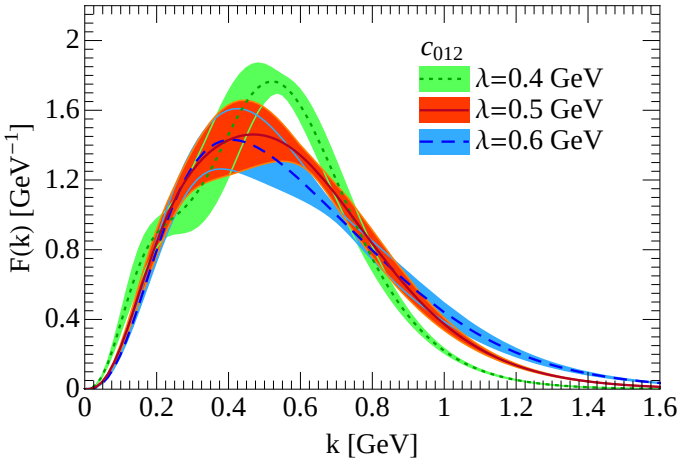
The fitted $|C_7^{\text{incl}}\ V_{tb}\ V_{ts}^*|$ values are compared with the NLO Standard Model prediction using $|V_{tb}\ V_{ts}^*| = 40.68^{+0.4}_{-0.5}$

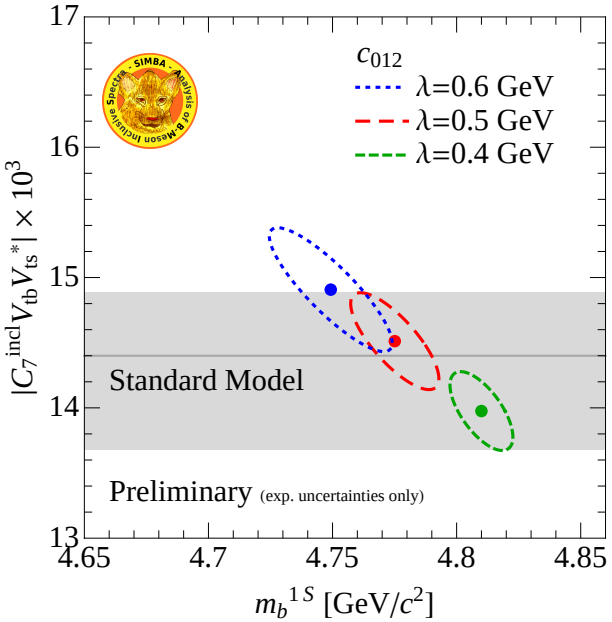
⇒ Exp. uncertainties underestimate model dependence



III.a Basis independence

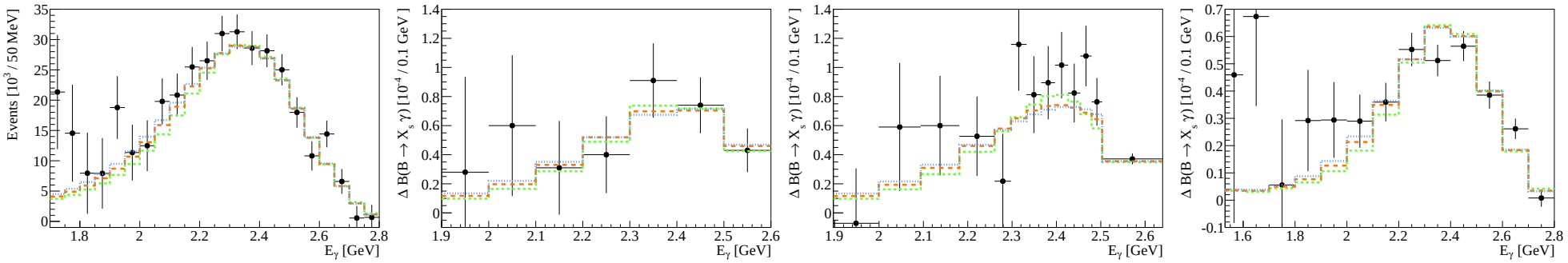
- Fit with three basis functions (c_{012}):





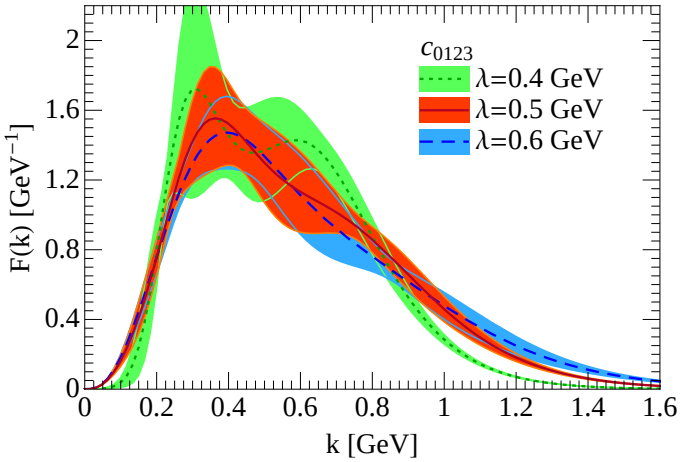
- χ^2/ndf: 46.5/49; 42.5/49; 41.6/49

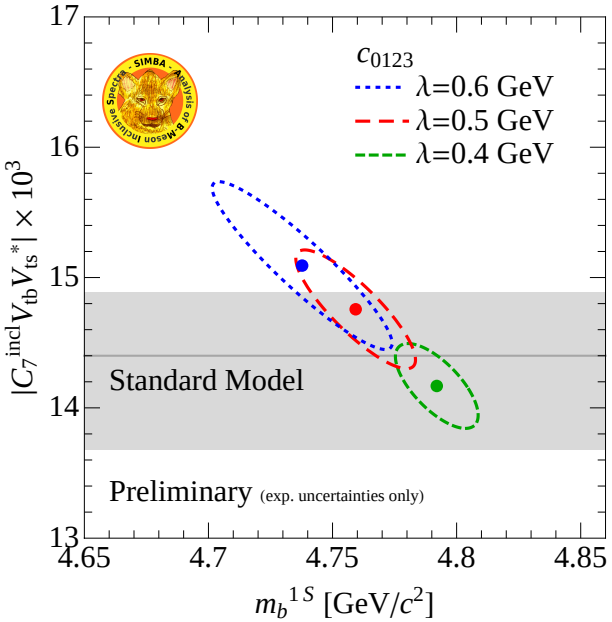
The fitted $|C_7^{incl}\ V_{tb}\ V_{ts}^*|$ values are compared with the NLO Standard Model prediction using $|V_{tb}\ V_{ts}^*| = 40.68^{+0.4}_{-0.5}$



III.a Basis independence

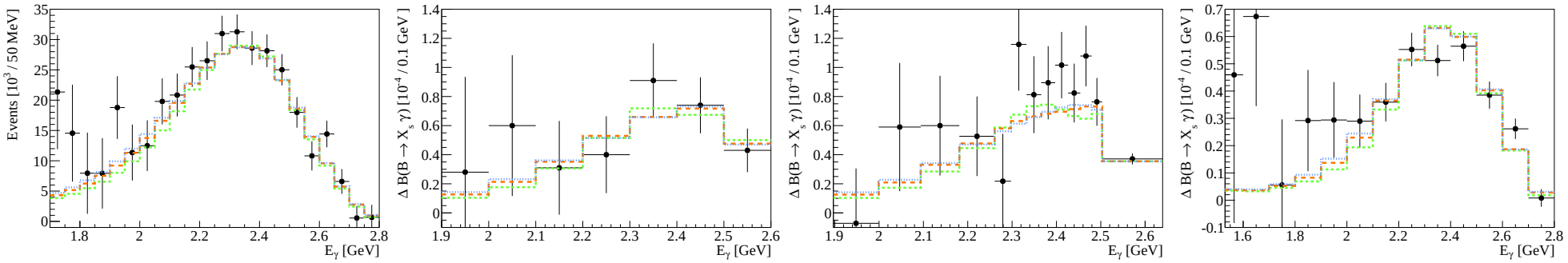
- Fit with four basis functions (c_{0123}):





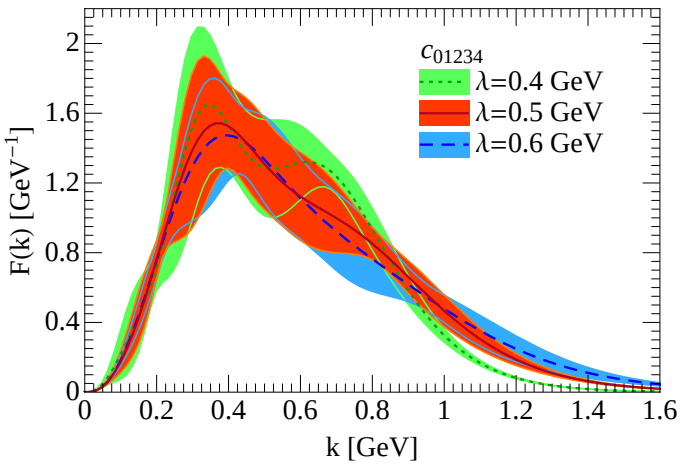
The fitted $|C_7^{incl}\ V_{tb}\ V_{ts}^*|$ values are compared with the NLO Standard Model prediction using $|V_{tb}\ V_{ts}^*| = 40.68^{+0.4}_{-0.5}$

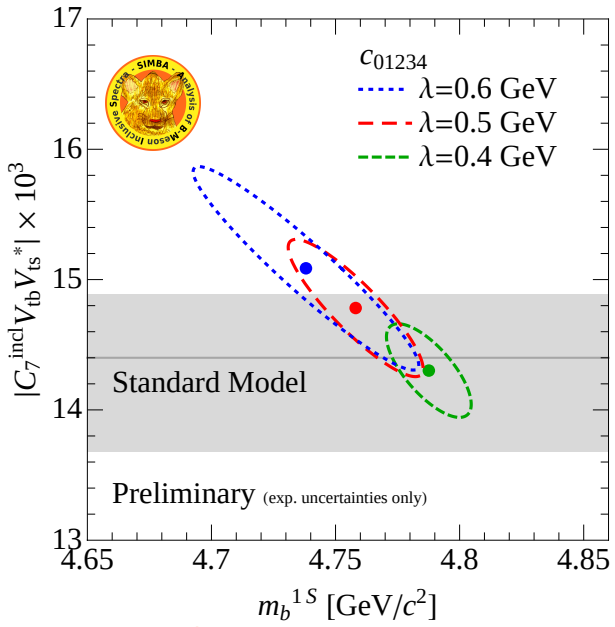
- χ^2/ndf: 43.7/48; 41.7/48; 41.4/48



III.a Basis independence

- Fit with five basis functions (c_{01234}):

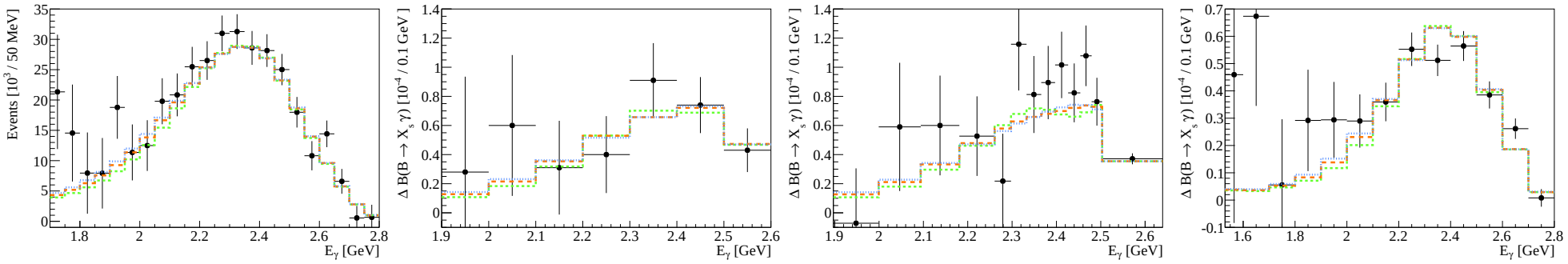




The fitted $|C_7^{incl}\ V_{tb}\ V_{ts}^*|$ values are compared with the NLO Standard Model prediction using $|V_{tb}\ V_{ts}^*| = 40.68^{+0.4}_{-0.5}$

- χ^2/ndf: 43.0/47; 41.6/47; 41.4/47

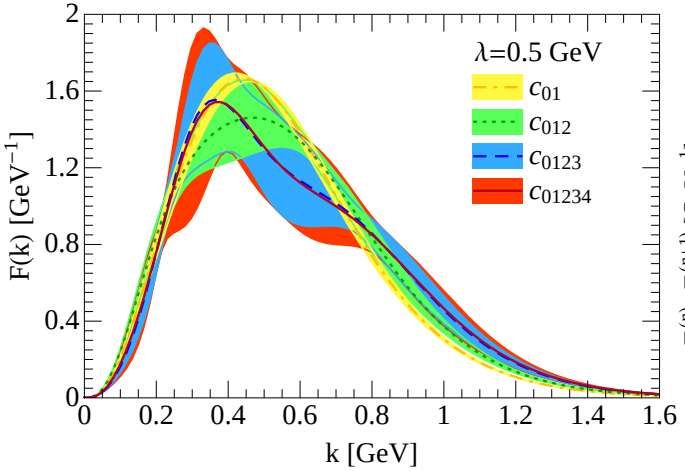
⇒ With enough coeff., results agree within uncert. and become basis (= model) independent

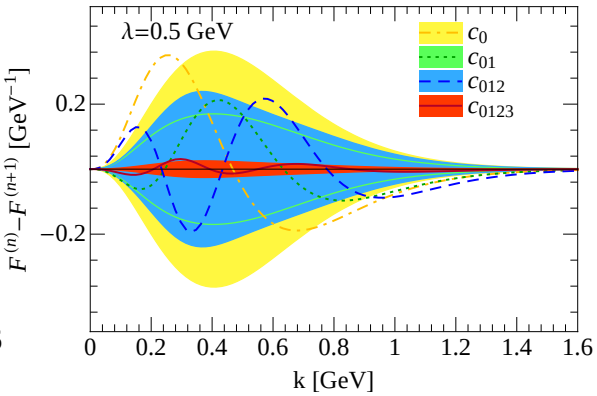


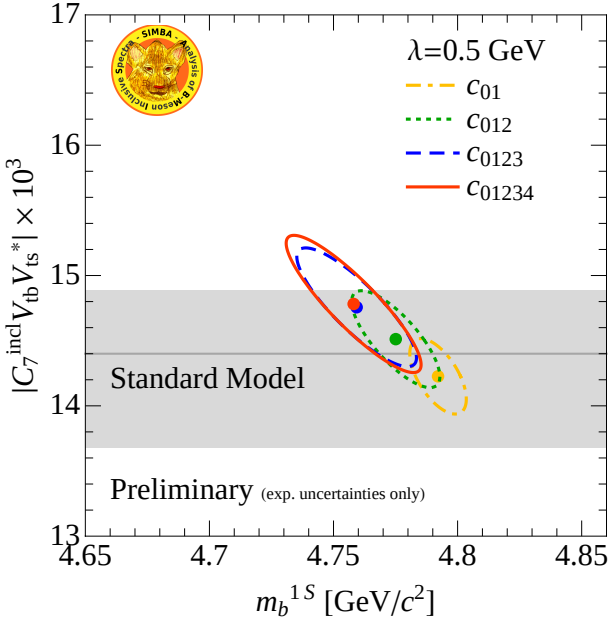
III.b Fit result for $\lambda = 0.5$ GeV

- Fits with 2,3,4 & 5 basis functions: ($c_{01}, c_{012}, c_{0123}, c_{01234}$)
- Shape function and estimated basis dependence

  determined from $n+1$ coefficient and envelop from first basis function







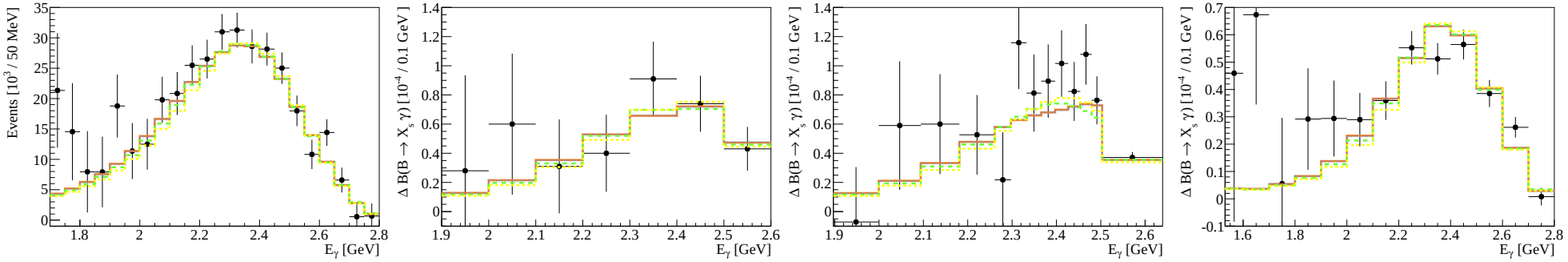
Fitted values of $|C_7^{incl}\ V_{tb}\ V_{ts}^*|$ are compared with the NLO Standard Model prediction using $|V_{tb}\ V_{ts}^*| = 40.68^{+0.4}_{-0.5}$

⇒ Uncertainties underestimated with too few coeff.

→ would need to include additional uncertainty due to truncation

⇒ Very little change by including 5*th* coefficient (c_4)

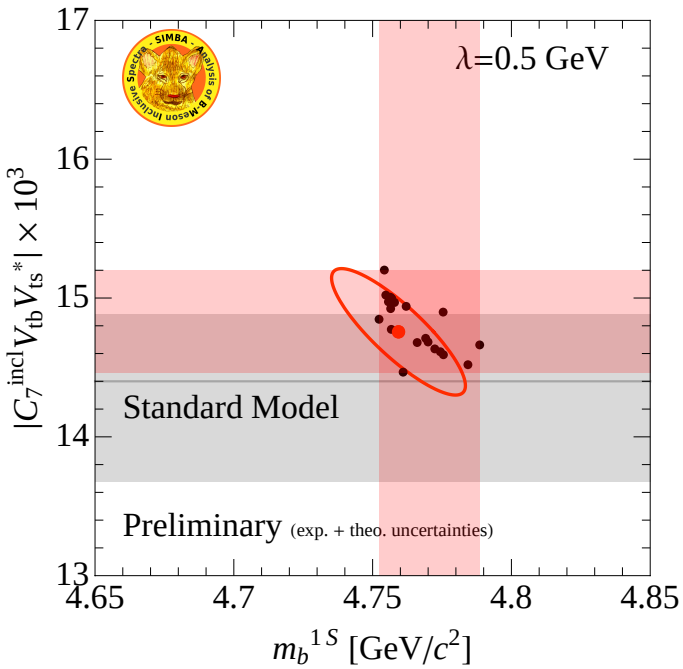
→ truncation uncertainty negligible compared to other uncertainties



IV. Theory Uncertainties

- Largest theory uncert. from higher order pert. theory.
- Evaluated by varying SCET scales: μ_h; μ_j; μ_s; μ_{NS}
- Probe contour with 22 variations and repeat fits:

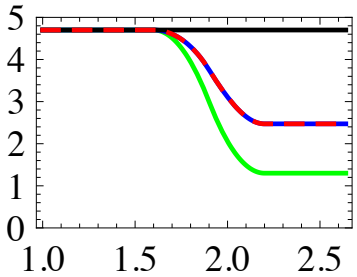
  Use fit with $\lambda = 0.5$ GeV and c_{0123}



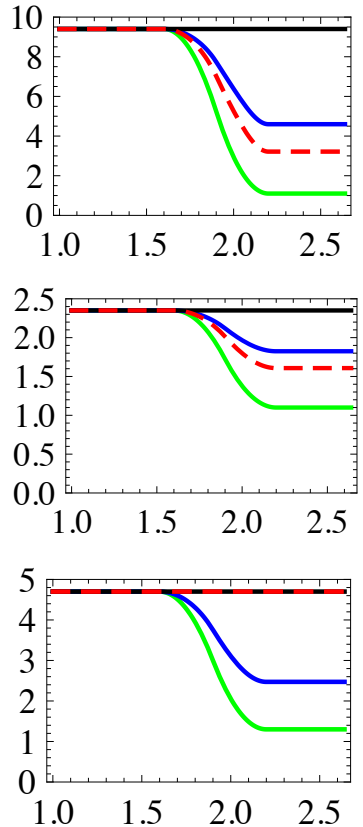
The red shaded region shows the largest extend of the probed variations

⇒ Shift central value scales to middle of contour results in symmetric theory uncert. interval.

Curent central value scales:

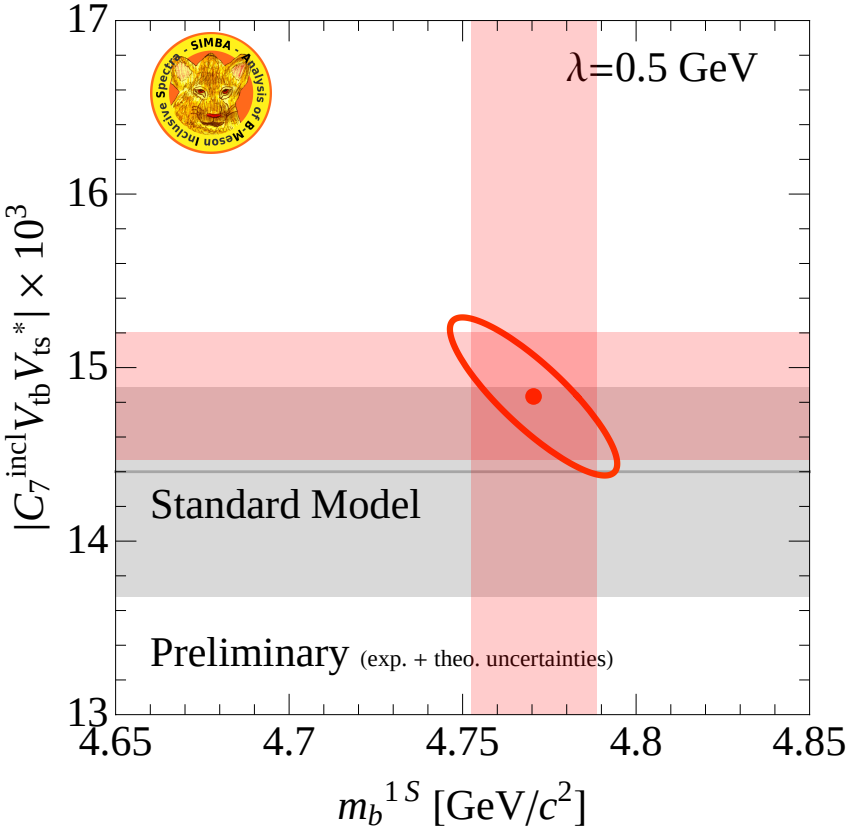


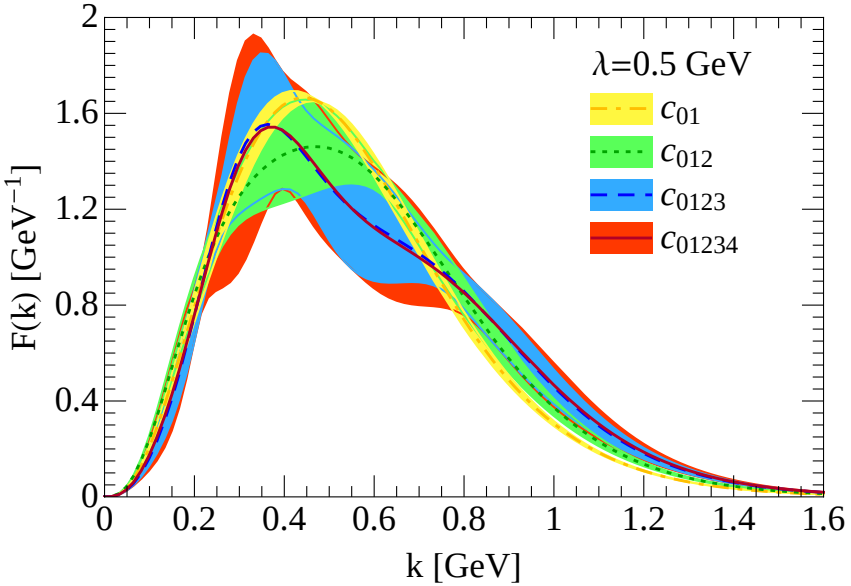
3 example variations:



V. Summary and Conclusion

- Obtained value of C_7^{incl} which is very good agreement with Standard Model
- Non-perturbative shape function (with abs. $1/m_b$ corrections) determined by data

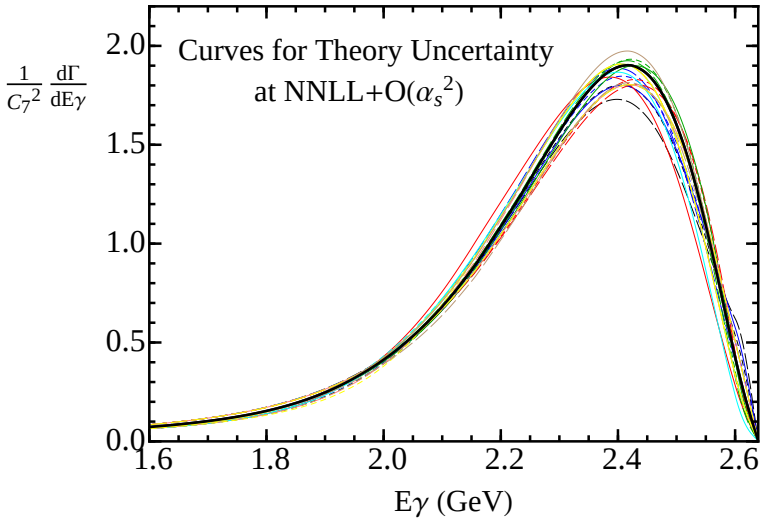


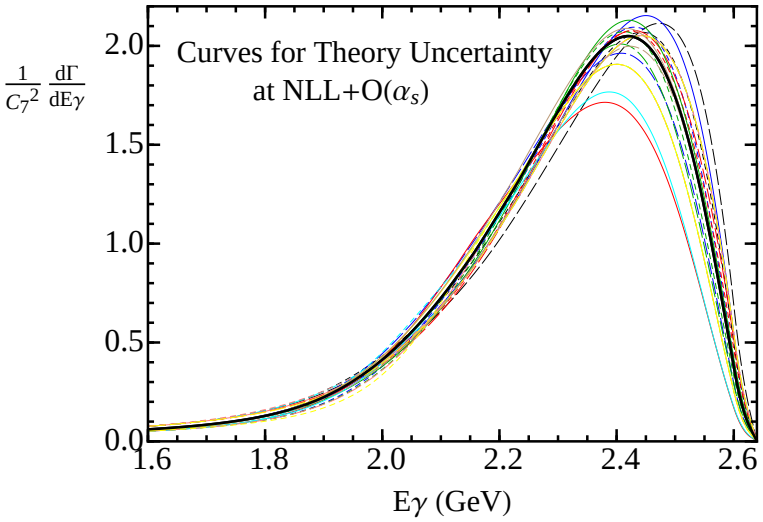


⇒ Test of Standard Model with negligible model uncertainties from non-perturbative QCD effects

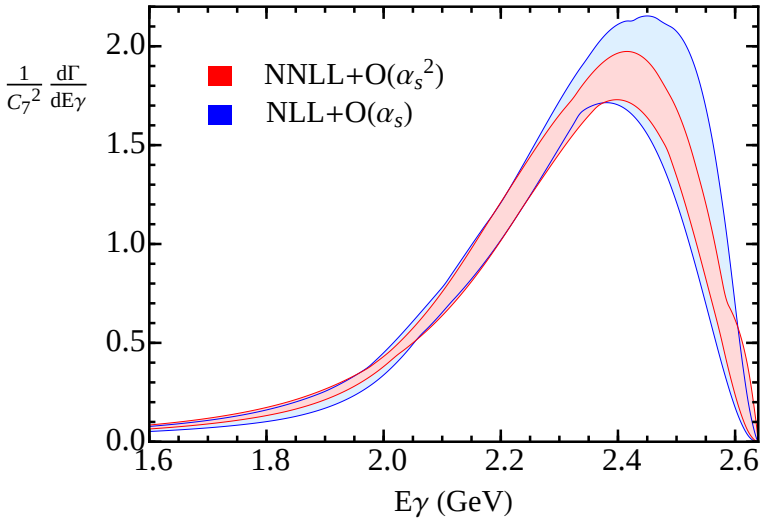
Backup

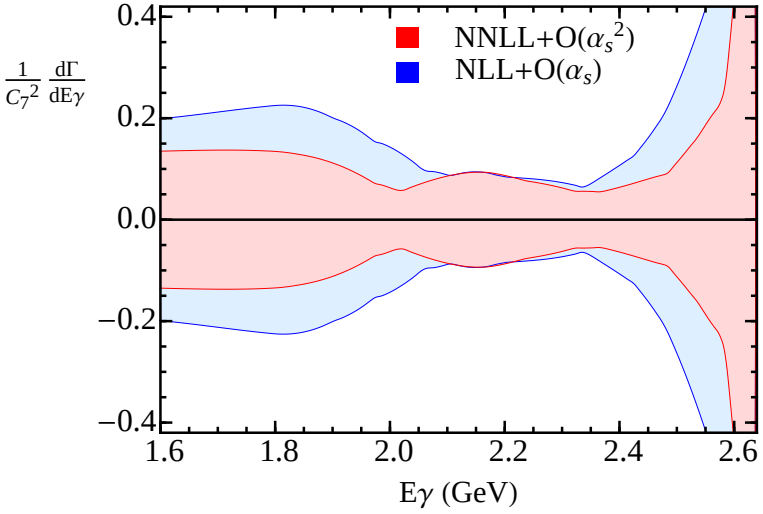
A.a Differential theory uncertainty





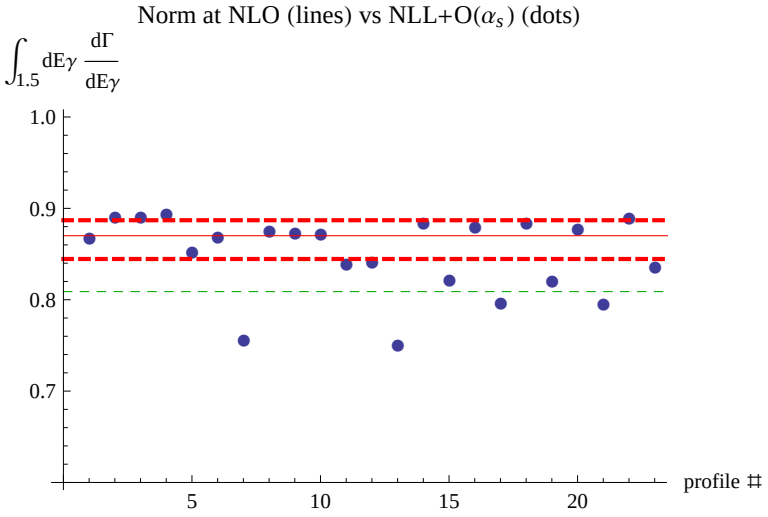
top: the impact on the scale variations on the differential spectra at NLL and NNLL are shown

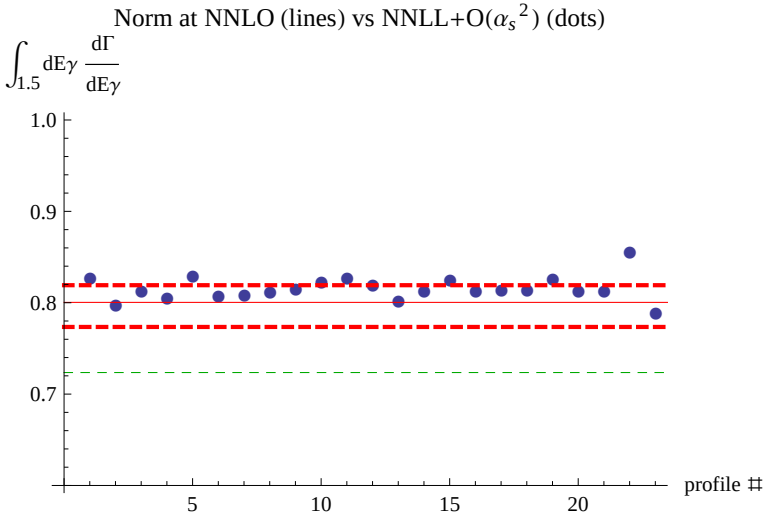




bottom: the resulting envelope and normalized envelopes at NLL and NNL are shown

A.b Differential theory uncertainty

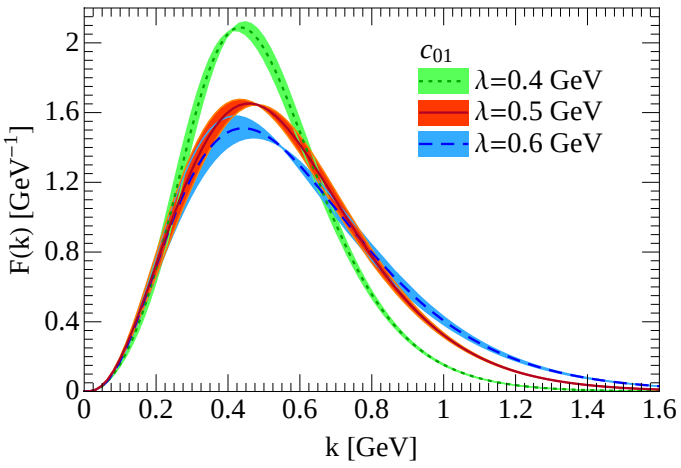


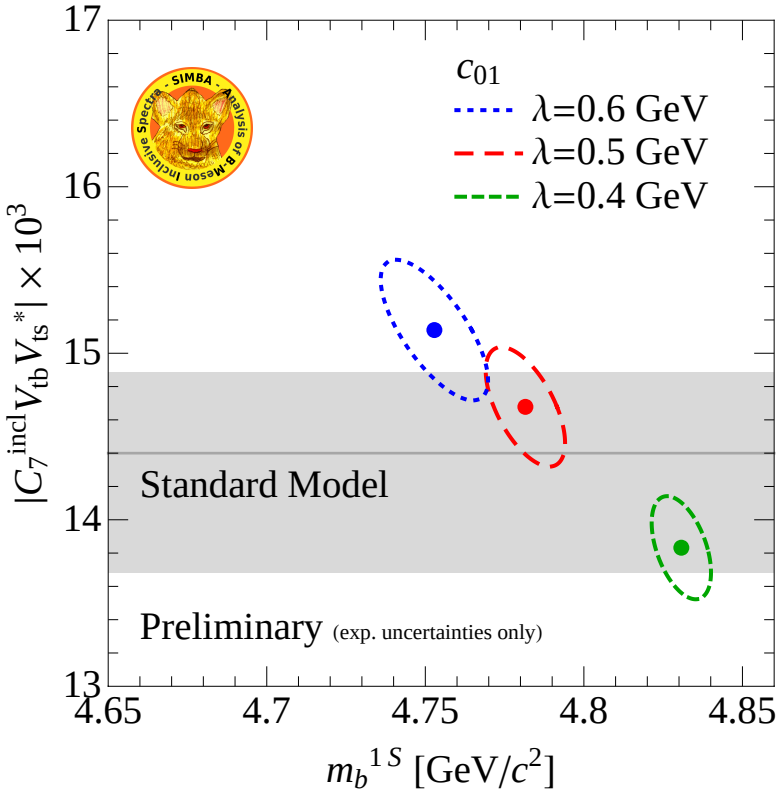


left and right: The fixed order theory uncertainty (at NLO and NNLO) is compared with the estimated uncertainty of the resumed NNLL/NNLO calculation used in this work: the red solid line corresponds to the fixed order result with a scale of $\mu = 4.7$, the red upper and lower dashed lines correspond to a variation of $\mu = 9.4$ and $\mu = 2.35$, respectively. The green line corresponds to the chosen scale of Misiak et al. [PRL:98:022002] (which uses a different definition of C_7 than this work). The blue dotes correspond to the chosen scale variations of the resumed NNLL/NNLO calculation. Our profiles have reasonable agreement with the fixed order results and also taking the range of dots as an uncertainty in this integral, our NNLL and NLL norms agree within uncertainties.

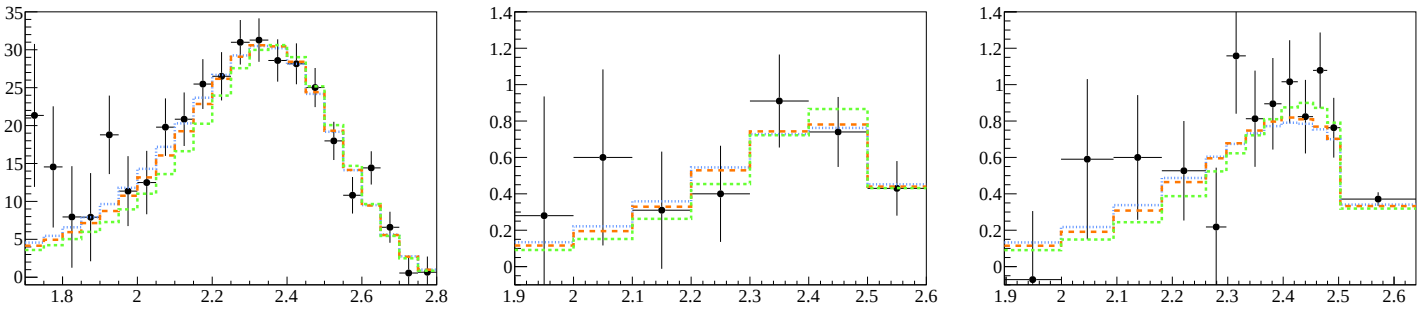
B. Result without *BaBar* incl. spectrum

- Fit with two basis functions (c_{01}):



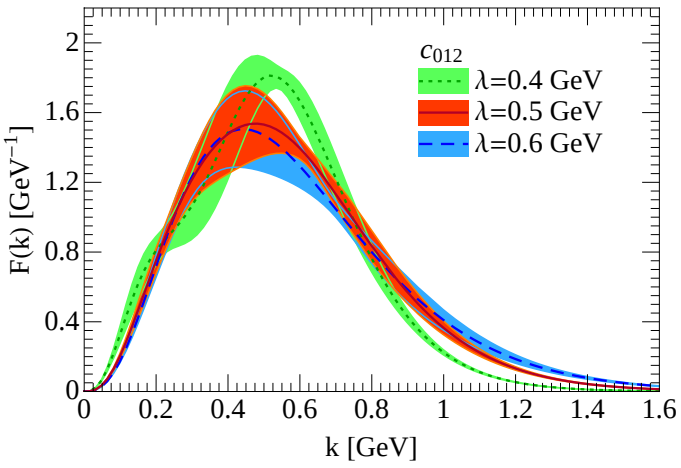


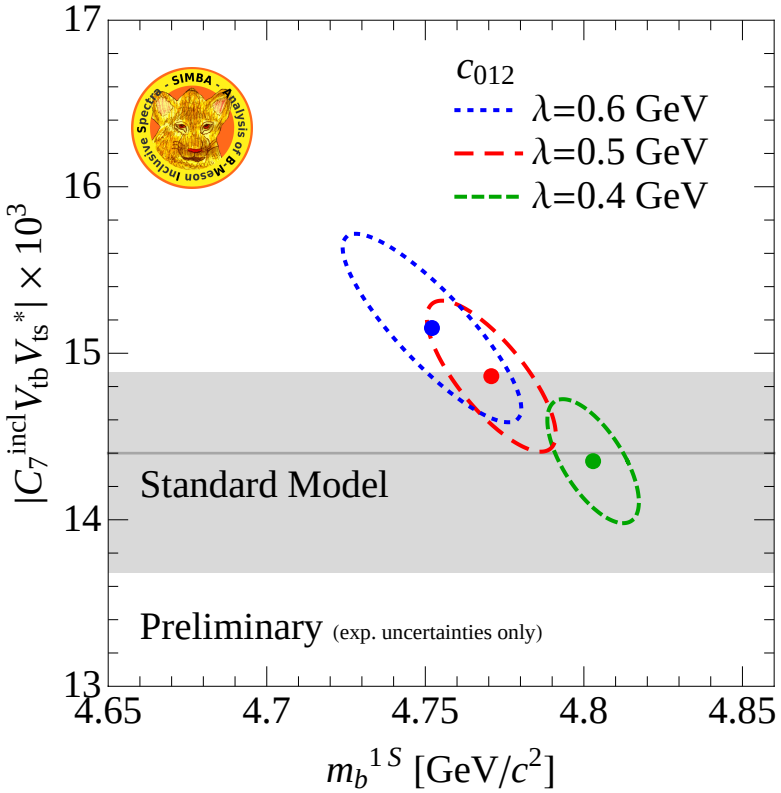
- χ^2/ndf: 53.8/50; 44.0/50; 42.3/50



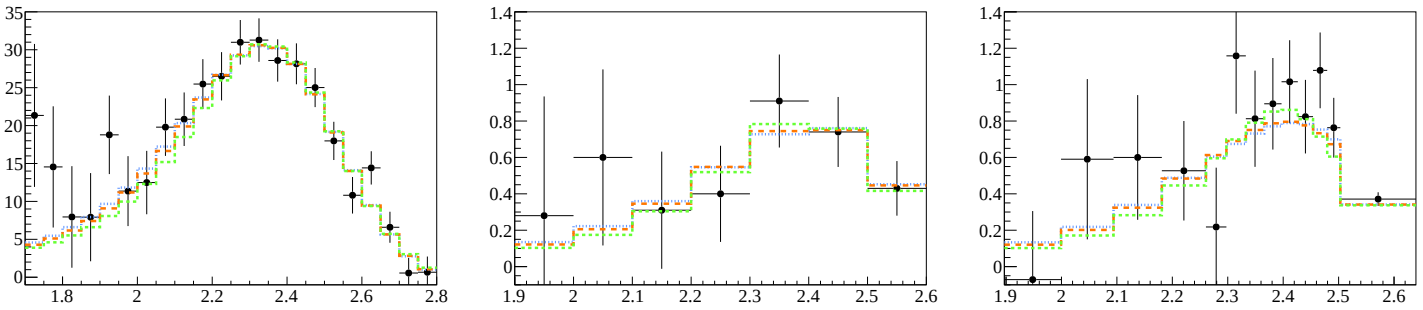
B.a Basis independence

- Fit with three basis functions (c_{012}):



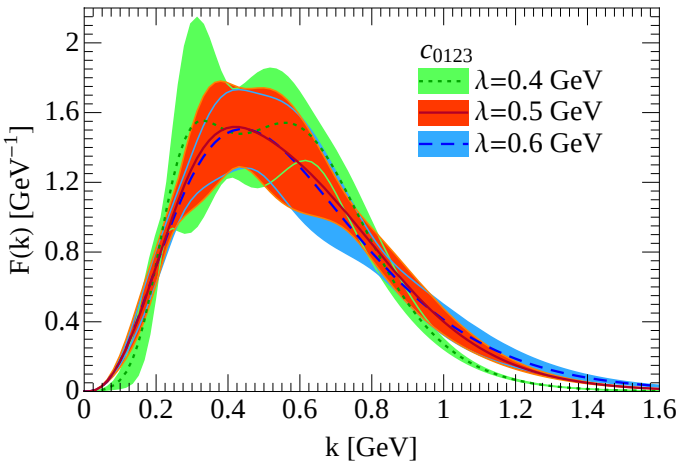


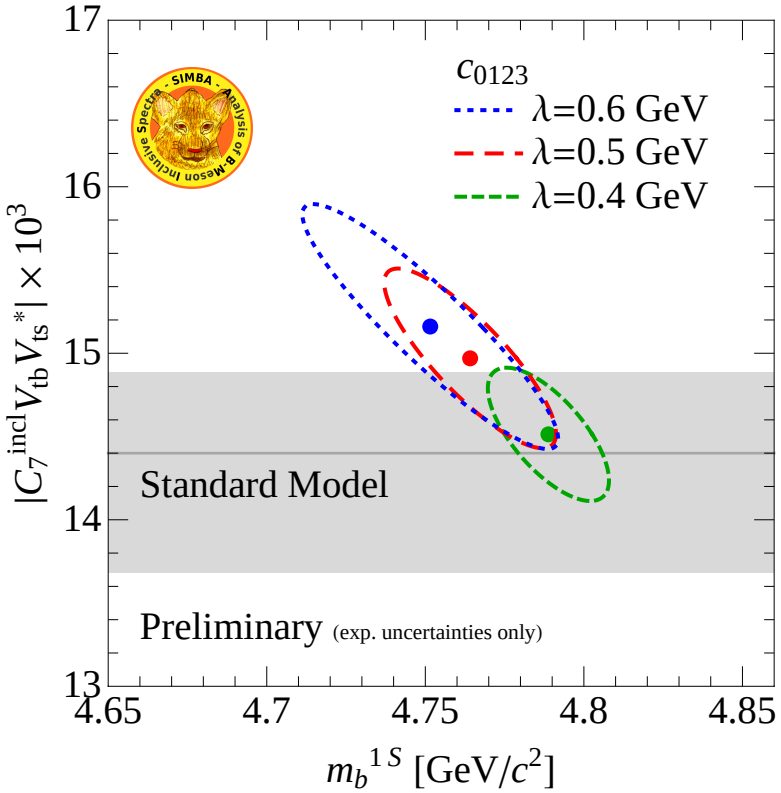
- χ^2/ndf: 53.8/50; 44.0/50; 42.3/50



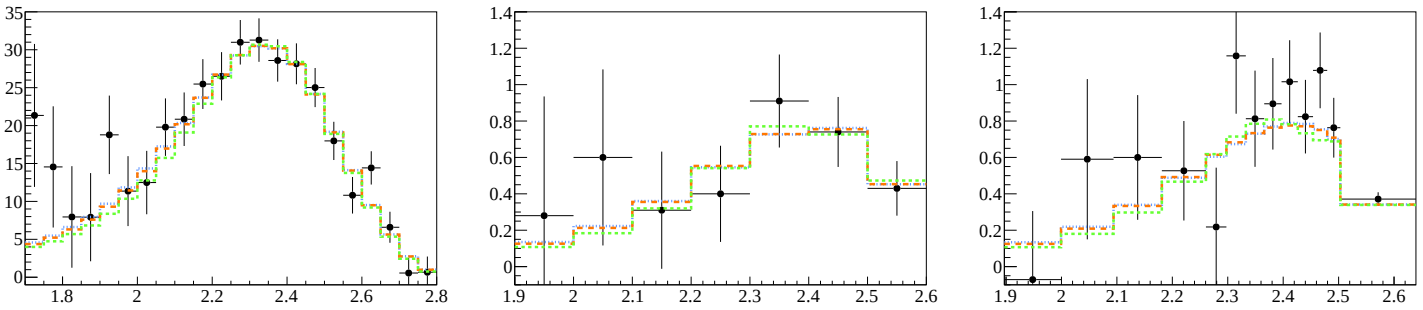
B.a Basis independence

- Fit with four basis functions (c_{0123}):



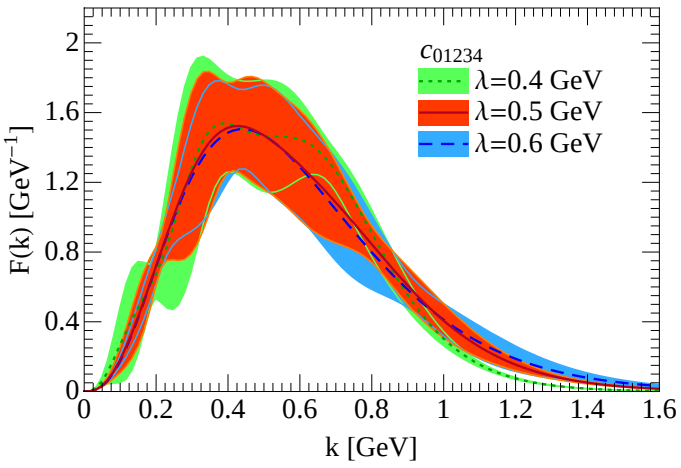


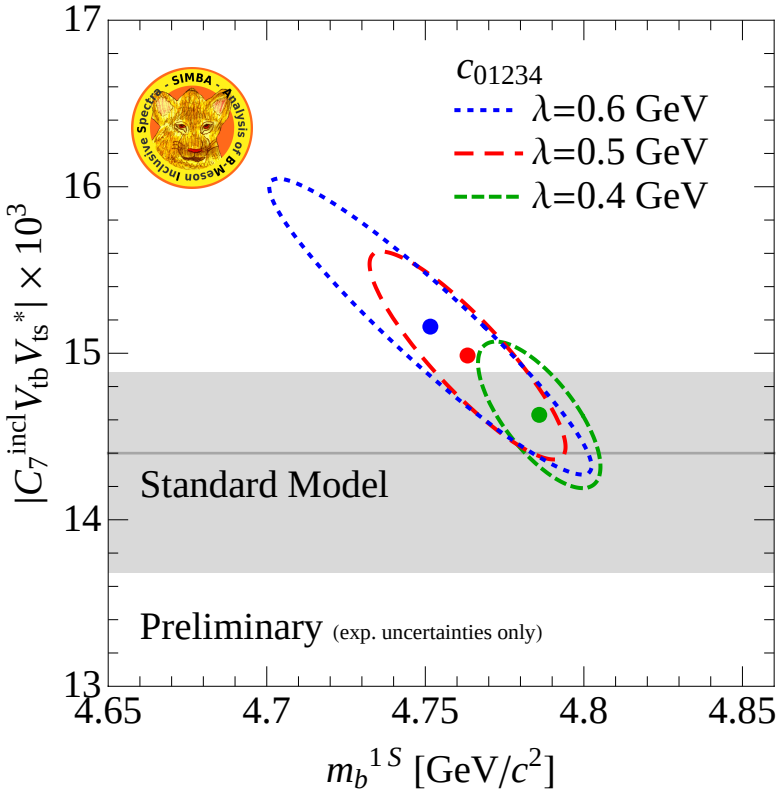
- χ^2/ndf: 53.8/50; 44.0/50; 42.3/50



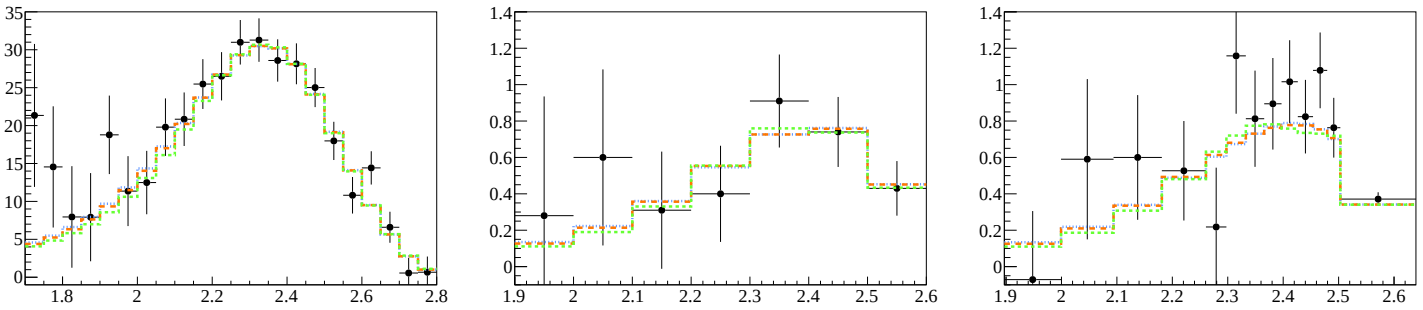
B.a Basis independence

- Fit with five basis functions (c_{01234}):





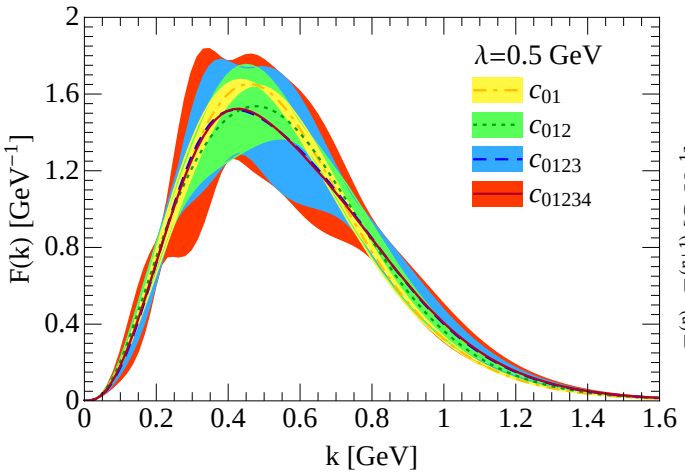
- χ^2/ndf: 53.8/50; 44.0/50; 42.3/50

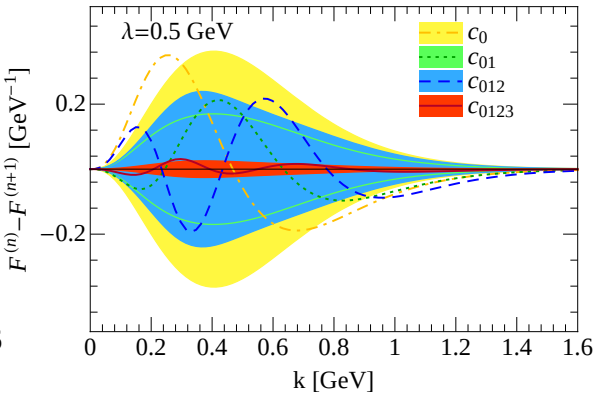


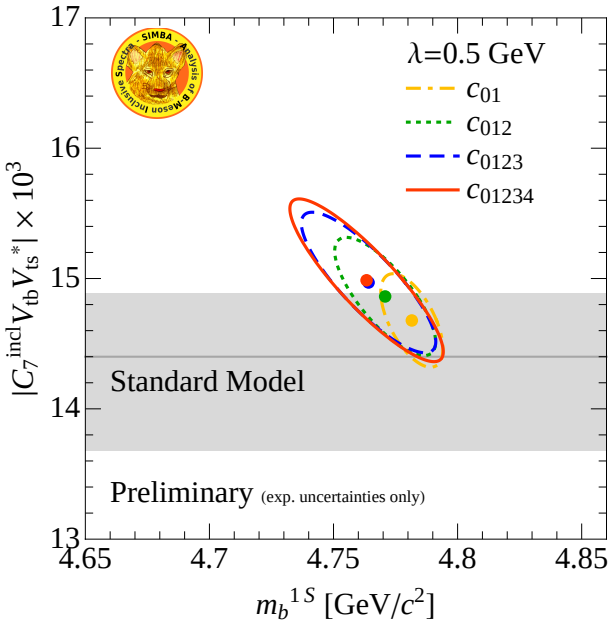
B.b Fit result for $\lambda = 0.5$ GeV

- Fits with 2,3,4 & 5 basis functions: ($c_{01}, c_{012}, c_{0123}, c_{01234}$)
- Shape function and estimated basis dependence

  determined from $n+1$ coefficient and envelop from first basis function







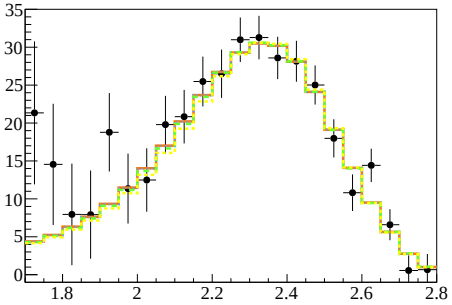
Fitted values of $|C_7^{\rm incl}\ V_{tb}\ V_{ts}^*|$ are compared with the NLO Standard Model prediction using $|V_{tb}\ V_{ts}^*| = 40.68^{+0.4}_{-0.5}$

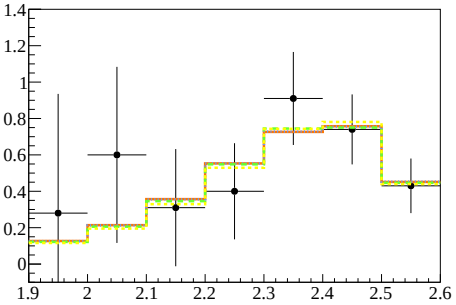
⇒ Uncertainties underestimated with too few coeff.

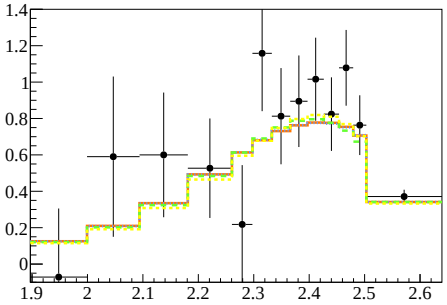
→ would need to include additional uncertainty due to truncation

⇒ Very little change by including 5*th* coefficient (c_4)

→ truncation uncertainty negligible compared to other uncertainties







B.c Theory uncertainty and results

- Obtained value of C_7^{incl} which is very good agreement with Standard Model
- Non-perturbative shape function (with abs. $1/m_b$ corrections) determined by data