A model independent determination of the $B \rightarrow X_s \gamma$ decay rate

SIMBA Collaboration:

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Talk overview

I. Introduction

II. Analyzed $B \rightarrow X_s \gamma$ measurements and Fit

III. Determination of the shape function and $C_7^{incl}$

IV. Theory uncertainties

V. Summary and Conclusion
I.a Introduction

- $B \to X_s \gamma$ very promising to probe Flavor sector for new physics

- Most precise measurements at high $E_\gamma$

  Theory most precise with low $E_\gamma$ cut

- Rising $E_\gamma$ cut $\leftrightarrow$ dependence on parton distribution function of $b$-quark ($\equiv$ Shape function)

- HFAG extrapolates $\Delta B$ to a lower cut $E_\gamma > 1.6$

<table>
<thead>
<tr>
<th>Reference</th>
<th>$\Delta B(E_\gamma &gt; 1.6 \text{ GeV})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HFAG [arXiv:1010.1589]</td>
<td>$(3.55 \pm 0.24 \pm 0.09) \times 10^{-4}$</td>
</tr>
<tr>
<td>Misiak et al. [PRL:98:022002]</td>
<td>$(3.15 \pm 0.23) \times 10^{-4}$</td>
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</tbody>
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⇒ **SIMBA** tests Standard Model without need of extrapolations
1.b Shape function and Master formulae

- Treat unknown shape function $\hat{F}(k)$ as expansion of set of basis functions:

$$\hat{F}(k) = \frac{1}{\lambda} \left[ \sum_n c_n f_n(k) \right]^2 \quad \text{with} \quad \int_0^\infty dk \hat{F}(k) = \sum_n c_n^2 = 1$$

$\lambda$ is a parameter of the basis, $f_n$ are the basis functs. with coeff. $c_n$.

- Non-perturbative physics in coefficients $c_n \rightarrow$ determine from measured differential $E_\gamma$ spectra

- finite exp. input $\leftrightarrow$ series must be truncated

Aim negligible model dependence w.r.t. exp. uncert.

- Master formula for differential decay rate:

$$d\Gamma_s \propto |V_{tb} V_{ts}^*|^2 m_b^2 \left\{ \left| C_7^{\text{incl}} \right|^2 \left[ (\hat{W}_{77}^{\text{sing}} + \hat{W}_{77}^{\text{nons}}) \otimes \hat{F} + \sum_n W_{77,n} F_n^{\text{subl}} \right] \right. \\
+ \sum_{i,j\neq 7} \left[ \Re(C_7^{\text{incl}}) 2C_i \hat{W}_{7i}^{\text{nons}} + C_i C_j \hat{W}_{ij}^{\text{nons}} \right] \otimes \hat{F} + \ldots \left\}$$

$C_7^{\text{incl}}$ sums all contributions creating same effective $b \rightarrow s\gamma$ vertex prop. to $C_7$. Included at full NNLL+NNLO. $C_{i\neq 7}$ fixed at SM values.

- Absorb sub-leading $1/m_b$ corrections: $\hat{F}(k) = \hat{F}(k) + \frac{1}{m_b} \sum_n F_n^{\text{subl}}$
1.3 Truncation uncertainty and basis

- finite exp. input $\leftrightarrow$ series must be truncated

$$\hat{F}(k) = \frac{1}{\lambda} \left[ \sum_{n} c_n f_n(k) \right]^2$$

$\Rightarrow$ Induces residual basis (= model) dependence

- Truncation error scales with truncation order $N$

$$1 - \sum_{n=0}^{N} c_n$$

- Optimal $N$ and $\lambda$ (= basis) determined from data

  $\Rightarrow$ Choose $\lambda$ so series converges quickly

  $\Rightarrow$ Choose $N$ so truncation error is small w.r.t. exp. uncert.

  $\Rightarrow$ Add more terms with more precise data

$\Rightarrow$ Must be careful not to 'overtune' things

Truncation error with $N = 2$:

![Graph showing truncation error for $N = 2$]

Truncation error with $N = 4$:

![Graph showing truncation error for $N = 4$]

[PRD:78:114014]
II. Experimental input and Fit

- Analyze four $E_\gamma$ spectra from $BABAR$ and $Belle$
  a $Belle$ inclusive (in $\Upsilon(4S)$ frame): Inclusive $E_\gamma$ spectrum using 605 fb$^{-1}$ and a leptonic tag. ⇒ Use eff. corrected spectrum and smear theory
  b $BABAR$ with hadronic tags (in $B$ frame): $E_\gamma$ spectrum using 210 fb$^{-1}$ and a hadronic tag.
  c $BABAR$ sum-over-exclusive modes (in $B$ frame): $E_\gamma$ spectrum is recon. using the had. mass $m_X$ using 82 fb$^{-1}$.
  d $BABAR$ inclusive (in $\Upsilon(4S)$ frame): Inclusive $E_\gamma$ spectrum using 347 fb$^{-1}$ ⇒ see previous talk
     ⇒ Use eff. corrected and resol. unfolded spectrum

**Fit Procedure:** Use a $\chi^2$ fit

- Float $C_7^{incl}$ and number of $c_n$ coefficients ($C_i\neq7$ fixed at SM values)
- Evaluate model dependence for several bases:
  Different Bases $\leftrightarrow \lambda = 0.4 - 0.6$ GeV
- Pick an expansion with negligible model dependence w.r.t. experimental uncertainty
III.a Basis independence

- Fit with two basis functions ($c_{01}$):
  - Equivalent to fixed model with fitted 1st moment
  - All fits with good $\chi^2/\text{ndf}$: 53.8/50; 44.0/50; 42.3/50

⇒ Exp. uncertainties underestimate model dependence
III.a Basis independence

- Fit with three basis functions ($c_{012}$):

  $c_0 = 0.4 \text{ GeV} \quad c_1 = 0.5 \text{ GeV} \quad c_2 = 0.6 \text{ GeV}$

- $\chi^2/\text{ndf}$: 46.5/49, 42.5/49, 41.6/49

The fitted $|C_7^{\text{incl}} V_{tb} V^*_{ts}|$ values are compared with the NLO Standard Model prediction using $|V_{tb} V^*_{ts}| = 40.68^{+0.4}_{-0.5}$.
III.a Basis independence

- Fit with four basis functions ($c_{0123}$):

$$c_0 l = 0.4 \text{ GeV}$$
$$c_1 l = 0.5 \text{ GeV}$$
$$c_2 l = 0.6 \text{ GeV}$$

- $\chi^2/\text{ndf}$: 43.7/48; 41.7/48; 41.4/48

The fitted $|C_7^{\text{incl}} V_{tb} V_{ts}^*|$ values are compared with the NLO Standard Model prediction using $|V_{tb} V_{ts}^*| = 40.68^{+0.4}_{-0.5}$
III.a Basis independence

- Fit with five basis functions (\(c_{01234}\)):

\[
\begin{align*}
c_0 &= 0.4 \text{ GeV} \\
c_1 &= 0.5 \text{ GeV} \\
c_2 &= 0.6 \text{ GeV}
\end{align*}
\]

\(-\chi^2/\text{ndf}: 43.0/47; 41.6/47; 41.4/47\)

⇒ With enough coeff., results agree within uncert. and become basis (= model) independent

The fitted \(\left|C_7^{\text{incl}} \, V_{tb} \, V_{ts}^*\right|\) values are compared with the NLO Standard Model prediction using \(\left|V_{tb} \, V_{ts}^*\right| = 40.68^{+0.4}_{-0.5}\)
III.b Fit result for $\lambda = 0.5$ GeV

- Fits with 2,3,4 & 5 basis functions: $(c_{01}, c_{012}, c_{0123}, c_{01234})$

- Shape function and estimated basis dependence
determined from $n + 1$ coefficient and envelop from first basis function

⇒ Uncertainties underestimated with too few coeff.
→ would need to include additional uncertainty due to truncation

⇒ Very little change by including 5th coefficient ($c_4$)
→ truncation uncertainty negligible compared to other uncertainties

Fitted values of $|C_7^{incl} V_{tb} V_{ts}^*|$ are compared with the NLO Standard Model prediction using $|V_{tb} V_{ts}^*| = 40.68^{+0.4}_{-0.5}$
IV. Theory Uncertainties

- Largest theory uncert. from higher order pert. theory.
- Evaluated by varying SCET scales: $\mu_h; \mu_j; \mu_s; \mu_{NS}$
- Probe contour with 22 variations and repeat fits:
  Use fit with $\lambda = 0.5$ GeV and $c_{0123}$

The red shaded region shows the largest extend of the probed variations

⇒ Shift central value scales to middle of contour results in symmetric theory uncert. interval.
V. Summary and Conclusion

- Obtained value of $C_7^{\text{incl}}$ which is very good agreement with Standard Model
- Non-perturbative shape function (with abs. $1/m_b$ corrections) determined by data

⇒ Test of Standard Model with negligible model uncertainties from non-perturbative QCD effects
Backup
### A.a Differential theory uncertainty

**top:** The impact on the scale variations on the differential spectra at NLL and NNLL are shown.

**bottom:** The resulting envelope and normalized envelopes at NLL and NNLL are shown.
A.b Differential theory uncertainty

**left and right:** The fixed order theory uncertainty (at NLO and NNLO) is compared with the estimated uncertainty of the resumed NNLL/NNLO calculation used in this work: the **red solid** line corresponds to the fixed order result with a scale of $\mu = 4.7$, the **red upper and lower dashed lines** correspond to a variation of $\mu = 9.4$ and $\mu = 2.35$, respectively. The **green line** corresponds to the chosen scale of Misiak et al. [PRL:98:022002] (which uses a different definition of $C_7$ than this work). The **blue dots** correspond to the chosen scale variations of the resumed NNLL/NNLO calculation. Our profiles have reasonable agreement with the fixed order results and also taking the range of dots as an uncertainty in this integral, our NNLL and NLL norms agree within uncertainties.
B. Result without $B_{ABAR}$ incl. spectrum

- Fit with two basis functions ($c_01$):

- $\chi^2/\text{ndf}$: 53.8/50; 44.0/50; 42.3/50
B.a Basis independence

- Fit with three basis functions ($c_{012}$):

- $\chi^2$/ndf: 53.8/50; 44.0/50; 42.3/50
B.a Basis independence

- Fit with four basis functions ($c_{0123}$):

- $\chi^2/\text{ndf}$: 53.8/50; 44.0/50; 42.3/50
B.a Basis independence

- Fit with five basis functions ($c_{01234}$):

- $\chi^2$/ndf: 53.8/50; 44.0/50; 42.3/50
B.b Fit result for $\lambda = 0.5$ GeV

- Fits with 2, 3, 4 & 5 basis functions: $(c_{01}, c_{012}, c_{0123}, c_{01234})$

- Shape function and estimated basis dependence determined from $n + 1$ coefficient and envelop from first basis function

|$\lambda = 0.5$ GeV
\[F(k) \text{[GeV}^{-1}]\]
\[F_{(n+1)}(m_{b}^{1S}) \text{[GeV}^{-1}]\]

$\Rightarrow$ Uncertainties underestimated with too few coeff.
→ would need to include additional uncertainty due to truncation

$\Rightarrow$ Very little change by including 5th coefficient ($c_4$)
→ truncation uncertainty negligible compared to other uncertainties

Fitted values of $|C_7^{\text{incl}} \ V_{tb} \ V_{ts}^*|$ are compared with the NLO Standard Model prediction using $|V_{tb} \ V_{ts}^*| = 40.68^{+0.4}_{-0.5}$
B.c Theory uncertainty and results

- Obtained value of $C_7^{\text{incl}}$ which is very good agreement with Standard Model
- Non-perturbative shape function (with abs. $1/m_b$ corrections) determined by data