Strong field effects on physics processes at the IP of a future Linear Collider

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Synopsis

- Definition of a strong field
- Why is it important in collider physics?
- What strong field processes have been calculated/simulated?
- Furry picture/Volkov solutions
- Furry picture Feynman diagram components and rules
- **IPstrong:** a new event generator to produce strong field events

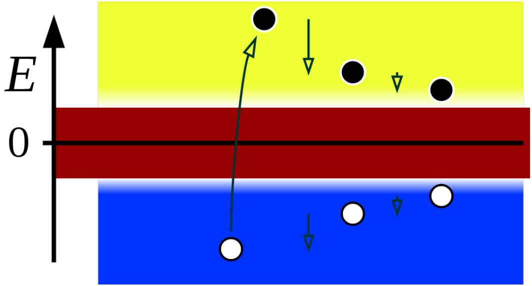
Motivation numero uno

We have a new particle to study!

A new linear collider....

- is a precision instrument to study the Higgs in fine detail
- is a high luminosity machine with very intense charge bunches
- has very strong fields associated with the collider bunches

Polarising the vacuum



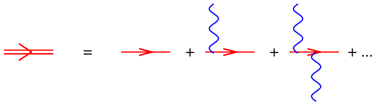
" In strong external fields the normal vacuum is unstable and decays into a new vacuum that contains real particles. "

Greiner and Muller, QED of Strong Fields

- The Schwinger limit ($E_{\text{cr}} = 10^{18}$ V/m)
- Particles in future linear colliders will see $E \to E_{\text{cr}}$
- How do we incorporate a strong external field in QFT?

Equiv Photon Approx and Perturbation expansion

- decompose external field into n equivalent photons



$$G^e = G + G\hat{V}G + G\hat{V}G\hat{V}G + \ldots$$

$$G = (p^2 - m^2)^{-1}$$

$$\hat{V} = 2eA^e \cdot p - e^2 A^{e\,2}$$

- within certain constraints the summation can be performed (Reiss Eberly 1966)
- reminiscent of cancellation of IR divergences

- It is highly suggestive that this is equivalent to the Furry/Feynman method...

(W.H.) Furry Picture

- Separate electromagnetic part into external and vacuum parts

$$\mathcal{L}_{\text{QED}}^{\text{Furry}} = \bar{\psi}^V (i\partial\!\!\!/ - eA\!\!\!/^e - m)\psi^V - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}^V A\!\!\!/ \psi^V$$

- require solutions ψ^V

$$(i\partial\!\!\!/ - eA\!\!\!/^e - m)\psi^V = 0$$

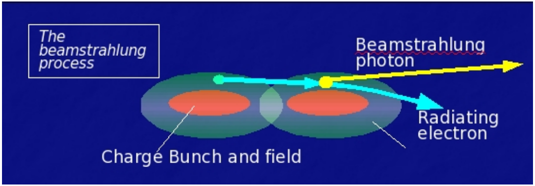
Exact solutions

- plane wave field
- Coulomb field
- collinear fields

Vacuum state stablity

- either unstable and non-vanishing vacuum current (tadpole diagrams)
- or stable with $E \to 0$ at $t = \pm\infty$

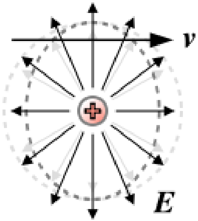
Strong fields at the collider Interaction Point



- relativistic particle p sees a plane-wave field of strength parameter Υ

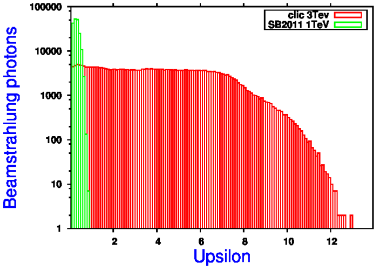
$$\Upsilon = \frac{e|\vec{a}|}{m E_{\text{cr}}} (k \cdot p)$$

- The scale of Υ depends on collider bunch parameters and the pinch effect



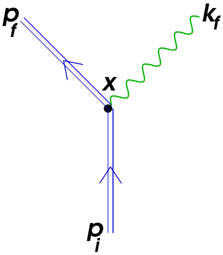
ILC and CLIC Strong field parameters

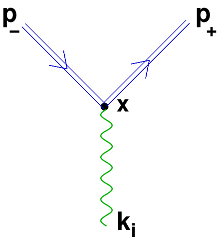
Parameter	ILC 1TeV	CLIC 3 TeV
$\mathcal{L}(\times 10^{34})$	4	3.6
N(incoh)	3.9e5	3.8e5
N(coh)	0	6.8e8
Υ (ave)	0.27	3.34
Υ (max)	0.94	10.9
δE_{bs}	10%	28%
$\langle\text{depol}\rangle_{\text{LW}}$	0.62%	3.5%



- CLIC far exceeds Schwinger critical field
- field strength varies from point to point through the beam collision
- depolarization due to spin precession and spin flip (and higher order processes)

Beamstrahlung, incoherent/coherent pair production



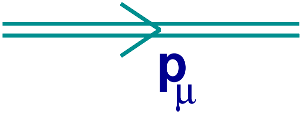


- IP beam-beam simulators - CAIN, Guinea-Pig
- beamstrahlung & coherent pair production calculated via quasi-classical approx
- incoherent pairs calculated with beamstrahlung photon and equivalent photon approx (EPA)
- **more exactly** these are 1st and 2nd order Furry picture processes

bkgd pairs	current	proposed
coherent	quasi-classical	1 vertex Furry picture
incoherent	EPA	2 vertex Furry picture

Lepton Volkov Solution

- Solution of the 2nd order Dirac equation with external potential



$$[D^2 + m^2 + \frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu}]\psi^{\mathsf{V}} = 0, \quad D_\mu = \partial_\mu + ieA^e_\mu$$

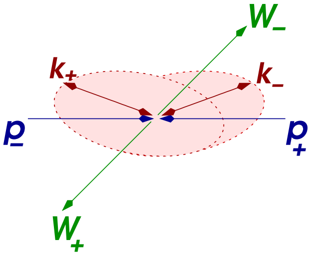
- Propose solution $\psi^{\mathsf{V}} = E_p\, e^{-ip\cdot x}\, u_p$

$$2i(kp)E'_p + [e^2 A^{e2} - 2e(A^e \cdot p) + ie\not{A}^{e\prime}\not{k}]E_p = 0$$

$$\therefore\ E_p = \exp\left[-\frac{1}{2(k\cdot p)}\left(e\not{A}^e\not{k} + i2e(A^e\cdot p) - ie^2A^{e2}\right)\right]$$

- Fourier Transform $\psi^{\mathsf{V}} = \int dr\, \exp(-ip.x - irk\cdot x - \mathcal{FT}(E_p))\, u_p$

New Volkov-type solutions



Two external fields

- both incoming bunches contribute external fields
- external field wavevectors are generally anti-collinear
- New results to be published shortly

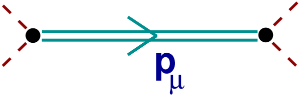
Spin 1 particle solutions, $W_\mu = E_p^W \, e^{-ip\cdot x} \, w_p$ where

$$E_p^W = \left(g_{\mu\nu} + \frac{e}{k\cdot p} \int F_{\mu\nu} - \frac{e^2}{2(k\cdot p)^2} A^{e2} k_\mu k_\nu \right)$$

$$.\exp\left[-\frac{i}{2(k\cdot p)} \left(2e(A^e \cdot p) - e^2 A^{e2} \right) \right]$$

Strong field propagator

- Look for Green's function solution



$$(\not{D}(x) - m)G^{\mathsf{V}}(x, x') = \delta(x - x')$$

- ψ^V are orthogonal and complete (Ritus, Ann Phys **69** 555, 1970)
- Solution is the fermion propagator flanked by Volkov E_p functions

$$G^{\mathsf{V}}(x, x') = \int d^4p\; E_p(x) \frac{\not{p} + m}{p^2 - m^2 + i\epsilon} \bar{E}_p(x')\, e^{ip\cdot(x'-x)}$$

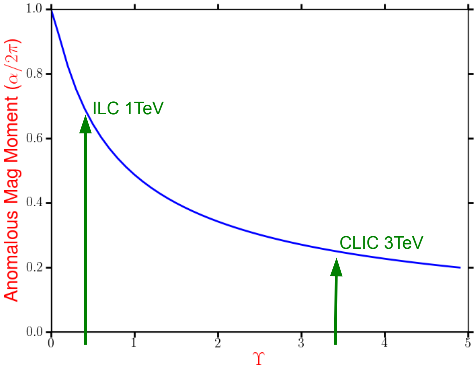
$$\psi^{\mathsf{V}} \equiv E_p(x) e^{-ip\cdot x}\, u_p$$

- photon propagator remains unchanged

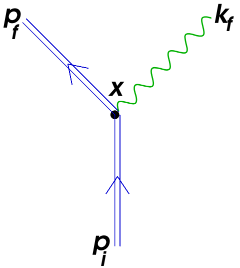
Loop corrections in an external field

- anomalous magnetic moment (one-loop) in a charge bunch field (V Baier, V.I. Ritus)

$$\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{2\pi}\int_0^\infty \frac{2\pi\,dx}{(1+x)^3}\left(\frac{x}{\Upsilon}\right)^{1/3}\mathrm{Gi}\left(\frac{x}{\Upsilon}\right)^{1/3}$$



Modified Feynman Rules



- double fermion lines are Volkov-type solutions
- conservation of momentum allows 1 vertex diagrams
- Volkov E_p functions as adjacent to the vertex

$$\gamma^e_\mu = \int d^4x\, \bar{E}_{p_f}(x)\gamma_\mu E_{p_i}(x)\, e^{i(p_f - p_i + k_f)\cdot x}$$

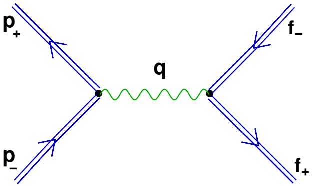
- momentum space vertex has contribution rk from external field

$$\gamma^e_\mu = (2\pi)^4 \int dr\; \bar{E}_{p_f}(r)\gamma_\mu E_{p_i}(r)\delta^4(p_f + k_f - p_i - rk)$$

Generic two vertex Furry picture S channel

$$M_{fi} = g_1 g_2 \int dr_1 dr_2 \; \bar{v}_{p_+} \gamma^{e\mu} \, u_{p_-} \, \bar{\epsilon}_{f_+} \gamma^e_\mu \, \epsilon_{f_-} \; \frac{\delta(F - I - (r_1 + r_2)(k_- + k_+))}{(I + (r_1 + r_2)(k_- + k_+))^2}$$

- final states momentum $F \equiv f_- + f_+$
  initial state momentum $I \equiv p_- + p_+$
- usual coupling constants and spinors/polarisation
- modified (Furry) vertices γ^e
- r_1, r_2 momentum contribution from (two) external fields



$$\frac{|M_{fi}|^2}{VT} = (g_1 g_2)^2 \int dr_1 dr_2 \;\; \mathrm{Tr}[..r_1..r_2..] \, \frac{d\vec{f_-} d\vec{f_+}}{4\omega_{f_-}\omega_{f_+}} \, \frac{\delta(F - I - (r_1 + r_2)(k_- + k_+))}{(I + (r_1 + r_2)(k_- + k_+))^2}$$

Furry picture phase integrals

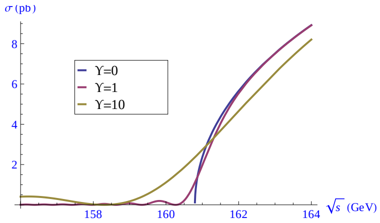
- usually, 6 integrations and 4 delta functions leaves $d\Omega$

$$\int dr_1 dr_2 \;\; \mathrm{Tr}(r_1, r_2) \frac{d\vec{f_-} d\vec{f_+}}{4\omega_{f_-}\omega_{f_+}} \frac{\delta(F - I - (r_1 + r_2)(k_- + k_+))}{(I + (r_1 + r_2)(k_- + k_+))^2}$$

- two extra integrations mean that process threshold is smeared

$$\int \frac{dr\, d\Omega}{8} \left[1 - \frac{m_f^2}{(E - r\,((\Upsilon_+ + \Upsilon_-)/E)^{1/3})^2}\right]^{1/2} \int dr_2 \;\; \mathrm{Tr}(r - r_2, r_2)$$

- eg. W pair production



Requirements for a strong field event generator

REQUIREMENTS:

To simulate a charge bunch collision and calculate the field strength at each point of production

To have a finely scaled simulation in order to accurately model disruption, hour glass effect etc.

To perform a relatively complex cross-section calculation at each point of production

To have full spin tracking

To be flexible enough to include new higher order processes

SOLUTION:

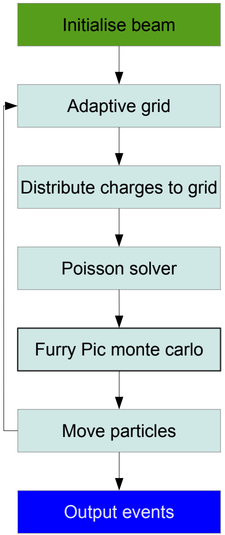
A PIC code using an efficient method for modeling the electrodynamics – crosscheck with CAIN/GP

MPI using openMPI or GPU programming

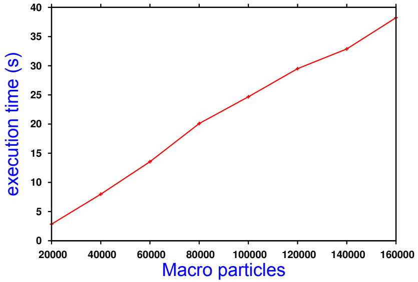
T-BMT with higher order corrections to AMM, Sokolov-Ternov and higher order helicity amplitudes

Allow new processes to be loaded externally

IPstrong - towards a strong field generator



- Fortran 2003 with openMPI (Fortran 2008 has inbuilt gpu)
- 3D electrostatic poisson solver (MPI)
- Furry picture processes replace all other processes
- output in multiple formats (stdhep, lcio)
- cross-checks with existing programs



Summary

- A linear collider is a precision instrument to study new physics
- We need to understand the effects of the strong IP bunch fields
- Physics processes can be calculated within the Furry picture which takes into account the strong field *exactly*
- We are developing new exact solutions for charged particles in two external fields and the cross-sections of generic 2-vertex Furry picture processes
- Leading term analysis indicate particle threshold energies are smeared. Continue analysis to understand all effects
- A new EM solver/generic event generator, **IPstrong** is being developed to model these strong field processes

Thanks for your attention